

# Mixed Map Labeling

Maarten Löffler  
Frank Staals

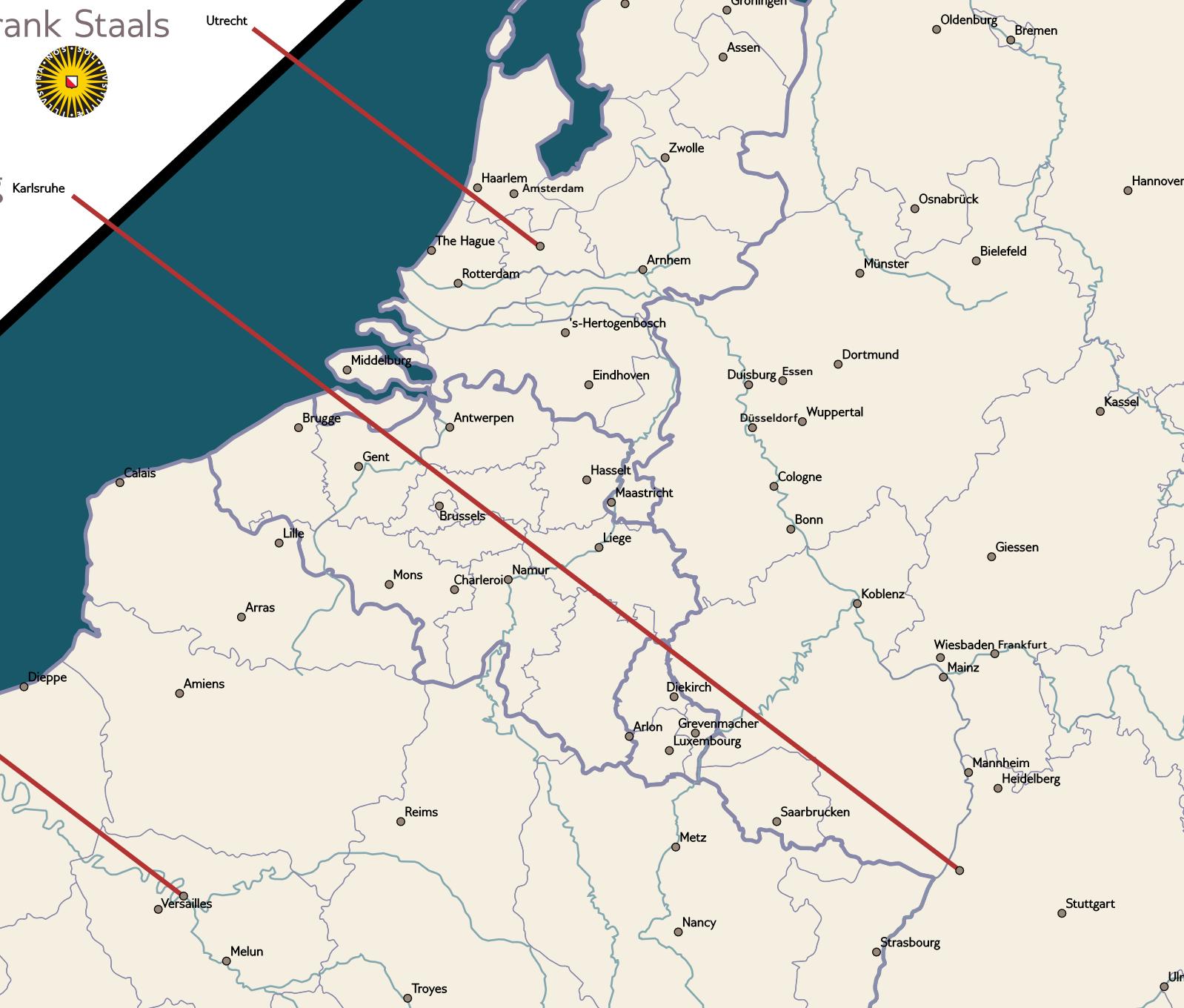


Martin Nöllenburg



CIAC 2015

Paris



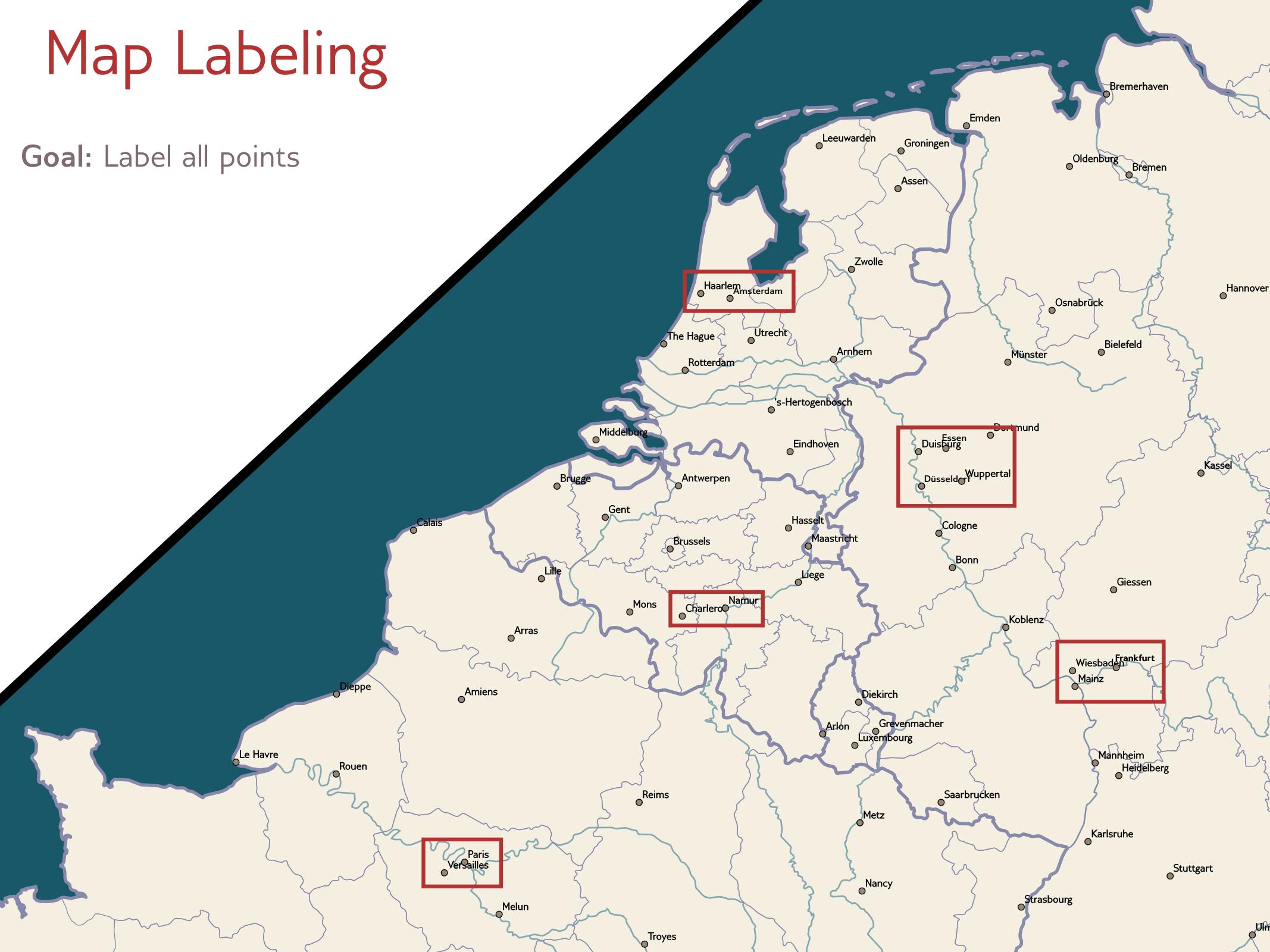
# Map Labeling

Goal: Label all points



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Goal: Maximize #points labeled



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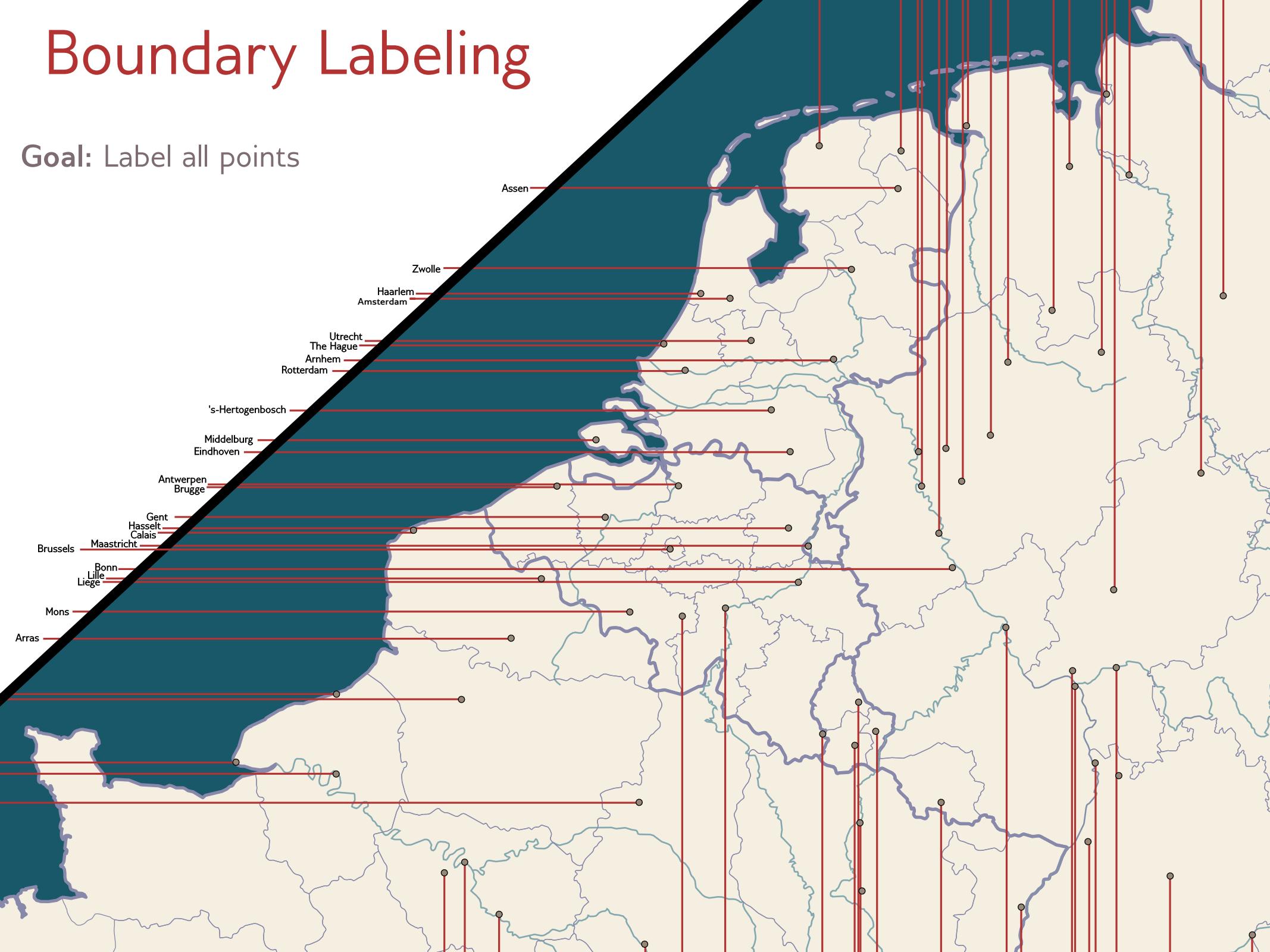
Different versions depending on where we may place the label.

Most versions are NP-hard



# Boundary Labeling

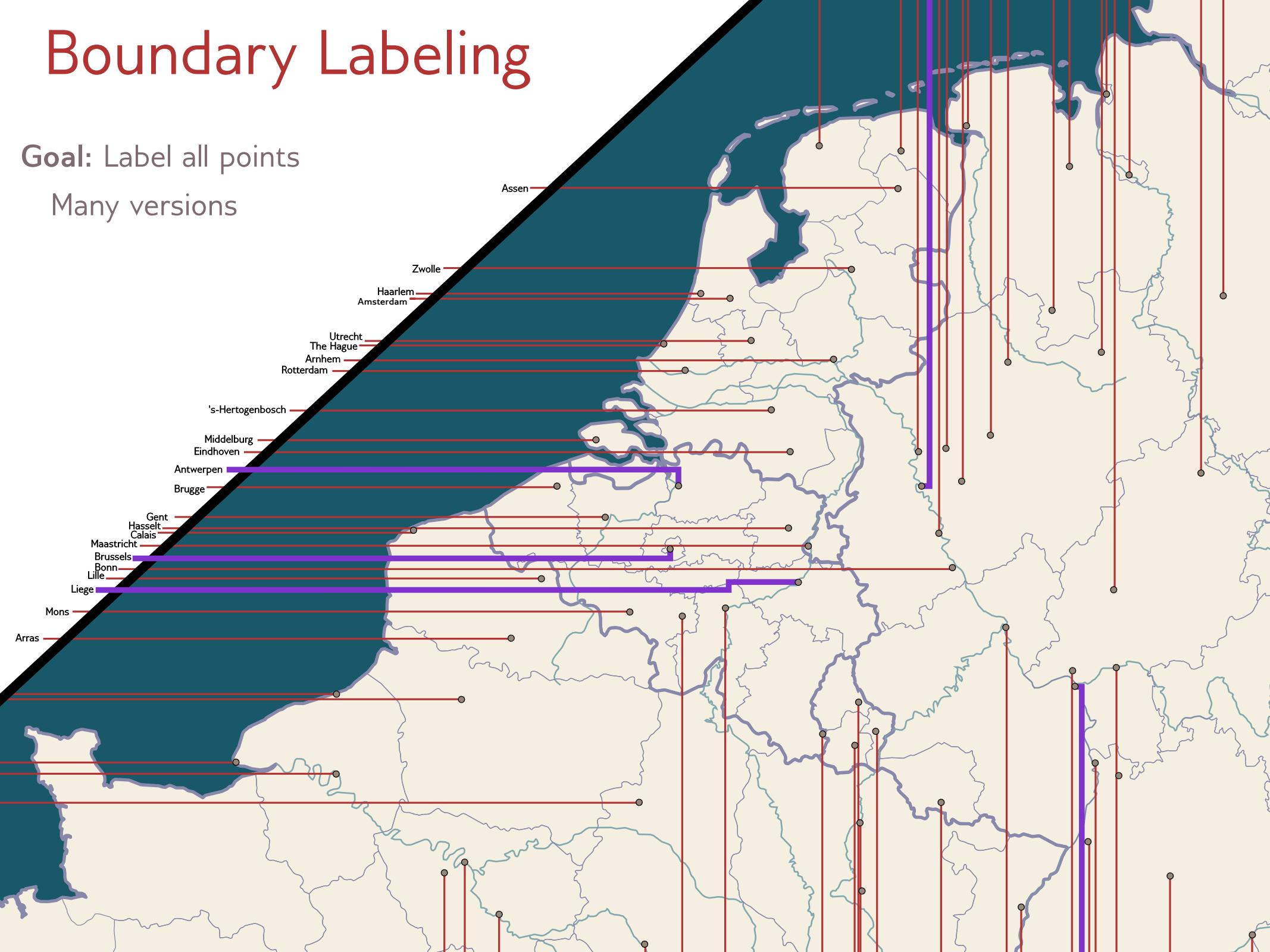
Goal: Label all points



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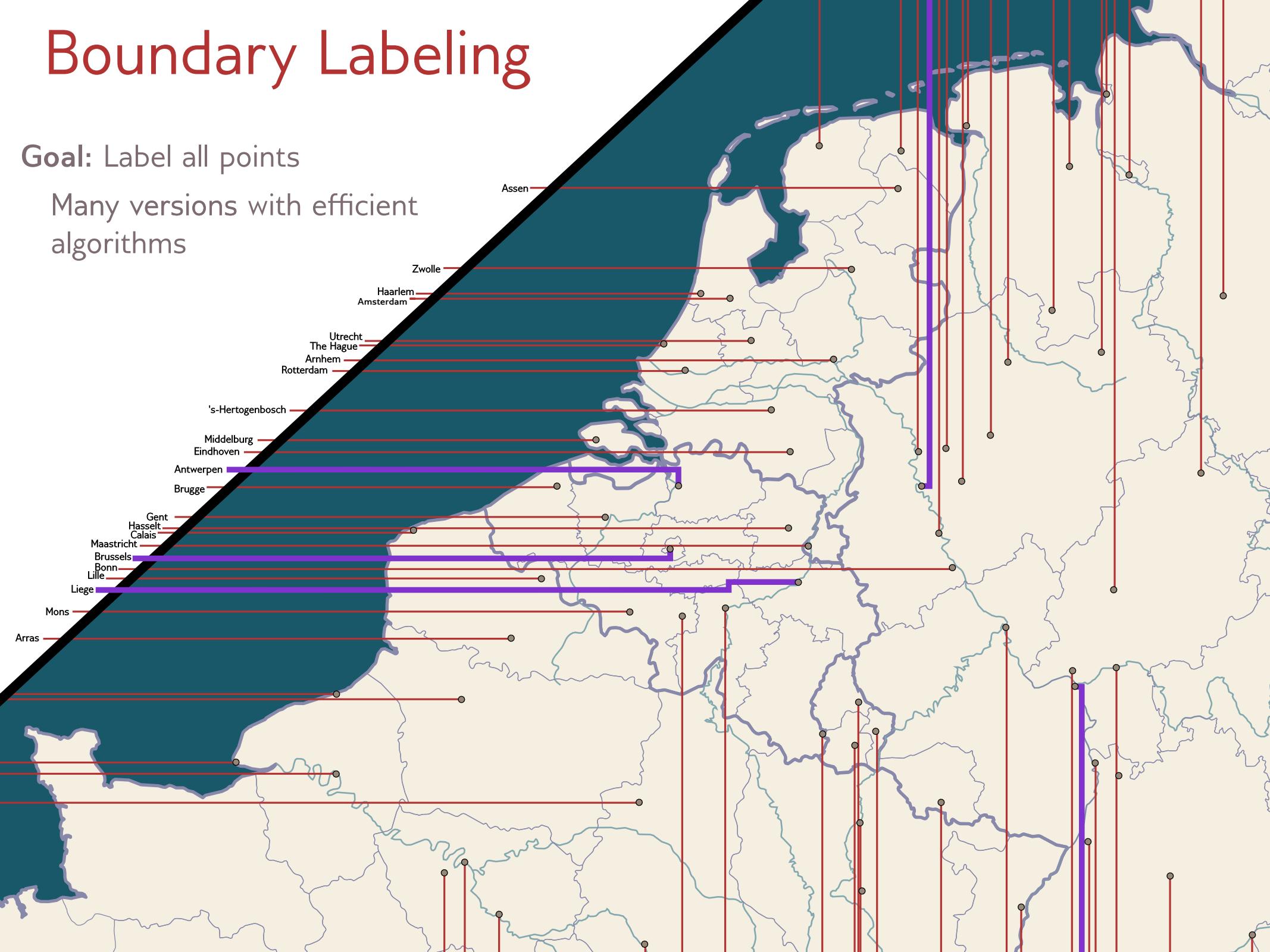
Many versions



# Boundary Labeling

Goal: Label all points

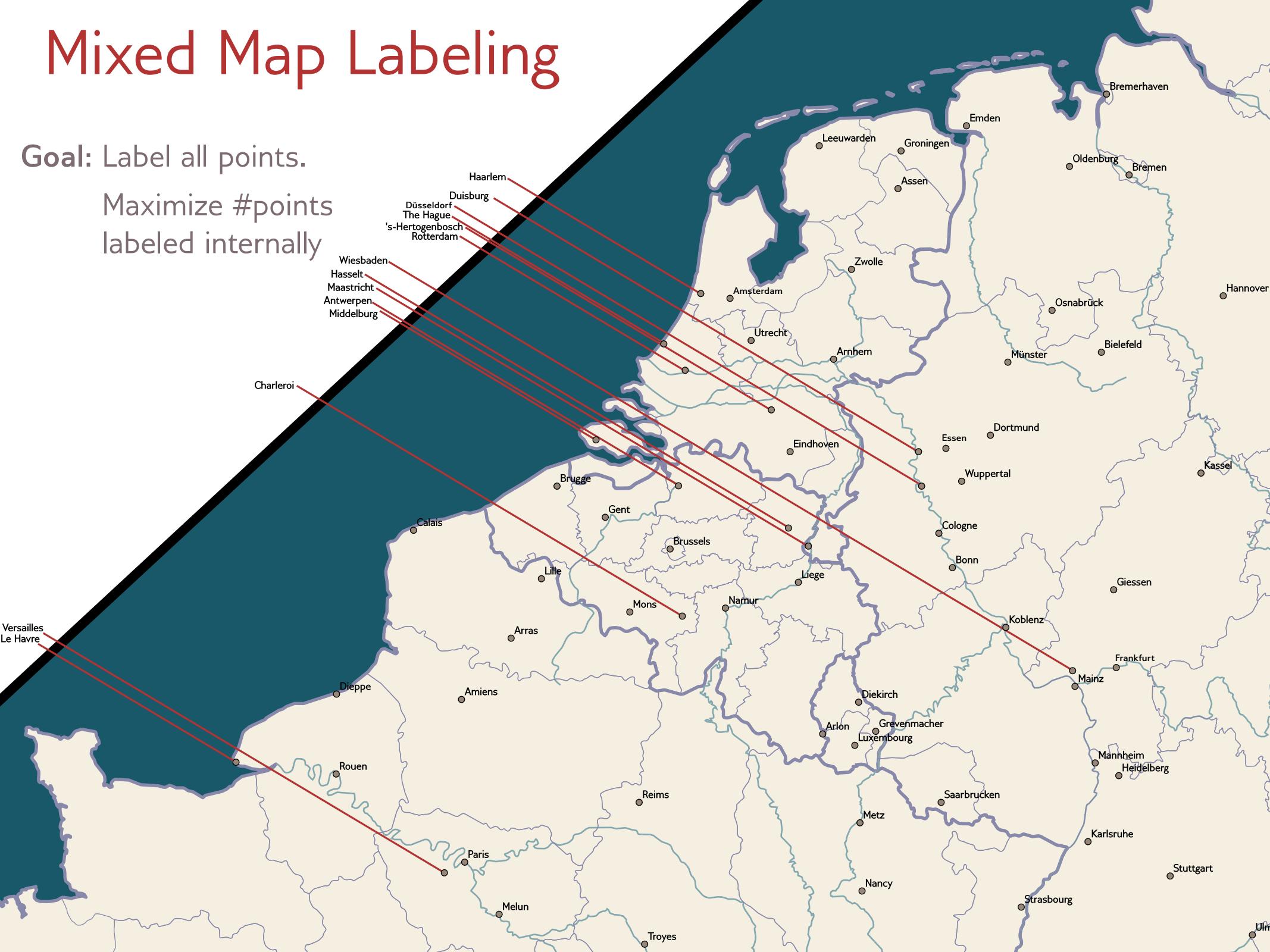
Many versions with efficient  
algorithms



# Mixed Map Labeling

Goal: Label all points.

Maximize #points  
labeled internally

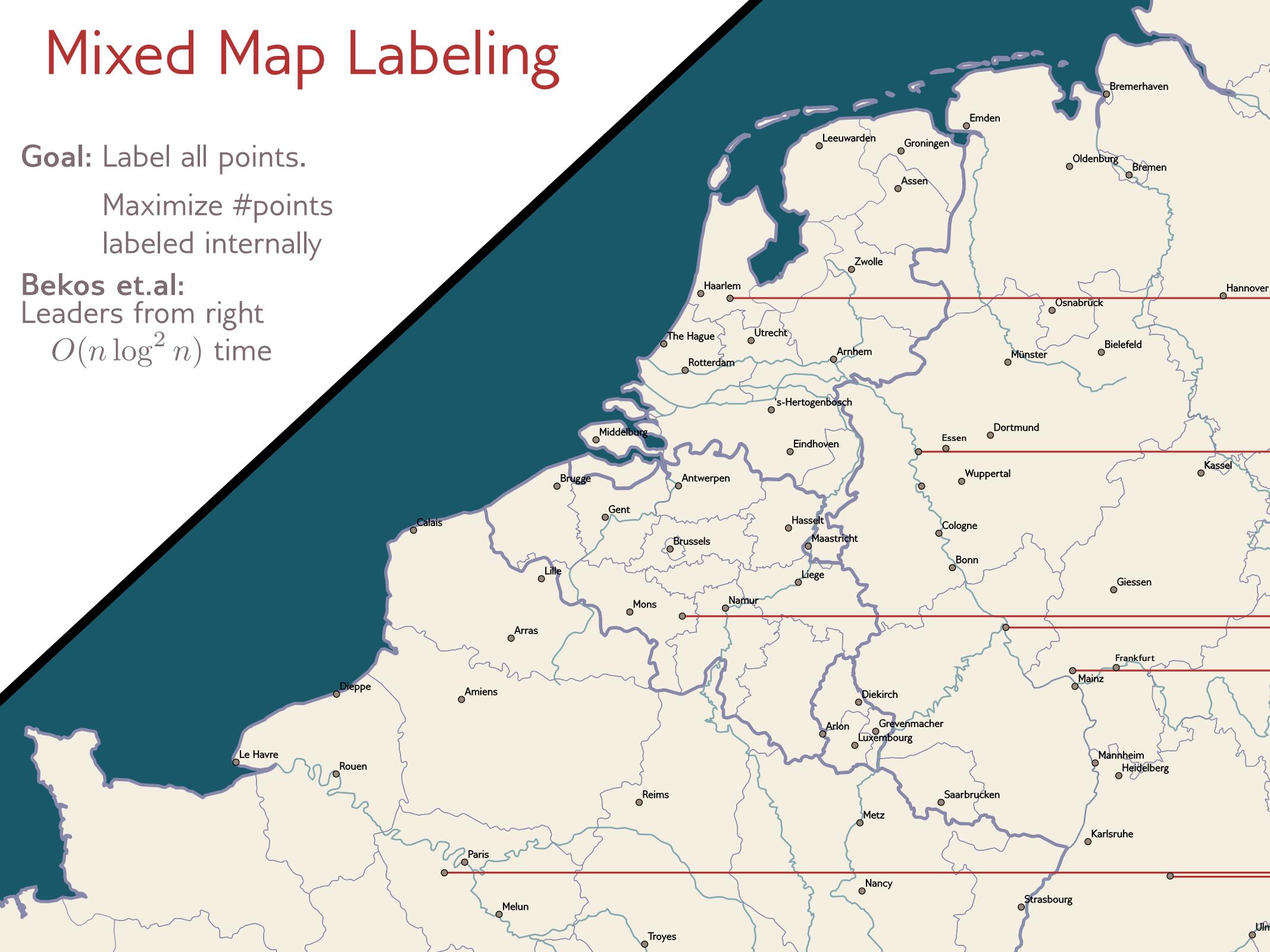


# Mixed Map Labeling

Goal: Label all points.

Maximize #points  
labeled internally

Bekos et.al:  
Leaders from right  
 $O(n \log^2 n)$  time



# Mixed Map Labeling

# Goal: Label all points.

## Maximize #points labeled internally

## Bekos et.al: Leaders from right

$O(n \log^2 n)$  time  
readers from left  
 $O(n^{3+\log n})$  time

10 of 10

1

1

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10

10 of 10

1

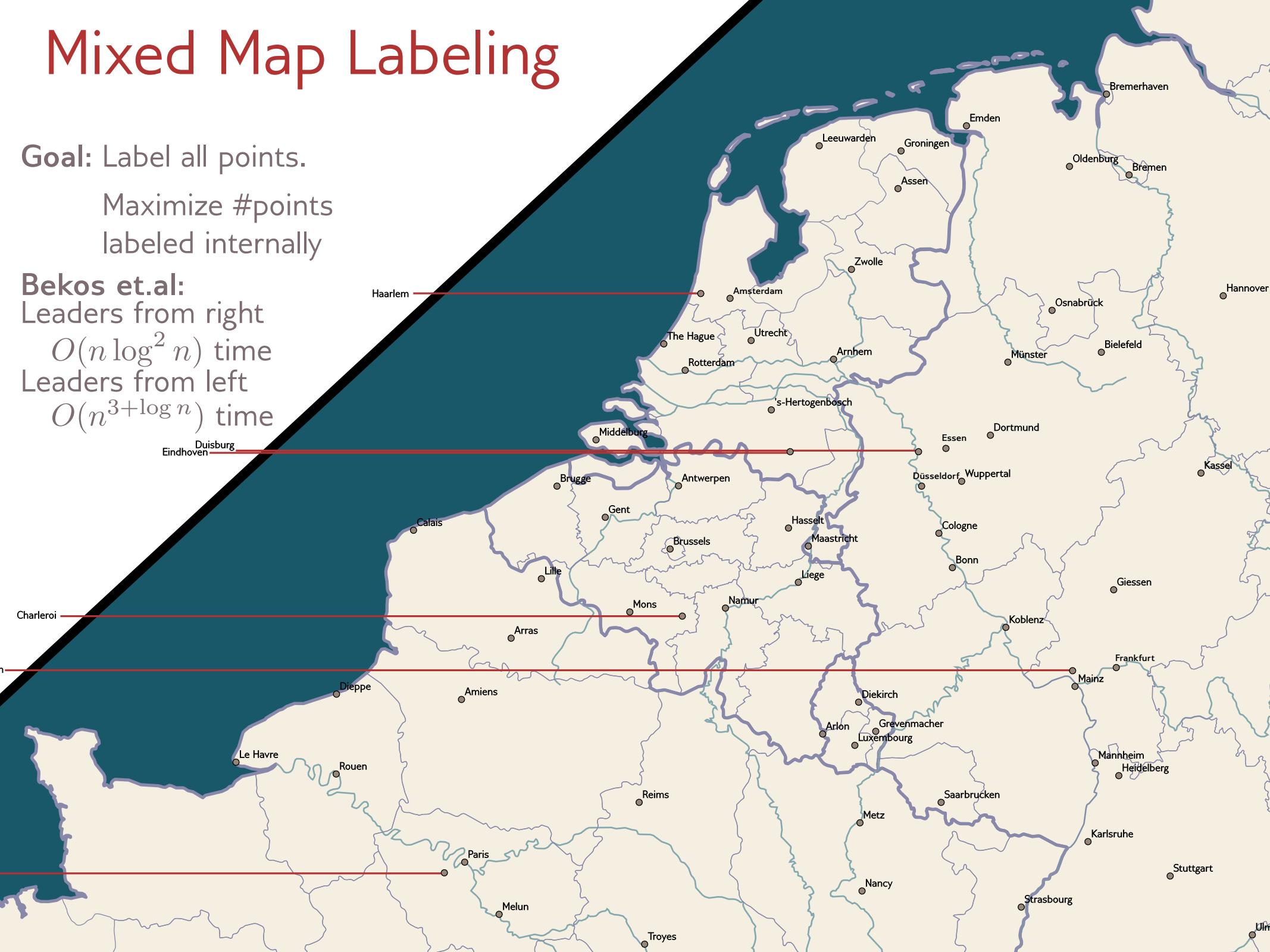
1

100

10

卷之三

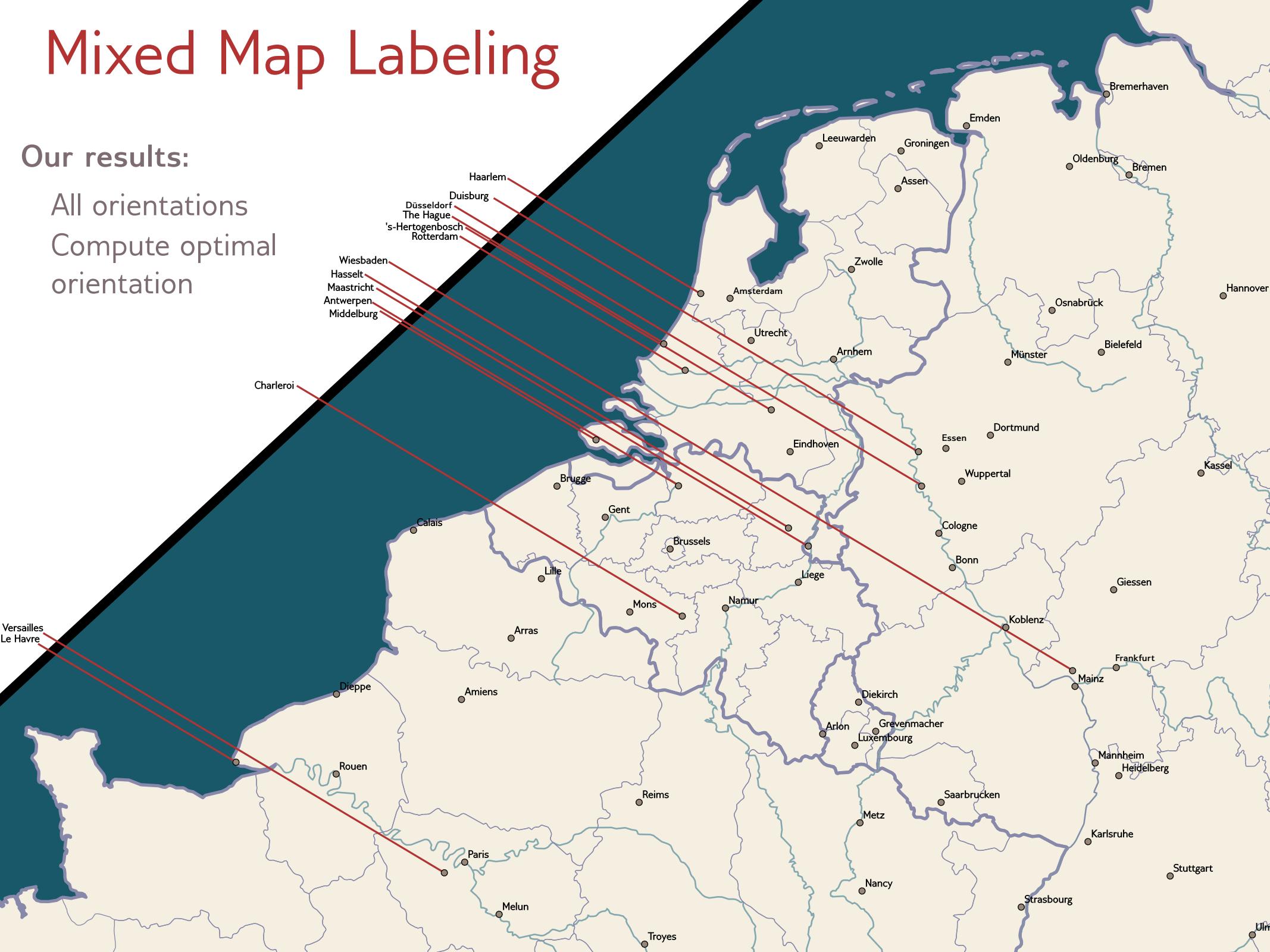
1



# Mixed Map Labeling

Our results:

All orientations  
Compute optimal  
orientation



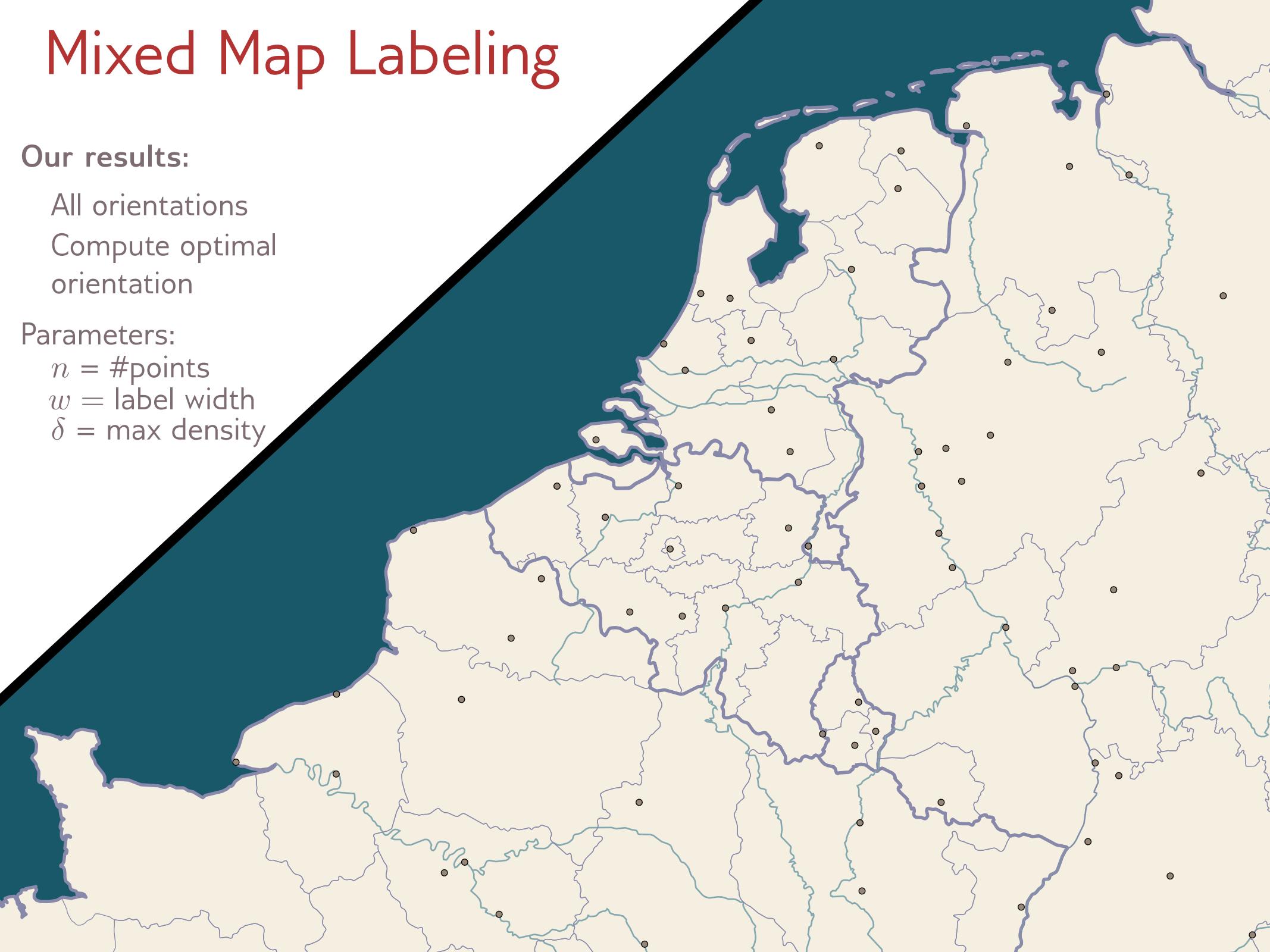
# Mixed Map Labeling

Our results:

- All orientations
- Compute optimal orientation

Parameters:

$n$  = #points  
 $w$  = label width  
 $\delta$  = max density



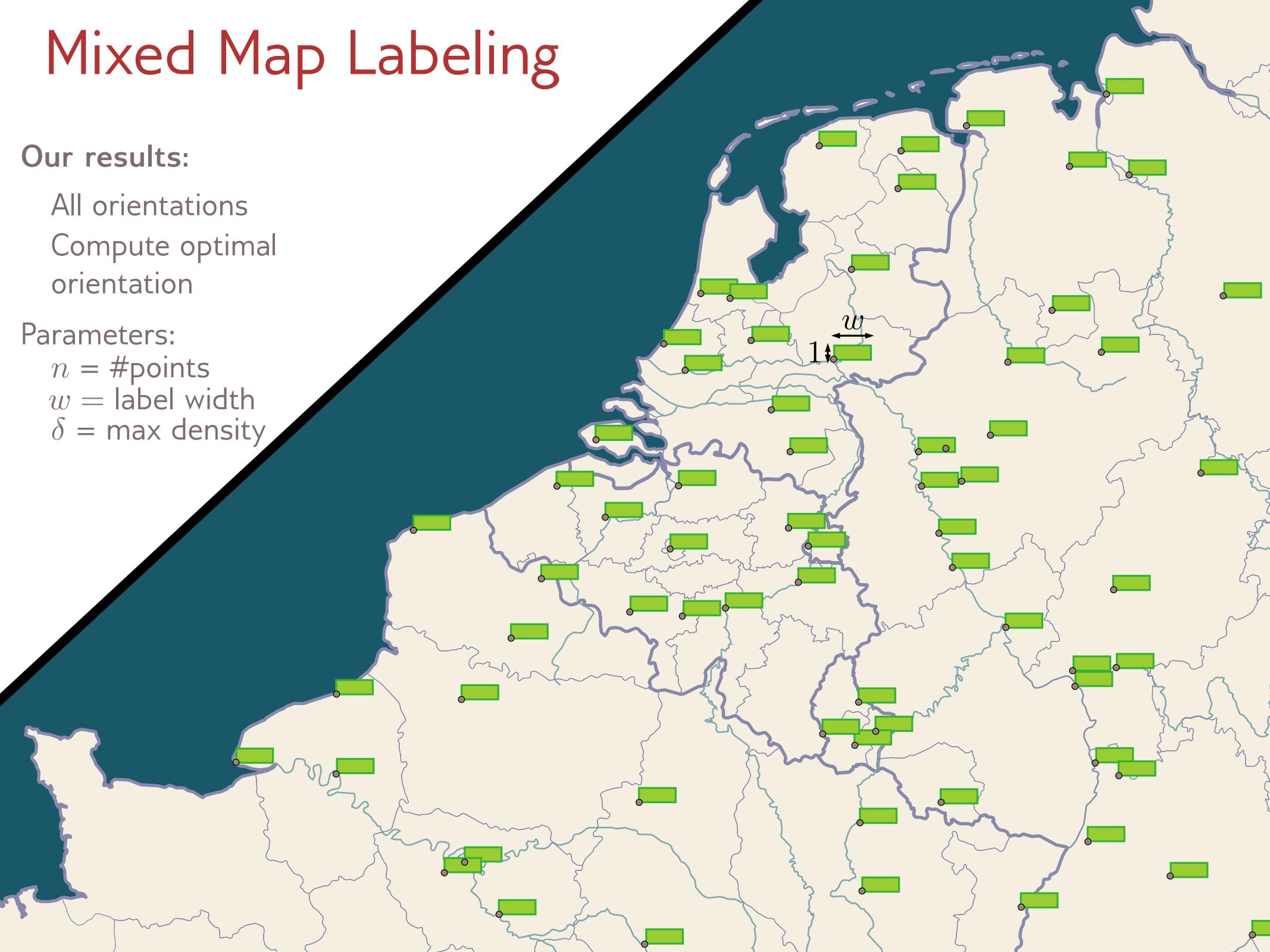
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# Mixed Map Labeling

Our results:

All orientations  
Compute optimal  
orientation

Parameters:

$n = \# \text{points}$   
 $w = \text{label width}$   
 $\delta = \min(n, \frac{1}{\min \|pq\|})$



# Mixed Map Labeling

Our results:

All orientations

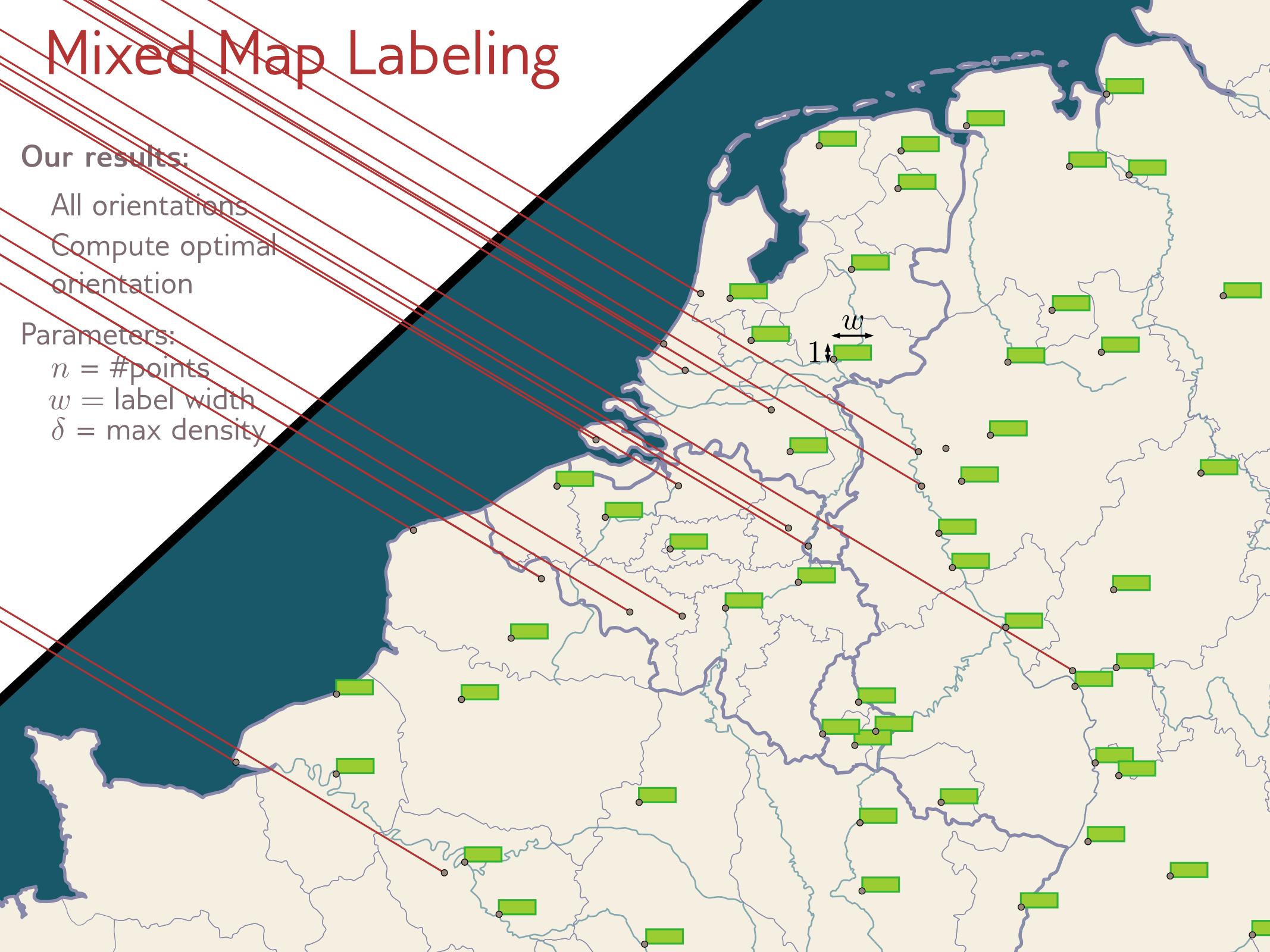
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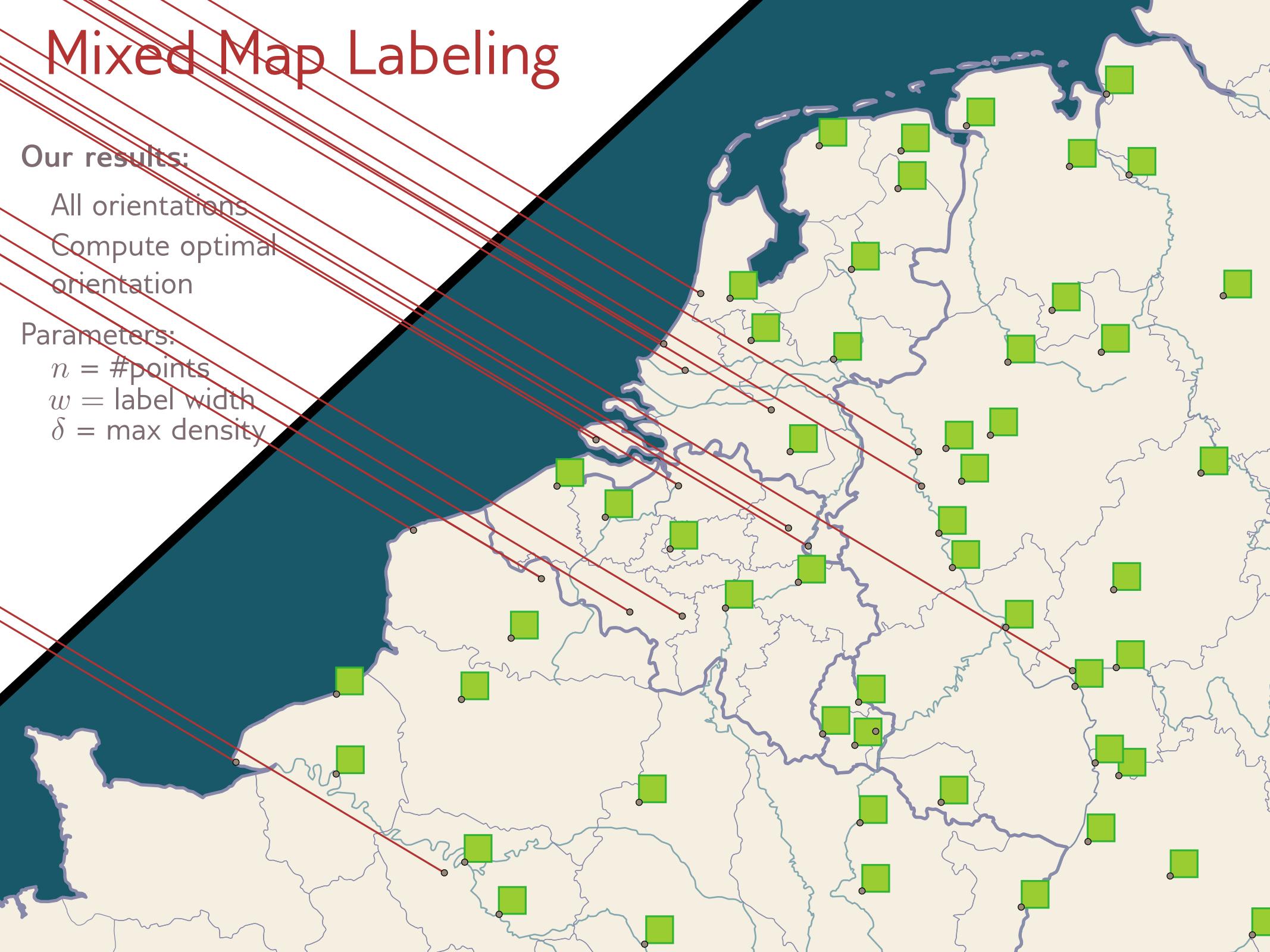
Compute optimal  
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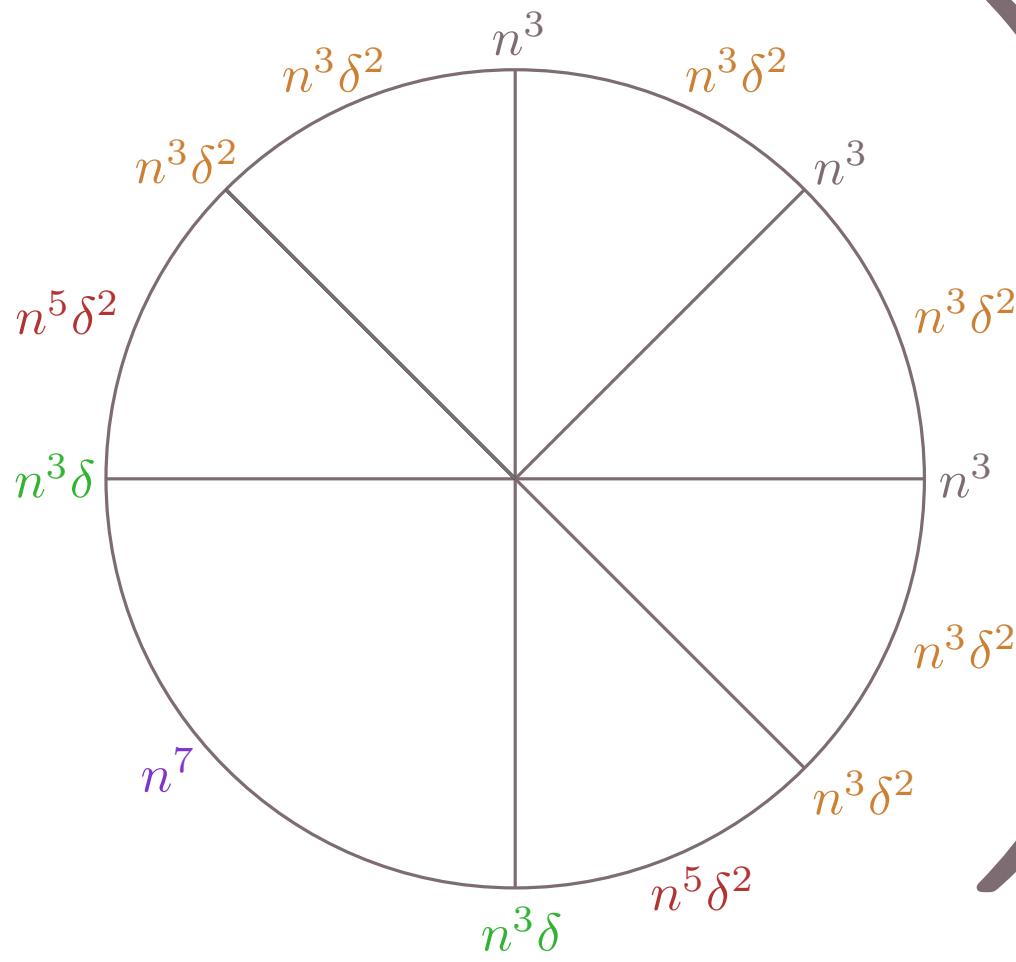
$w$  = label width

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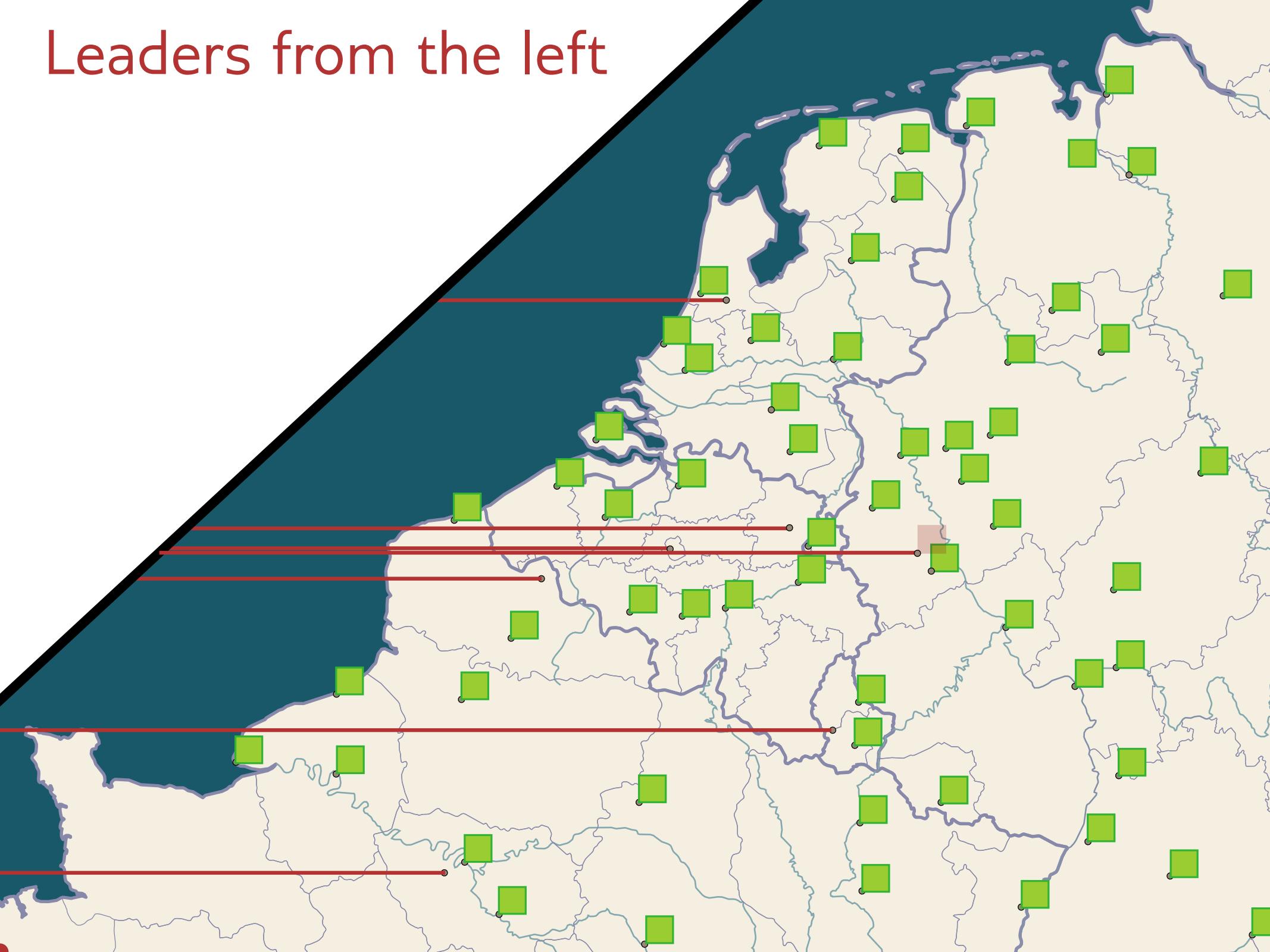


# Mixed Map Labeling

$n^3 \log n +$

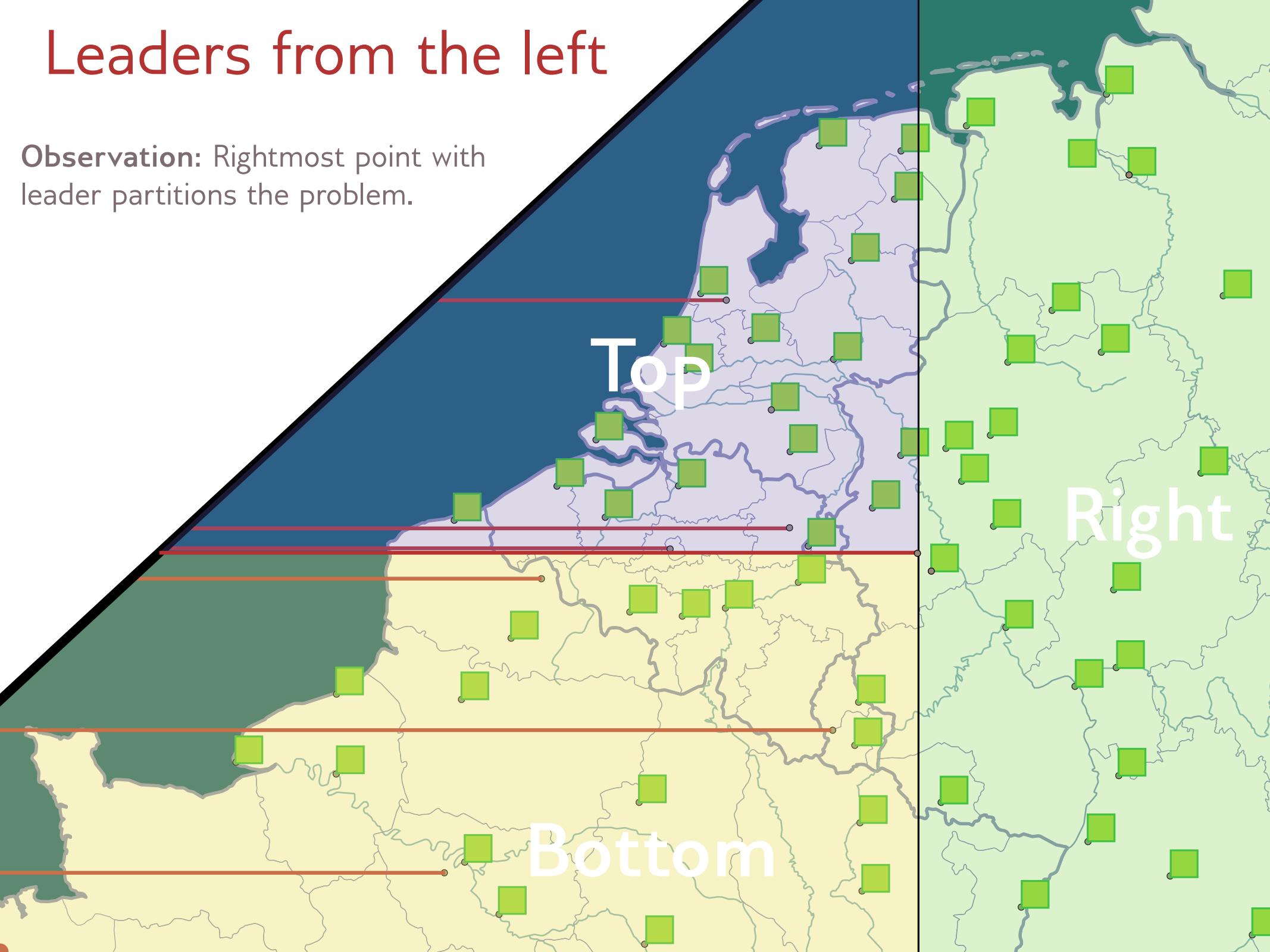


# Leaders from the left



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Observation: Rightmost point with leader partitions the problem.



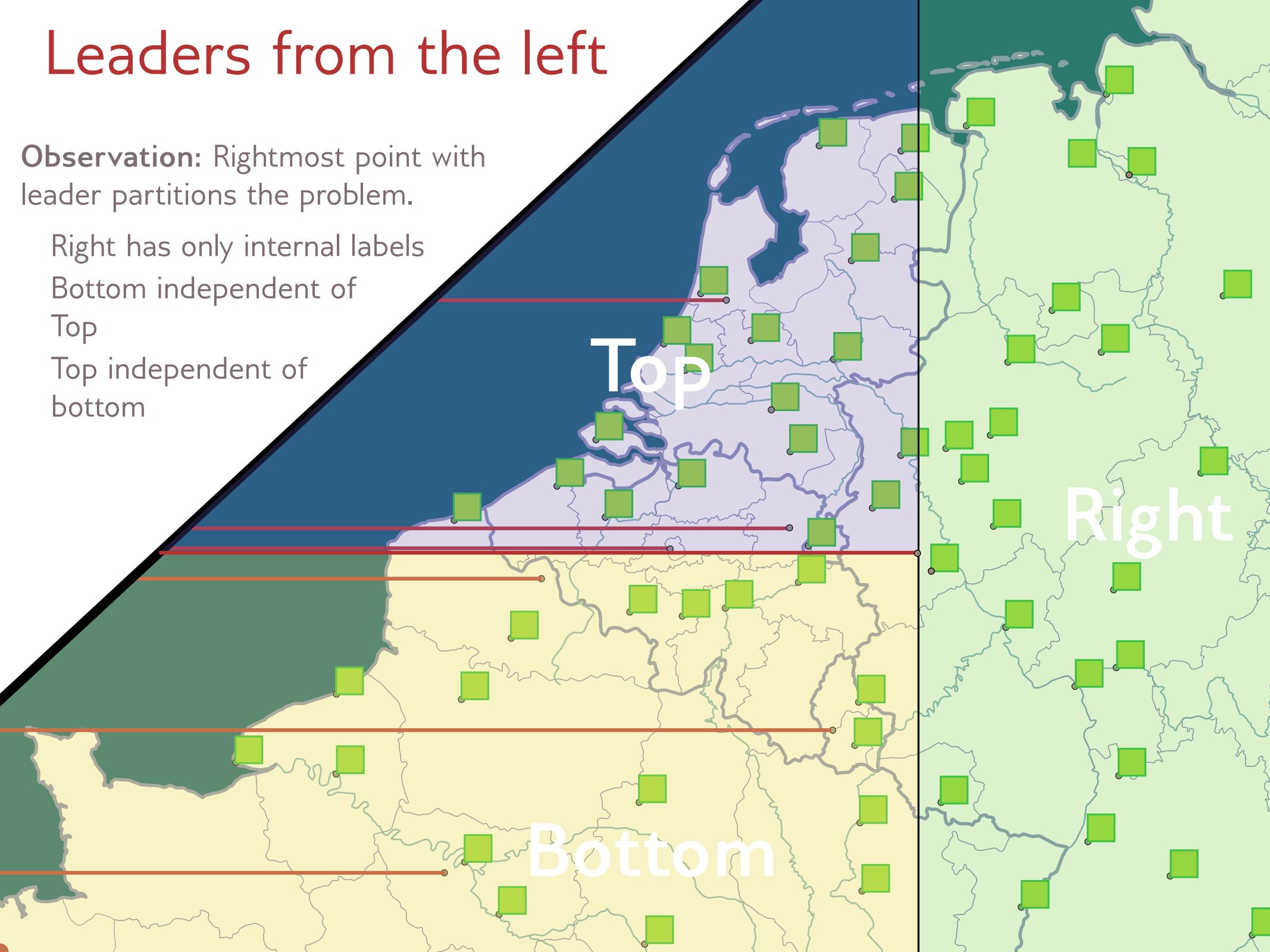
# Leaders from the left

Observation: Rightmost point with leader partitions the problem.

Right has only internal labels

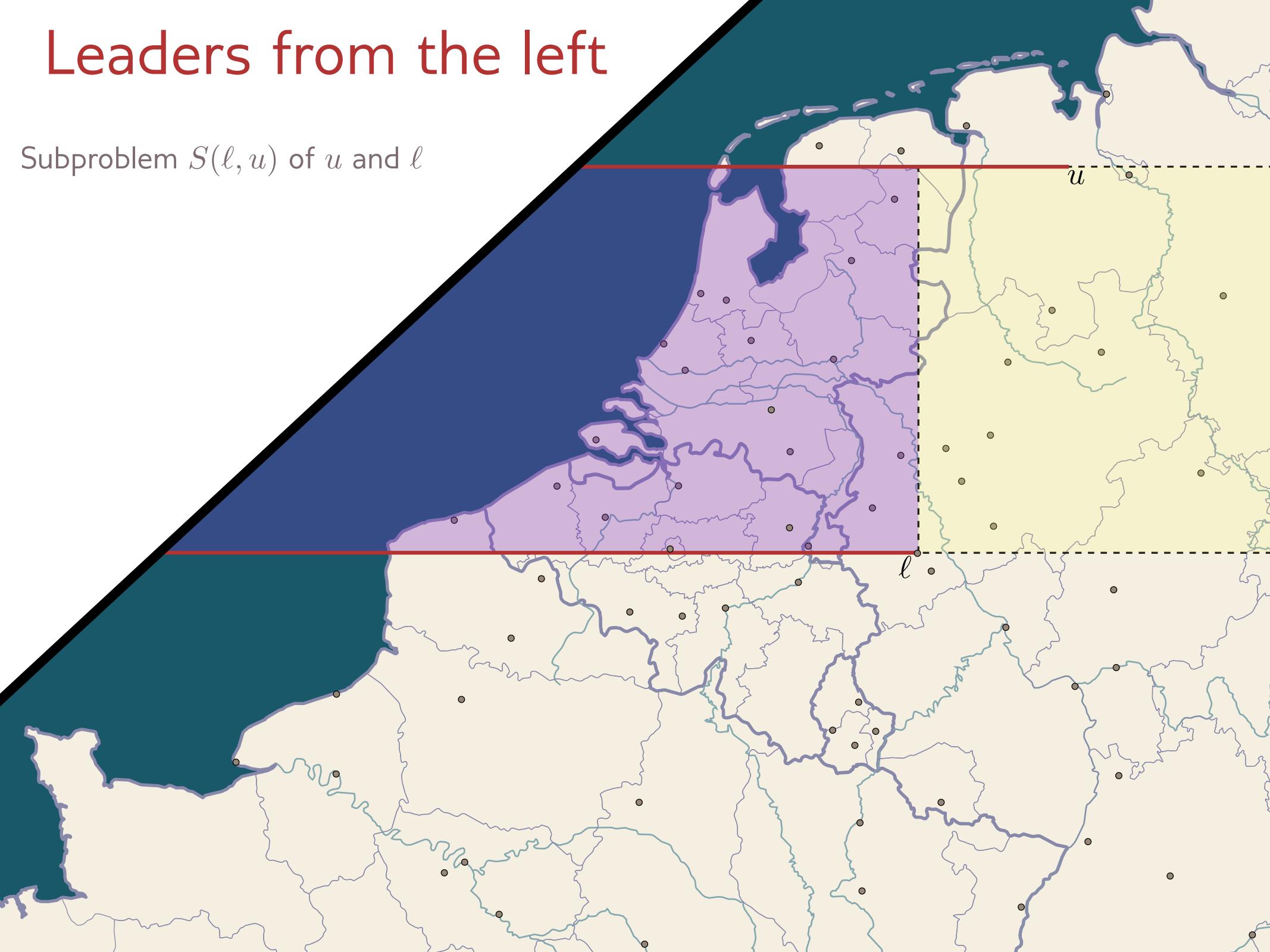
Bottom independent of  
Top

Top independent of  
bottom



# Leaders from the left

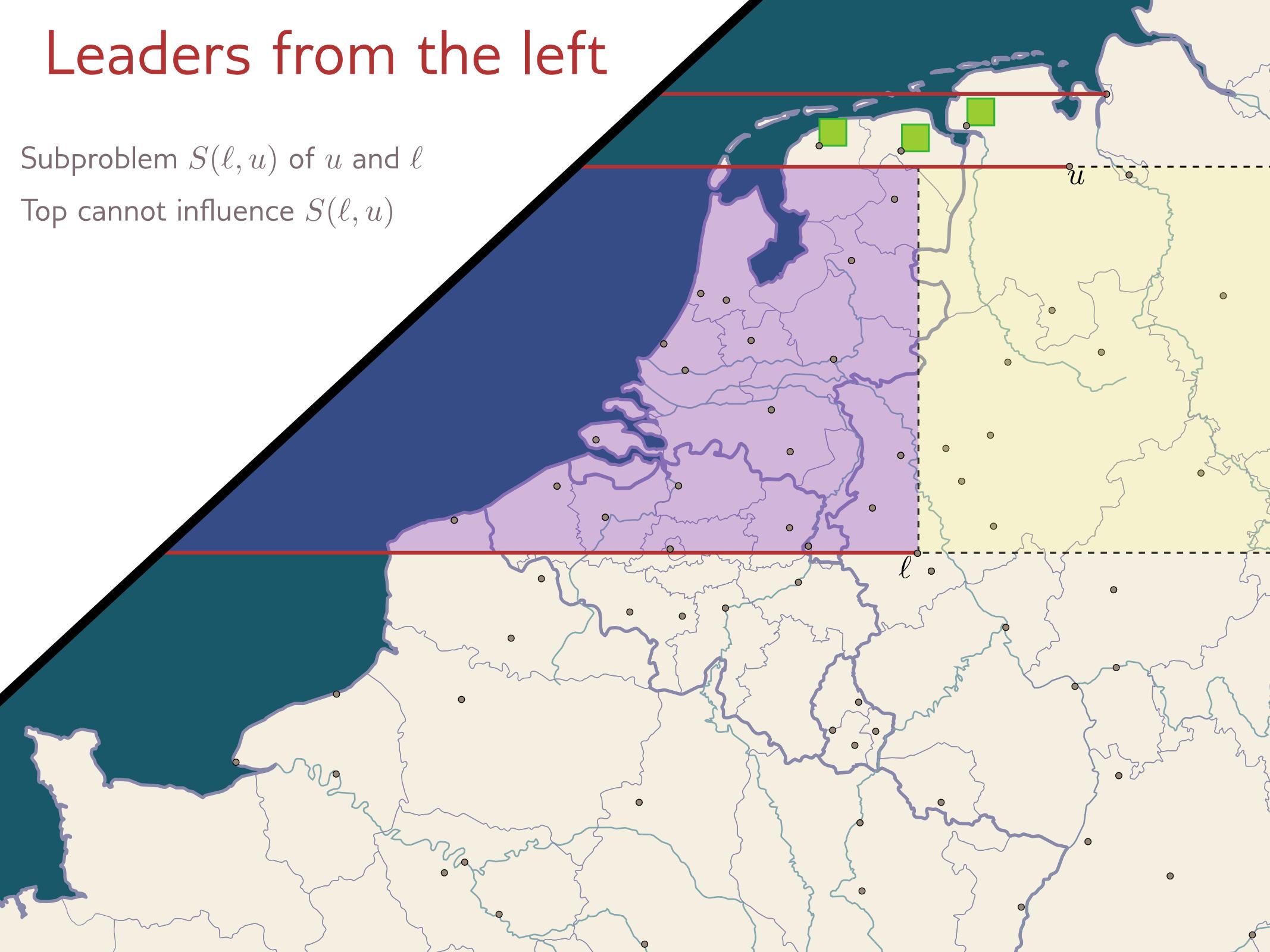
Subproblem  $S(\ell, u)$  of  $u$  and  $\ell$



# Leaders from the left

Subproblem  $S(\ell, u)$  of  $u$  and  $\ell$

Top cannot influence  $S(\ell, u)$

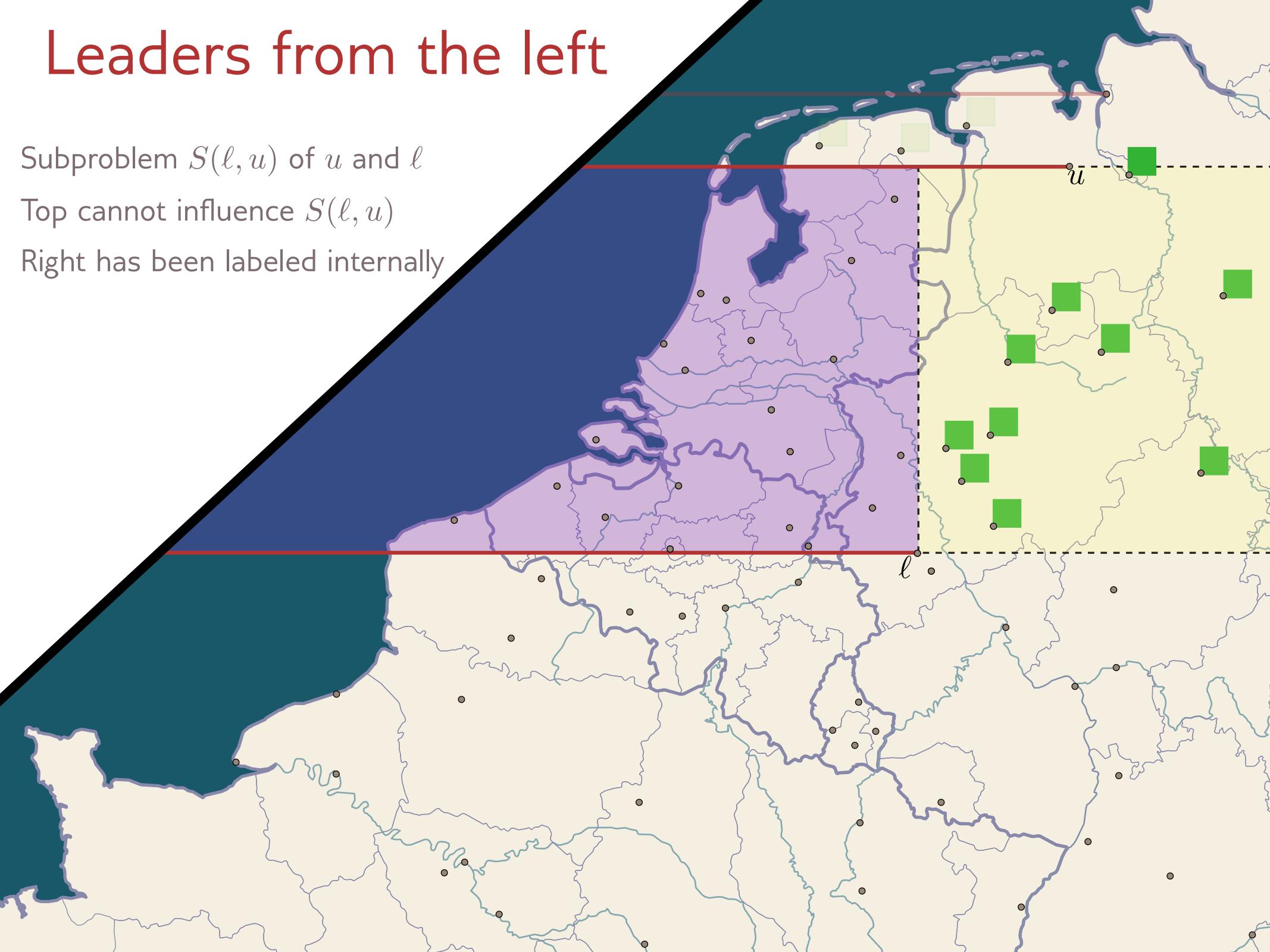


# Leaders from the left

Subproblem  $S(\ell, u)$  of  $u$  and  $\ell$

Top cannot influence  $S(\ell, u)$

Right has been labeled internally

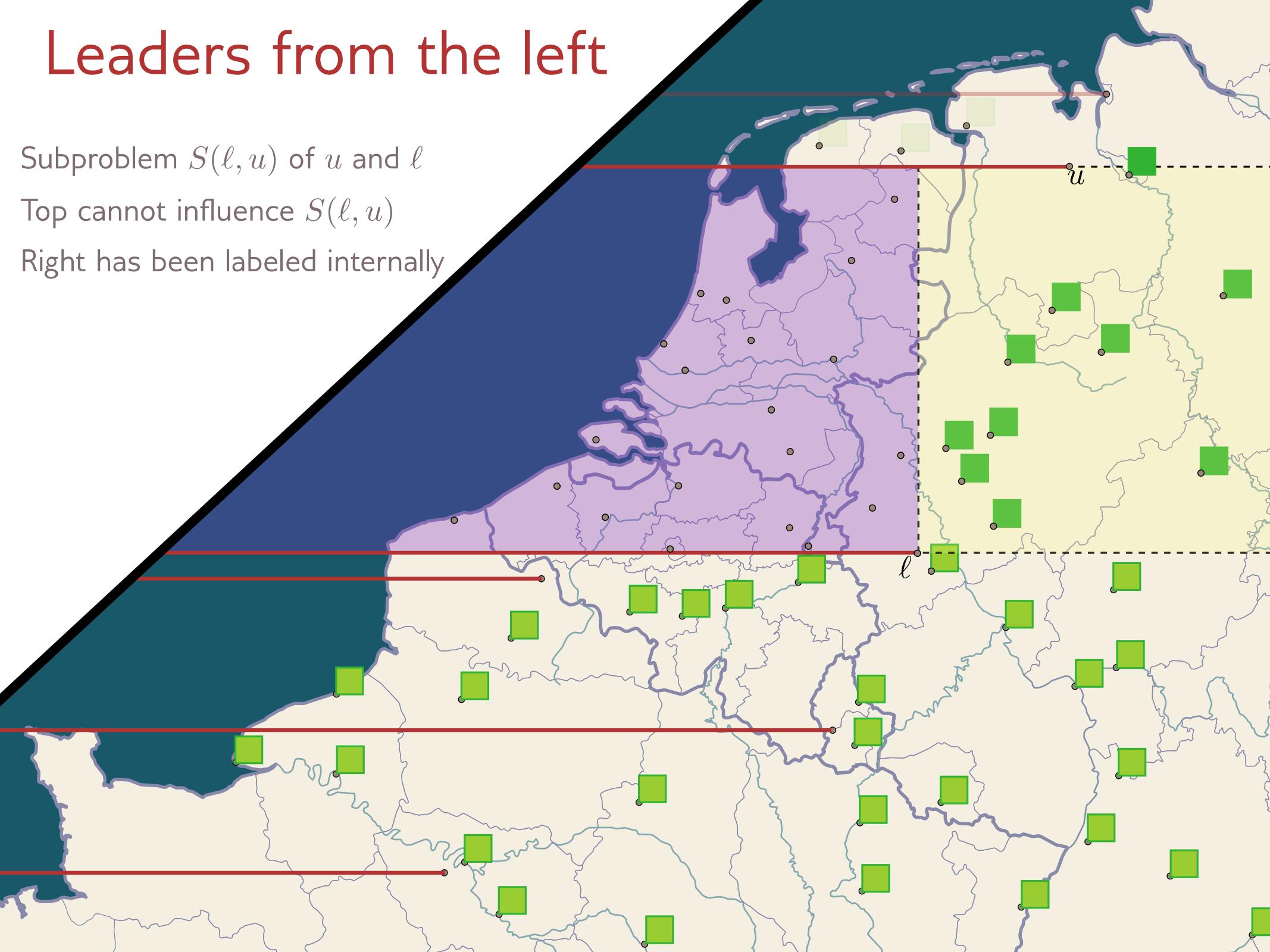


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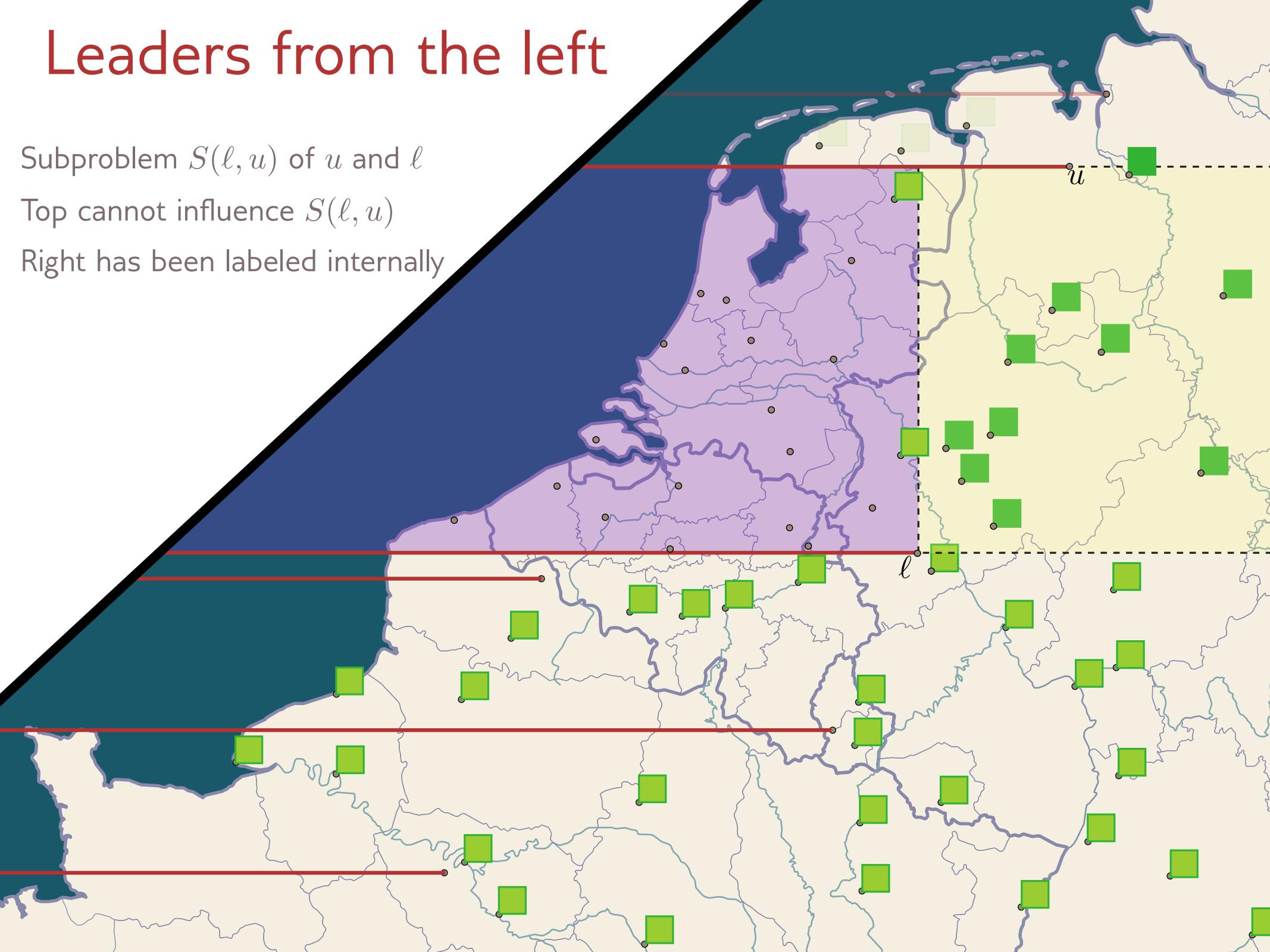


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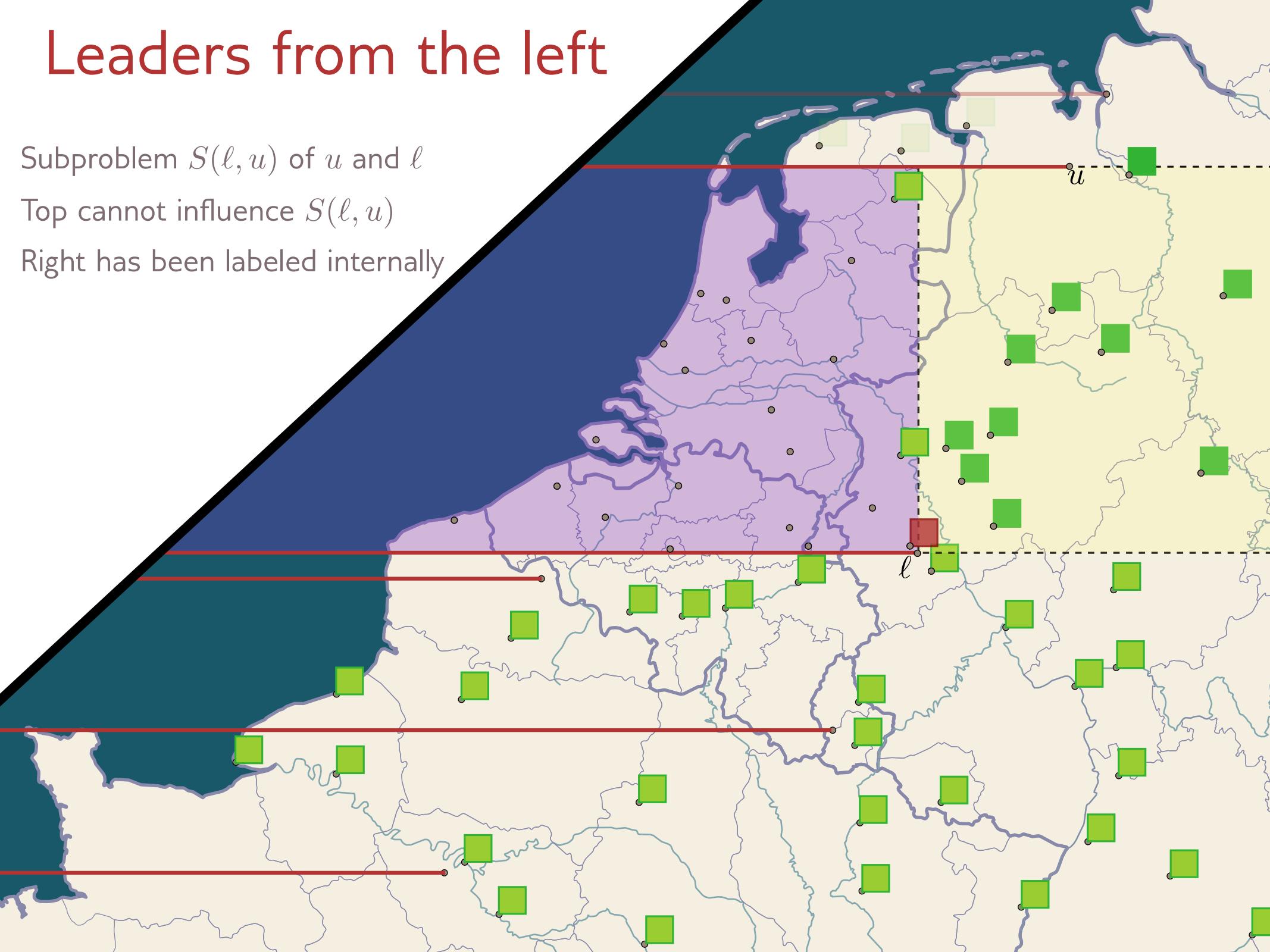


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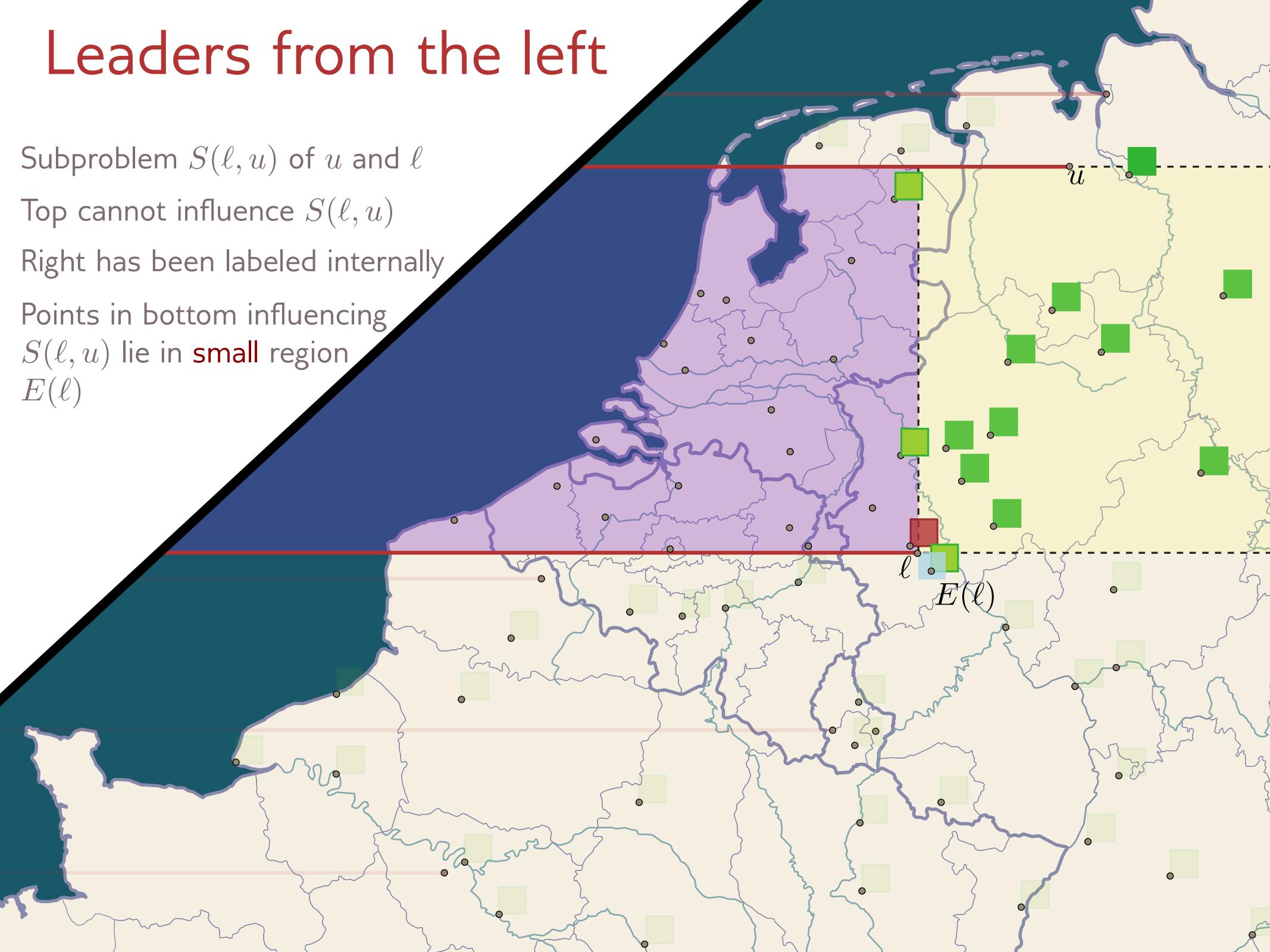
Top cannot influence  $S(\ell, u)$

Right has been labeled internally

Points in bottom influencing

$S(\ell, u)$  lie in **small** region

$E(\ell)$



# Leaders from the left

Subproblem  $S(\ell, u)$  of  $u$  and  $\ell$

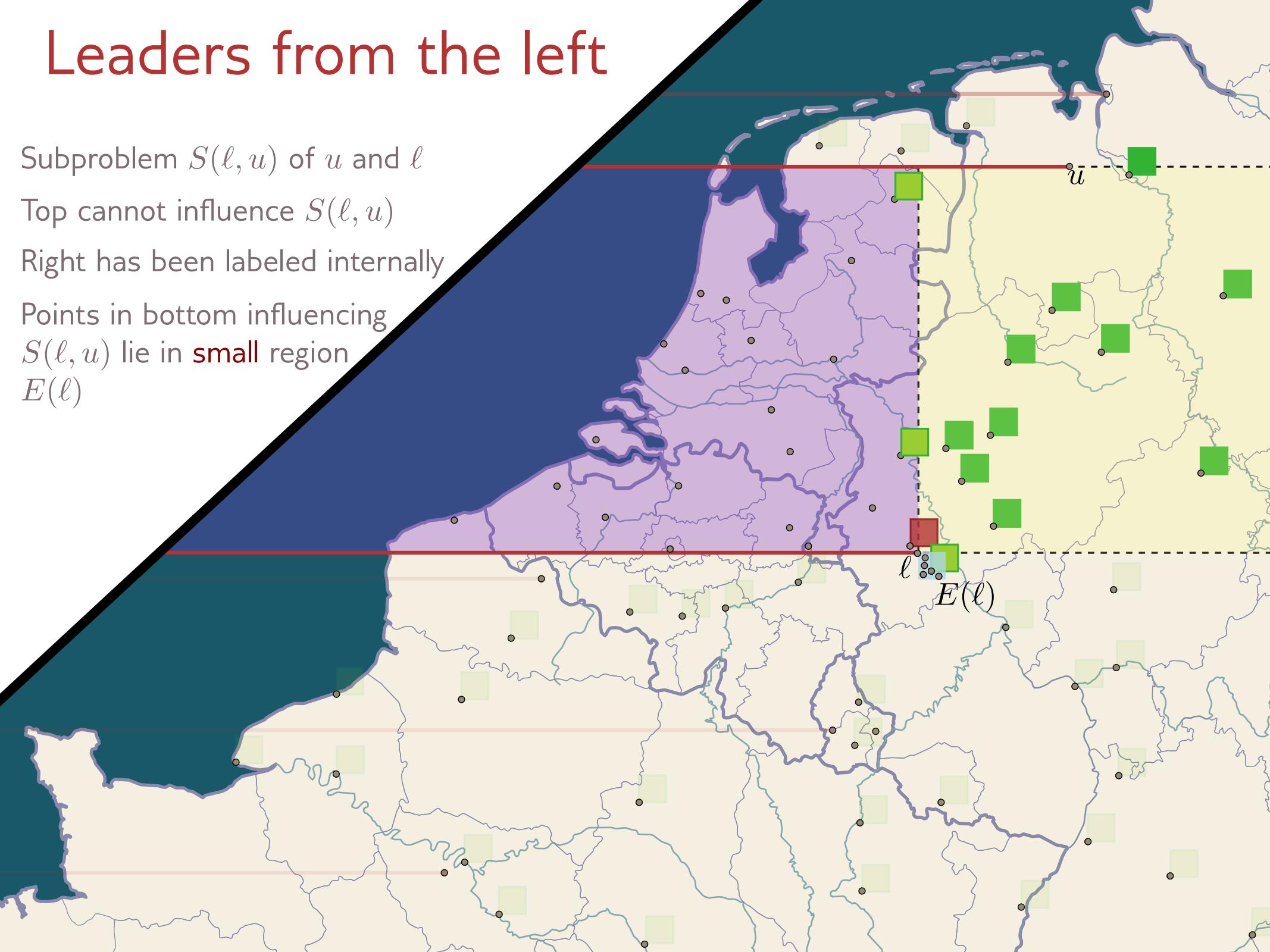
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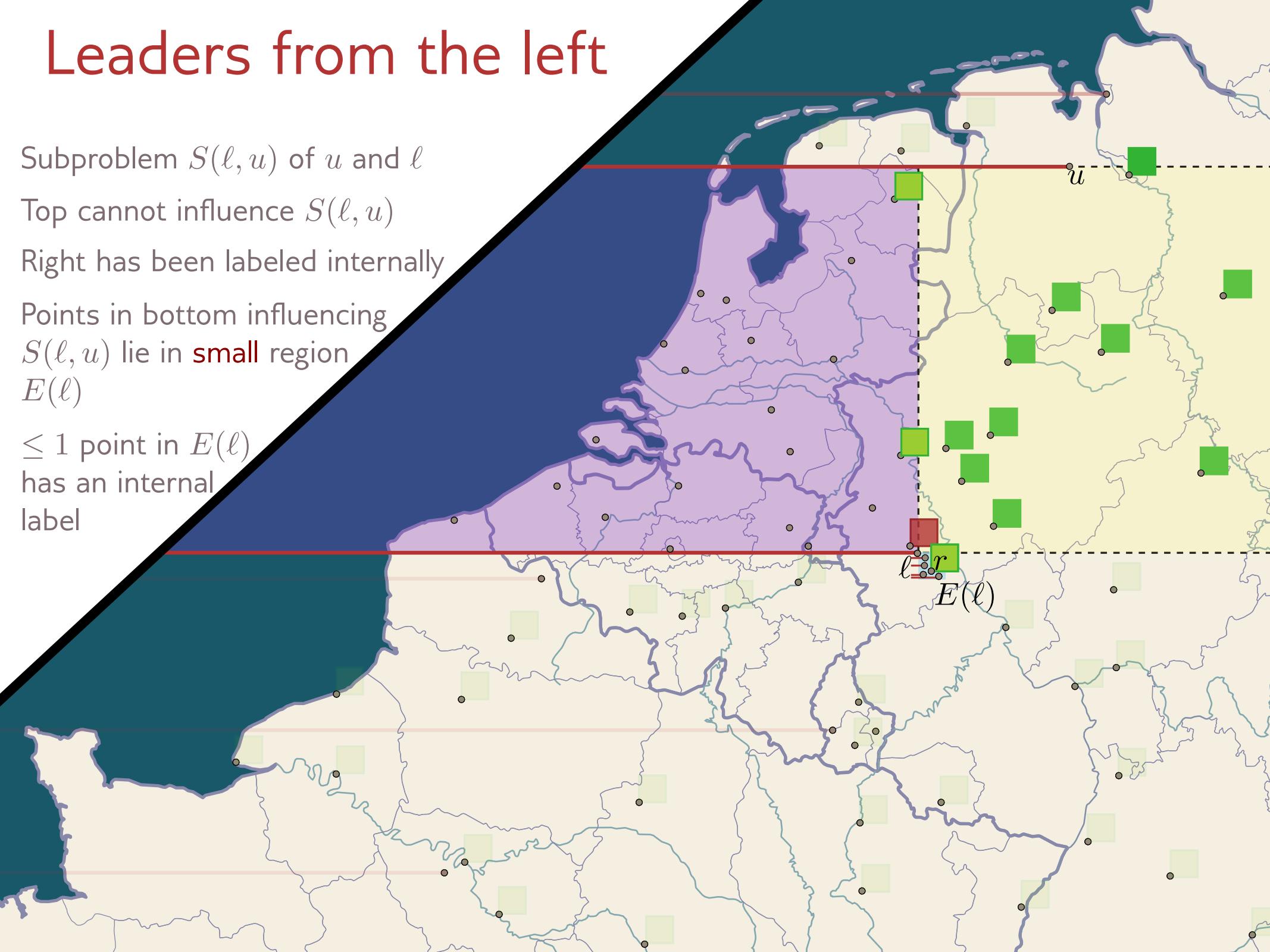
Subproblem  $S(\ell, u)$  of  $u$  and  $\ell$

Top cannot influence  $S(\ell, u)$

Right has been labeled internally

Points in bottom influencing  
 $S(\ell, u)$  lie in **small** region  
 $E(\ell)$

$\leq 1$  point in  $E(\ell)$   
has an internal  
label

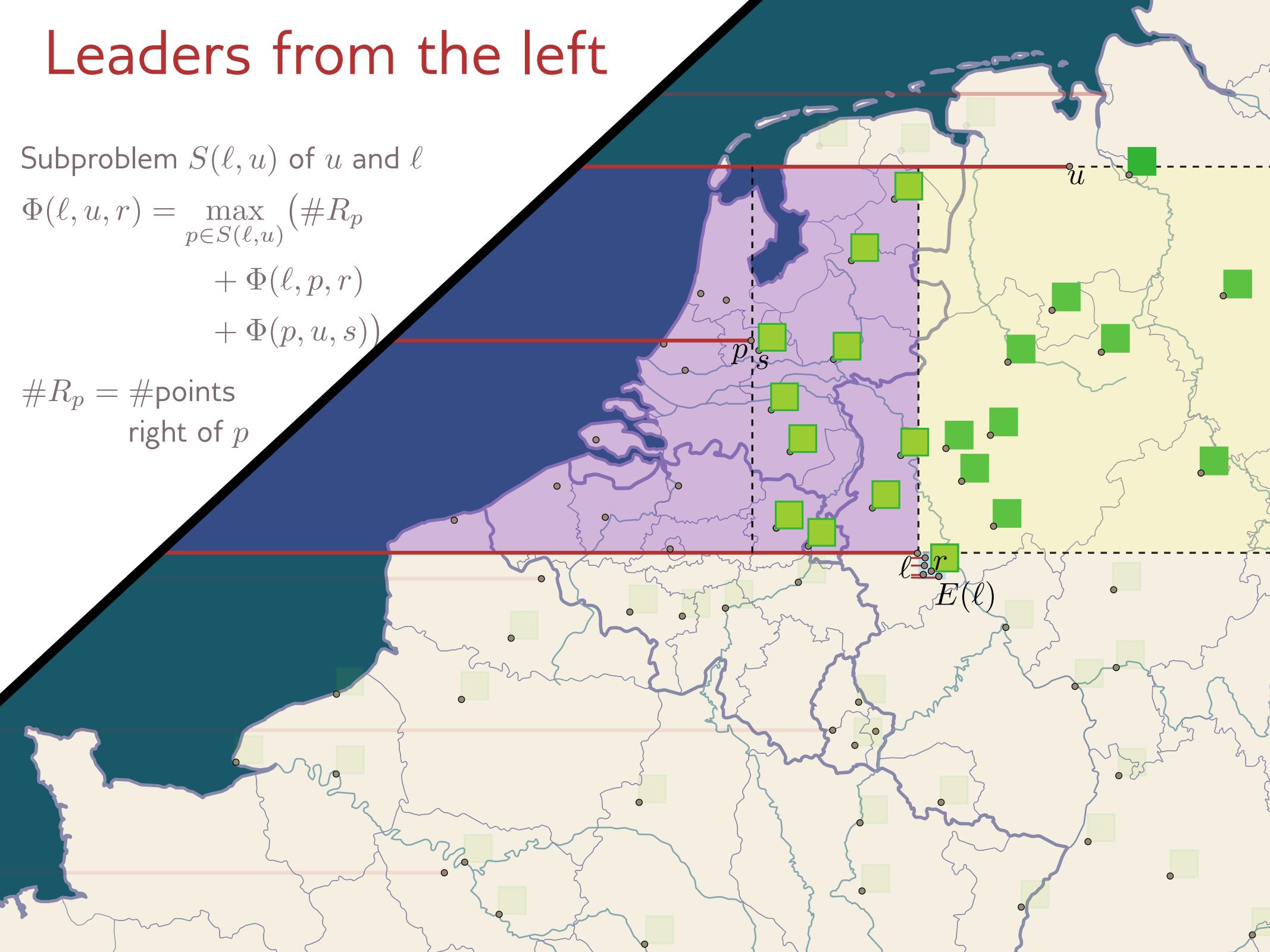


# Leaders from the left

Subproblem  $S(\ell, u)$  of  $u$  and  $\ell$

$$\Phi(\ell, u, r) = \max_{p \in S(\ell, u)} (\#R_p + \Phi(\ell, p, r) + \Phi(p, u, s))$$

$\#R_p = \#\text{points}$   
right of  $p$

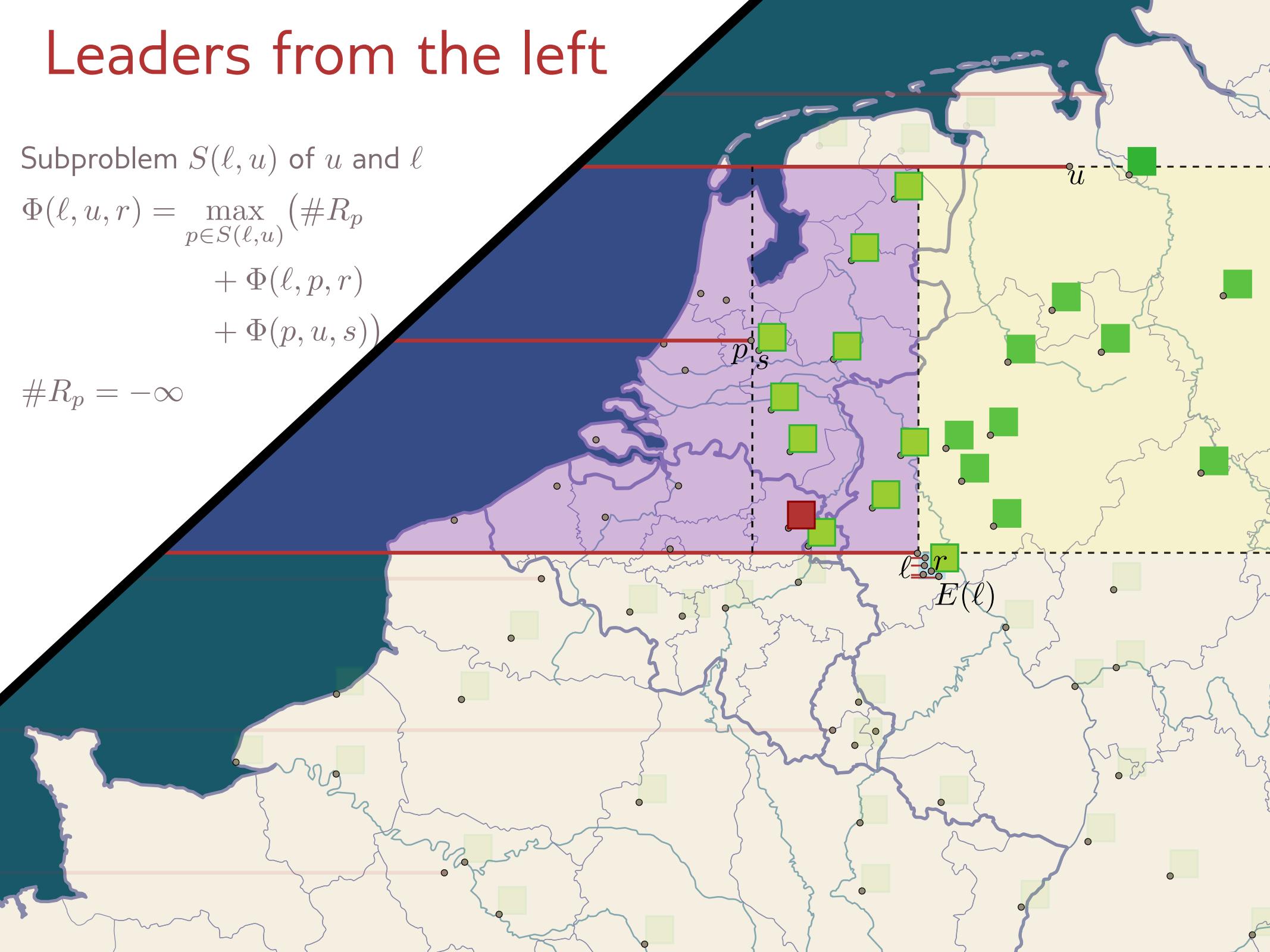


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$$\#R_p = -\infty$$

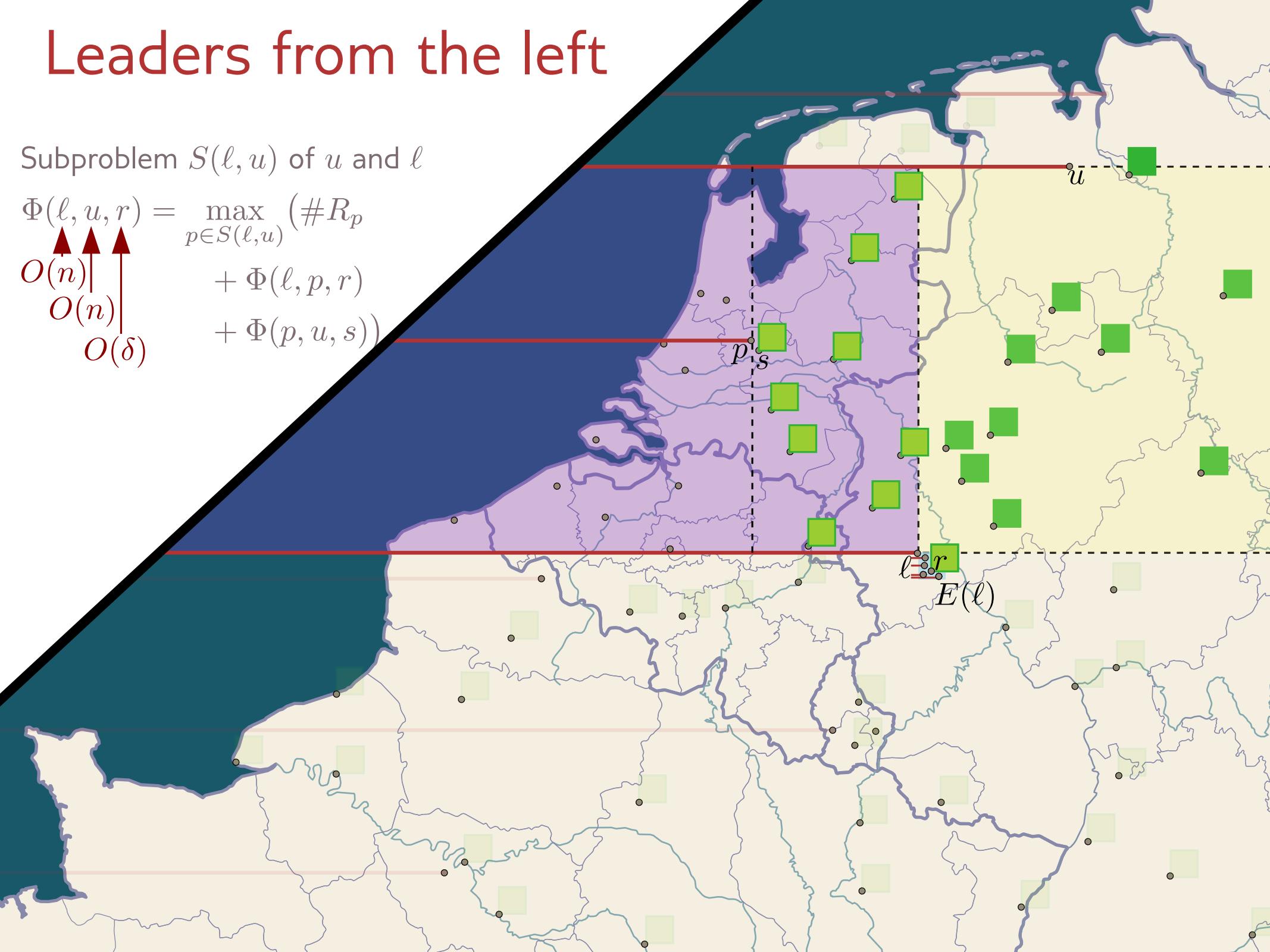


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$O(n)$   $O(n)$   $O(\delta)$



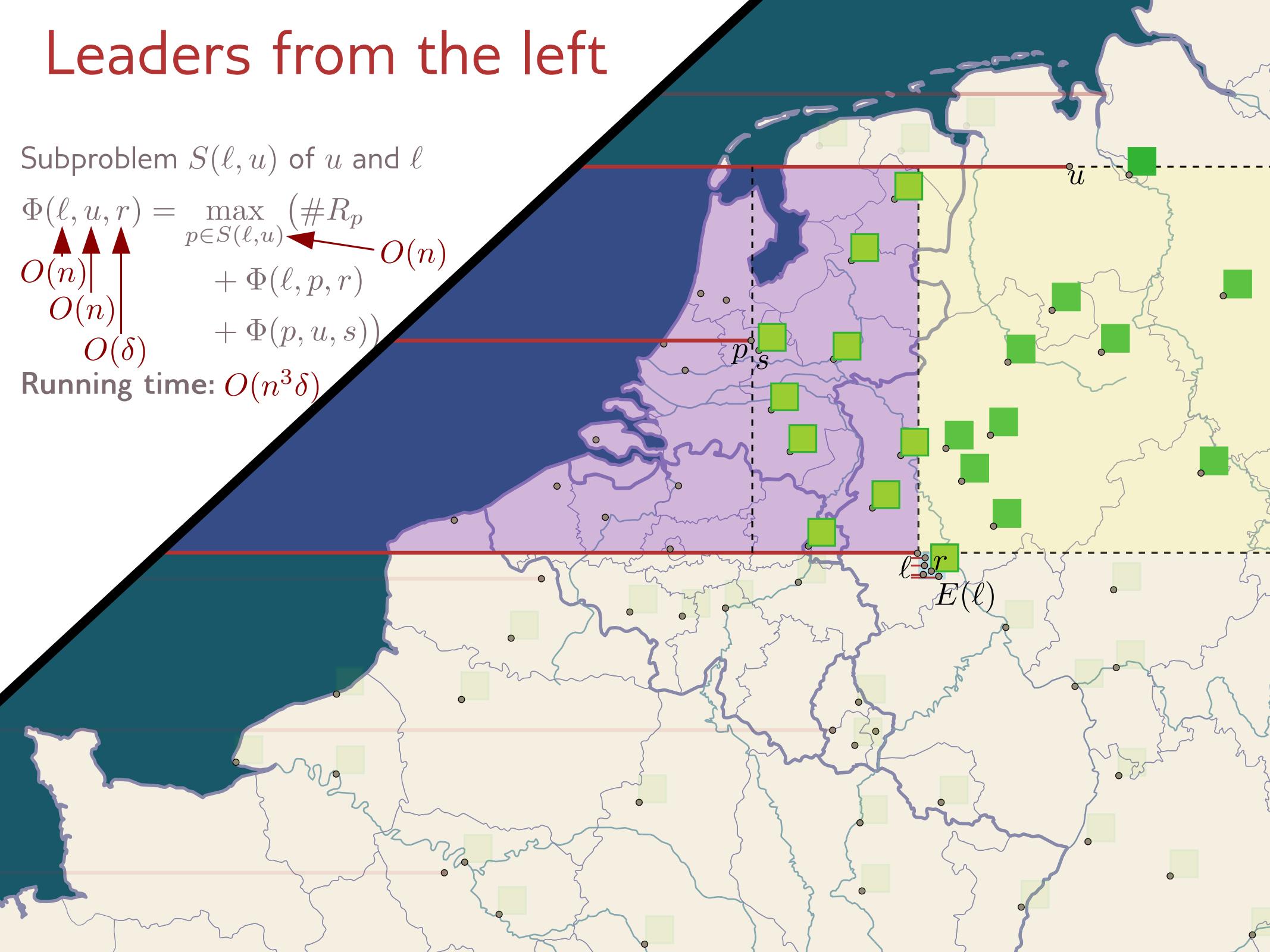
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$O(n)$  |  $O(n)$  |  $O(\delta)$

Running time:  $O(n^3\delta)$



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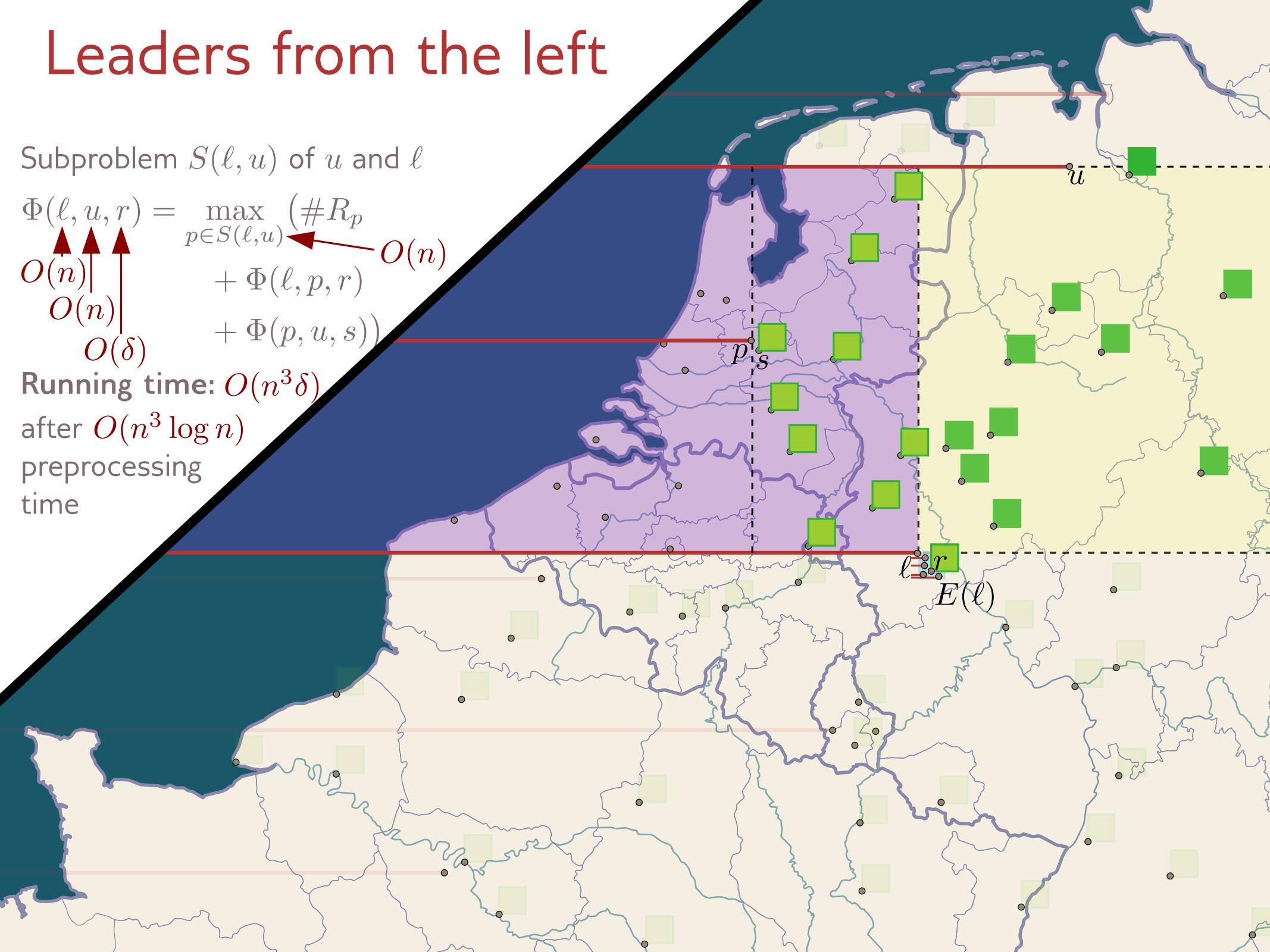
$$\Phi(\ell, u, r) = \max_{p \in S(\ell, u)} (\#R_p + \Phi(\ell, p, r) + \Phi(p, u, s))$$

$O(n)$  |  
 $O(n)$  |  
 $O(\delta)$

Running time:  $O(n^3\delta)$

after  $O(n^3 \log n)$

preprocessing  
time



# Other leader directions

Region  $E(\ell)$  has different shape



# Other leader directions

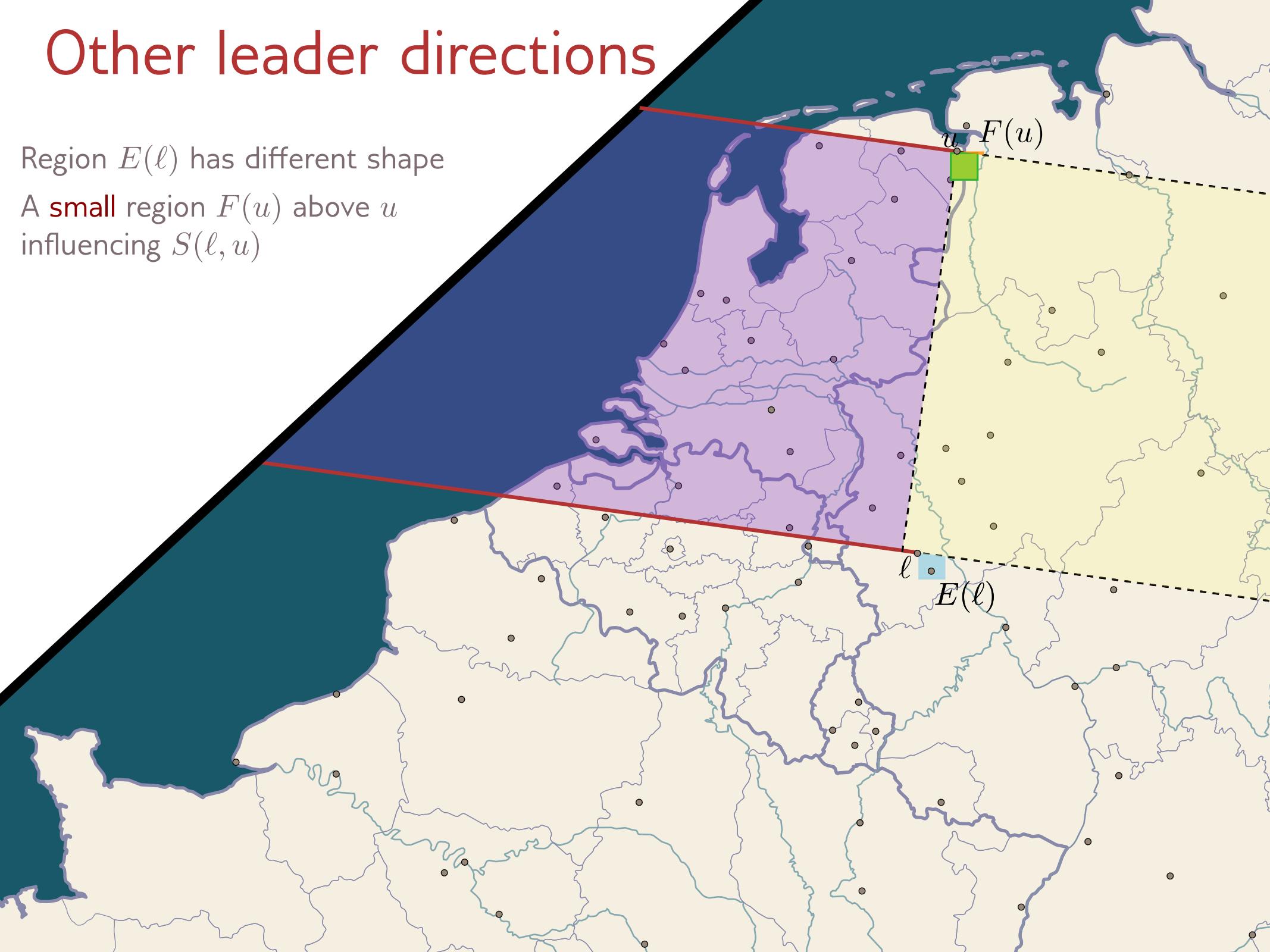
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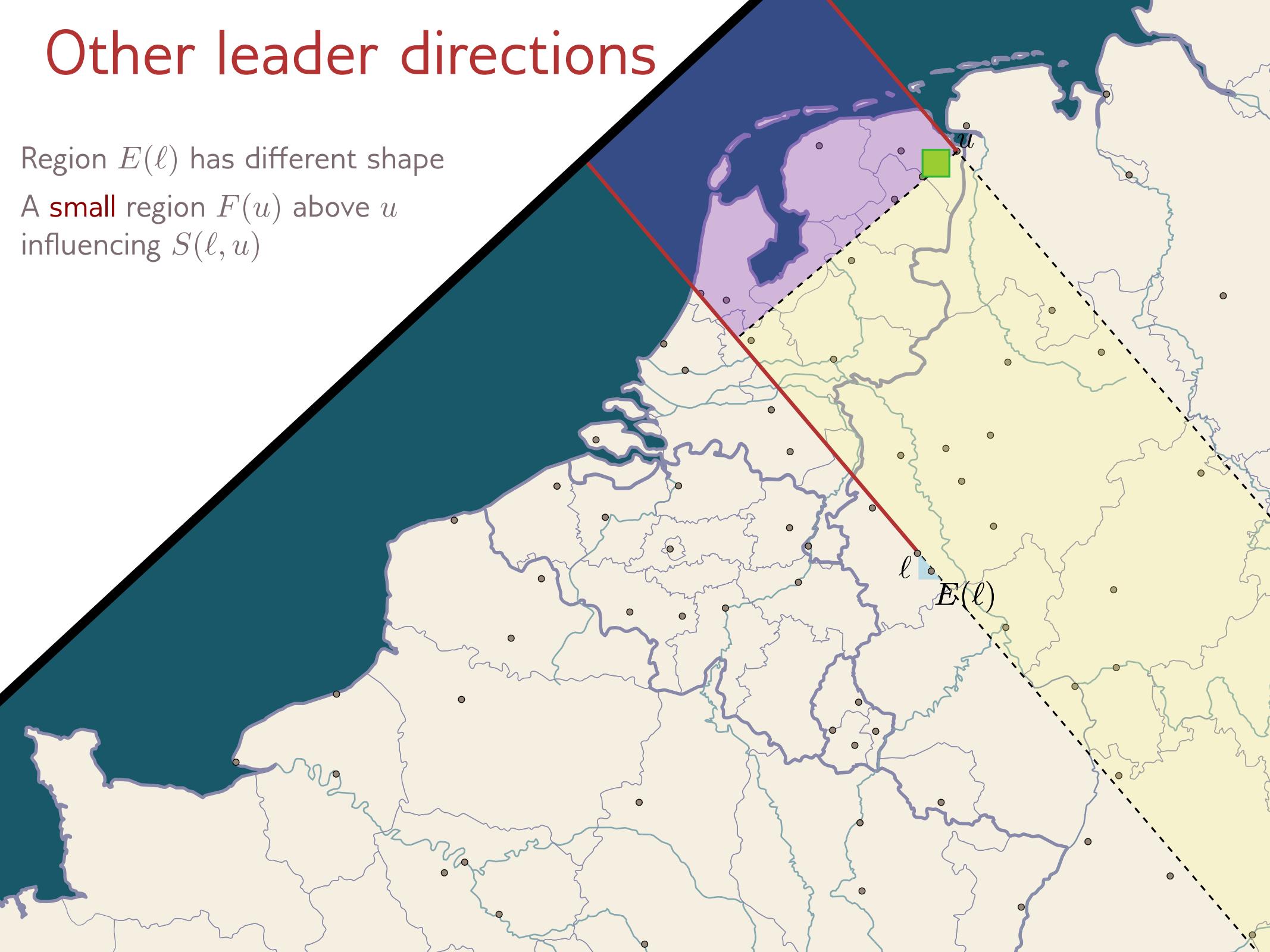
A **small** region  $F(u)$  above  $u$   
influencing  $S(\ell, u)$



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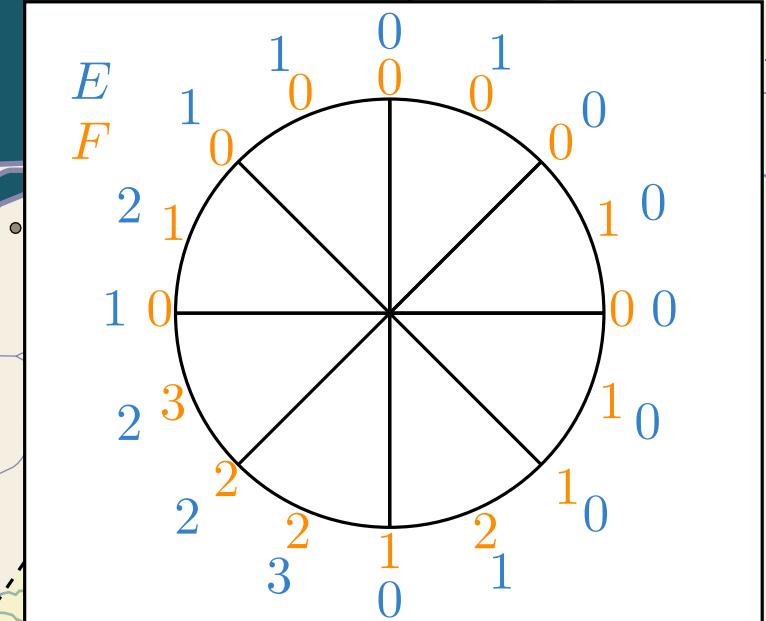
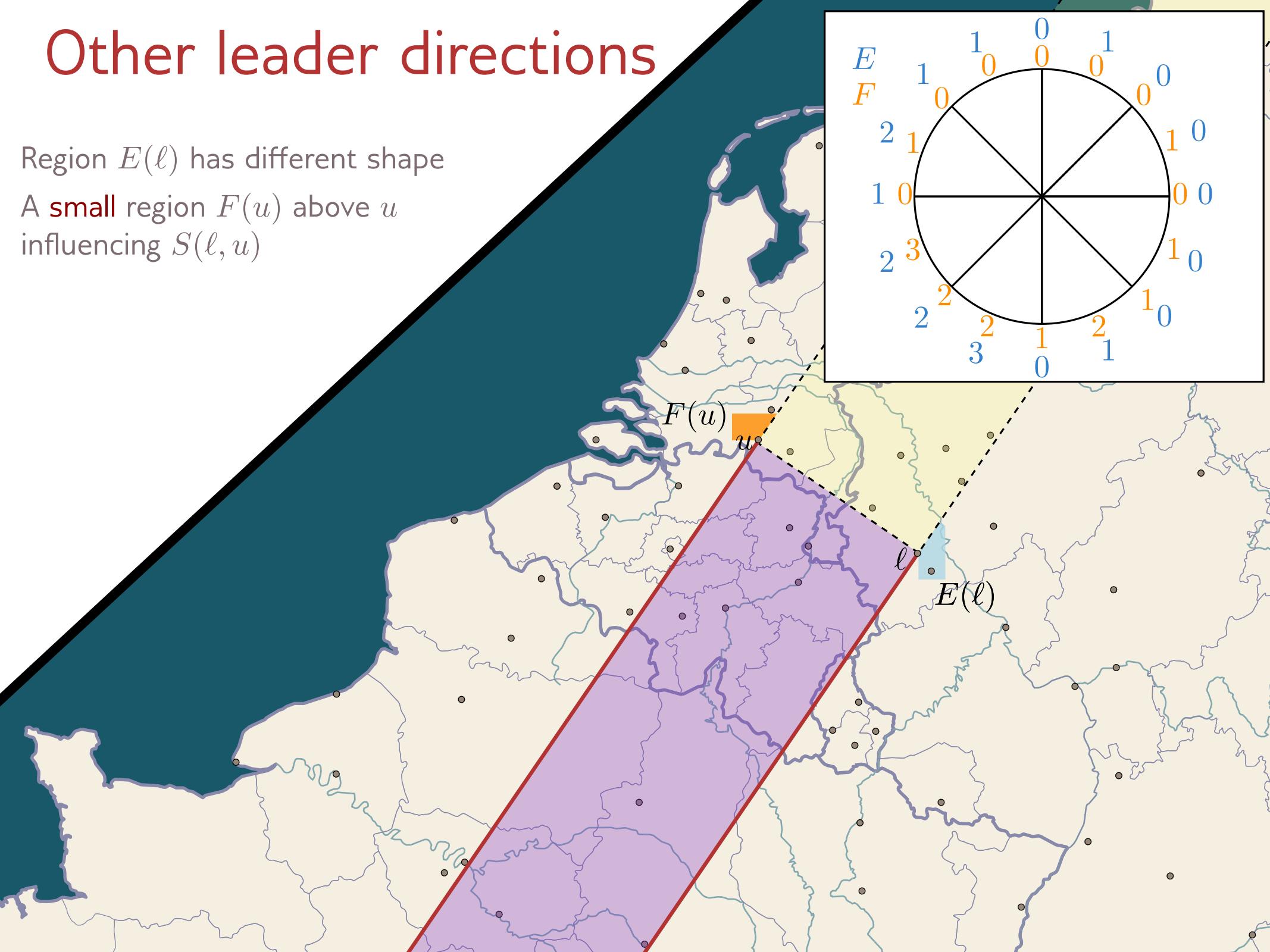
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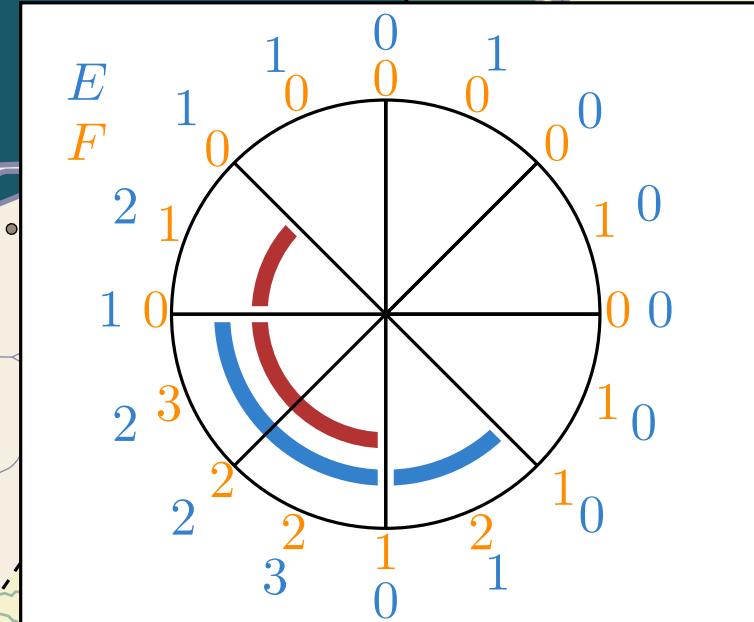
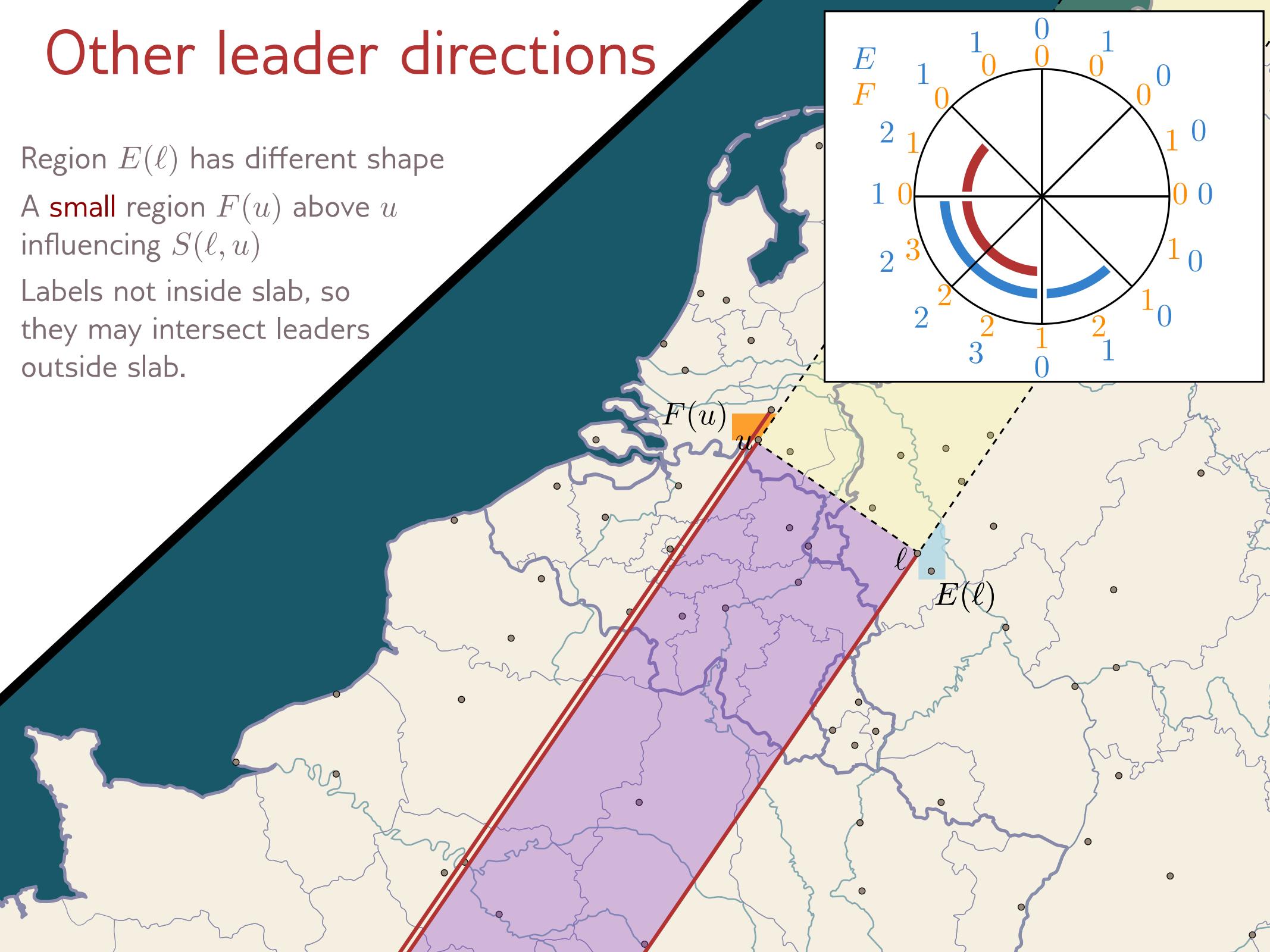


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Region  $E(\ell)$  has different shape

A **small** region  $F(u)$  above  $u$   
influencing  $S(\ell, u)$

Labels not inside slab, so  
they may intersect leaders  
outside slab.

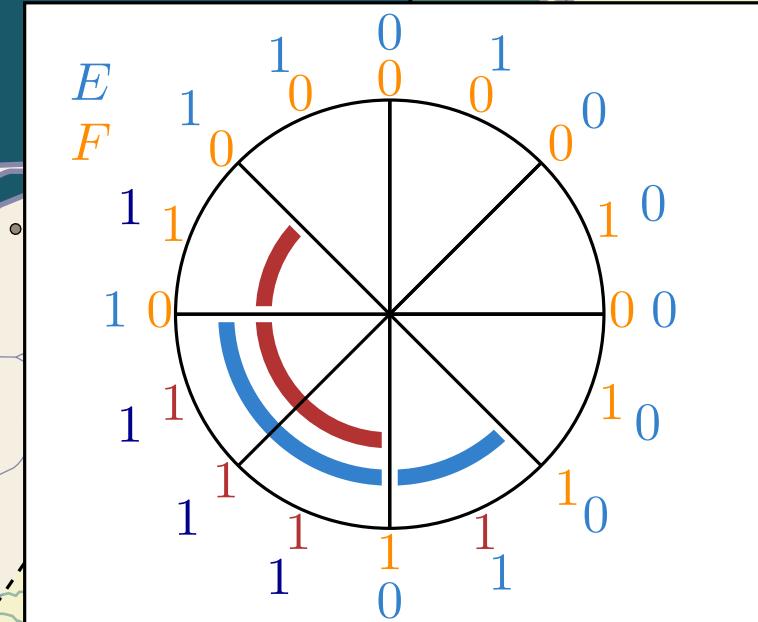
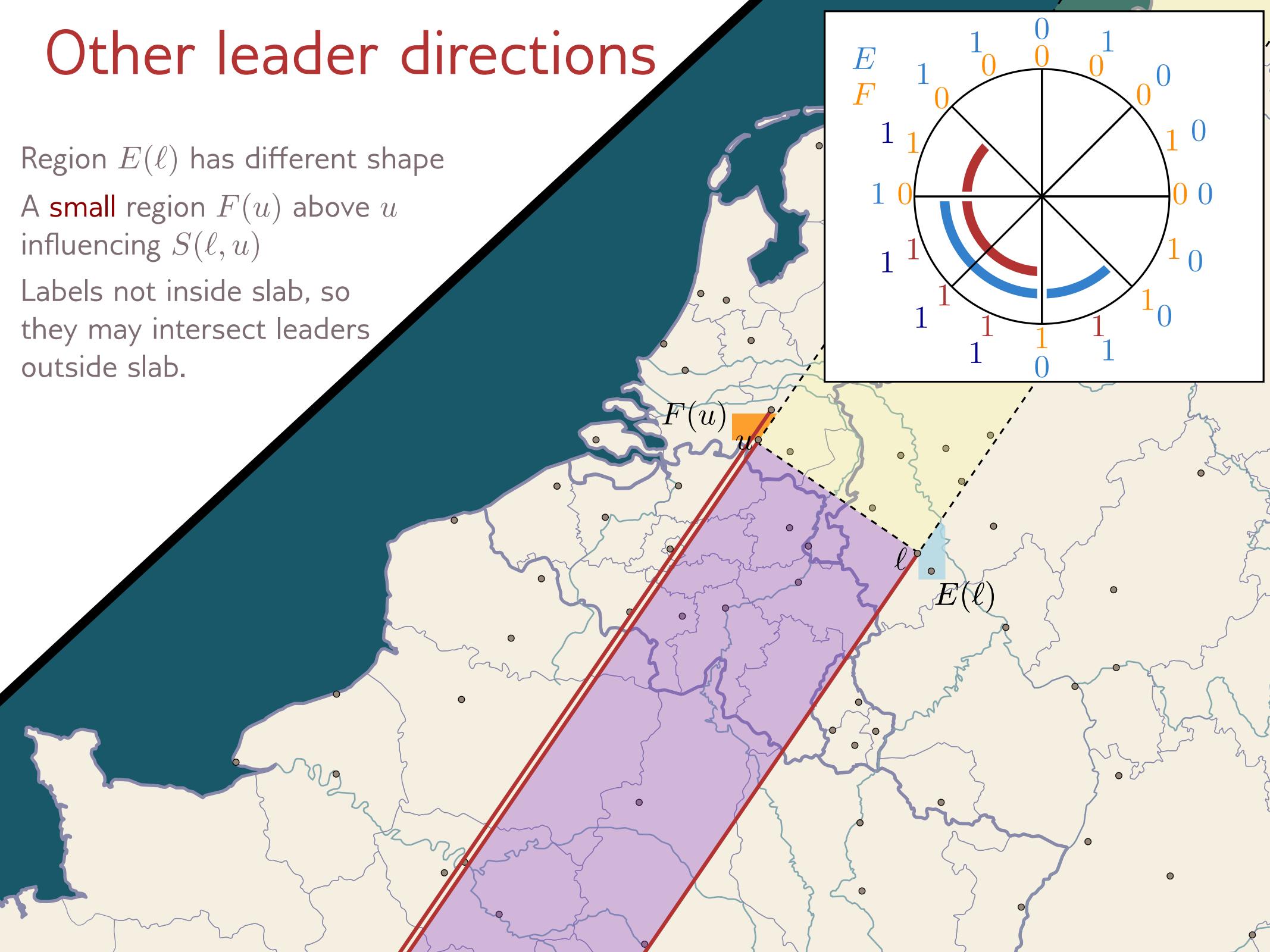


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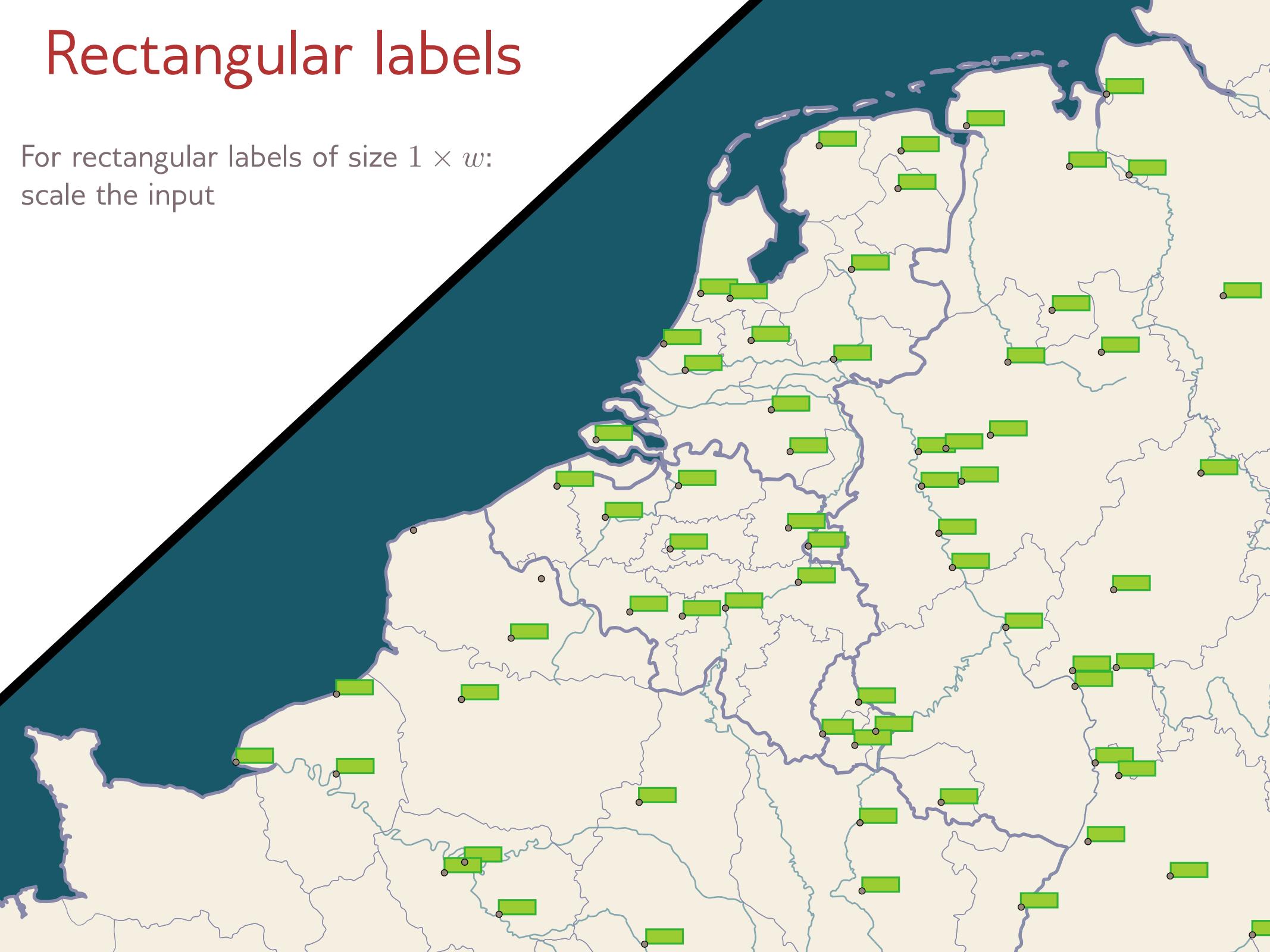
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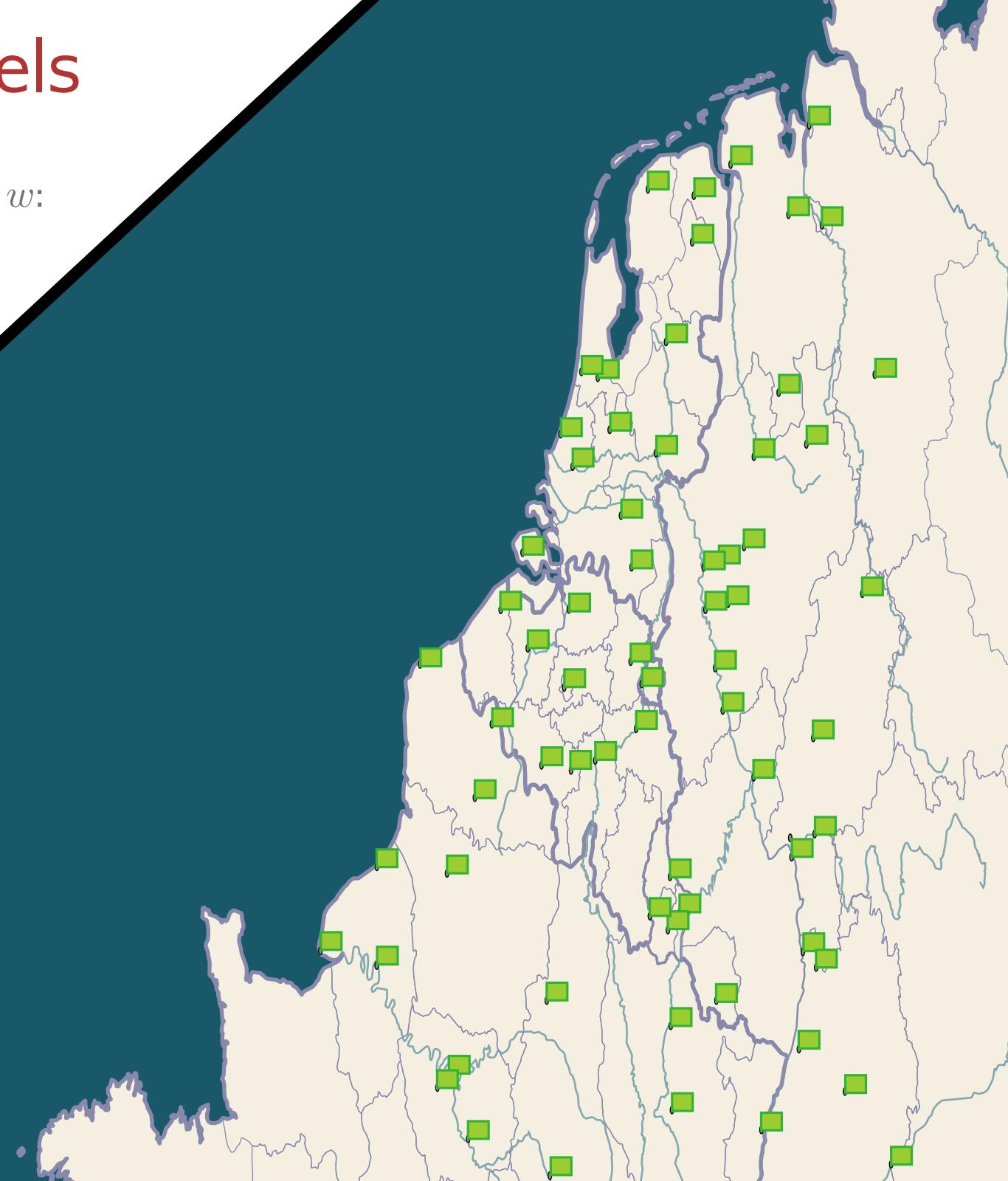
# Rectangular labels

For rectangular labels of size  $1 \times w$ :  
scale the input



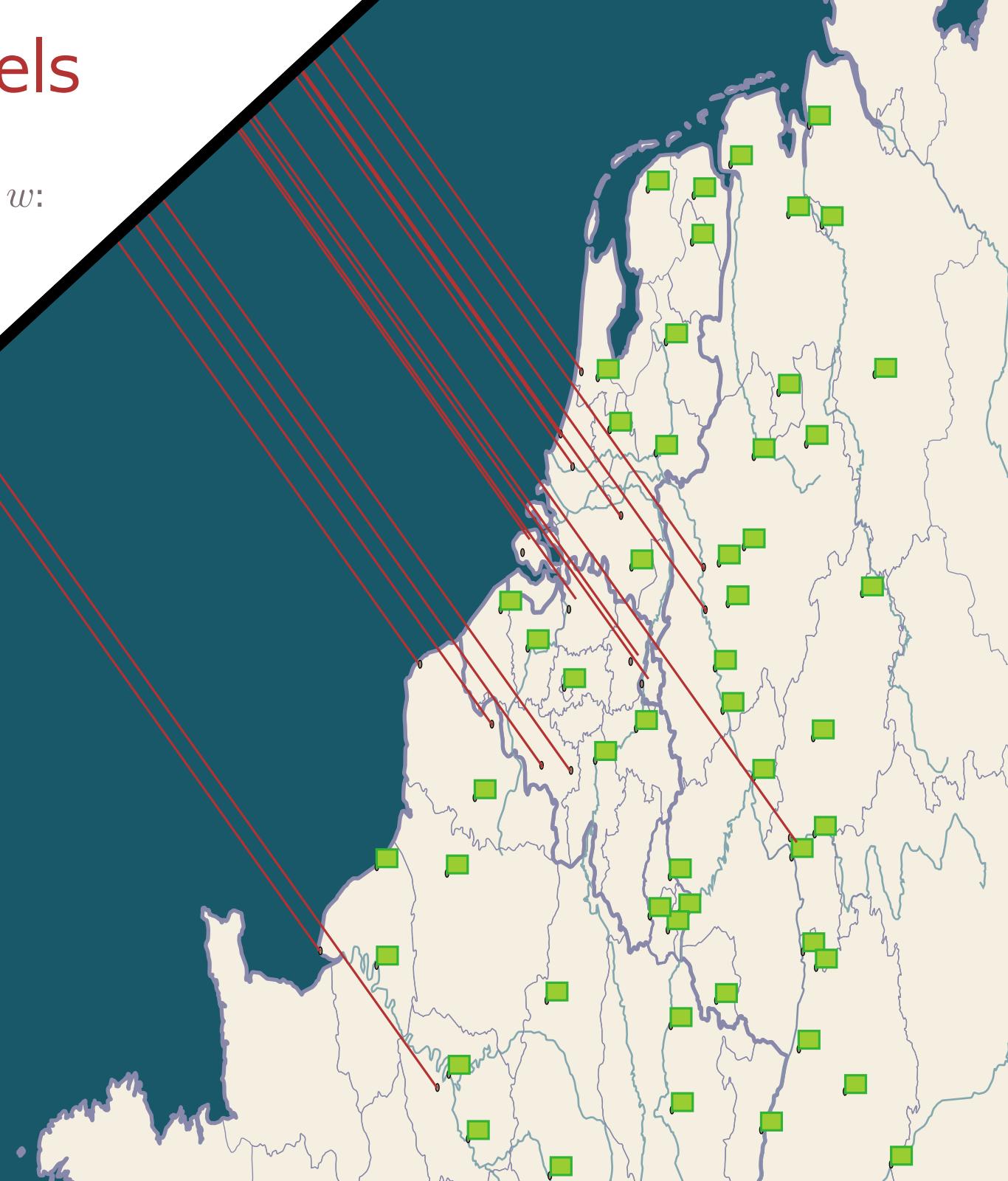
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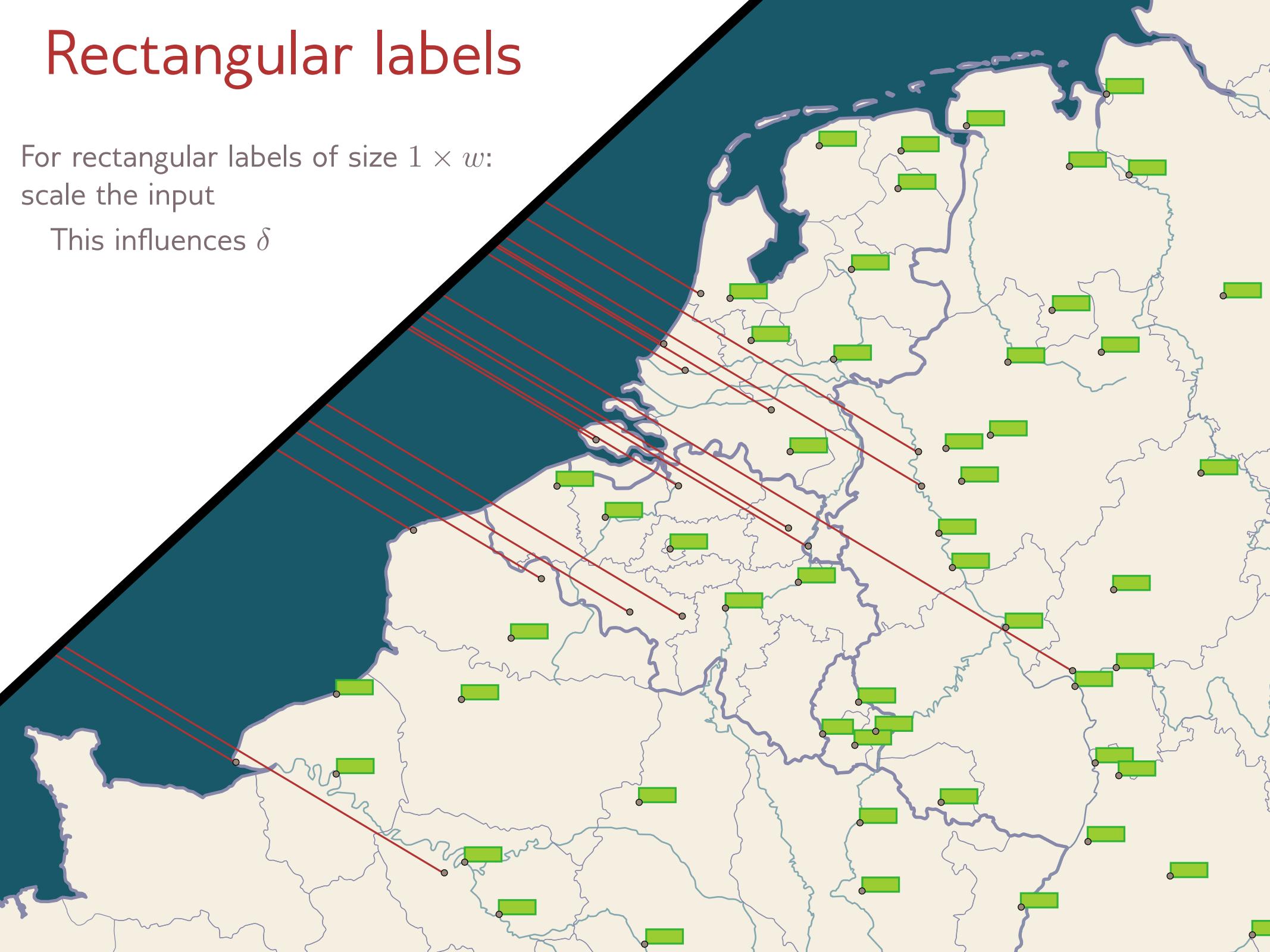
For rectangular labels of size  $1 \times w$ :  
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# Rectangular labels

For rectangular labels of size  $1 \times w$ :  
scale the input

This influences  $\delta$

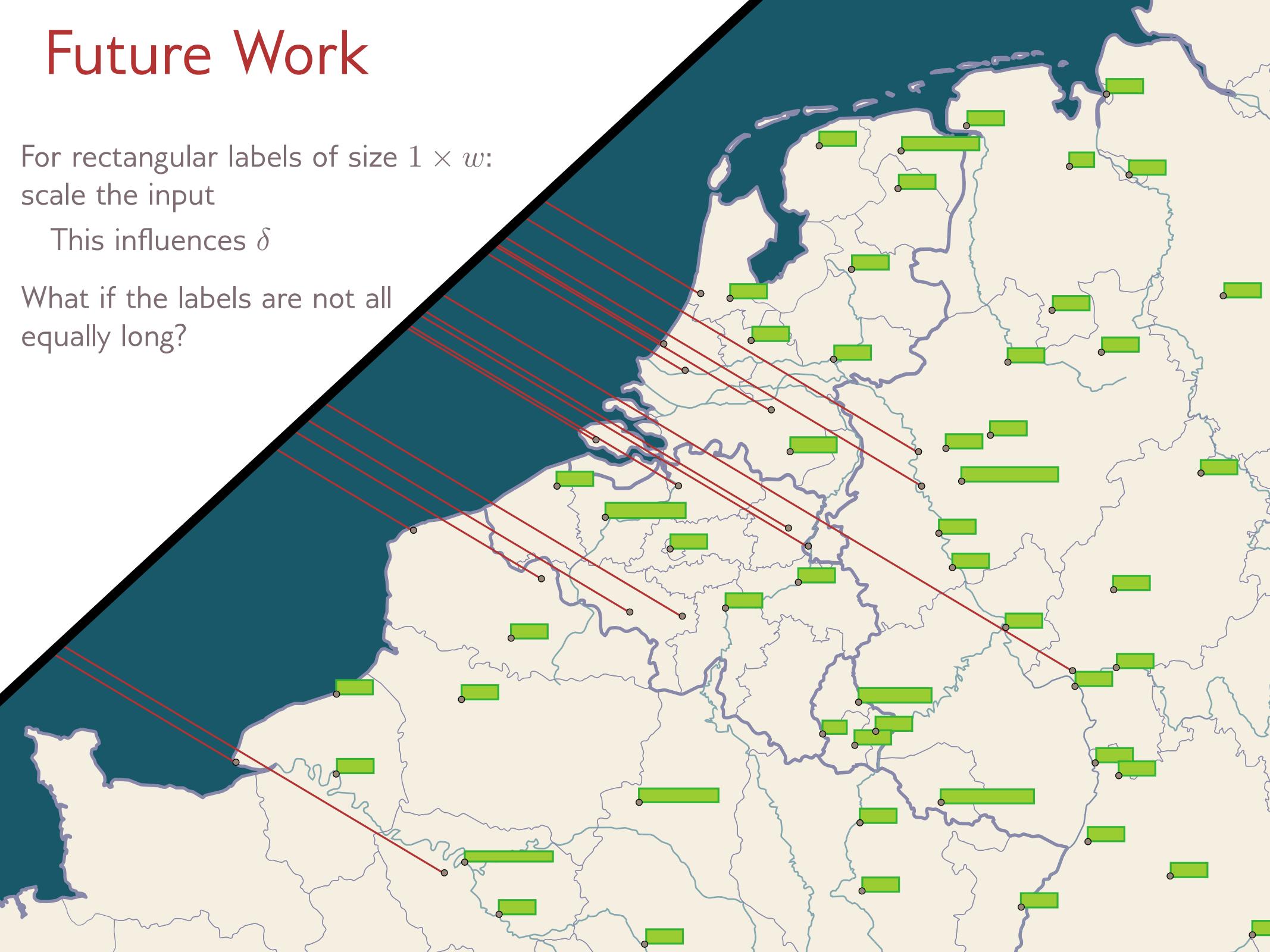


# Future Work

For rectangular labels of size  $1 \times w$ :  
scale the input

This influences  $\delta$

What if the labels are not all  
equally long?



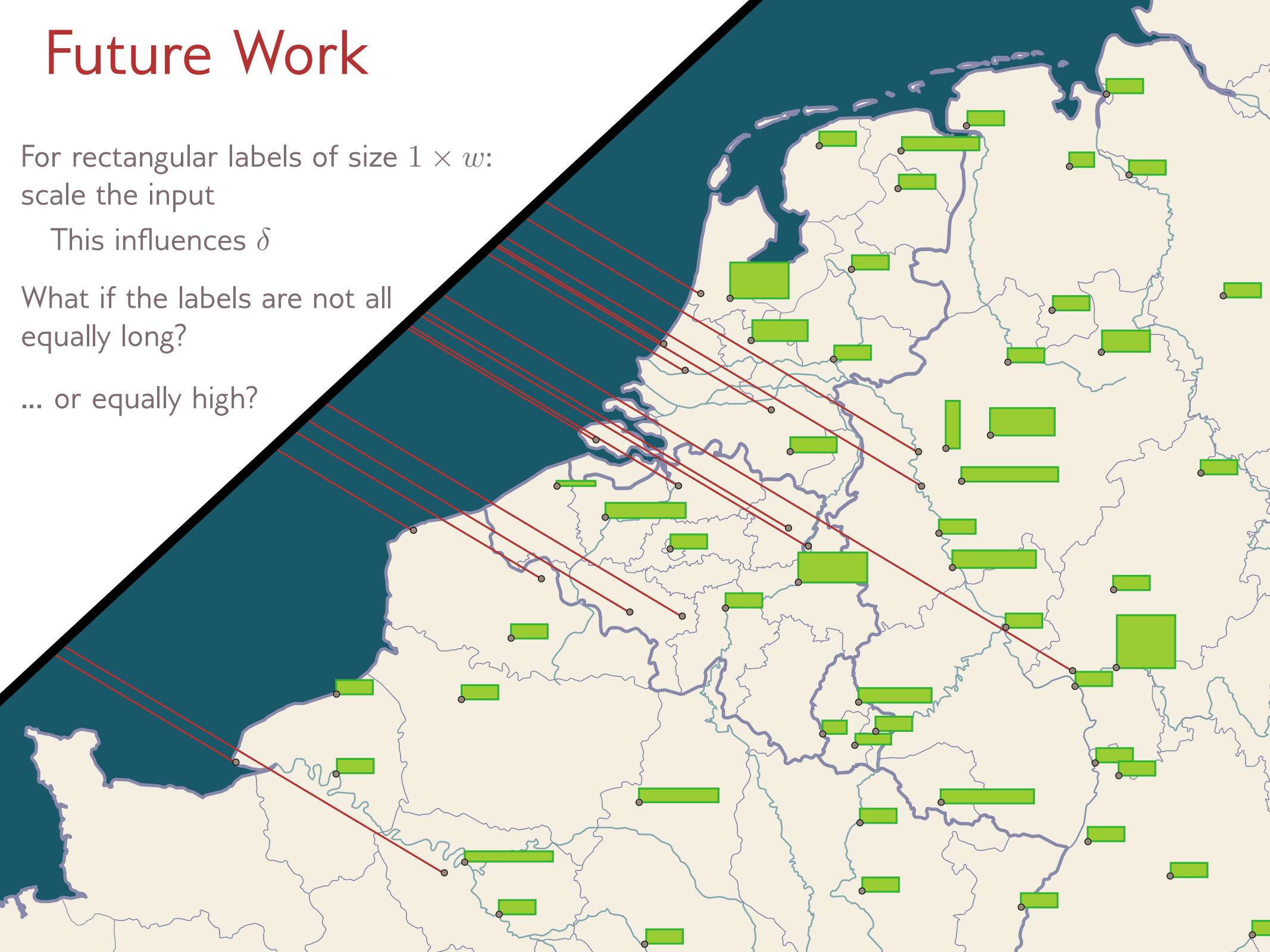
# Future Work

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This influences  $\delta$

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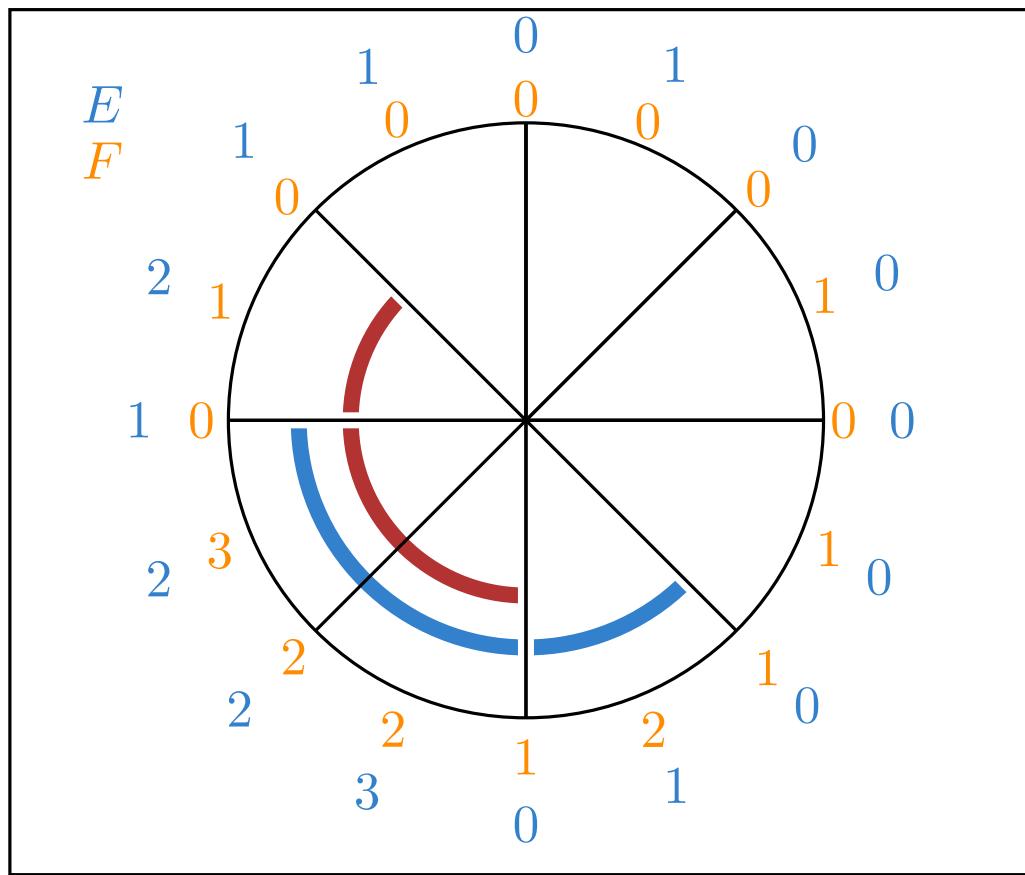
... or equally high?





# Thank You!





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All orientations  
Compute optimal  
orientation

Parameters:

$n = \# \text{points}$   
 $w = \text{label width}$   
 $\delta = \min(n, \frac{1}{\min \|pq\|})$

