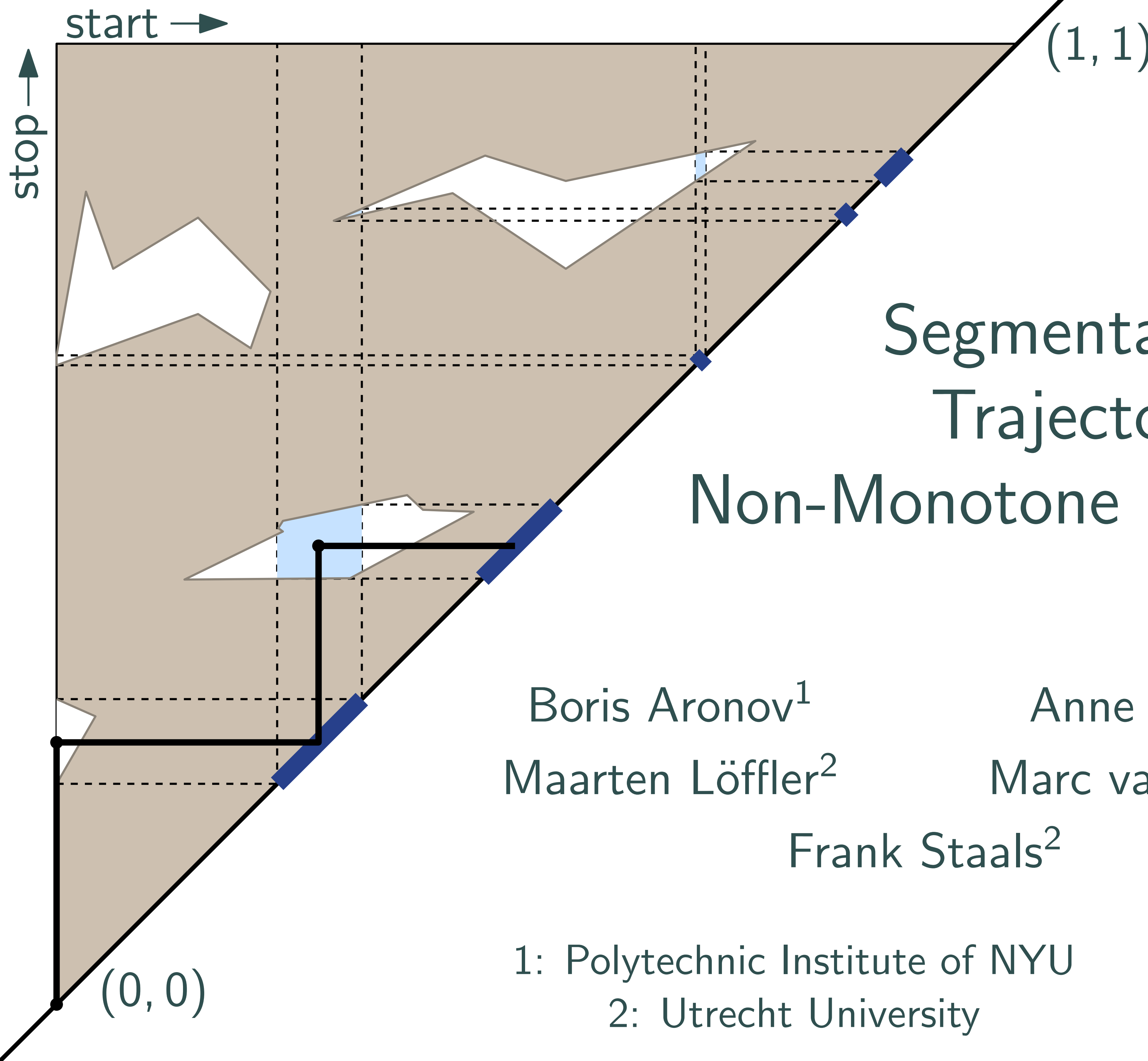
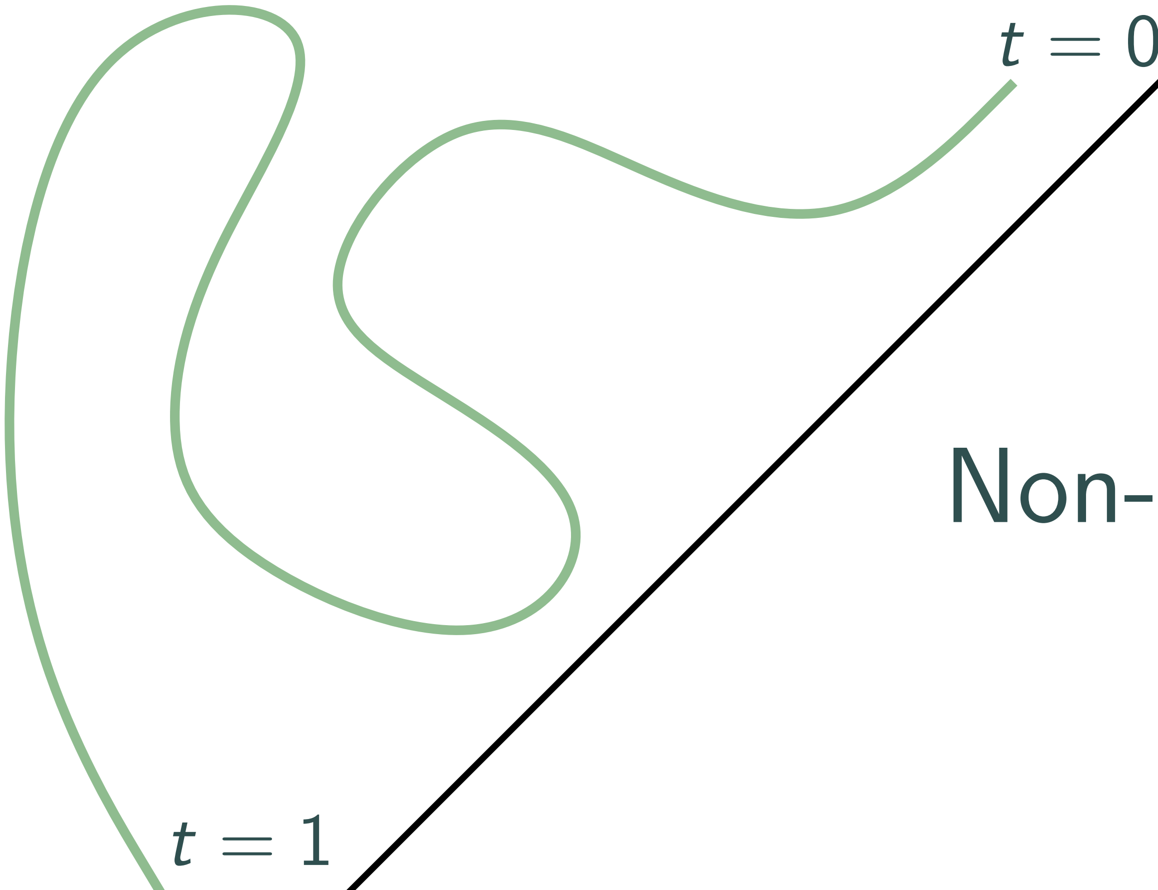



# Segmentation of Trajectories on Non-Monotone Criteria

Boris Aronov[1]
Anne Driemel[2]
Maarten Löffler[2]
Marc van Kreveld[2]
Frank Staals[2]

1: Polytechnic Institute of NYU
2: Utrecht University

$T : [0, 1] \to \mathbb{R}^2$

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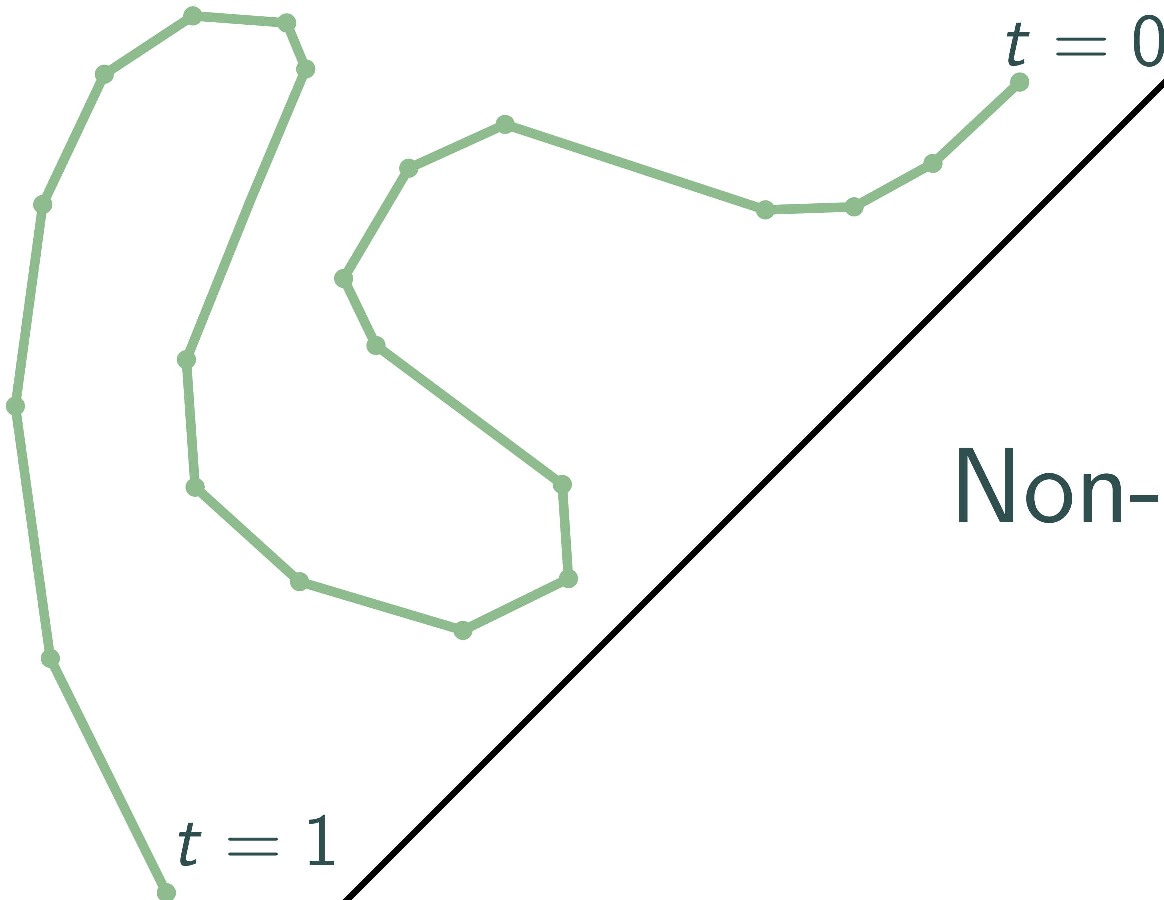


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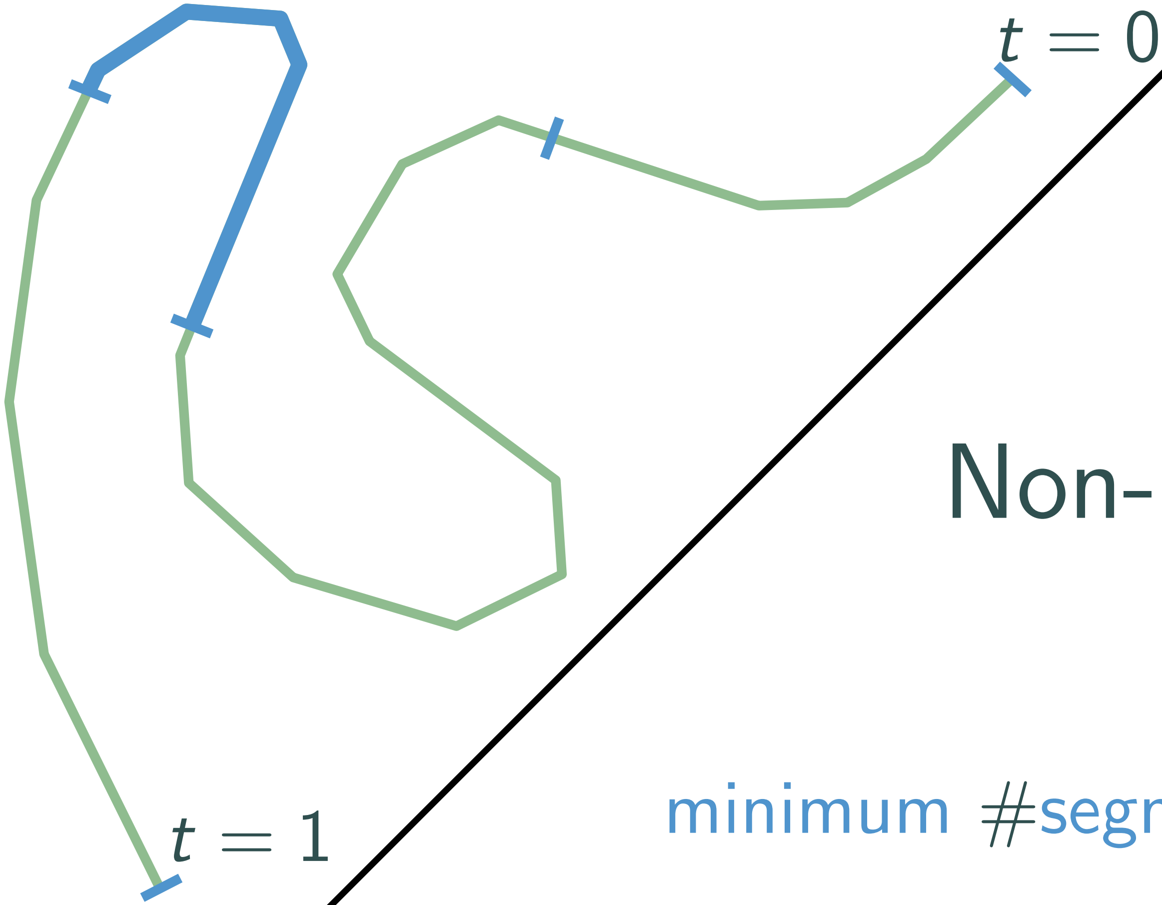
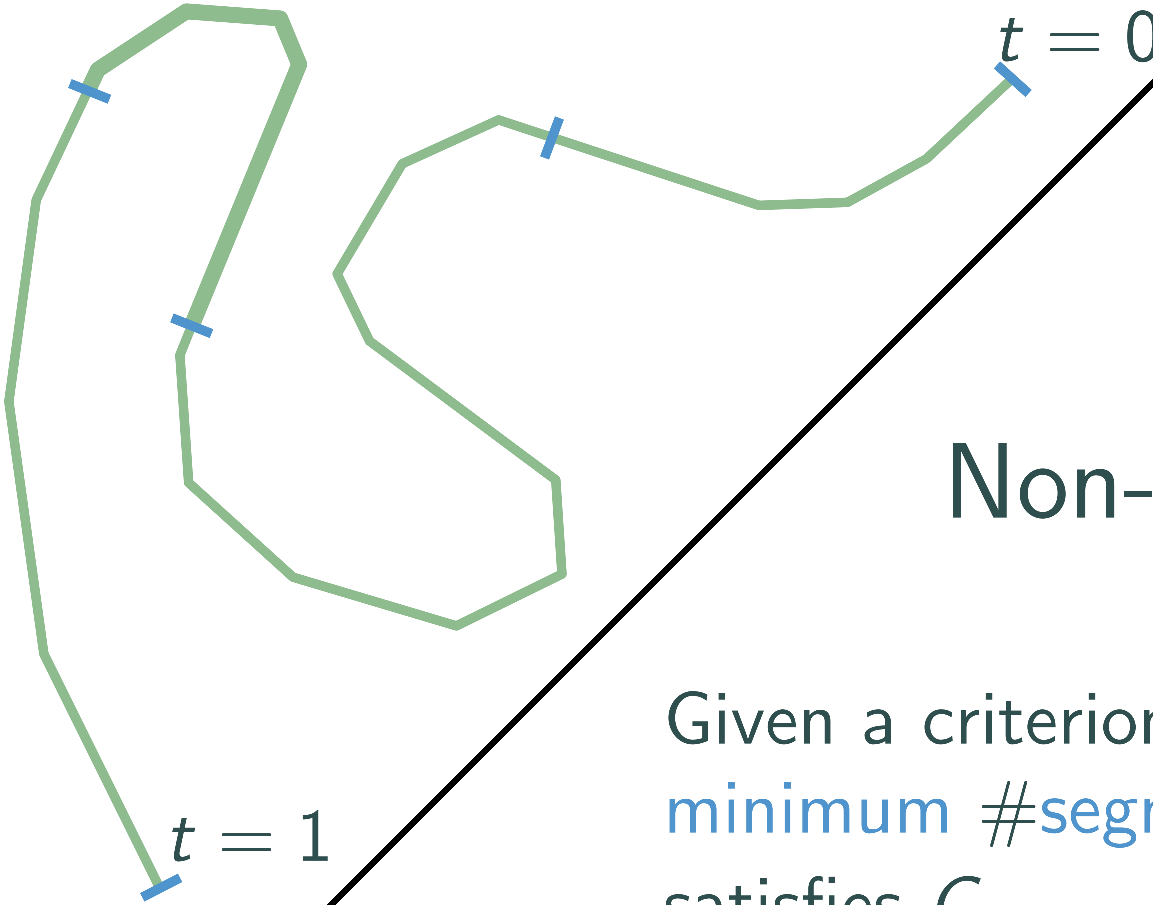



# Segmentation of Trajectories on Non-Monotone Criteria

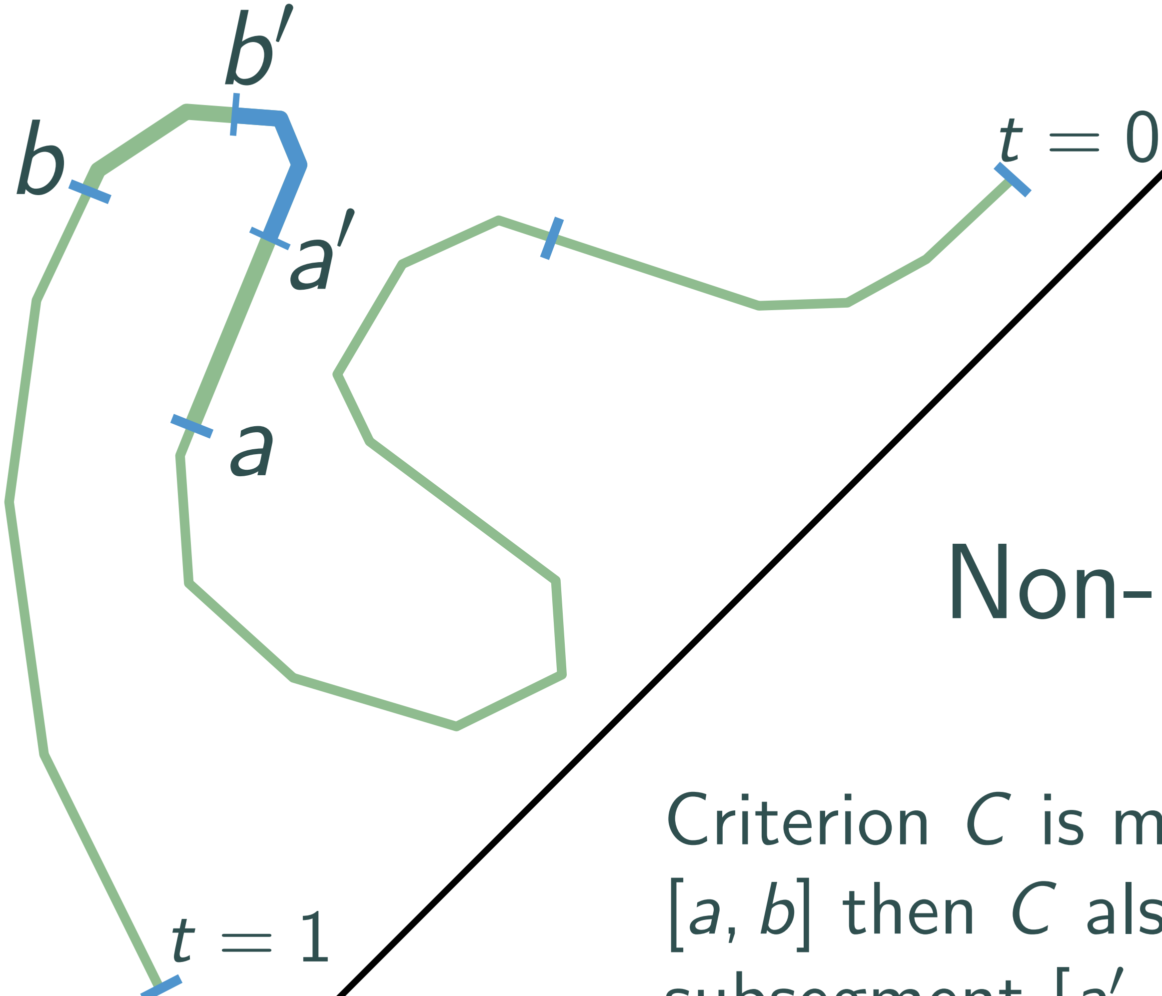
Partition $T$ into a minimum #segments,




# Segmentation of Trajectories on Non-Monotone Criteria

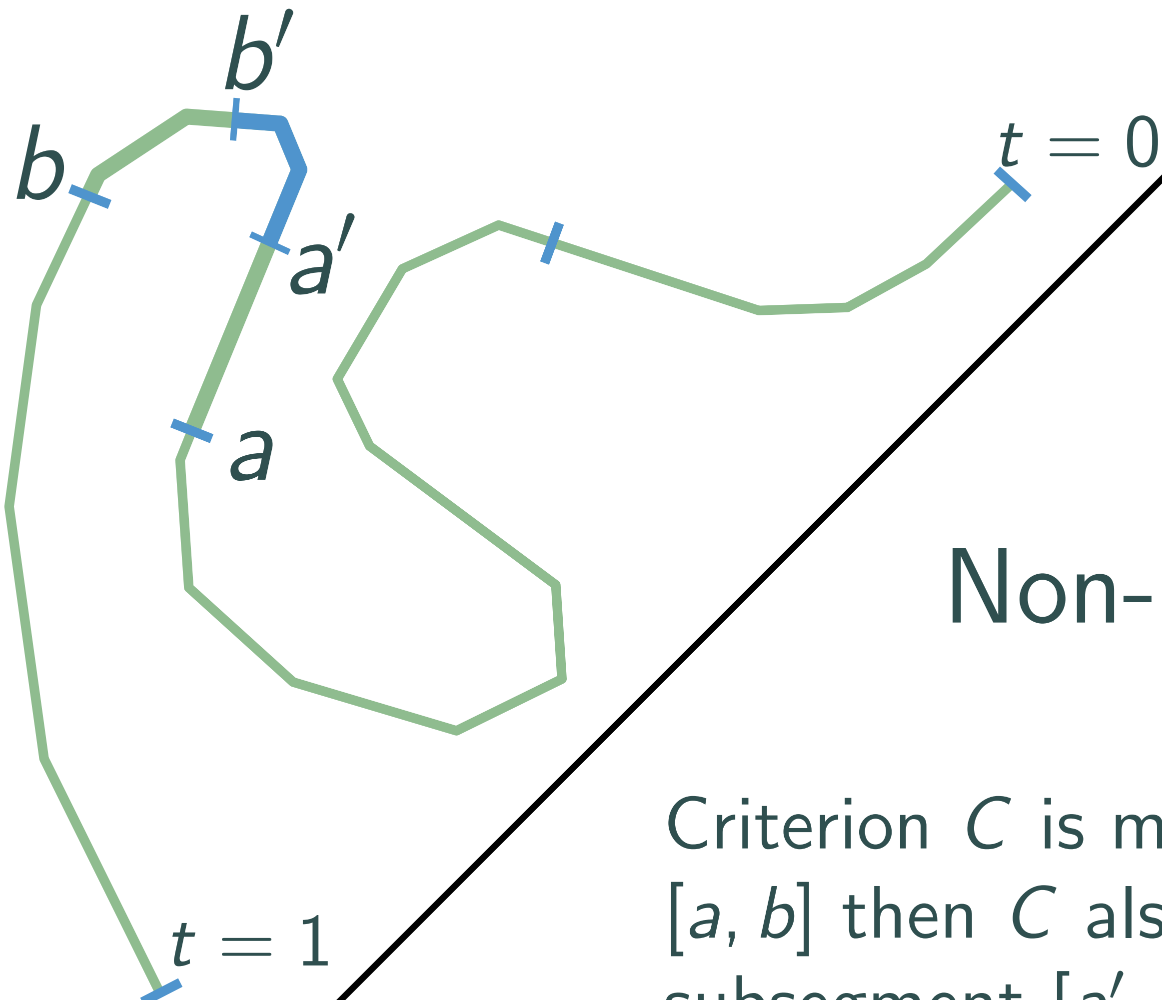
Given a criterion $C$. Partition $T$ into a minimum #segments, s.t. each segment satisfies $C$.

**For example:** The minimum and maximum speed differ by at most $x$.




# Segmentation of Trajectories on Non-Monotone Criteria

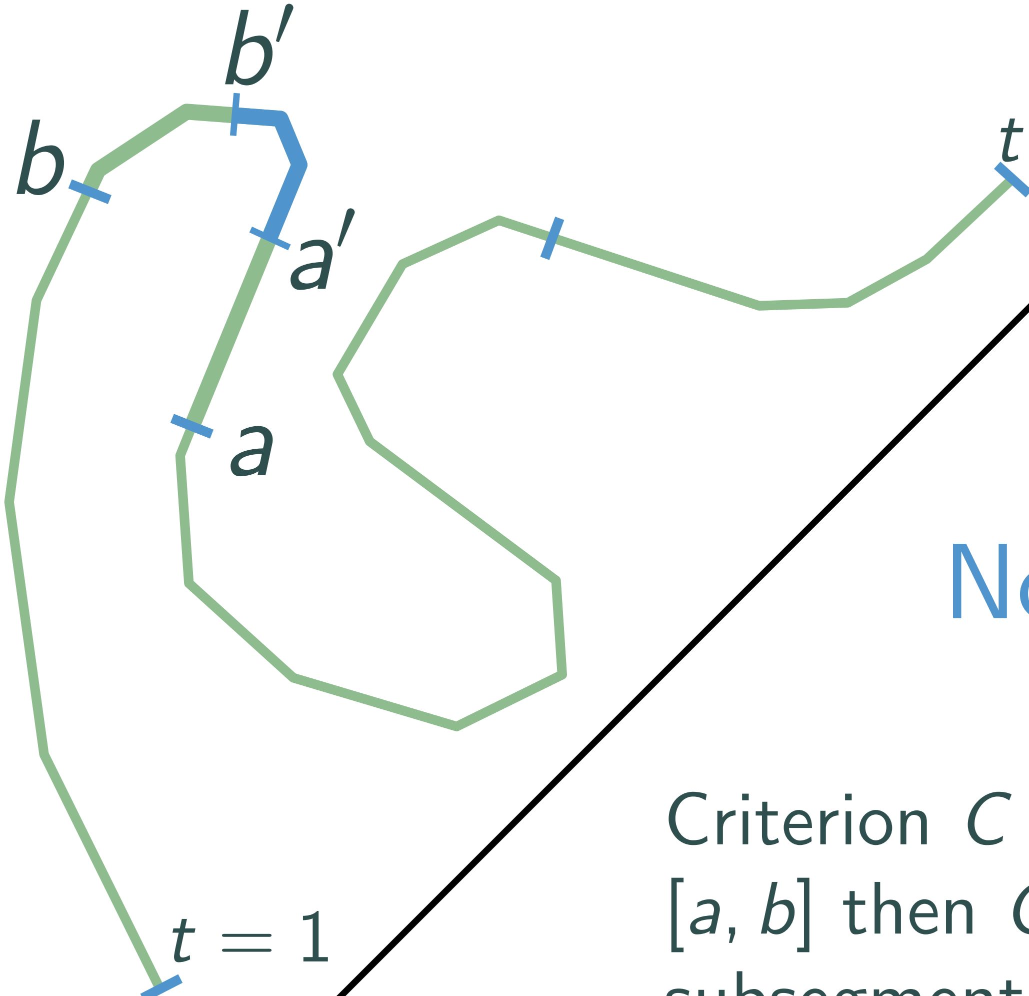
Criterion $C$ is monotone if $C$ holds on $[a, b]$ then $C$ also holds on any subsegment $[a', b'] \subseteq [a, b]$




# Segmentation of Trajectories on Non-Monotone Criteria

Criterion $C$ is monotone if $C$ holds on $[a, b]$ then $C$ also holds on any subsegment $[a', b'] \subseteq [a, b]$

Solvable in $O(n \log n)$ time [Buchin, Driemel, van Kreveld, Sacristan, 2011].

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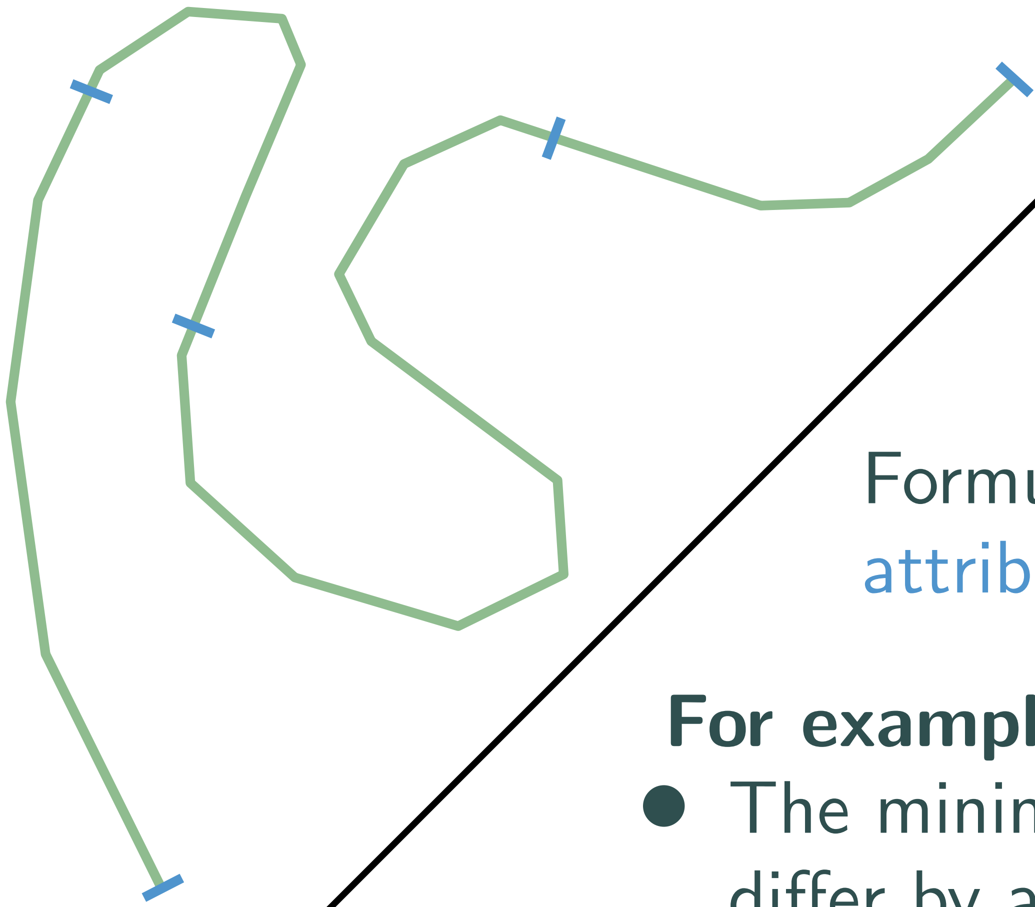


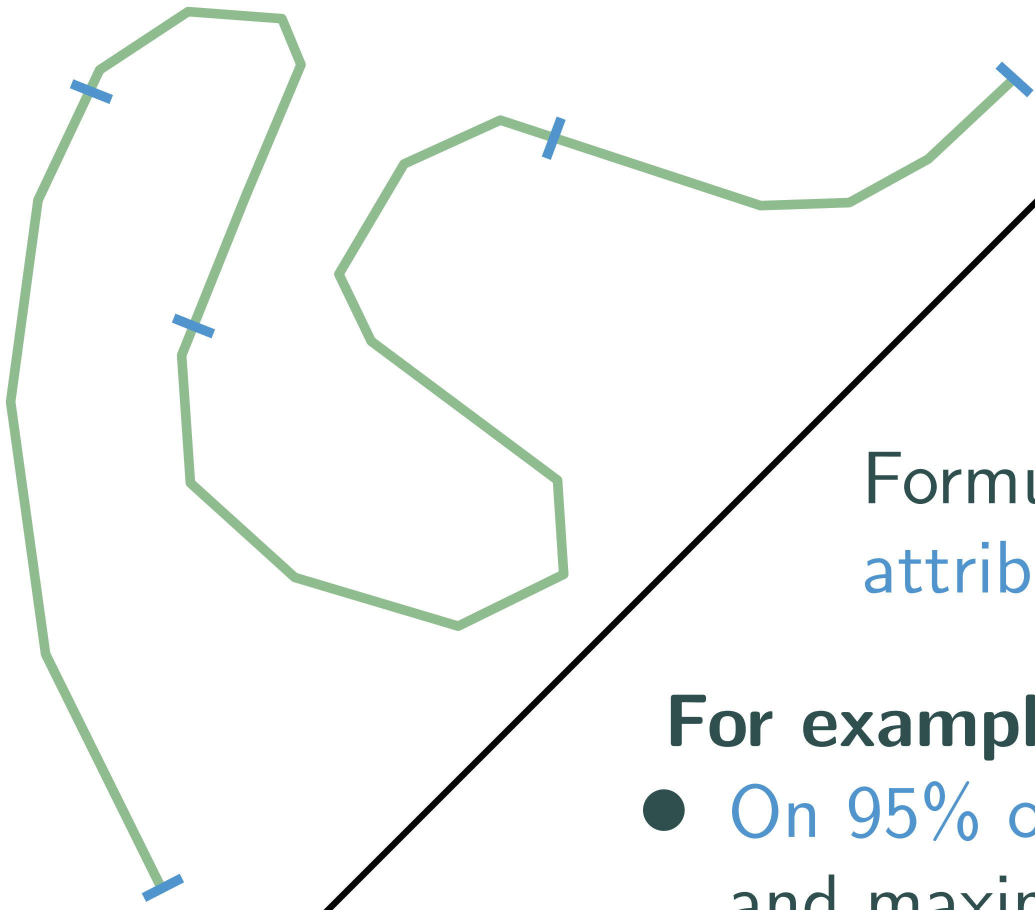
# Why?

To help analyze trajectory data, e.g. in animal trajectories.

Formulate the behaviour in terms of attributes like speed, heading, etc.

**For example:**

- The minimum and maximum speed differ by at most $x$ km/h.
- The direction of motion differs by at most 90°.



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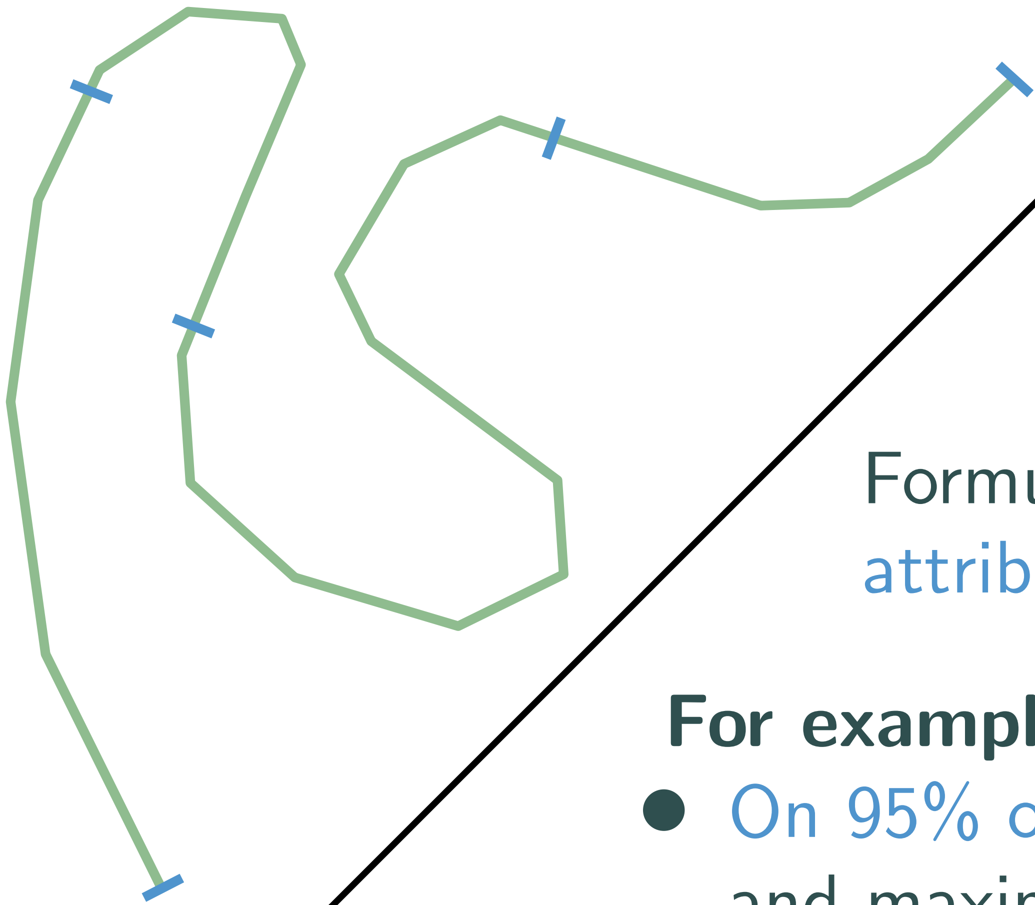
Formulate the behaviour in terms of attributes like speed, heading, etc.

**For example:**

- On 95% of the segment, the minimum and maximum speed differ by at most $h$.
- The standard deviation of the heading is at most $45^\circ$.



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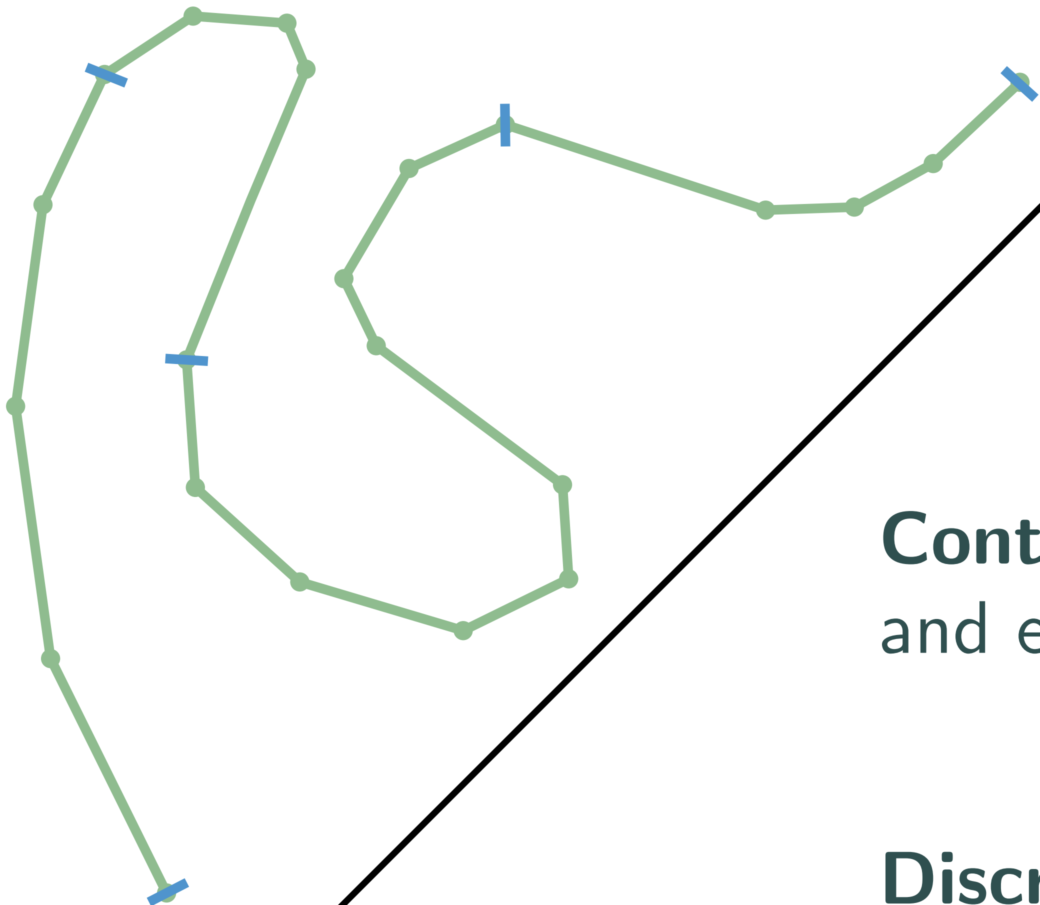
These critiria are non-monotone.

# Continuous vs Discrete

**Continuous:** Segments may start and end anywhere.

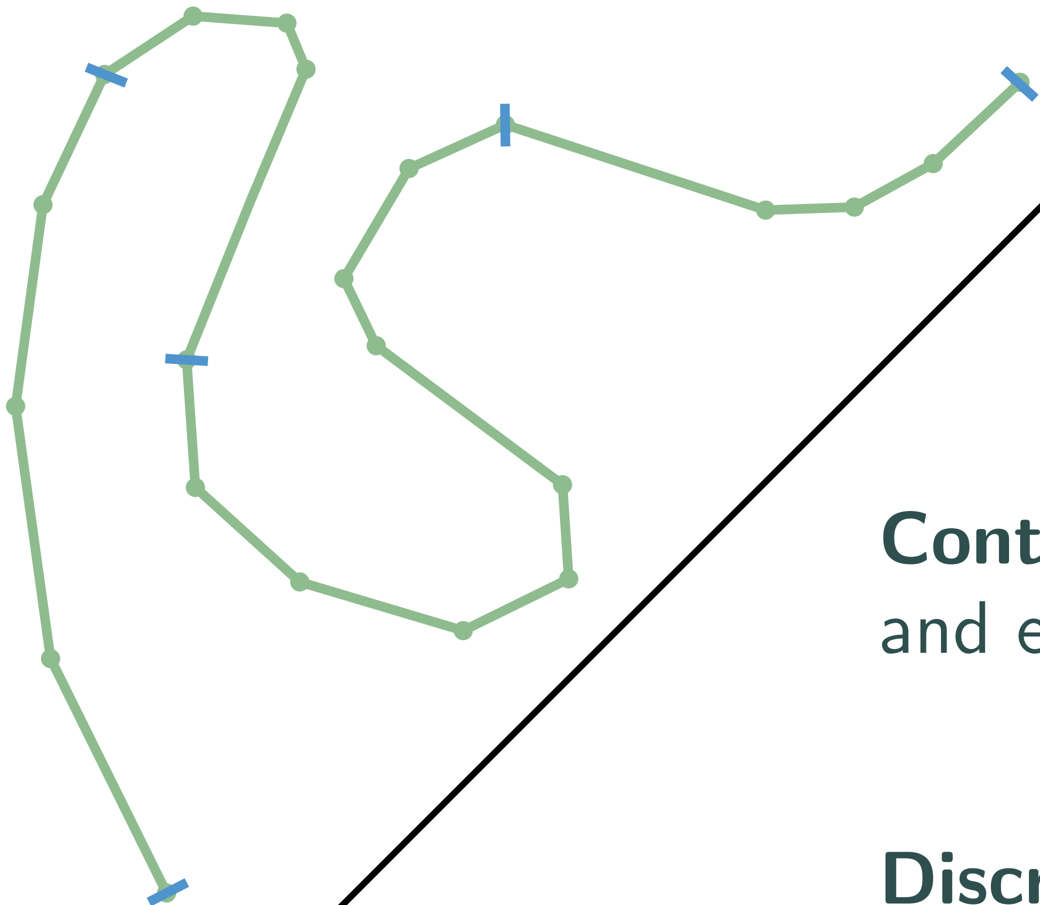


# Continuous vs Discrete

**Continuous:** Segments may start and end anywhere.

**Discrete:** Segments may start and end only at vertices.

# Continuous vs Discrete



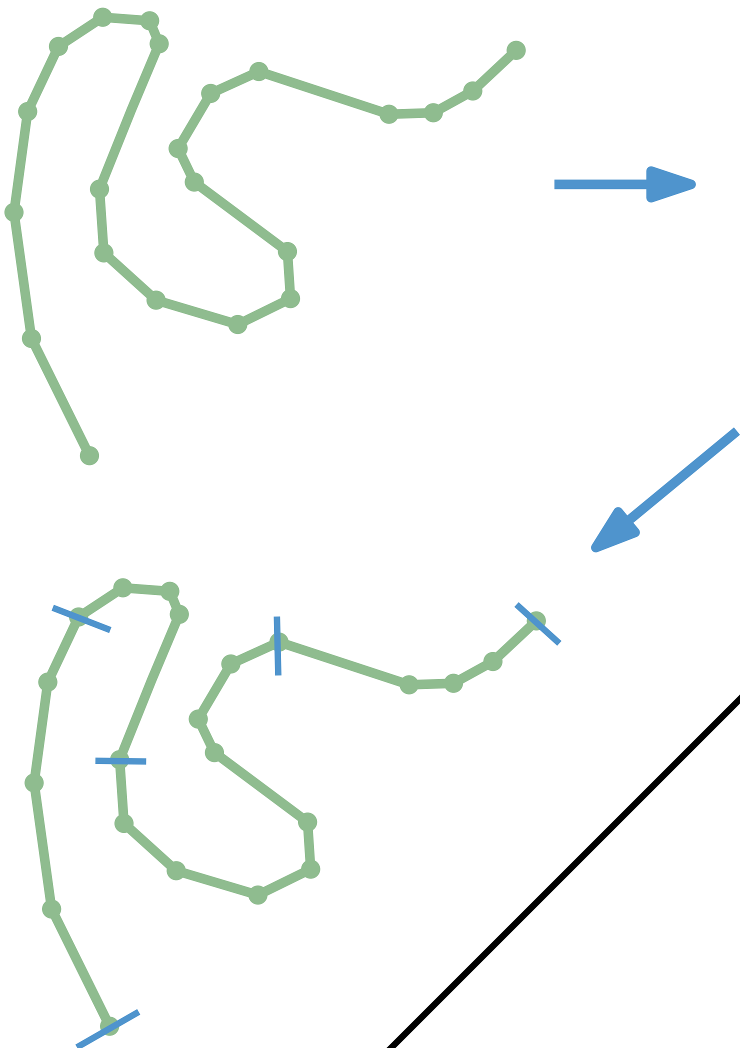
**Continuous:** Segments may start and end anywhere.

Hard

**Discrete:** Segments may start and end only at vertices.

Easy

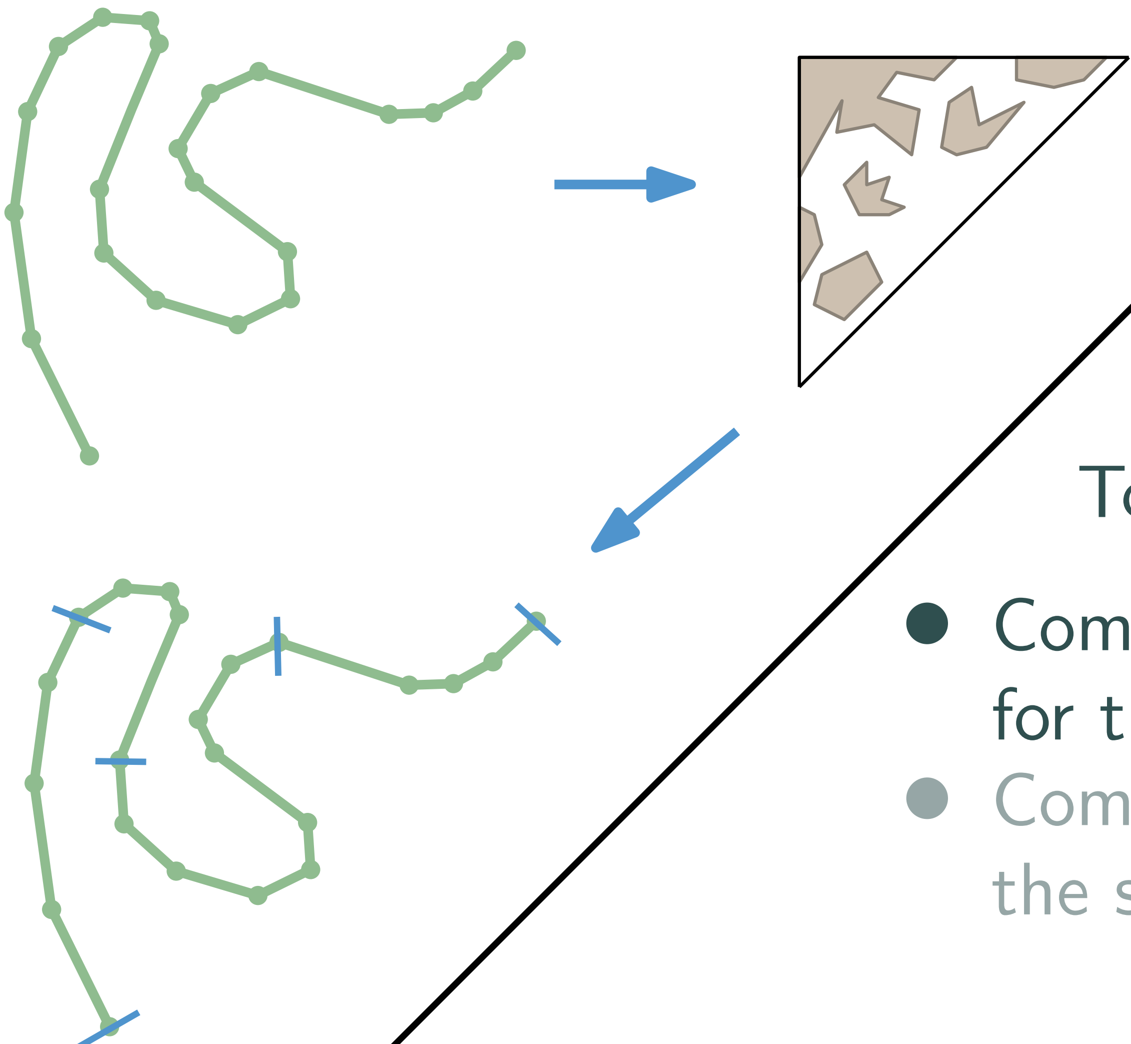
May result in more segments.



# Approach & Results

To segment $T$ we:

- Compute the start-stop diagram for the given criterion.
- Compute a segmentation using the start-stop diagram.



# Approach & Results

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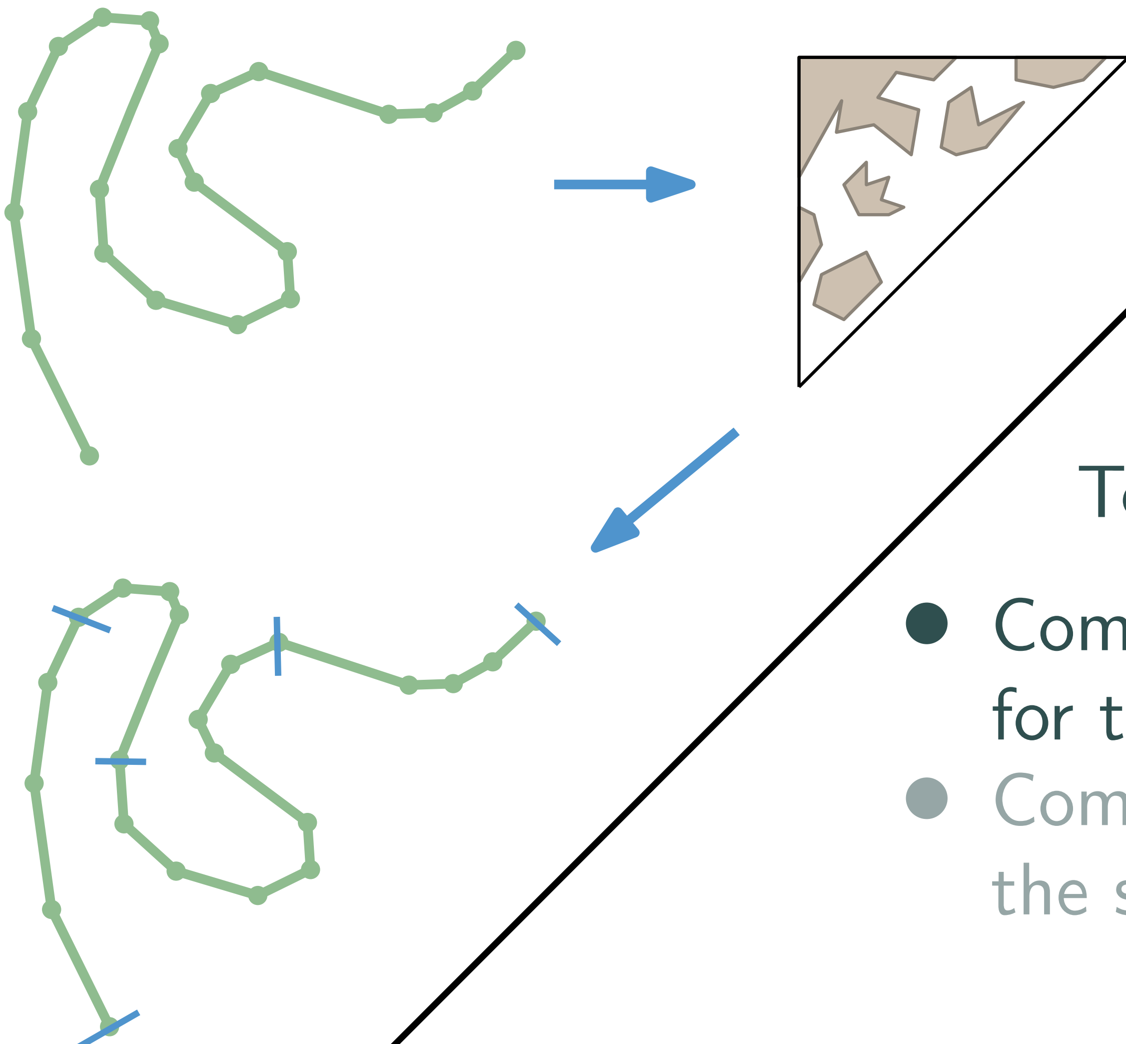
**Standard deviation criterion**:
The standard deviation $\sigma(a, b)$ on each segment $[a, b]$ is at most $h$.

$O(n^2)$ time.



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To segment $T$ we:

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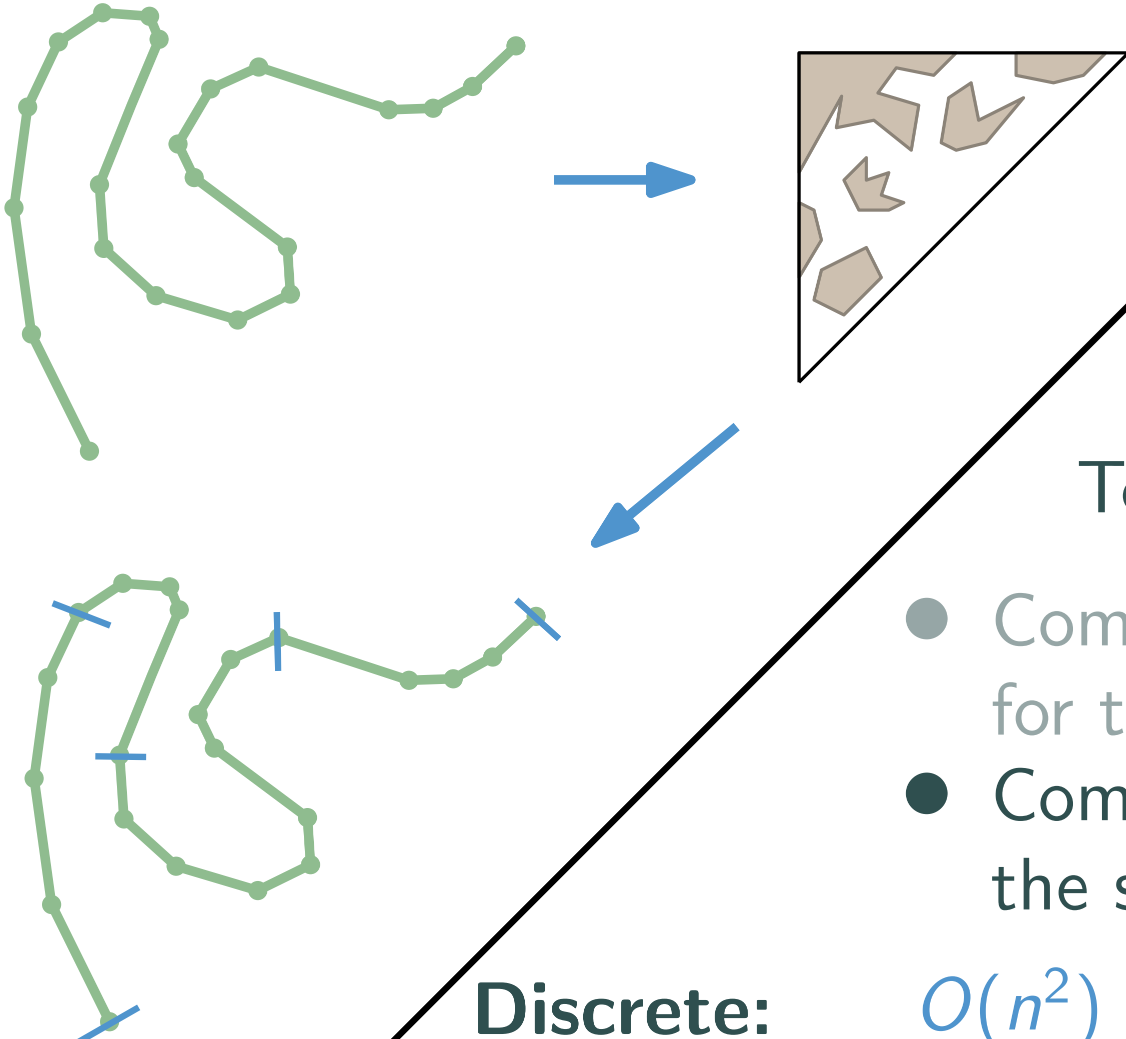
**Outlier-tolerant criterion**:
On a fraction $\rho$ of each segment $[a, b]$ the min and max value differ by at most $h$.

$O(n^2 \log n)$ time.



# Approach & Results

To segment $T$ we:

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**Discrete:** $O(n^2)$

**Continuous:** depends on the start-stop diagram:

Easy —— NP-hard

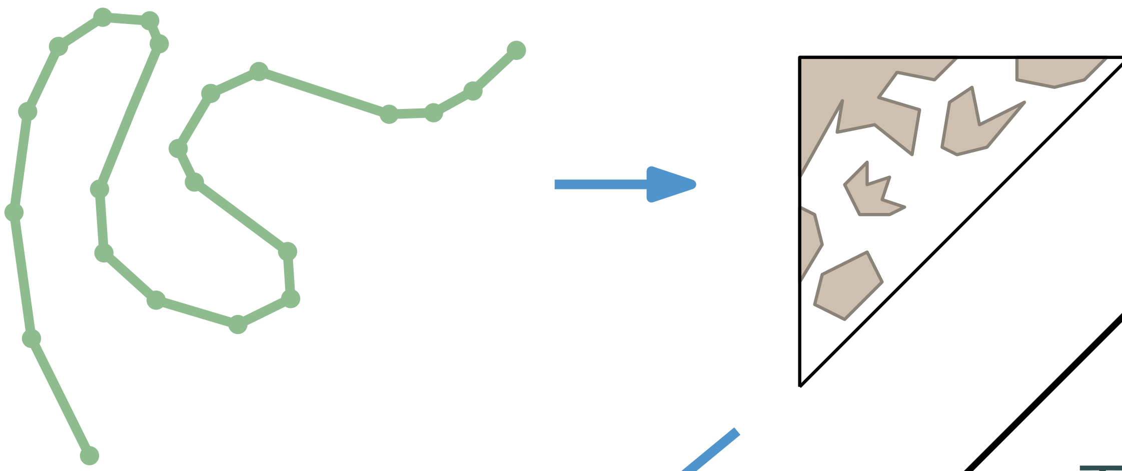
Monotone

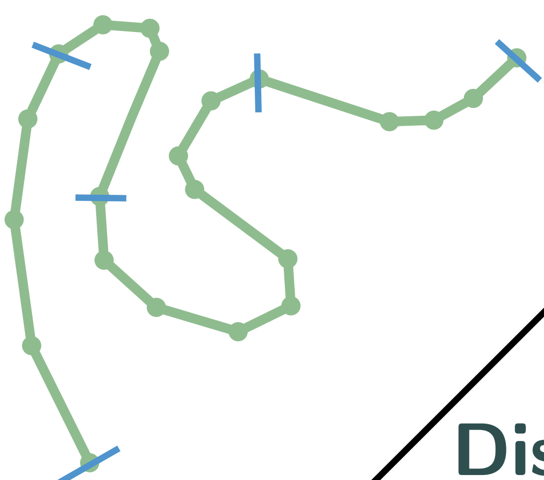
Verticonvex

Outlier

Standard Dev.

Multiple





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- Compute the start-stop diagram for the given criterion.
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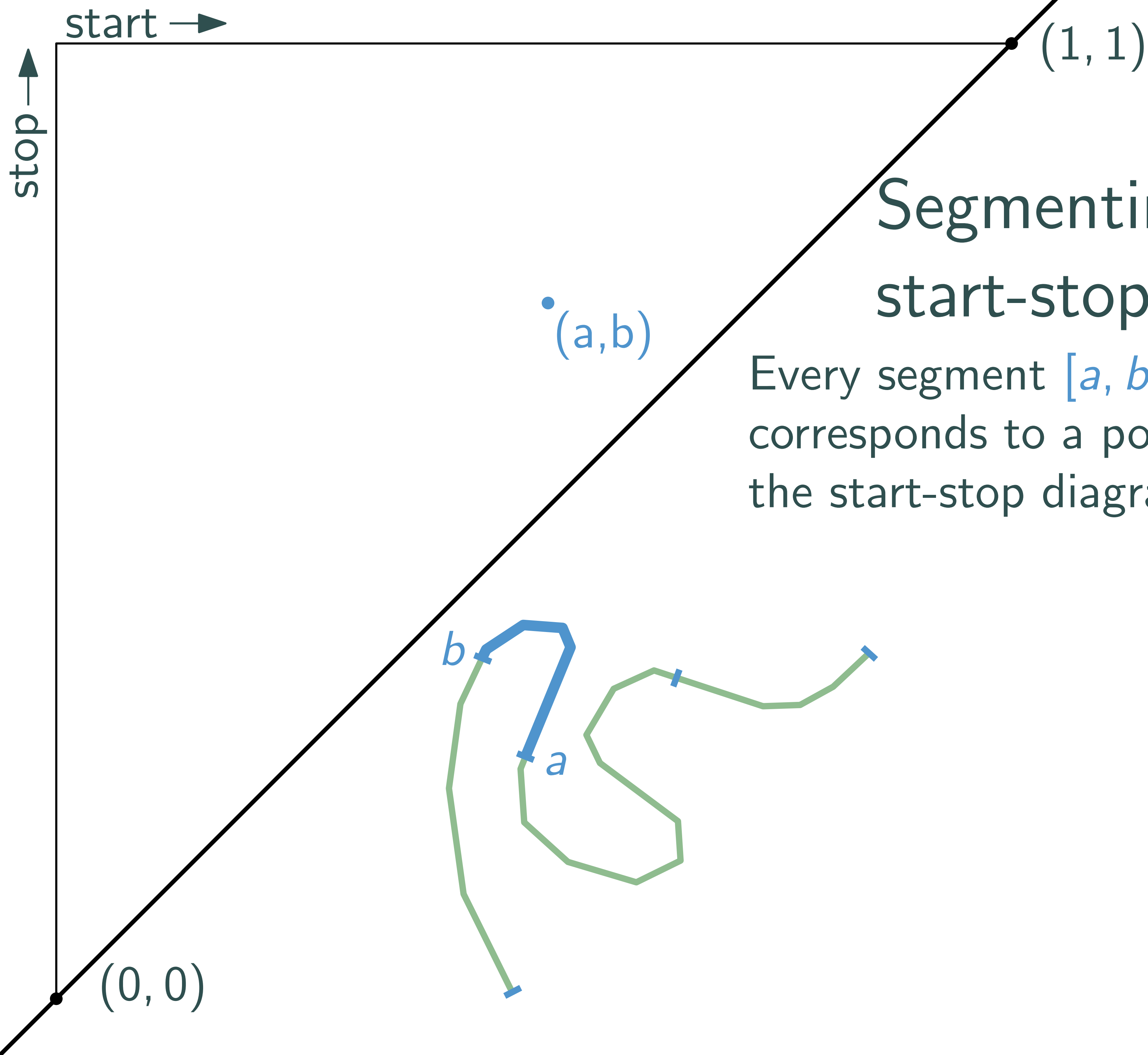
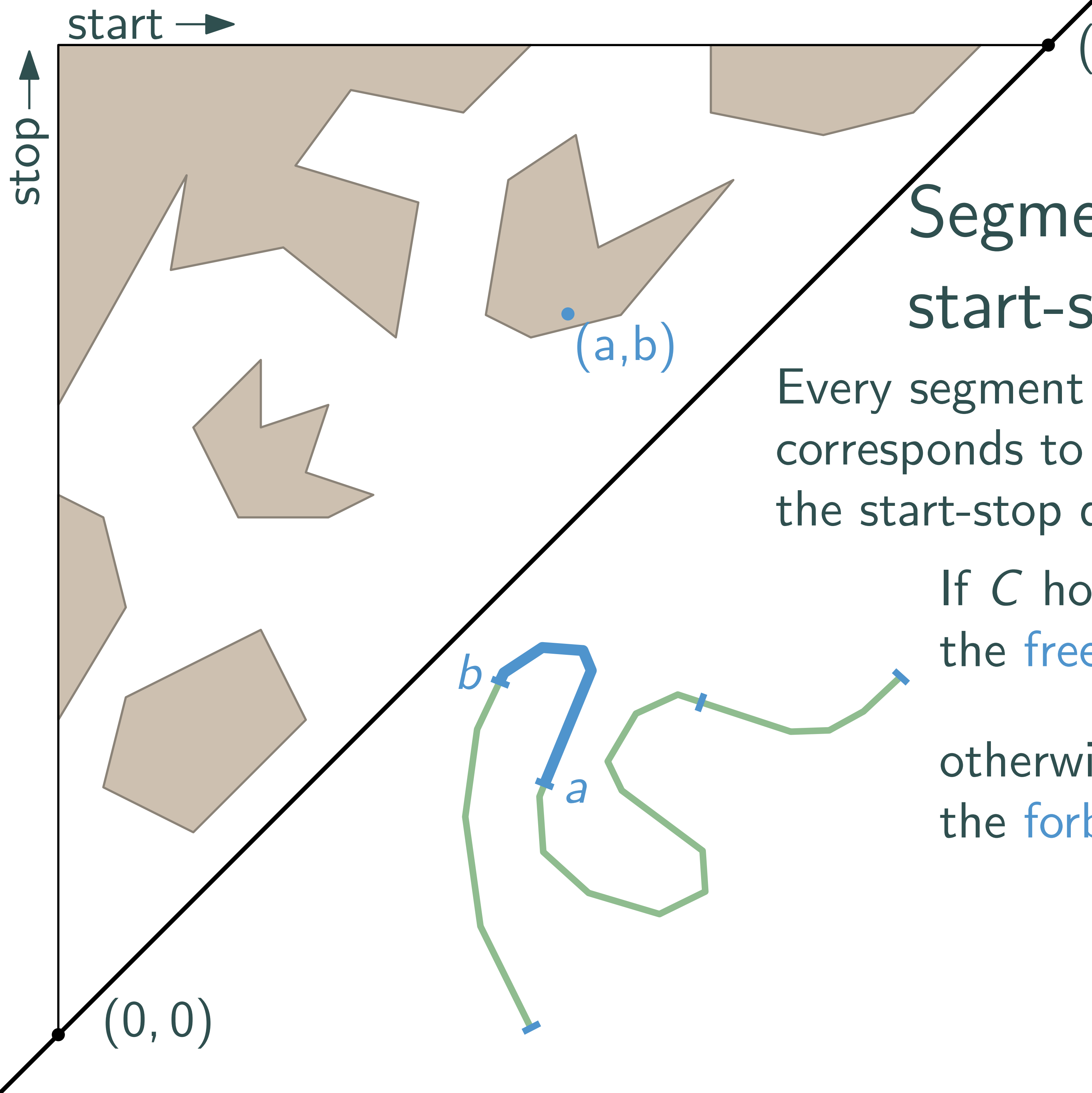
Easy ——— NP-hard

| Monotone | Verticonvex | Multiple |
|---|---|---|
| | $O(kn^2)$ | $O(k^2 n^2)$ |




# Segmenting the start-stop diagram

Every segment $[a, b]$ corresponds to a point $(a, b)$ in the start-stop diagram

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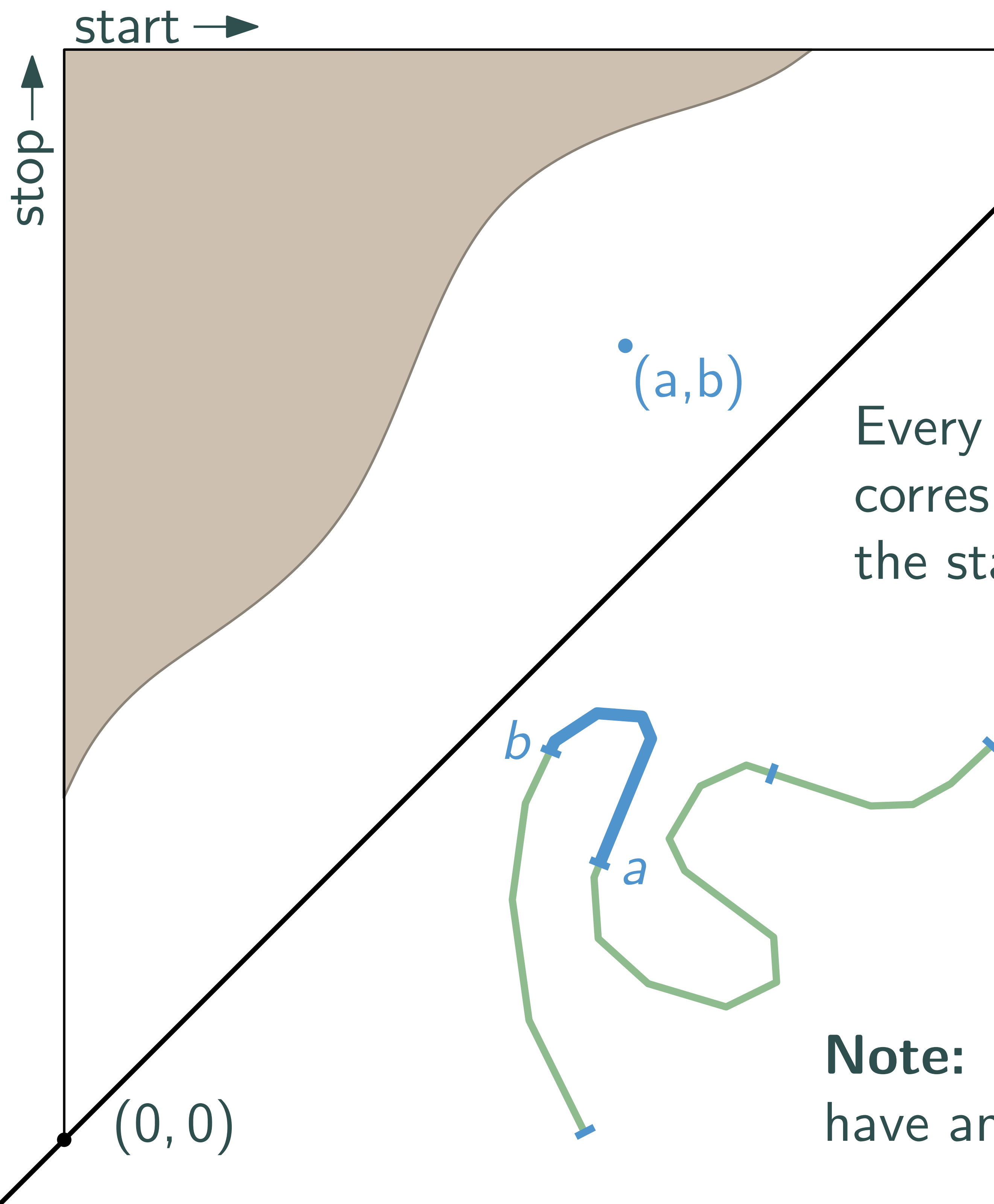


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Every segment $[a, b]$ corresponds to a point $(a, b)$ in the start-stop diagram

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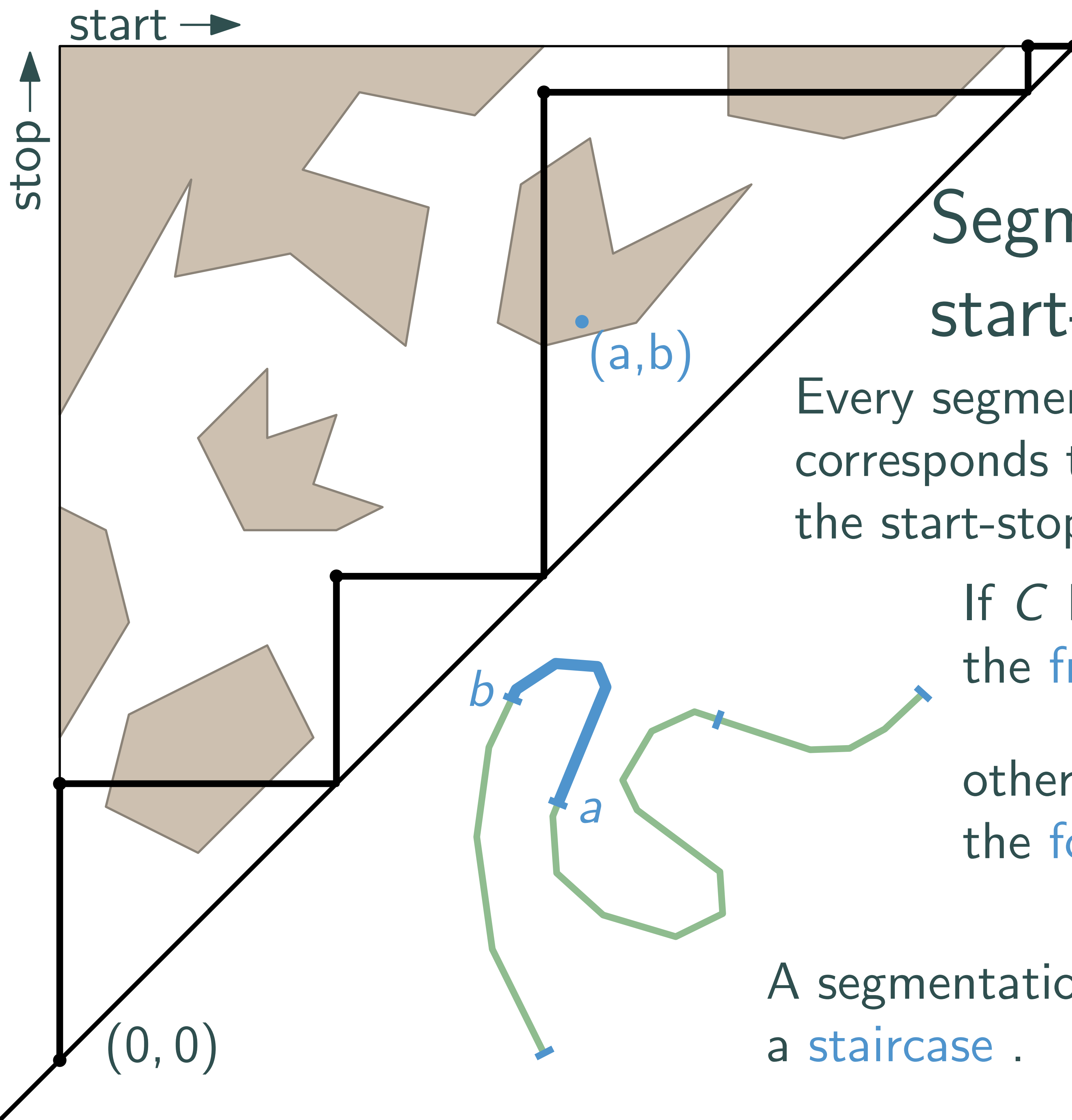
**Note:** If $C$ is monotone we have an *xy*-monotone curve

# Segmenting the start-stop diagram

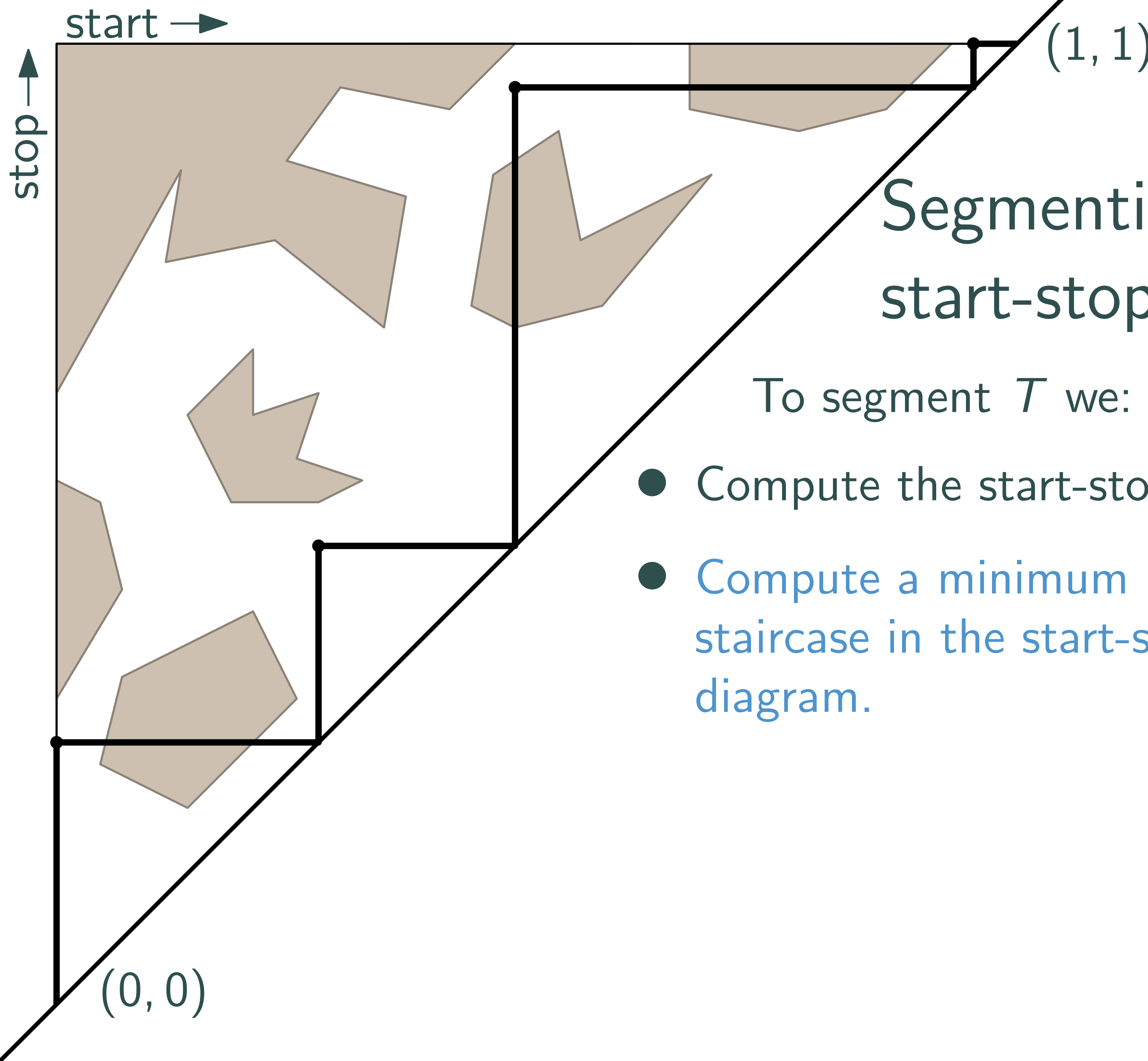



Every segment $[a, b]$ corresponds to a point $(a, b)$ in the start-stop diagram

If $C$ holds, $(a, b)$ is in the free space.

otherwise, $(a, b)$ is in the forbidden space.

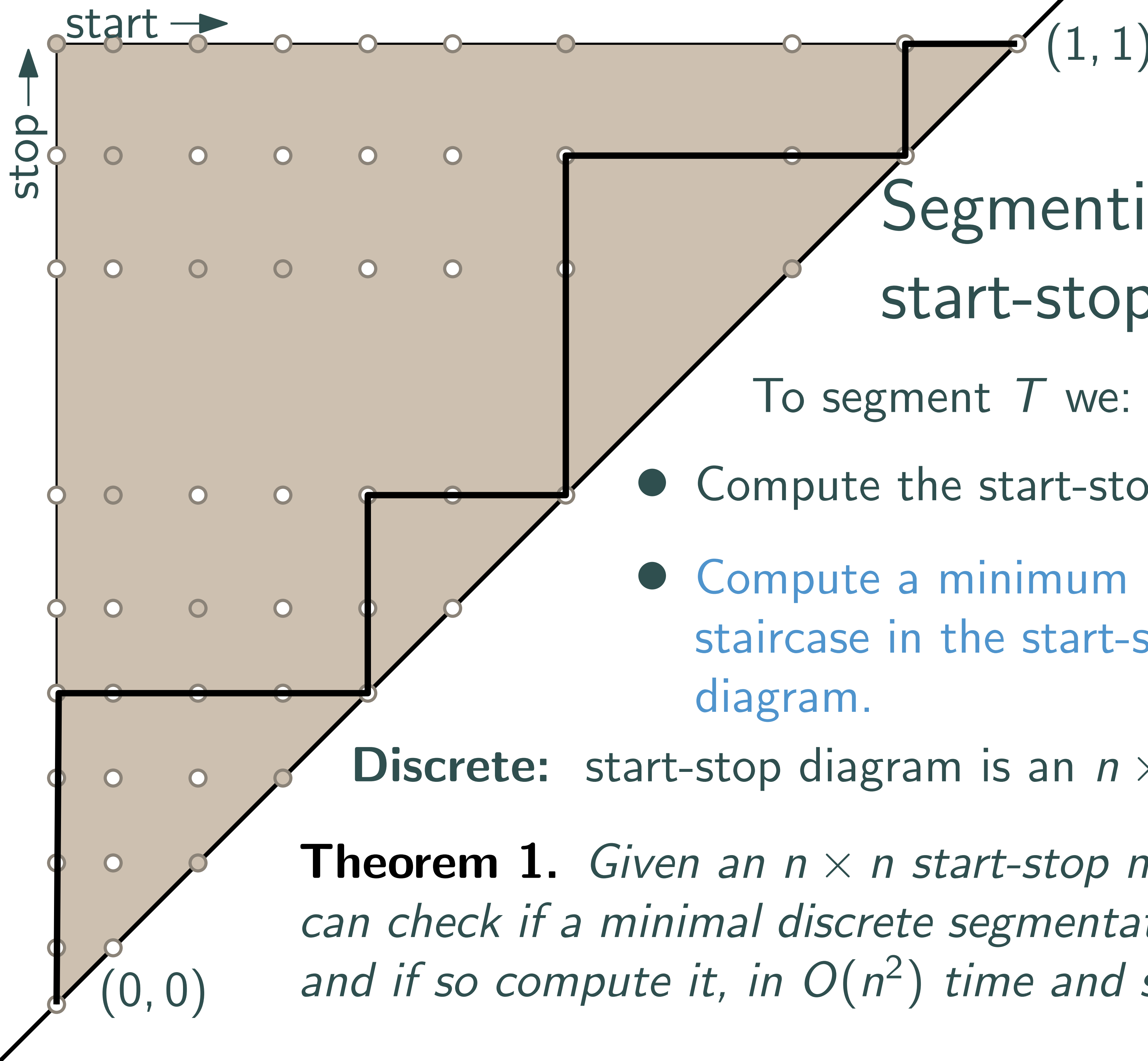
A segmentation corresponds to a staircase .




# Segmenting the start-stop diagram

To segment $T$ we:

- Compute the start-stop diagram.
- Compute a minimum link staircase in the start-stop diagram.



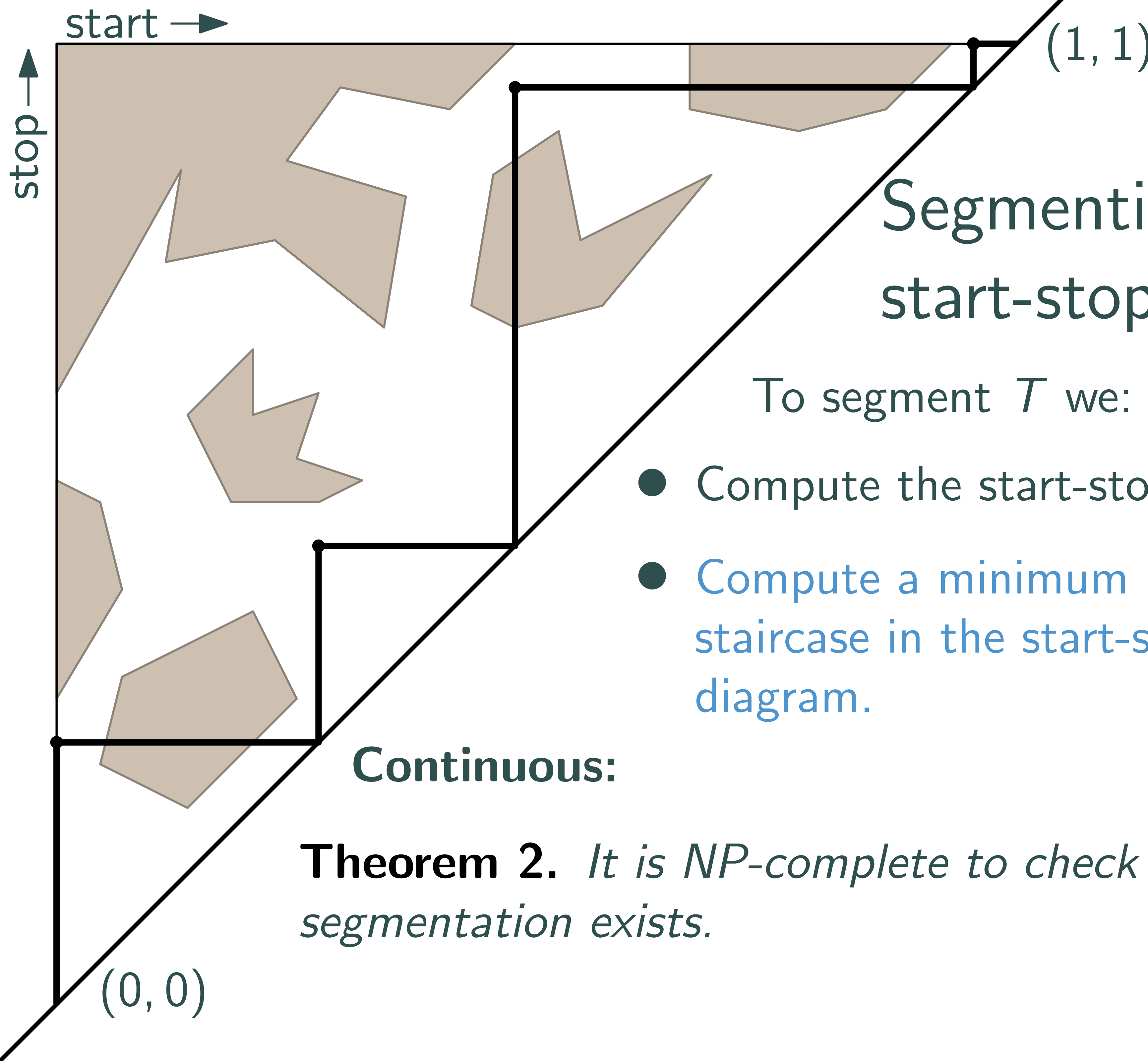

# Segmenting the start-stop diagram

To segment $T$ we:

- Compute the start-stop diagram.
- Compute a minimum link staircase in the start-stop diagram.

**Discrete:** start-stop diagram is an $n \times n$ matrix.

**Theorem 1.** *Given an $n \times n$ start-stop matrix, one can check if a minimal discrete segmentation exists, and if so compute it, in $O(n^2)$ time and space.*




# Segmenting the start-stop diagram

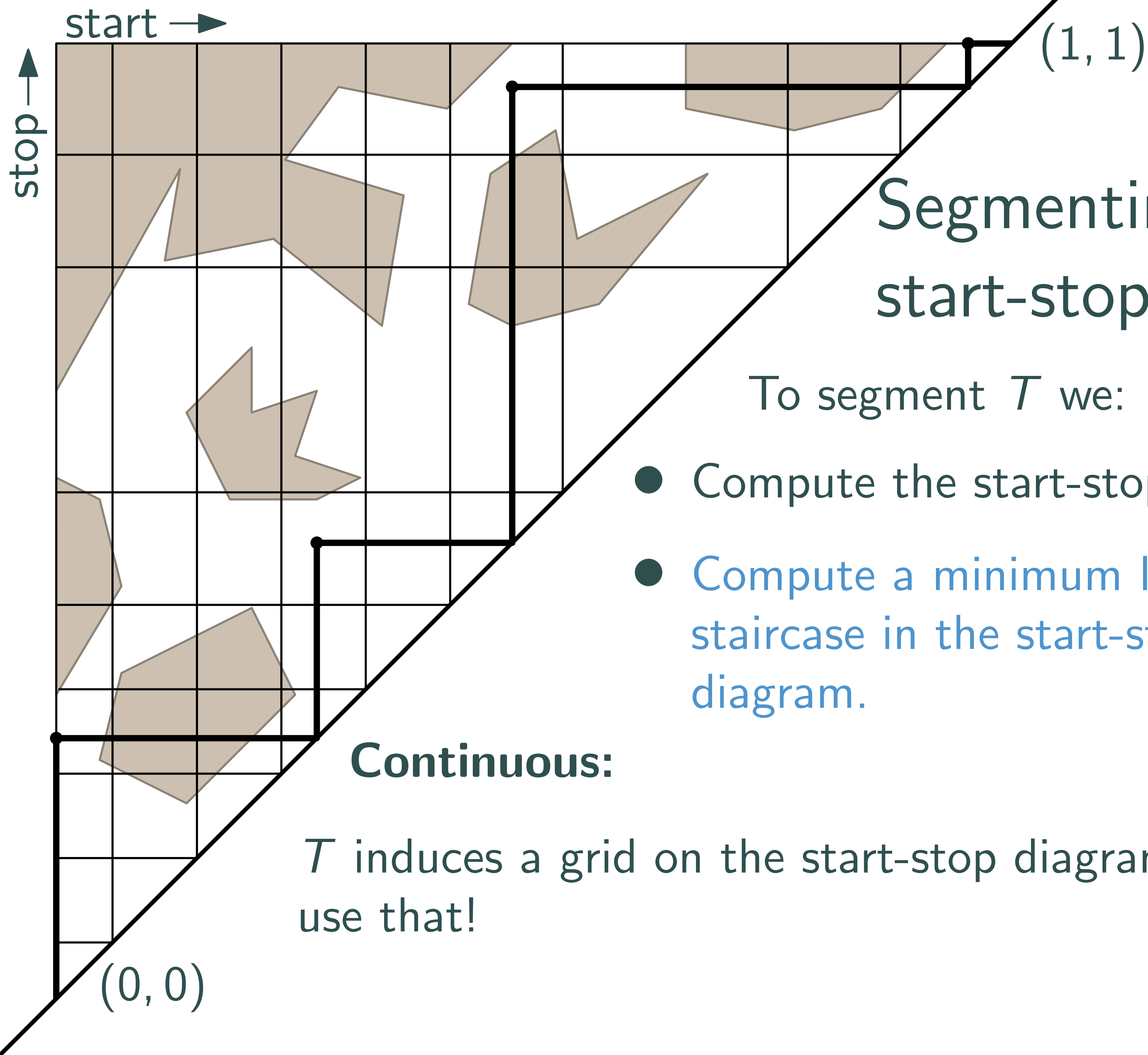
To segment $T$ we:

- Compute the start-stop diagram.
- Compute a minimum link staircase in the start-stop diagram.

**Continuous:**

**Theorem 2.** *It is NP-complete to check if a valid segmentation exists.*

# Segmenting the start-stop diagram



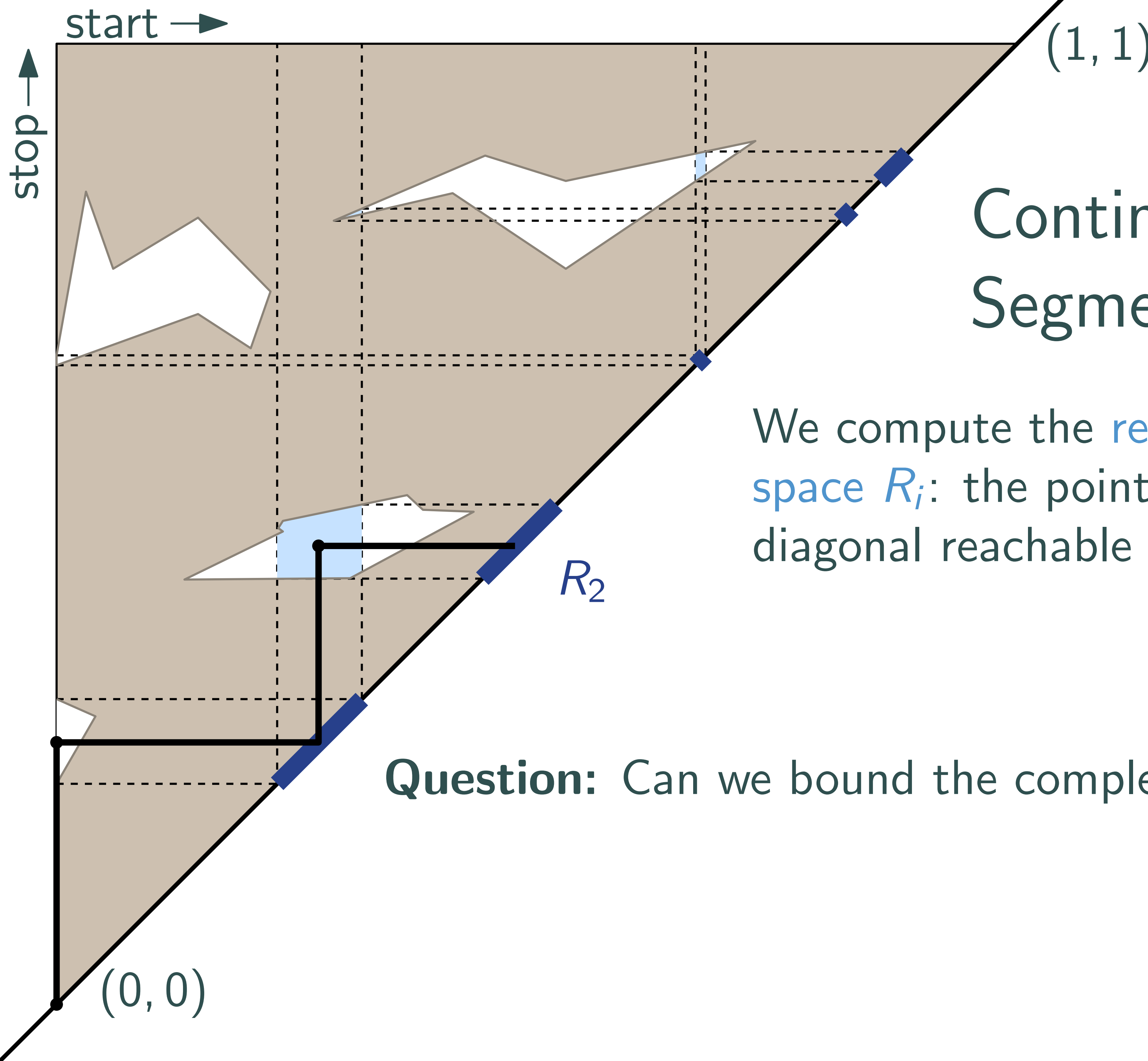

To segment $T$ we:

- Compute the start-stop diagram.
- Compute a minimum link staircase in the start-stop diagram.

**Continuous:**

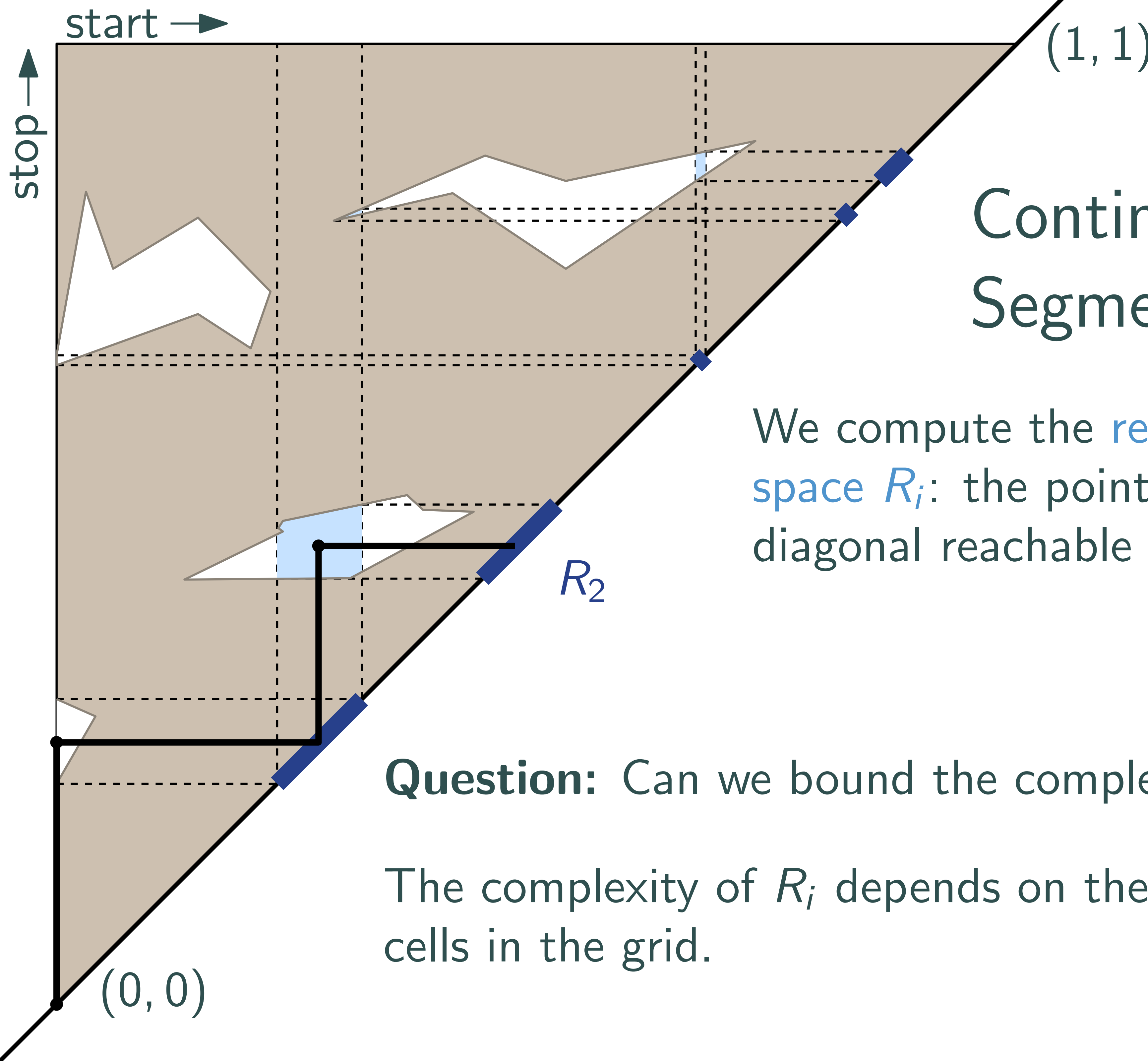
$T$ induces a grid on the start-stop diagram. We can use that!

# Continuous Segmentation




We compute the reachable space $R_i$: the points on the diagonal reachable in $\leq i$ steps.

**Question:** Can we bound the complexity of $R_i$?



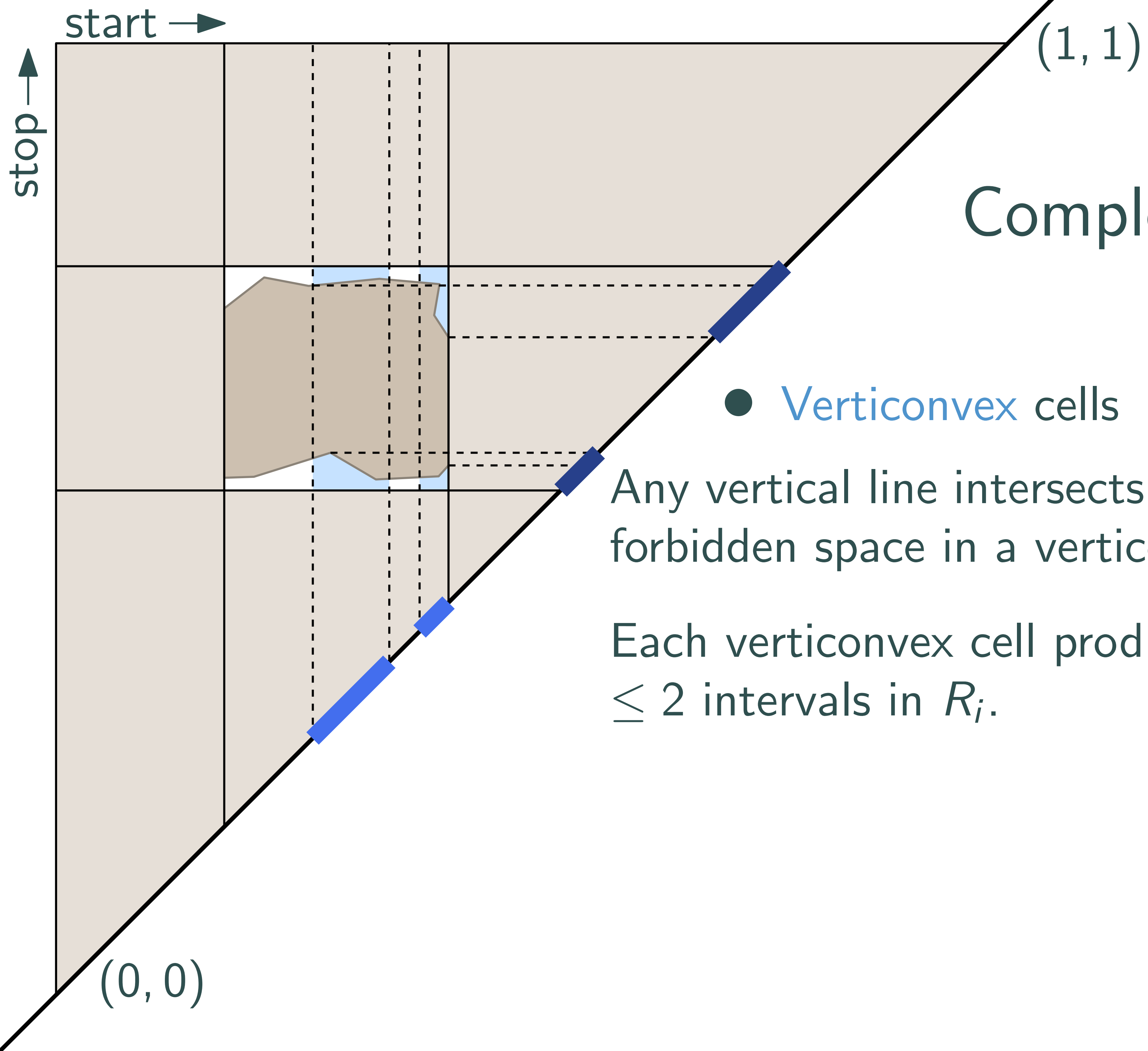

# Continuous Segmentation

We compute the reachable space $R_i$: the points on the diagonal reachable in $\leq i$ steps.

**Question:** Can we bound the complexity of $R_i$?

The complexity of $R_i$ depends on the type of cells in the grid.

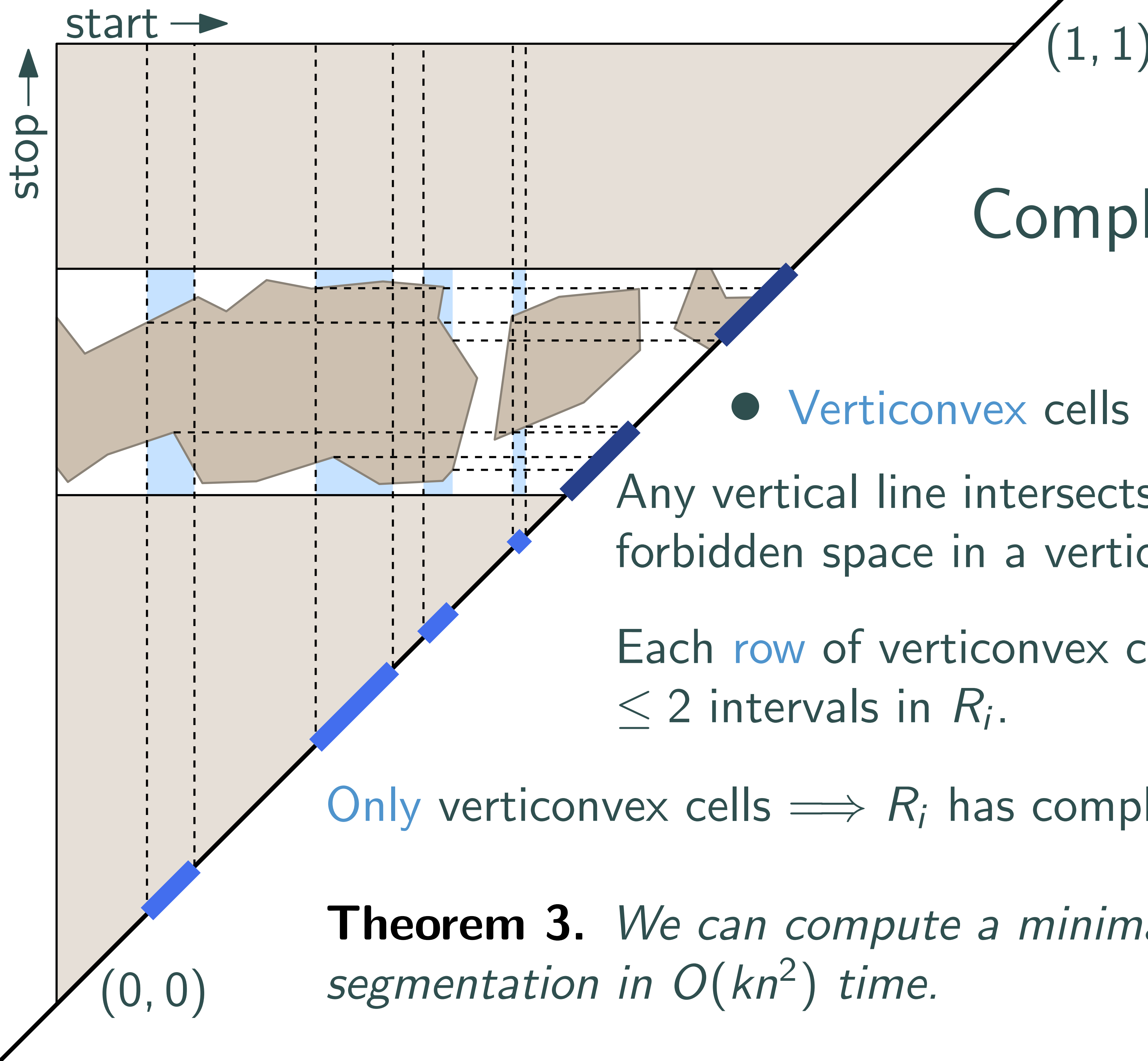
# Complexity $R_i$




- Verticonvex cells

Any vertical line intersects $\leq 1$ region forbidden space in a verticonvex cell.

Each verticonvex cell produces at $\leq 2$ intervals in $R_i$.




# Complexity $R_i$

- Verticonvex cells

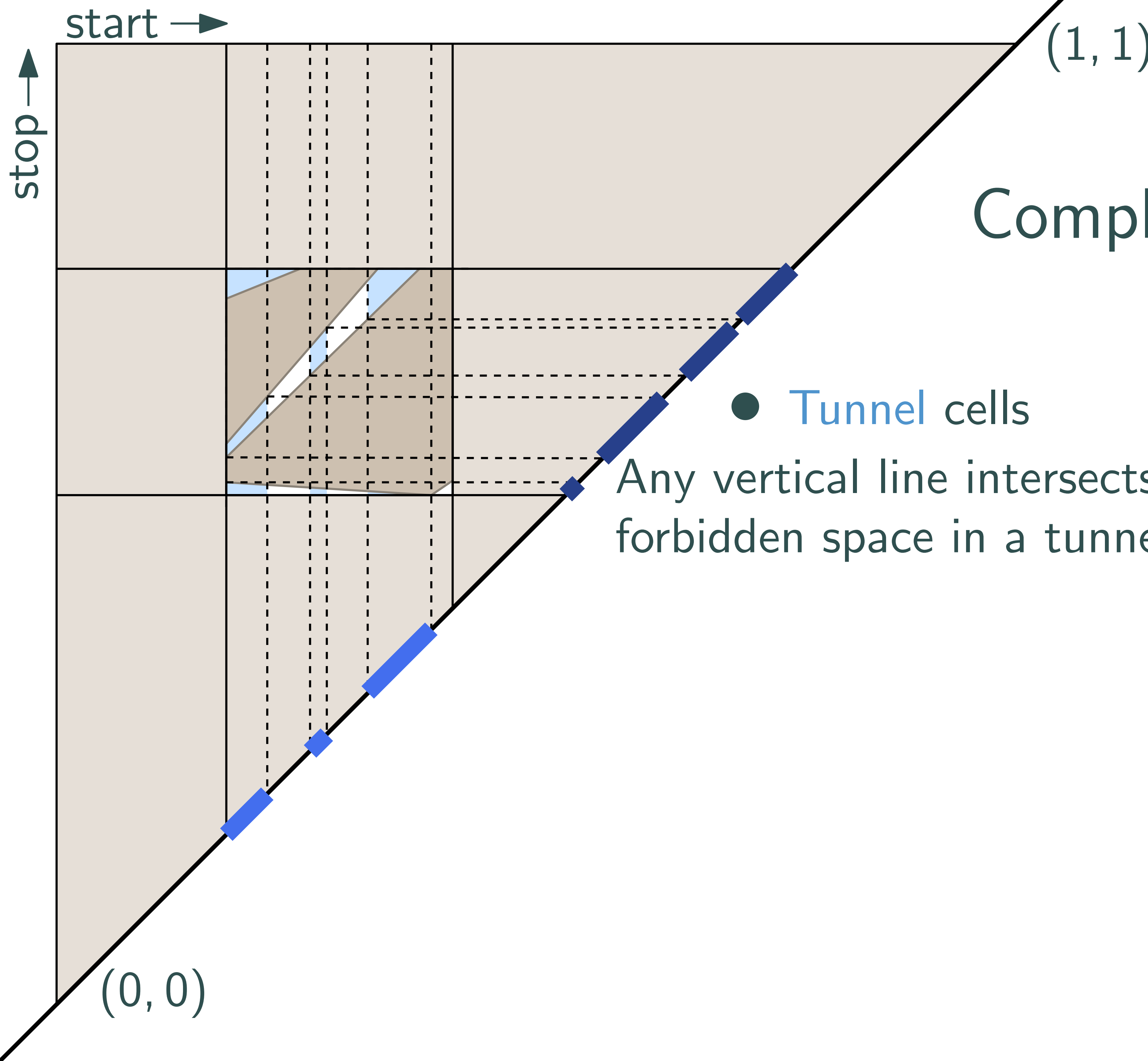
Any vertical line intersects $\leq 1$ region forbidden space in a verticonvex cell.

Each row of verticonvex cells produces $\leq 2$ intervals in $R_i$.

Only verticonvex cells $\Longrightarrow$ $R_i$ has complexity $O(n)$

**Theorem 3.** *We can compute a minimal segmentation in $O(kn^2)$ time.*

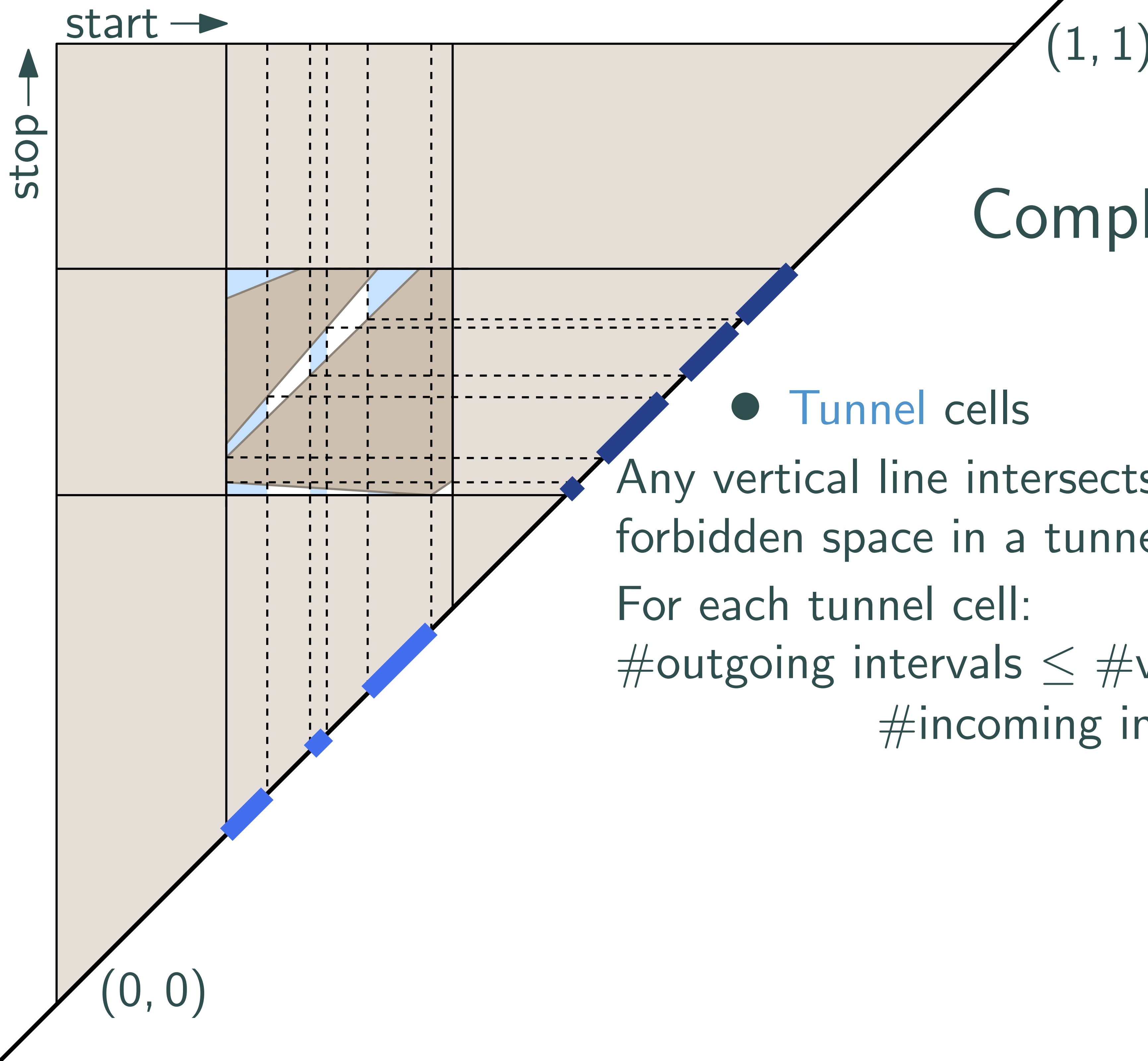
# Complexity $R_i$




- Tunnel cells

Any vertical line intersects $\leq 2$ regions forbidden space in a tunnel cell.

# Complexity $R_i$

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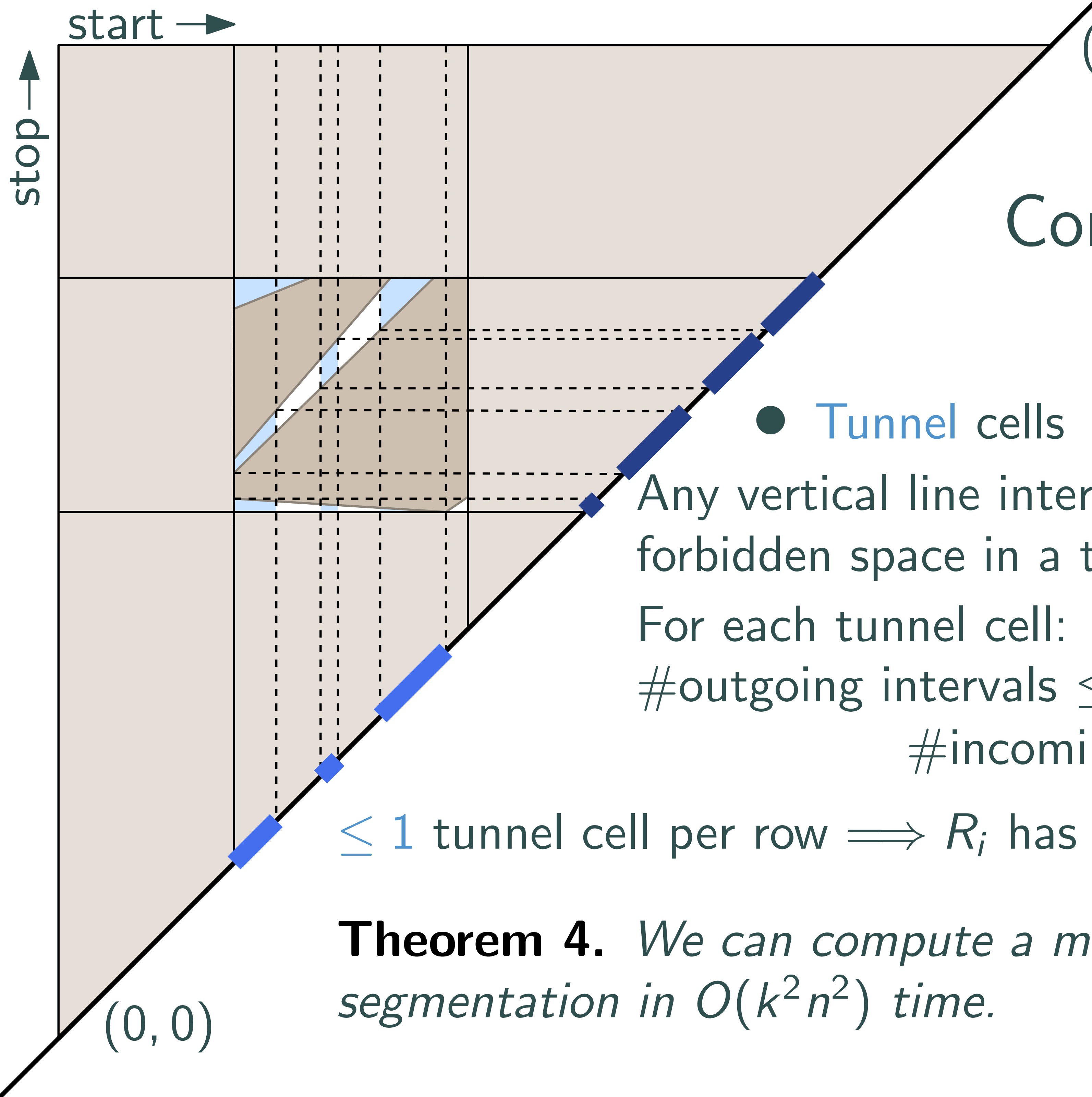
Any vertical line intersects $\leq 2$ regions forbidden space in a tunnel cell.

For each tunnel cell:
#outgoing intervals $\leq$ #vertices + #incoming intervals + 2

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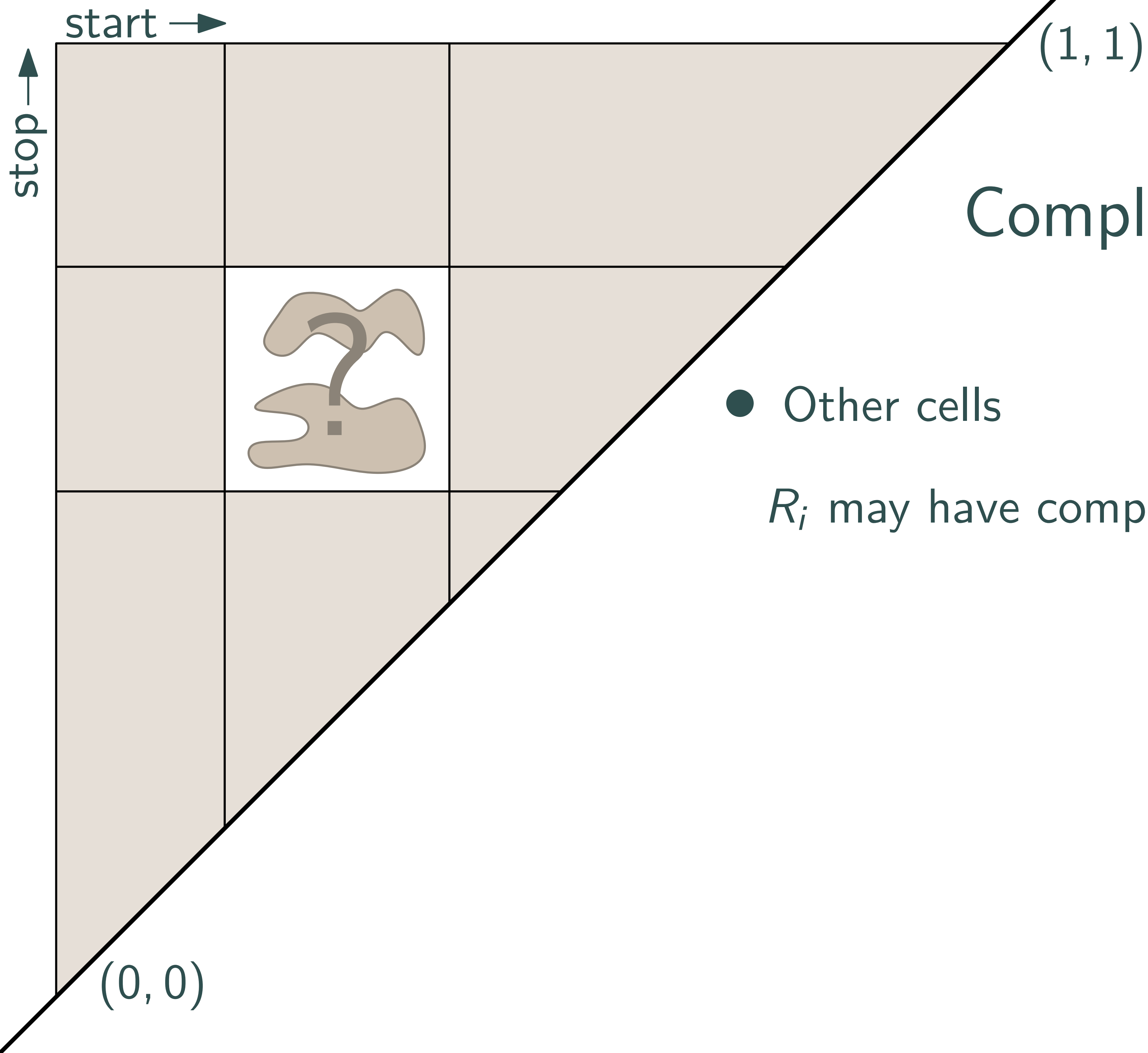
For each tunnel cell:

$$\#\text{outgoing intervals} \leq \#\text{vertices} + \#\text{incoming intervals} + 2$$

$\leq 1$ tunnel cell per row $\Longrightarrow$ $R_i$ has complexity $O(in)$

**Theorem 4.** *We can compute a minimal segmentation in $O(k^2 n^2)$ time.*

# Complexity $R_i$

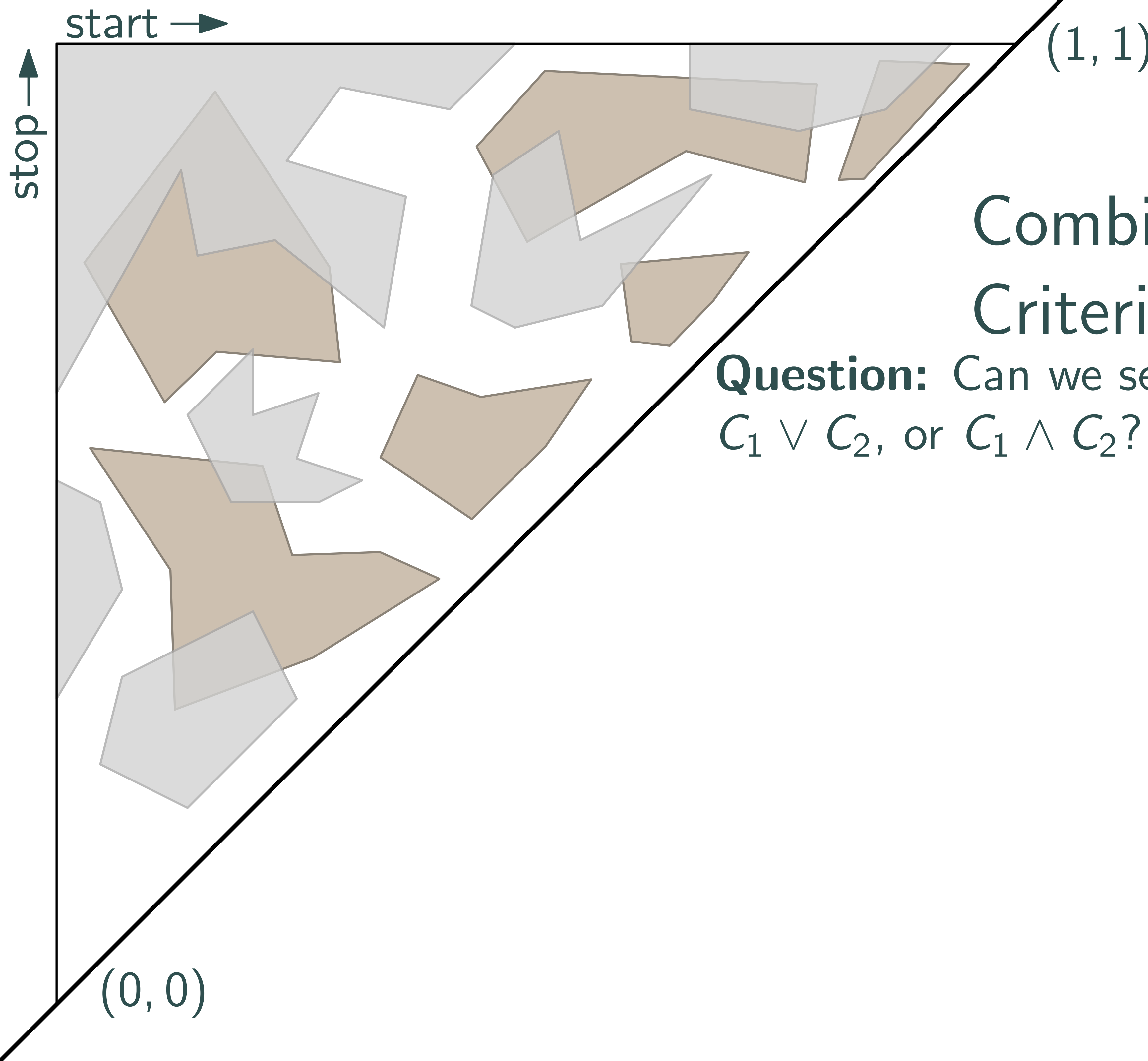


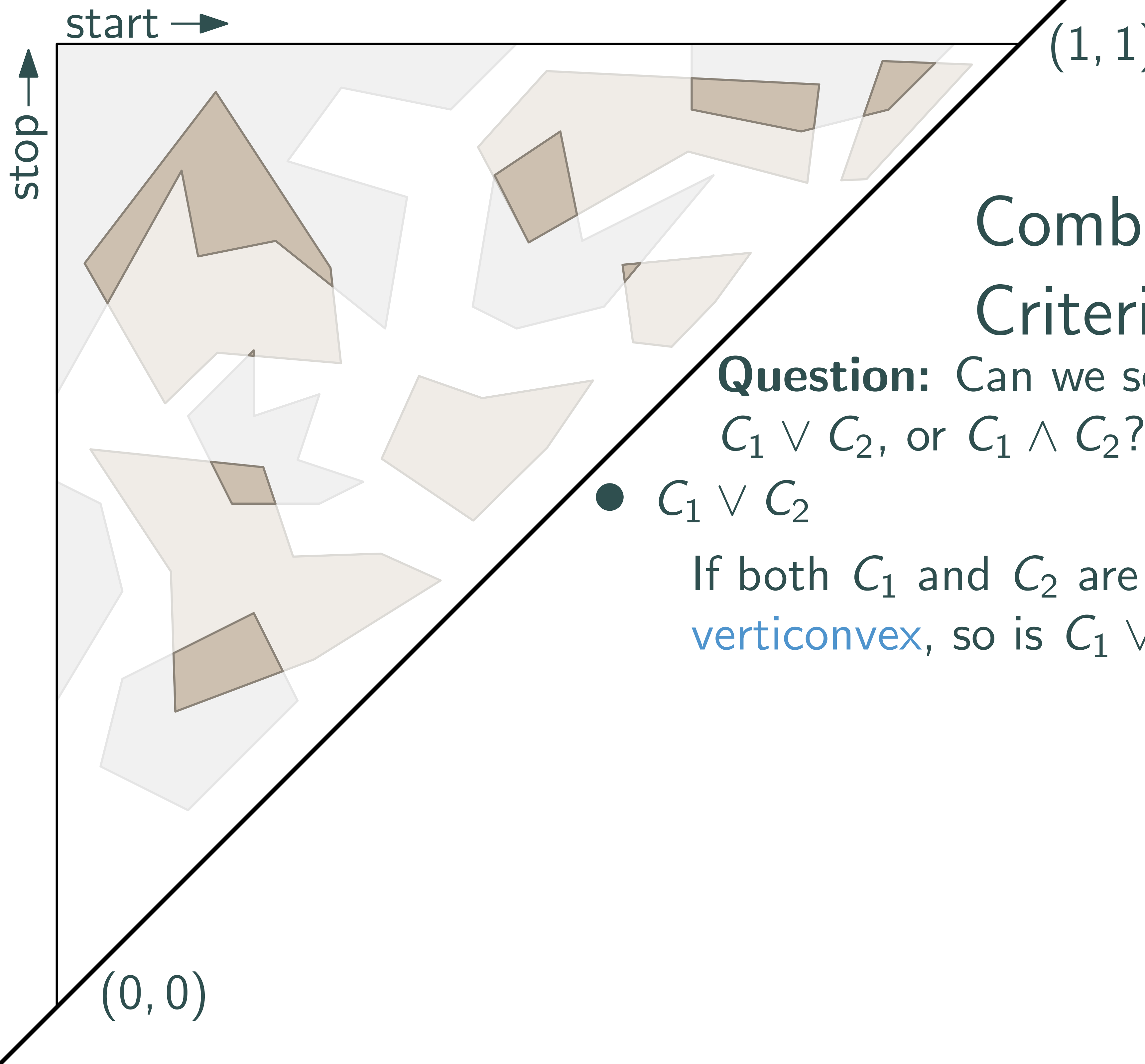

- Other cells

  $R_i$ may have complexity $\Omega(2^i)$

# Combining Criteria

**Question:** Can we segment on $C_1 \vee C_2$, or $C_1 \wedge C_2$?



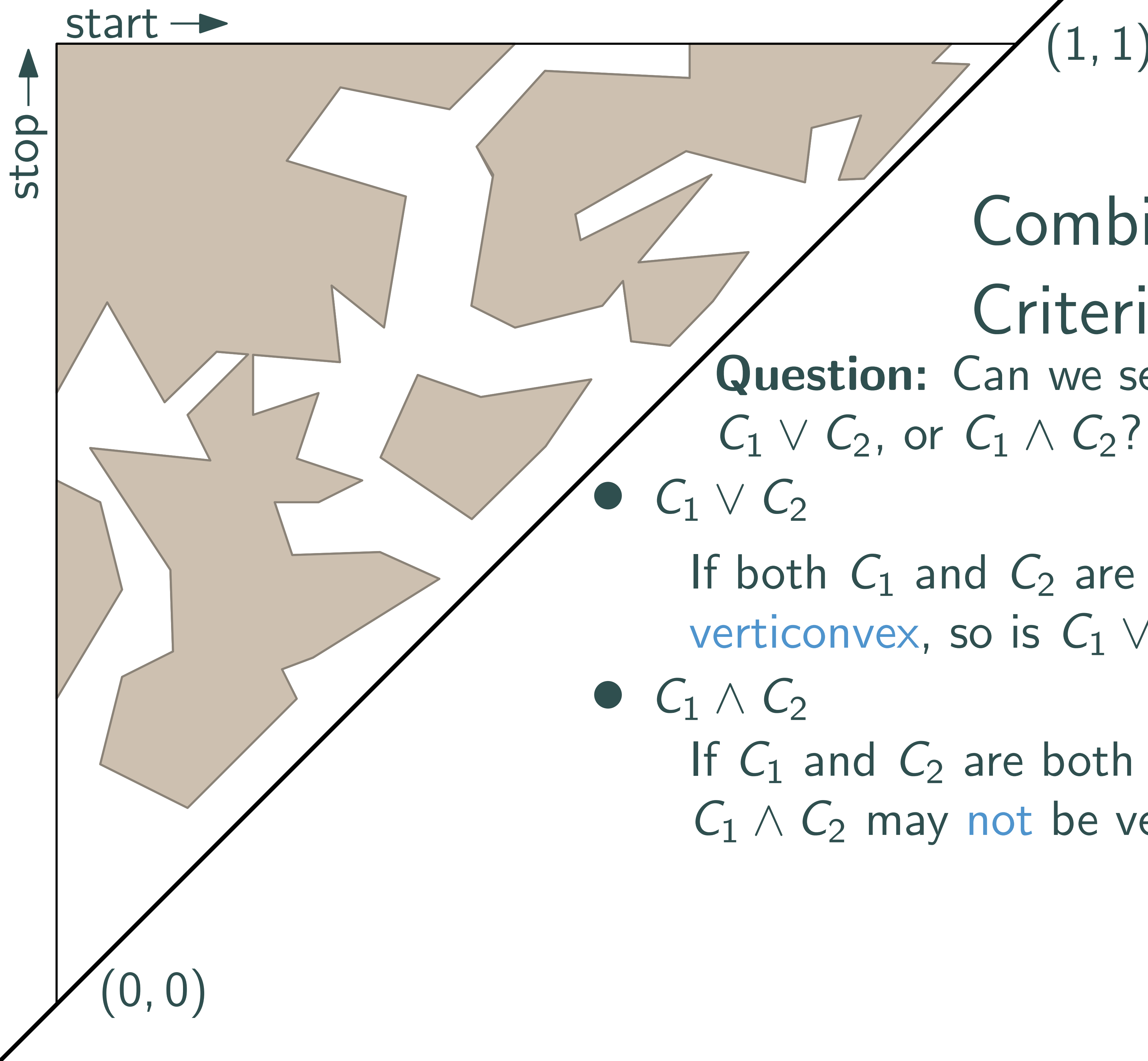



# Combining Criteria

**Question:** Can we segment on $C_1 \vee C_2$, or $C_1 \wedge C_2$?

- $C_1 \vee C_2$

  If both $C_1$ and $C_2$ are verticonvex, so is $C_1 \vee C_2$

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- $C_1 \wedge C_2$

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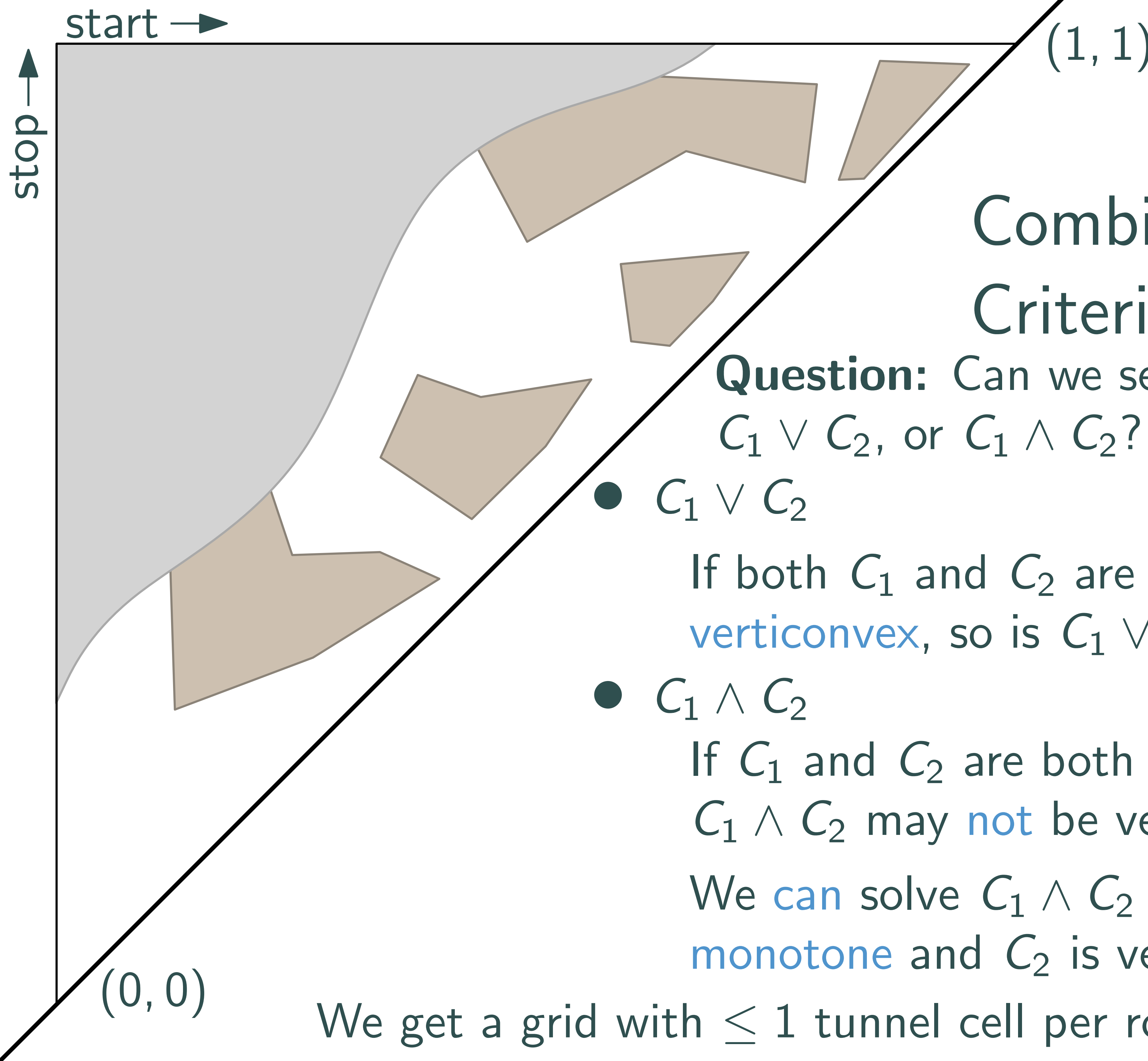


# Combining Criteria

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- $C_1 \vee C_2$

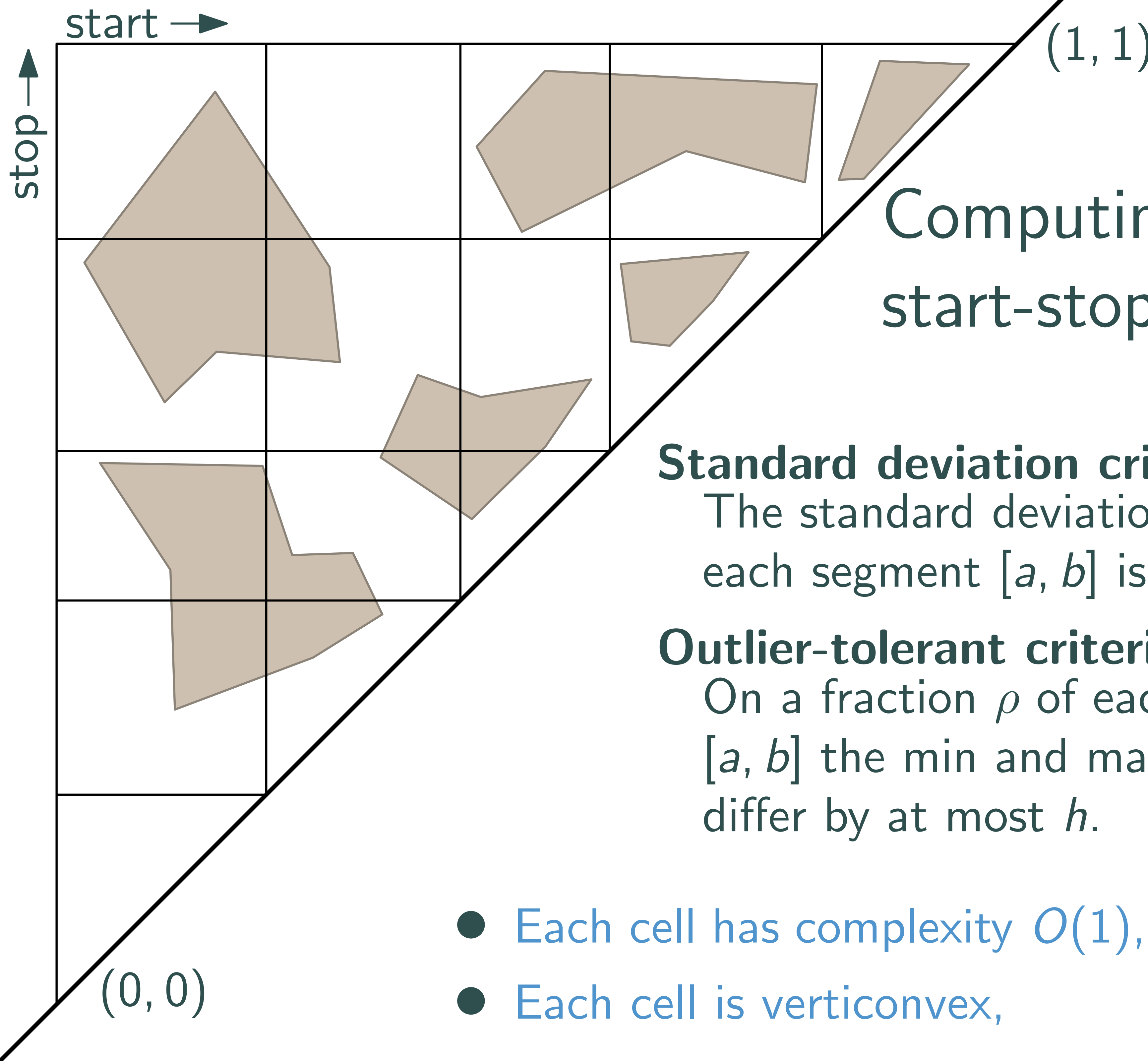
  If both $C_1$ and $C_2$ are verticonvex, so is $C_1 \vee C_2$

- $C_1 \wedge C_2$

  If $C_1$ and $C_2$ are both verticonvex, $C_1 \wedge C_2$ may not be verticonvex

  We can solve $C_1 \wedge C_2$ if $C_1$ is monotone and $C_2$ is verticonvex:

We get a grid with $\leq 1$ tunnel cell per row.

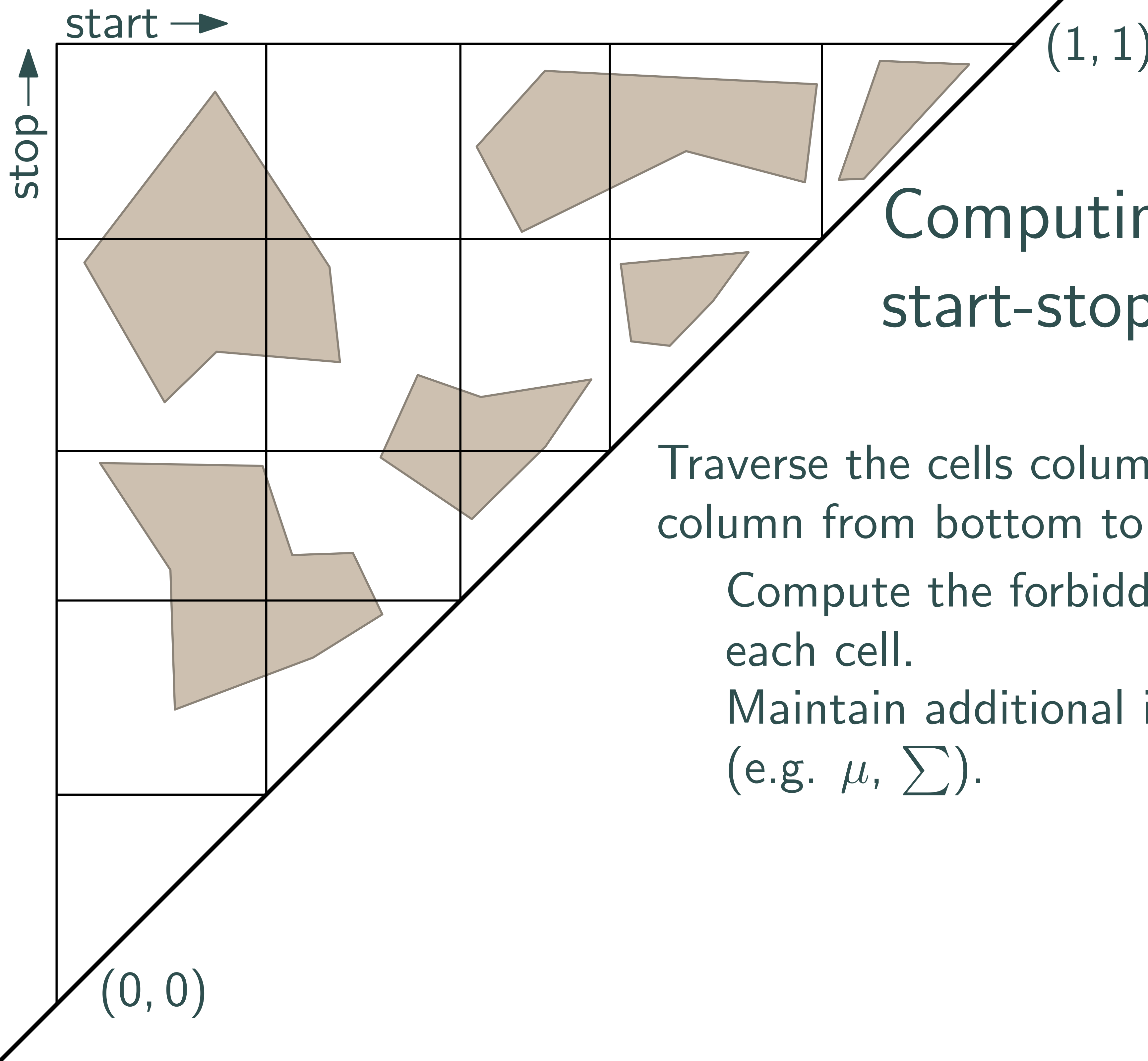



# Computing a start-stop diagram

**Standard deviation criterion**:
The standard deviation $\sigma(a, b)$ on each segment $[a, b]$ is at most $h$.

**Outlier-tolerant criterion**:
On a fraction $\rho$ of each segment $[a, b]$ the min and max value differ by at most $h$.

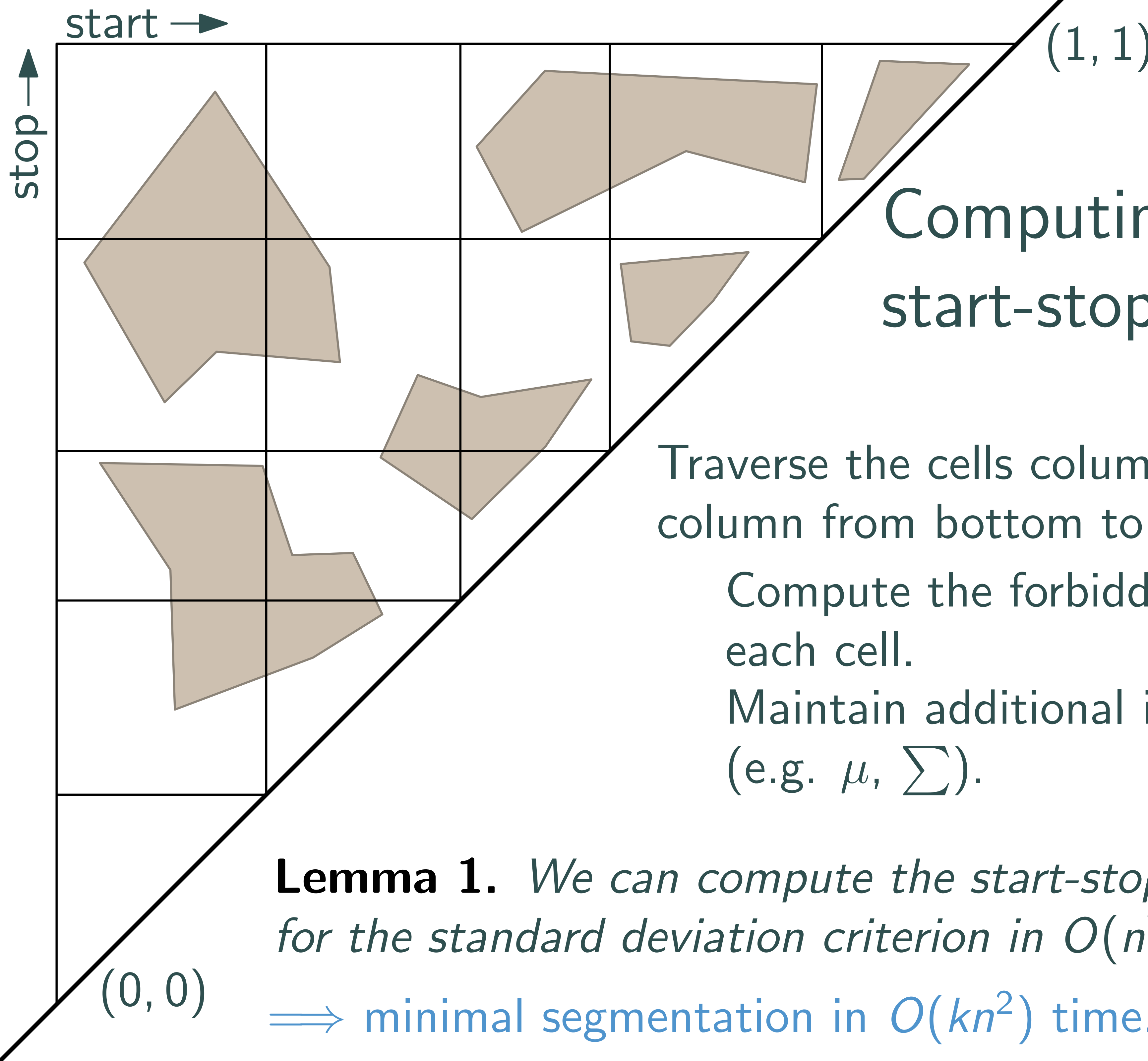
- Each cell has complexity $O(1)$,
- Each cell is verticonvex,




# Computing a start-stop diagram

Traverse the cells column by column from bottom to top.

- Compute the forbidden space in each cell.
- Maintain additional information (e.g. $\mu$, $\sum$).




# Computing a start-stop diagram

Traverse the cells column by column from bottom to top.

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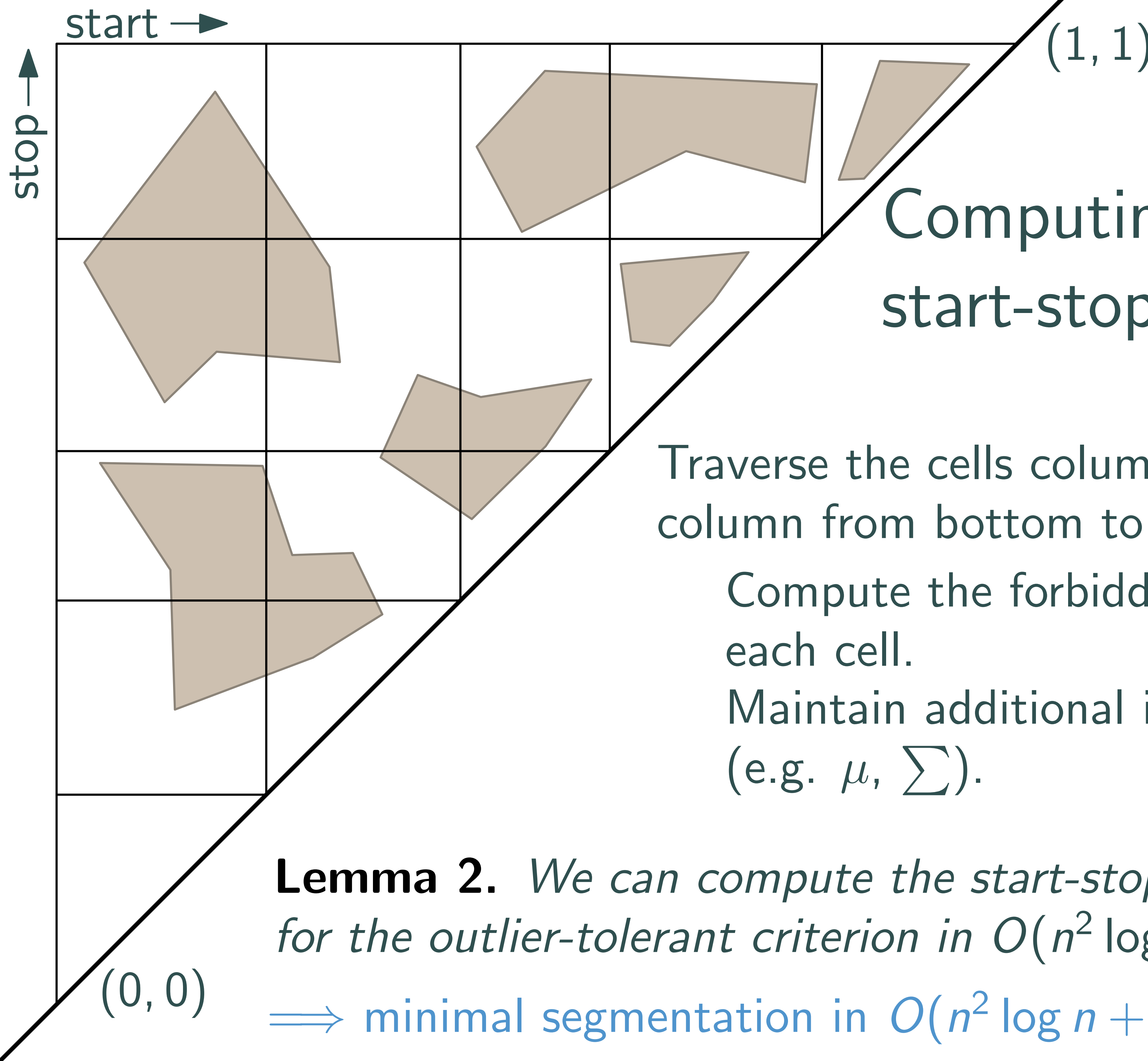
**Lemma 1.** *We can compute the start-stop diagram for the standard deviation criterion in $O(n^2)$ time.*

$\Longrightarrow$ minimal segmentation in $O(kn^2)$ time.



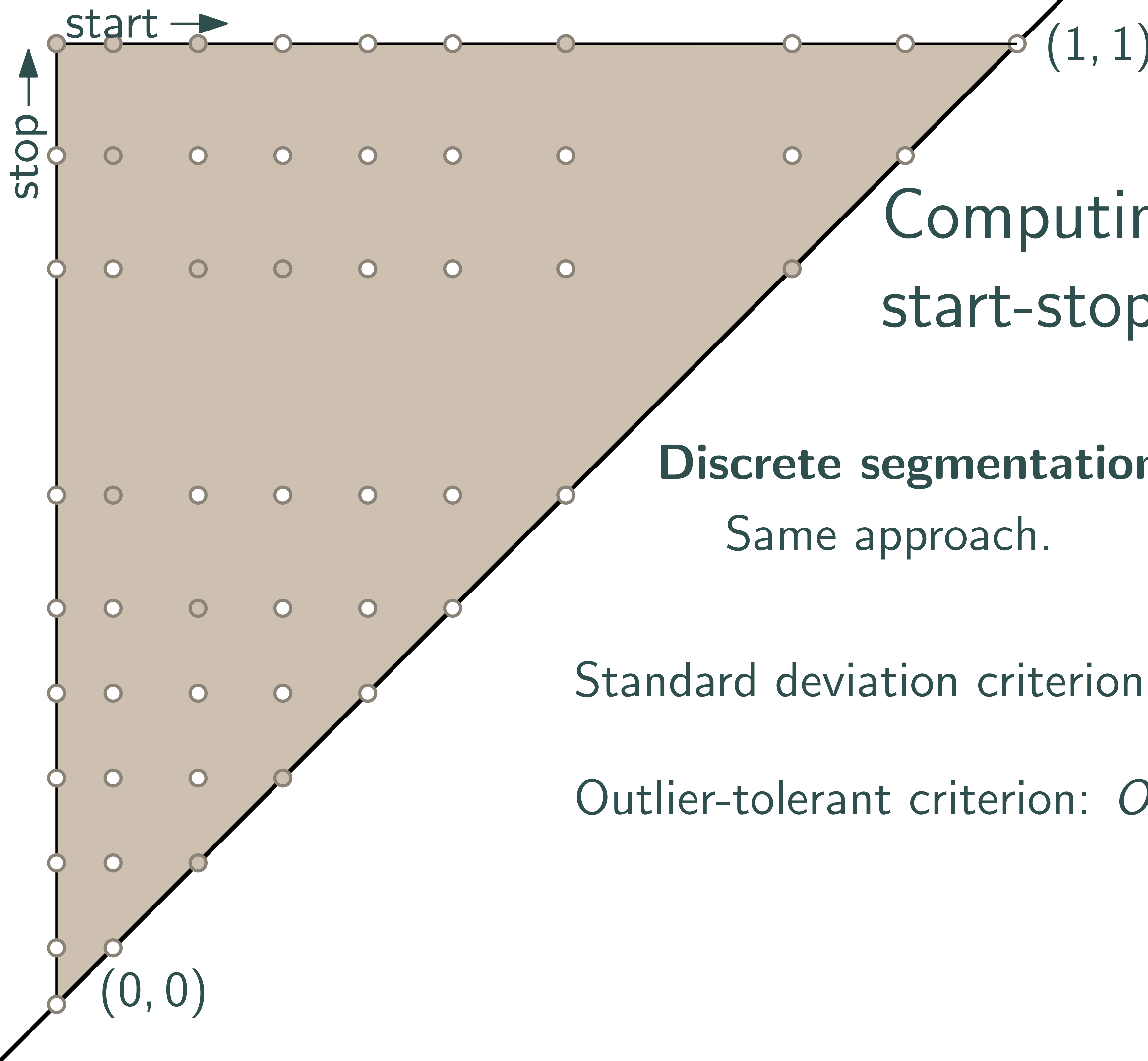

# Computing a start-stop diagram

Traverse the cells column by column from bottom to top.

- Compute the forbidden space in each cell.
- Maintain additional information (e.g. $\mu$, $\sum$).

**Lemma 2.** *We can compute the start-stop diagram for the outlier-tolerant criterion in $O(n^2 \log n)$ time.*

$\Longrightarrow$ minimal segmentation in $O(n^2 \log n + kn^2)$ time.



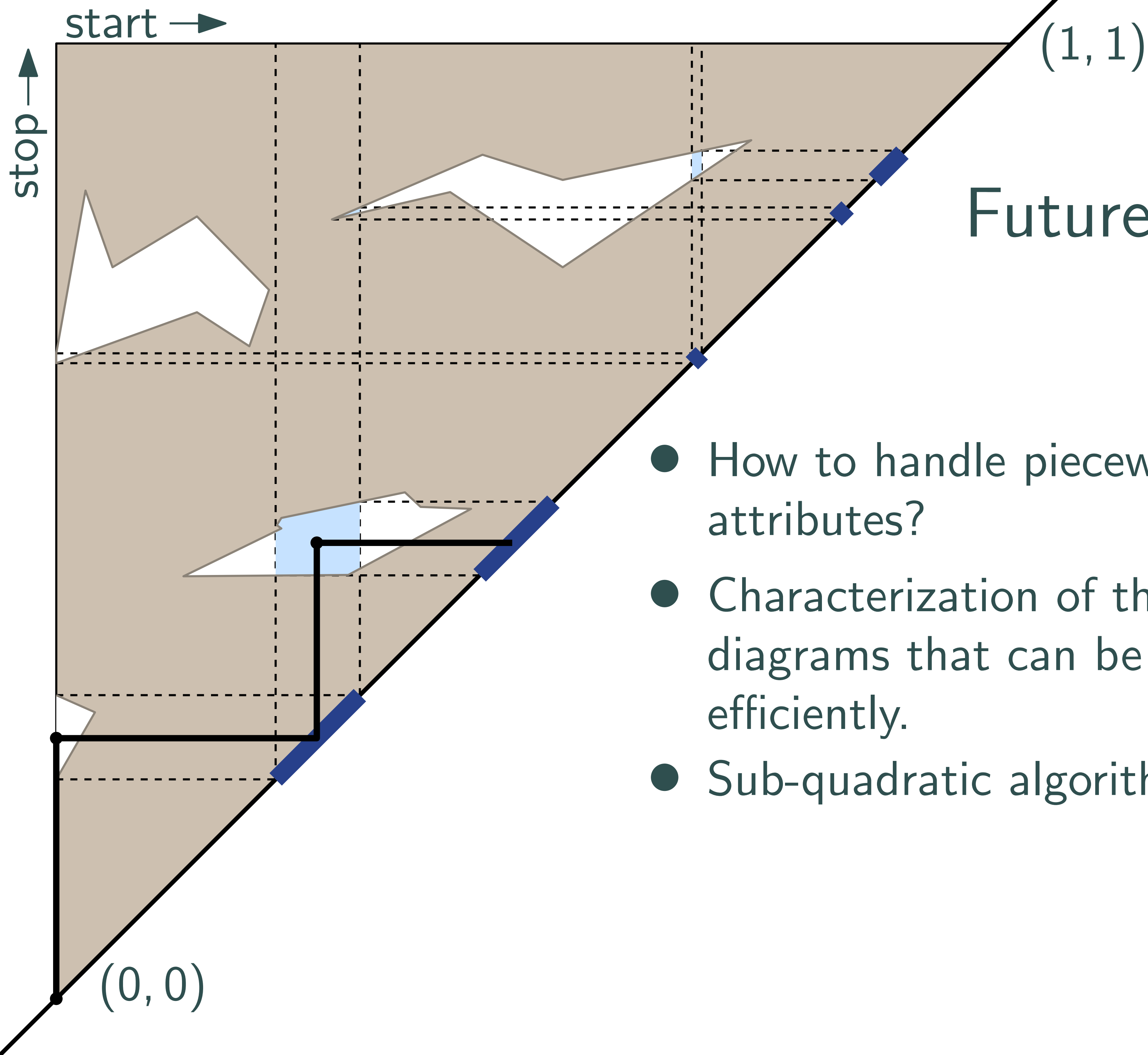

# Computing a start-stop diagram

**Discrete segmentation:**

Same approach.

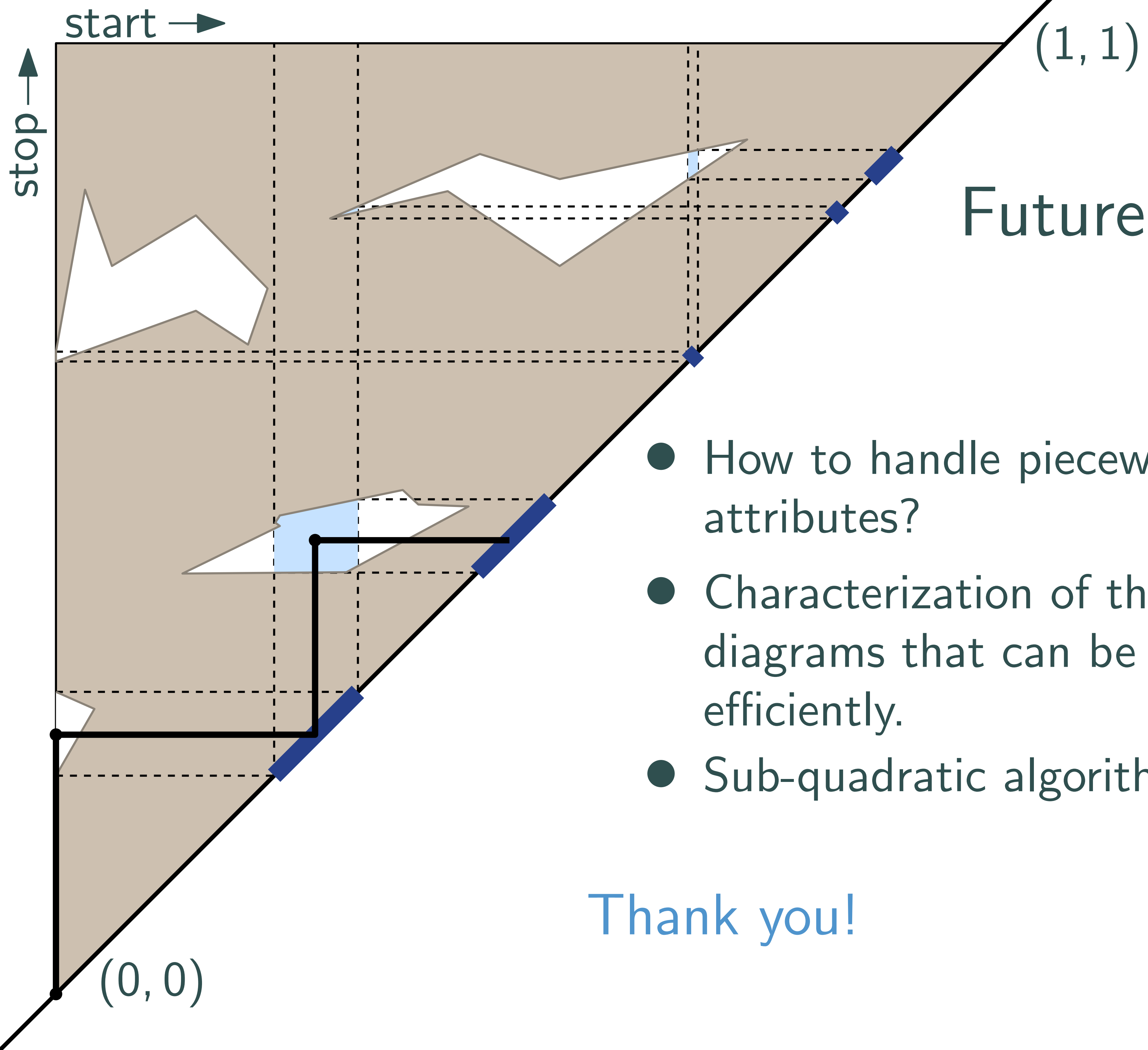
Standard deviation criterion: $O(n^2)$

Outlier-tolerant criterion: $O(n^2 \log n)$




# Future Work

- How to handle piecewise linear attributes?
- Characterization of the start-stop diagrams that can be handled efficiently.
- Sub-quadratic algorithms?

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Thank you!