

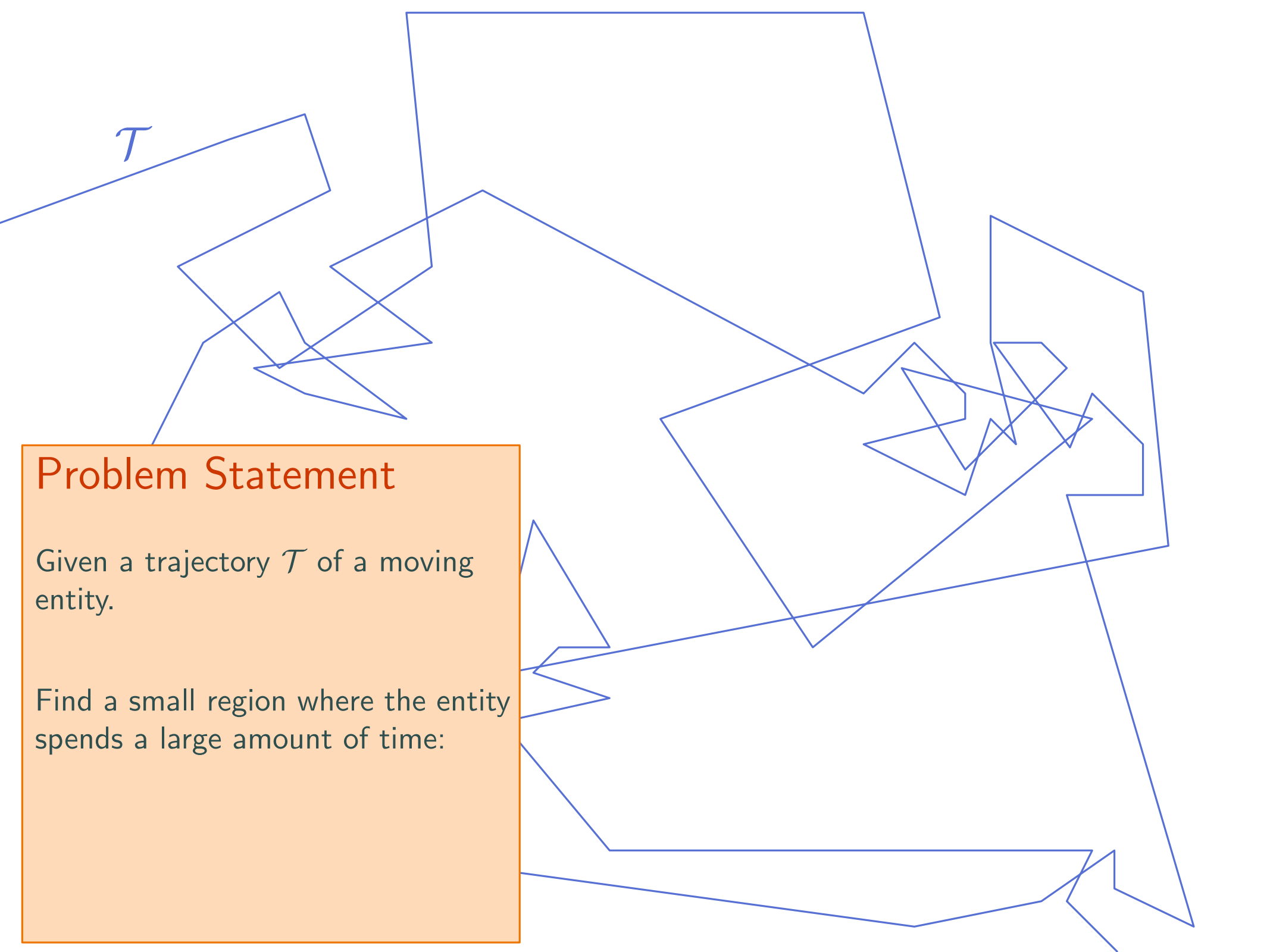
Algorithms for Hotspot Computation on Trajectory Data

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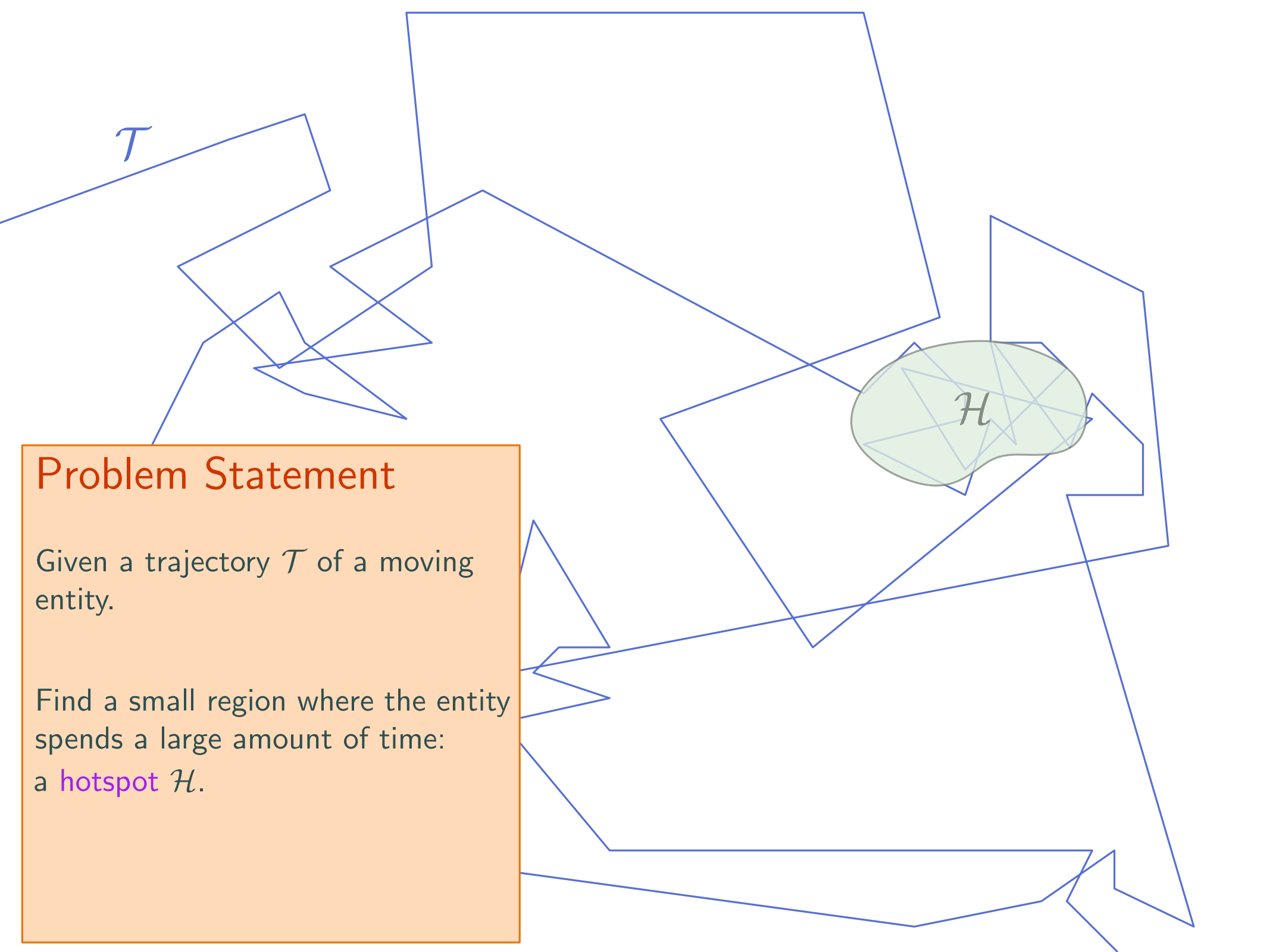


$\mathcal{T}$

Problem Statement

Given a trajectory $\mathcal{T}$ of a moving entity.

Find a small region where the entity spends a large amount of time:

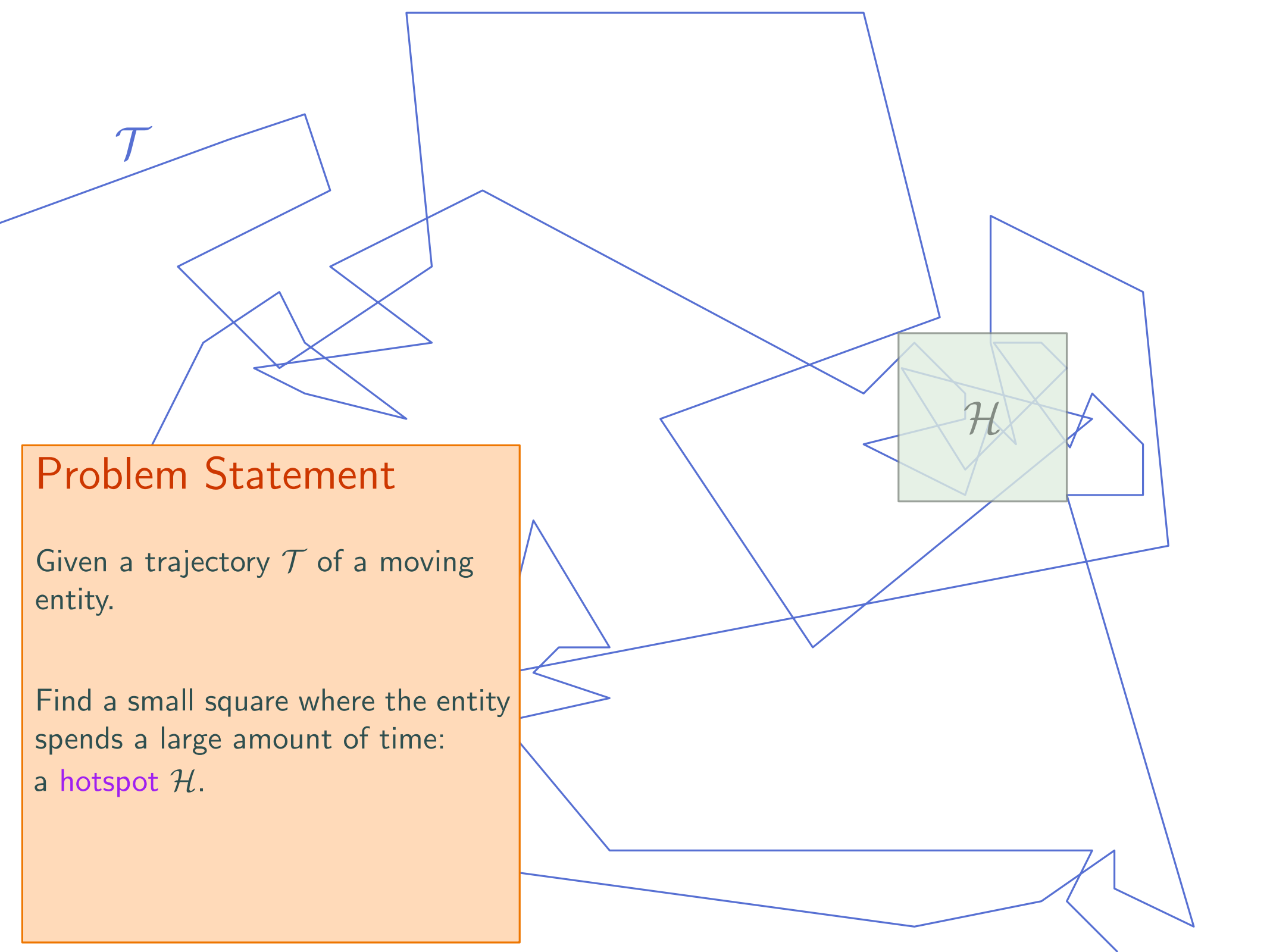


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Find a small region where the entity spends a large amount of time:
a hotspot $\mathcal{H}$.

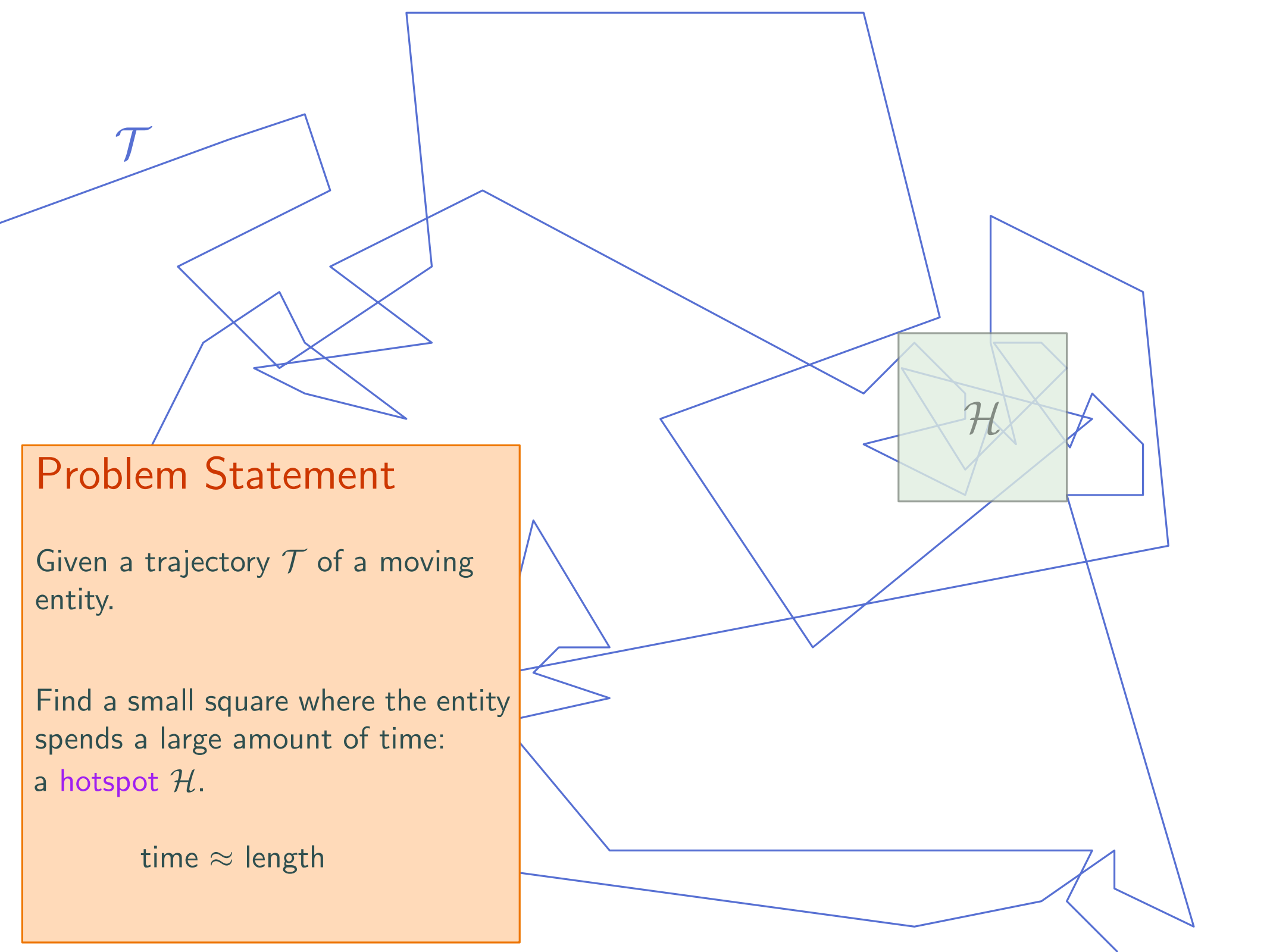


$\mathcal{T}$

Problem Statement

Given a trajectory $\mathcal{T}$ of a moving entity.

Find a small square where the entity spends a large amount of time:
a **hotspot** $\mathcal{H}$.



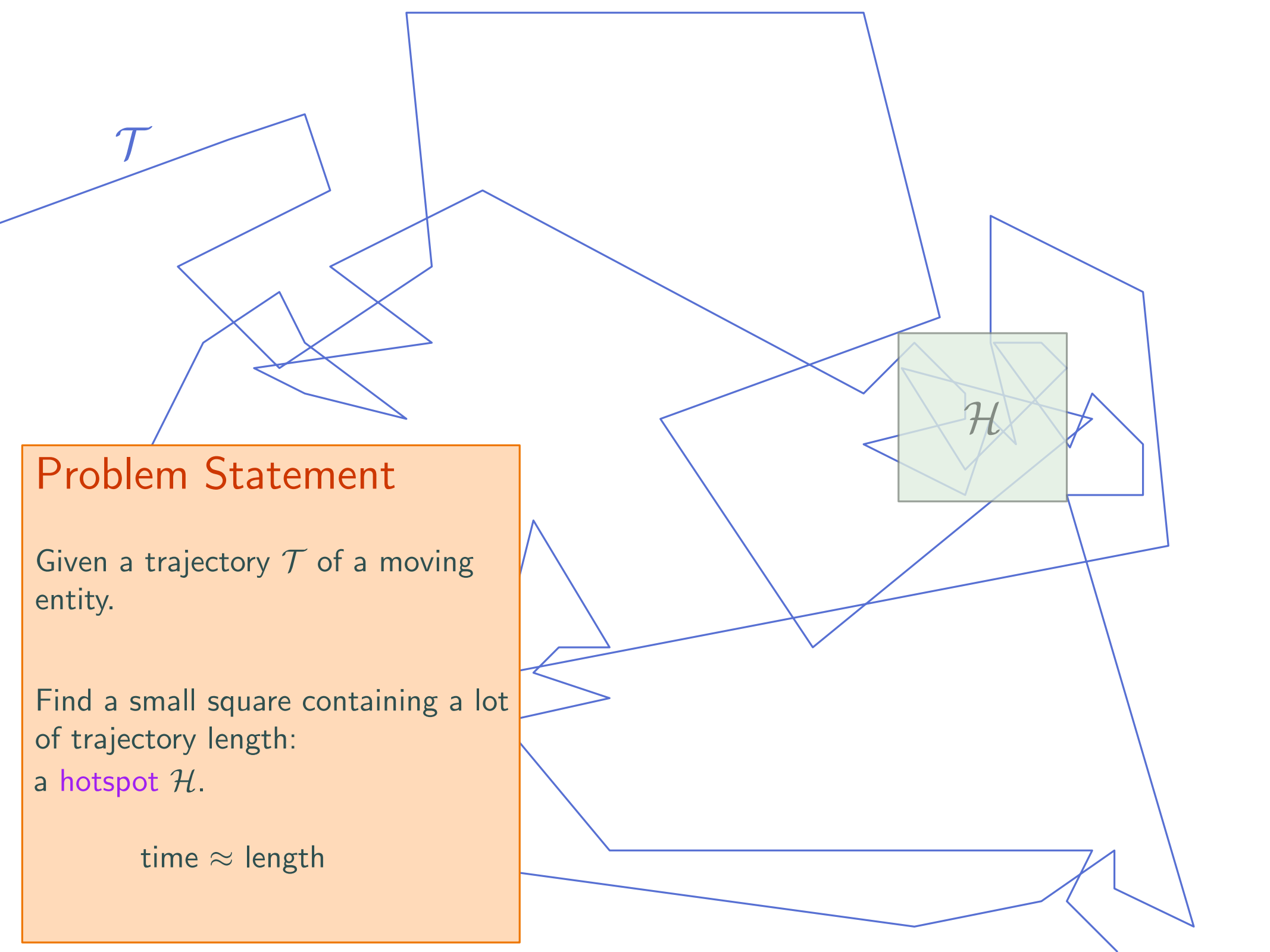
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Given a trajectory $\mathcal{T}$ of a moving entity.

Find a small square where the entity spends a large amount of time:
a **hotspot** $\mathcal{H}$.

time $\approx$ length



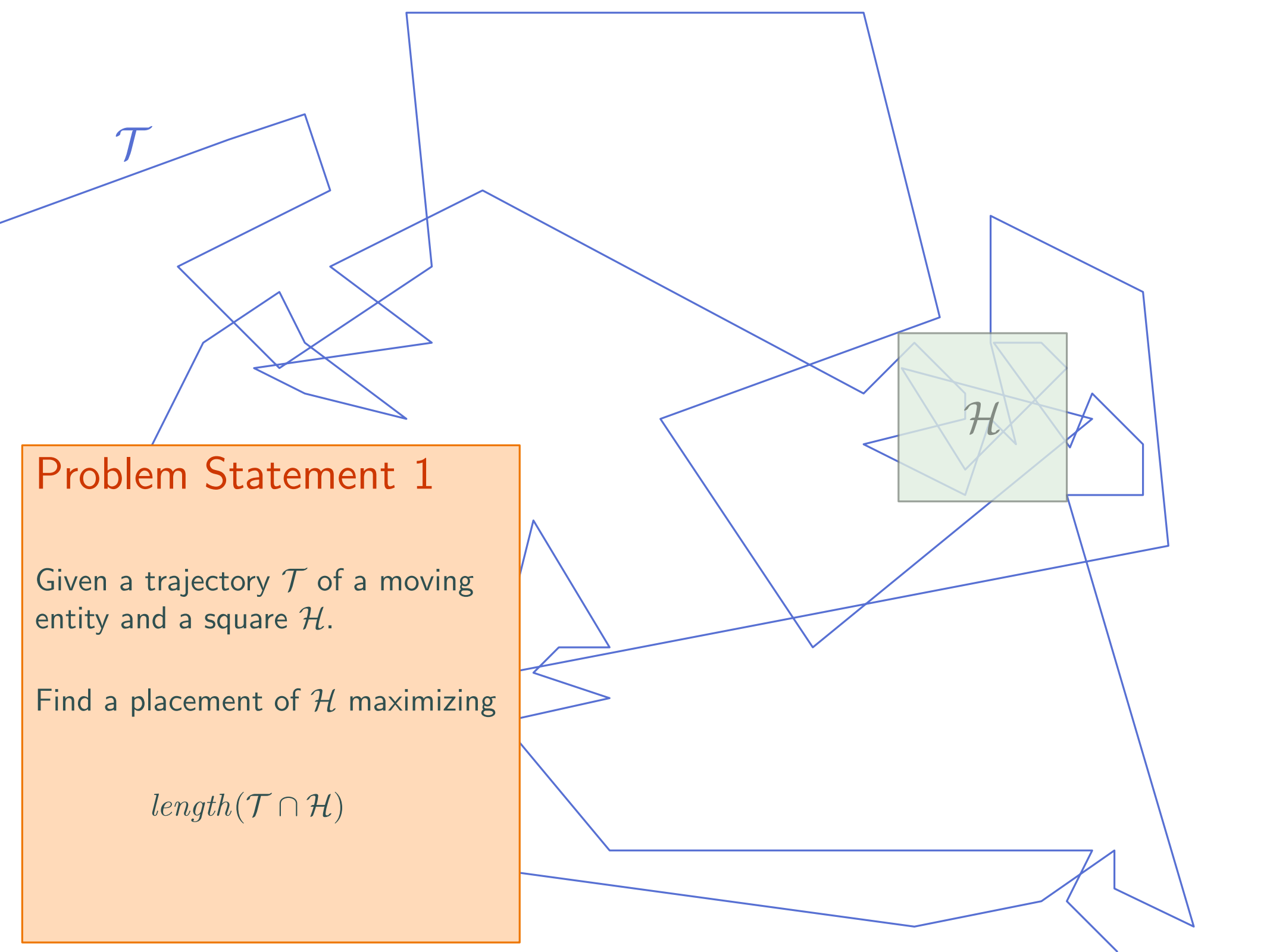
$\mathcal{T}$

Problem Statement

Given a trajectory $\mathcal{T}$ of a moving entity.

Find a small square containing a lot of trajectory length:
a **hotspot** $\mathcal{H}$.

time $\approx$ length



A diagram illustrating a geometric problem. A blue line, labeled $\mathcal{T}$, represents a trajectory. It is a complex, self-intersecting path that winds across the frame. In the middle-right portion of the image, there is a light green square, labeled $\mathcal{H}$. The trajectory $\mathcal{T}$ passes through the square $\mathcal{H}$, with several segments of the path contained within the square's boundaries. The label $\mathcal{T}$ is placed at the beginning of the trajectory on the left side, and the label $\mathcal{H}$ is placed inside the square.

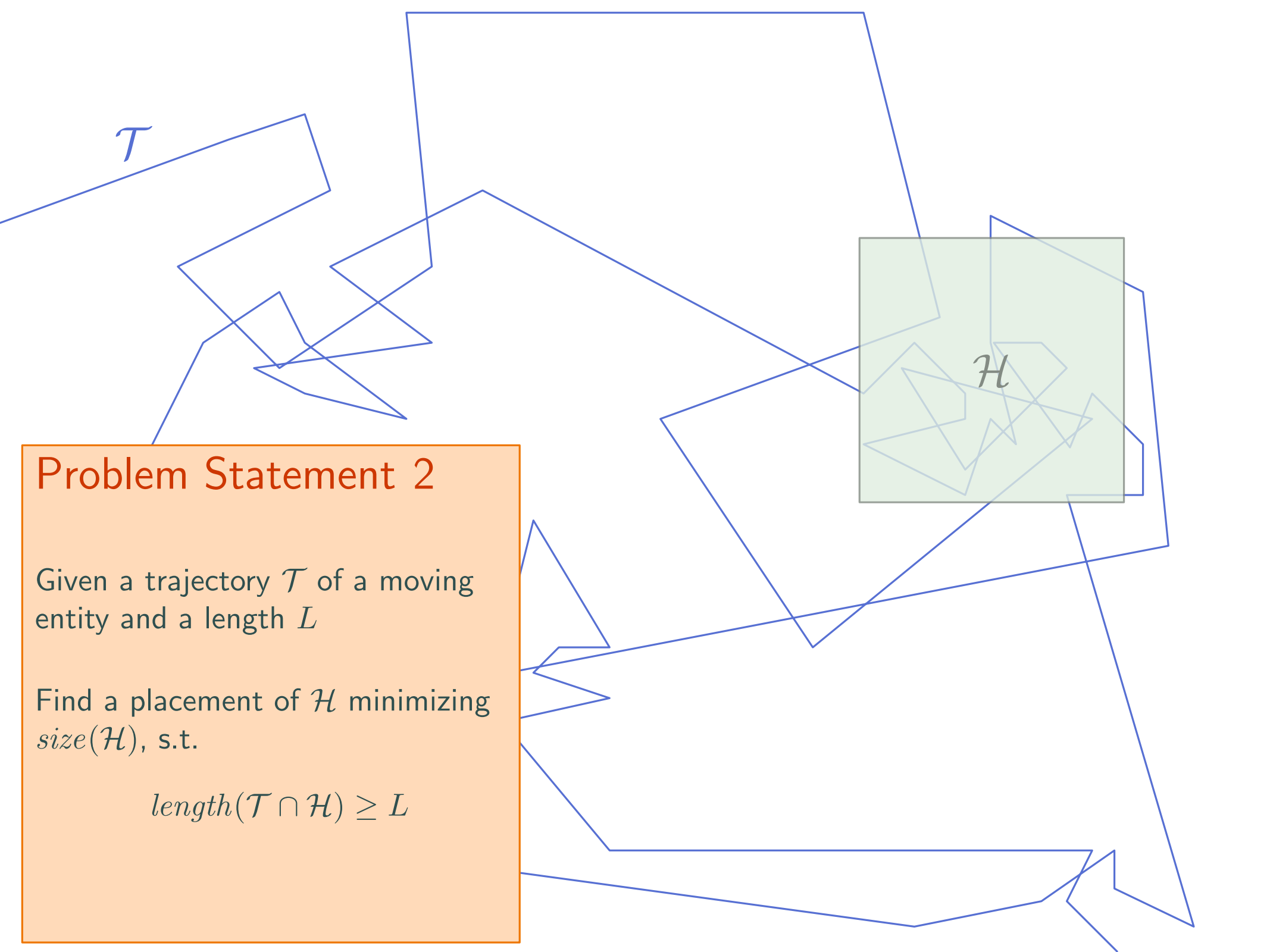
 $\mathcal{T}$

Problem Statement 1

Given a trajectory $\mathcal{T}$ of a moving entity and a square $\mathcal{H}$.

Find a placement of $\mathcal{H}$ maximizing

$$\text{length}(\mathcal{T} \cap \mathcal{H})$$



The diagram shows a blue line representing a trajectory $\mathcal{T}$ that moves across the frame. A light green rectangular region $\mathcal{H}$ is positioned on the right side. The trajectory $\mathcal{T}$ enters the region $\mathcal{H}$ from the top left and exits from the bottom right. The label $\mathcal{T}$ is placed near the start of the trajectory, and the label $\mathcal{H}$ is placed inside the green rectangle.

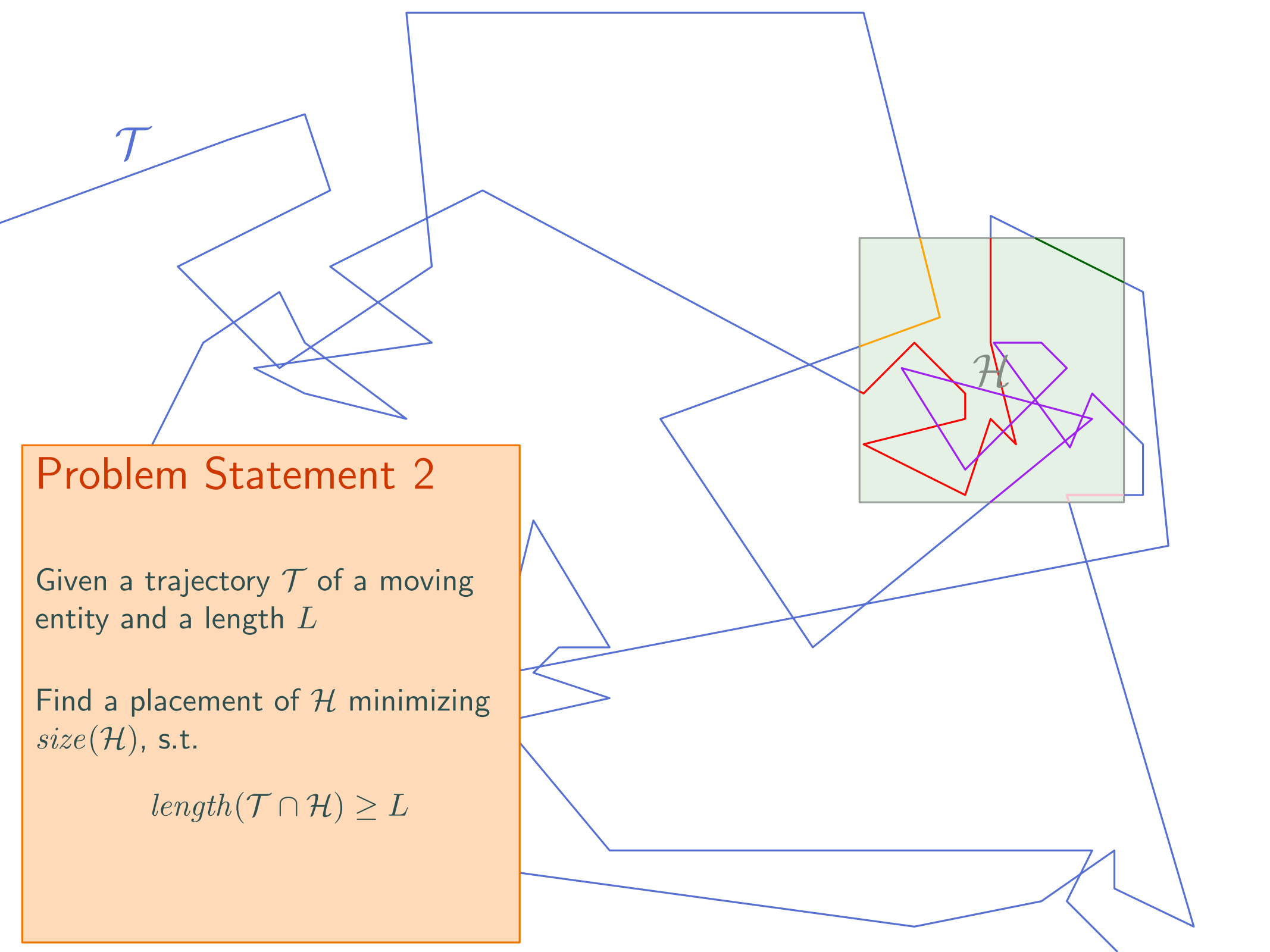
 $\mathcal{T}$

Problem Statement 2

Given a trajectory $\mathcal{T}$ of a moving entity and a length L

Find a placement of $\mathcal{H}$ minimizing $size(\mathcal{H})$, s.t.

$$length(\mathcal{T} \cap \mathcal{H}) \geq L$$

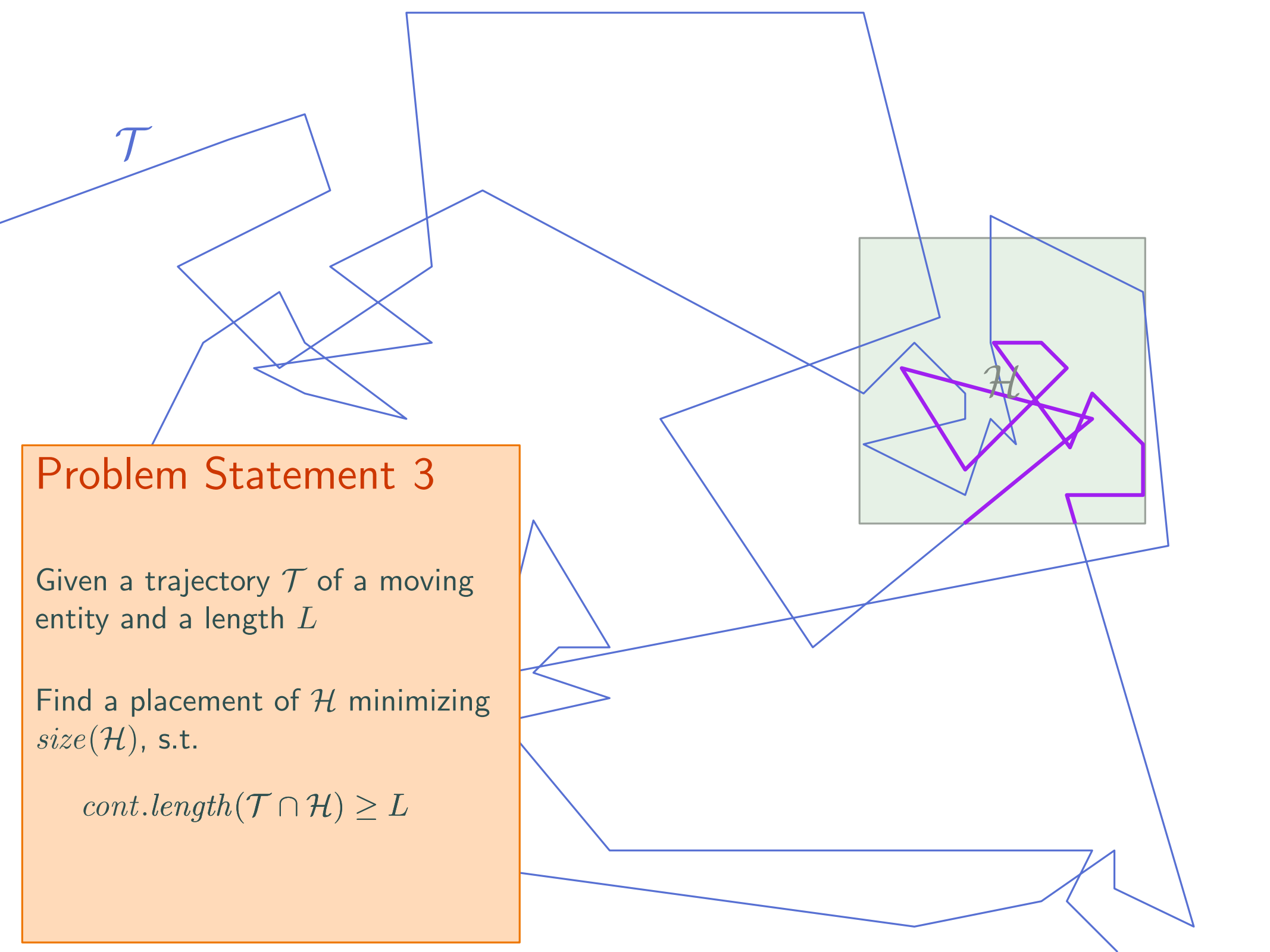


Problem Statement 2

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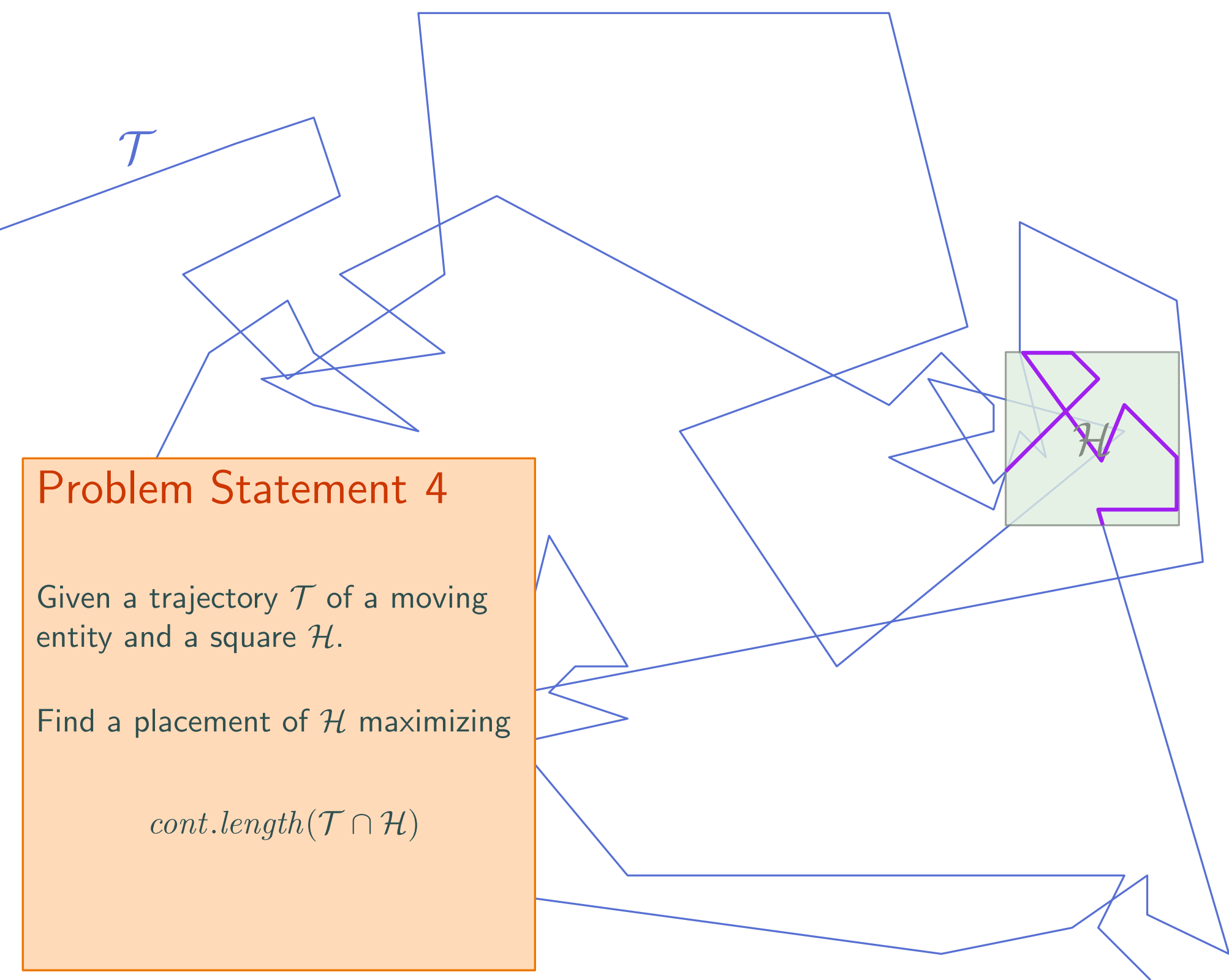


Problem Statement 3

Given a trajectory $\mathcal{T}$ of a moving entity and a length L

Find a placement of $\mathcal{H}$ minimizing $size(\mathcal{H})$, s.t.

$$cont.length(\mathcal{T} \cap \mathcal{H}) \geq L$$

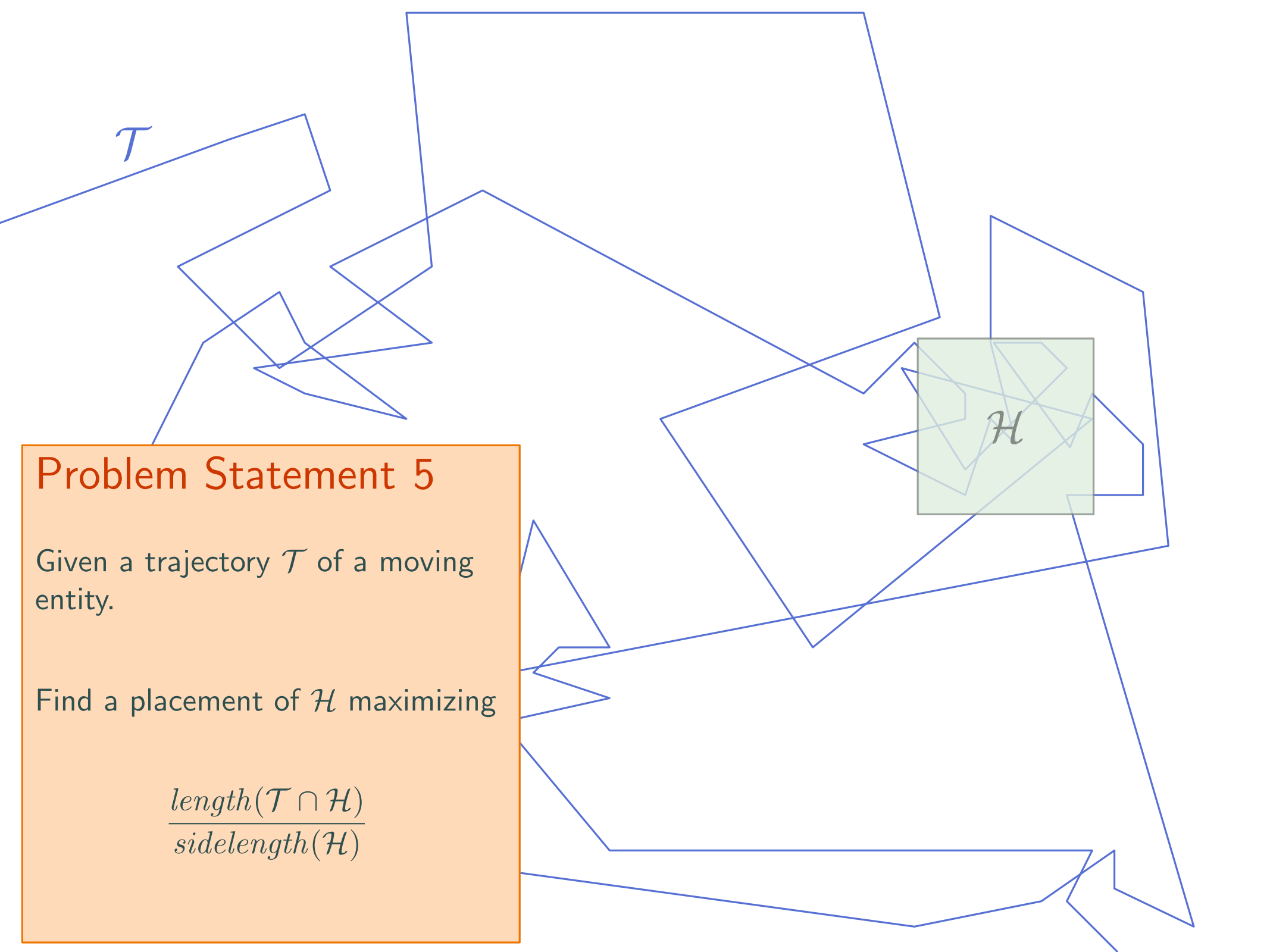


Problem Statement 4

Given a trajectory $\mathcal{T}$ of a moving entity and a square $\mathcal{H}$.

Find a placement of $\mathcal{H}$ maximizing

$$\text{cont.length}(\mathcal{T} \cap \mathcal{H})$$

A diagram illustrating a problem in geometry or optimization. It features a blue line representing a trajectory $\mathcal{T}$ that winds through a white space. A light green square region, labeled $\mathcal{H}$, is positioned on the right side of the image. The trajectory $\mathcal{T}$ enters the square $\mathcal{H}$ from the top-left corner and exits from the bottom-right corner. The label $\mathcal{T}$ is placed at the beginning of the trajectory line on the left. The label $\mathcal{H}$ is centered within the green square.

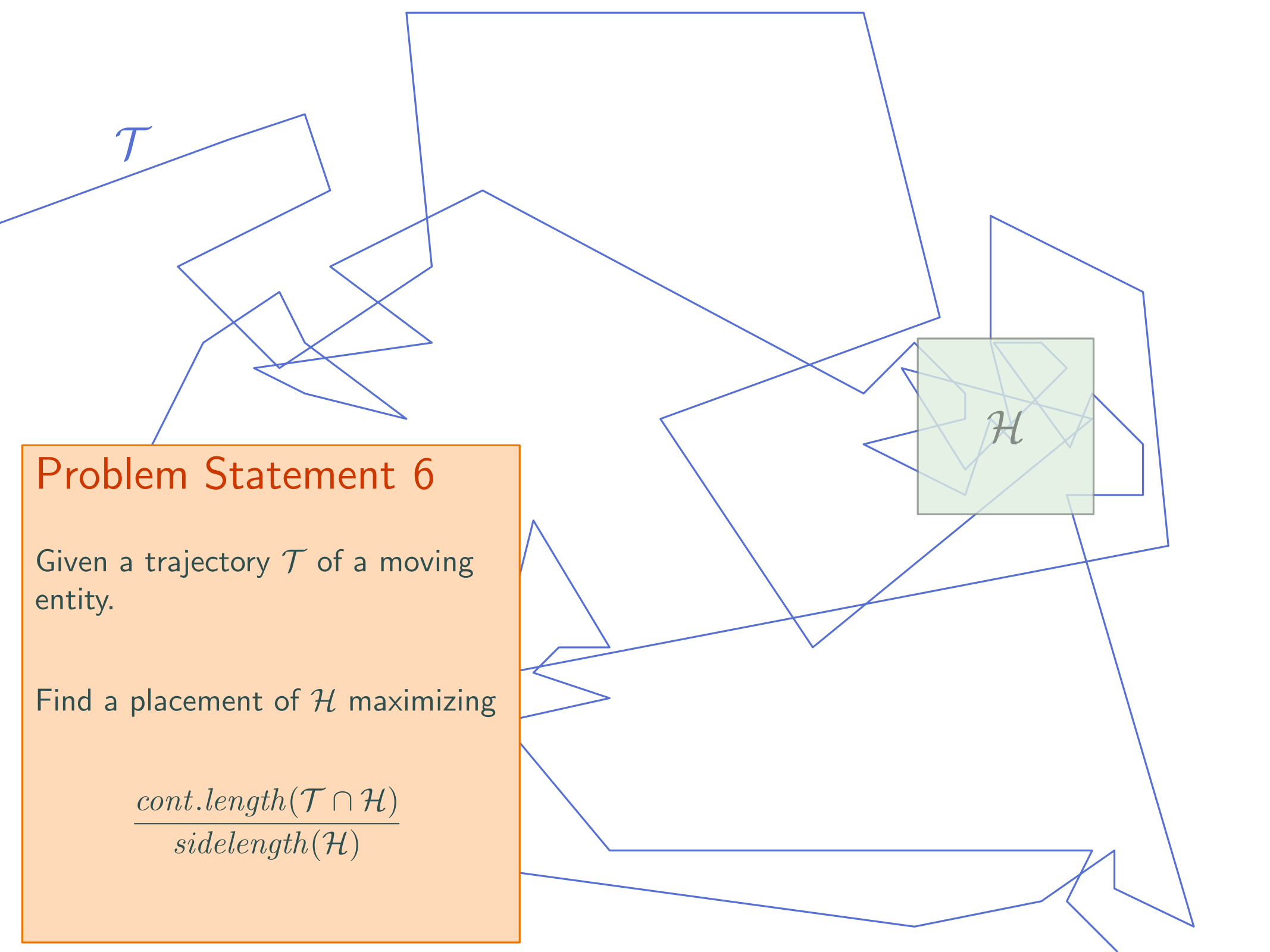
$\mathcal{T}$

Problem Statement 5

Given a trajectory $\mathcal{T}$ of a moving entity.

Find a placement of $\mathcal{H}$ maximizing

$$\frac{\text{length}(\mathcal{T} \cap \mathcal{H})}{\text{sidelength}(\mathcal{H})}$$



A diagram illustrating a problem in computational geometry. A blue line represents a trajectory $\mathcal{T}$, which is a continuous path that moves across the scene. A green square represents a region $\mathcal{H}$, which is a convex polygon. The trajectory $\mathcal{T}$ enters the region $\mathcal{H}$ from the left and exits from the bottom right. The intersection of the trajectory and the region is highlighted by a darker green color.

 $\mathcal{T}$

Problem Statement 6

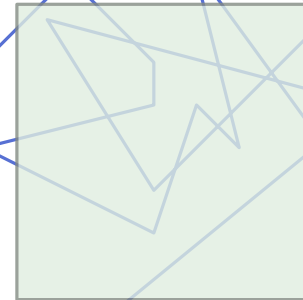
Given a trajectory $\mathcal{T}$ of a moving entity.

Find a placement of $\mathcal{H}$ maximizing

$$\frac{\text{cont.length}(\mathcal{T} \cap \mathcal{H})}{\text{sidelength}(\mathcal{H})}$$

Results

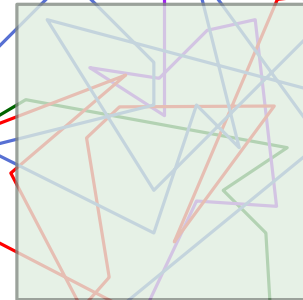
	Fixed Size	Fixed Length	Relative
<i>length</i> :	$O(n^2)$	$O(n^2 \log^2 n)$	$O(n^3)$
<i>cont.length</i> :	$O(n \log n)$	$O(n \log n)$	-



Results

	Fixed Size	Fixed Length	Relative
<i>length</i> :	$O(n^2)$	$O(n^2 \log^2 n)$	$O(n^3)$
<i>cont.length</i> :	$O(n \log n)$	$O(n \log n)$	-

Our algorithms also work for multiple trajectories,

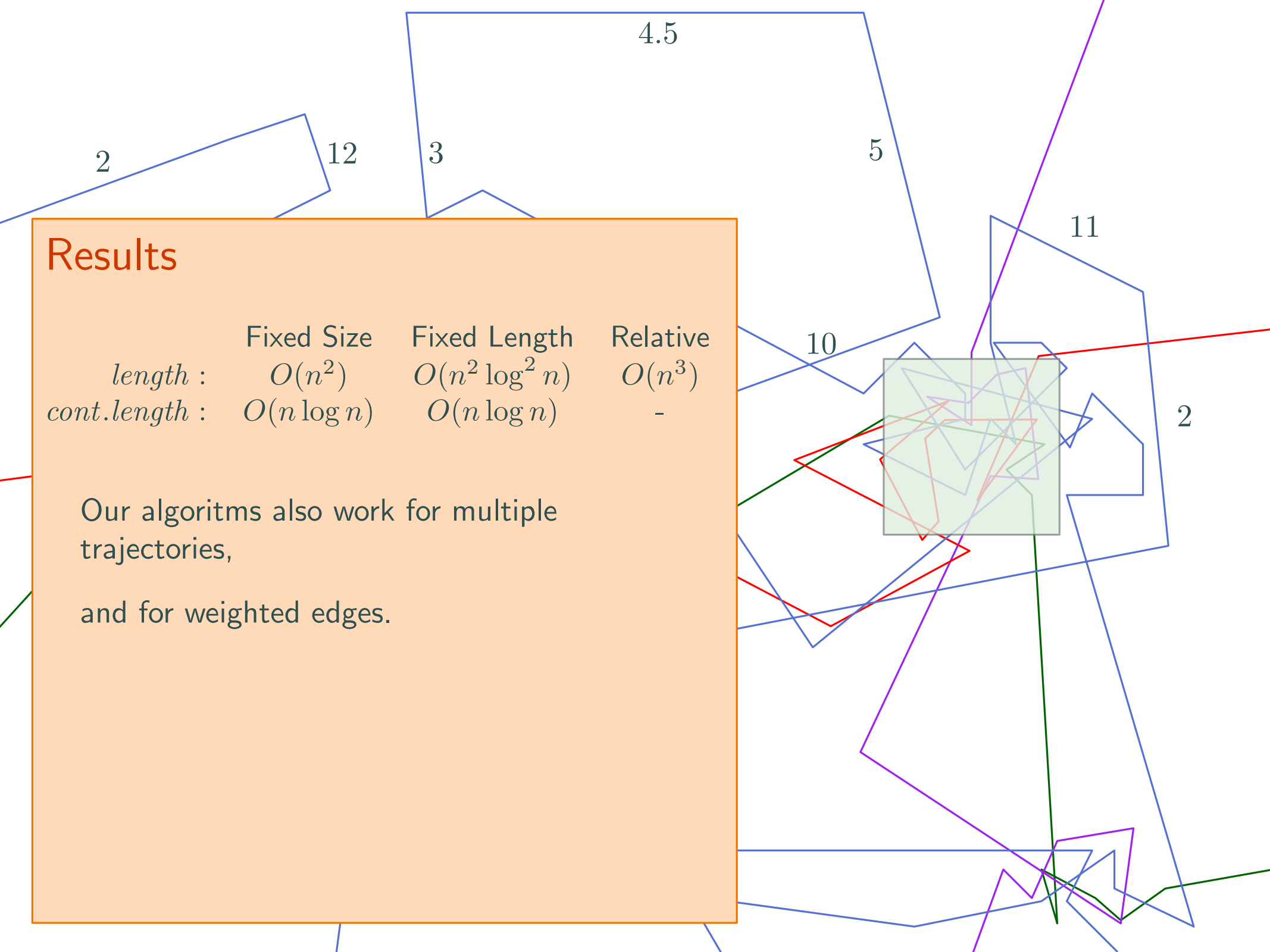


Results

	Fixed Size	Fixed Length	Relative
<i>length</i> :	$O(n^2)$	$O(n^2 \log^2 n)$	$O(n^3)$
<i>cont.length</i> :	$O(n \log n)$	$O(n \log n)$	-

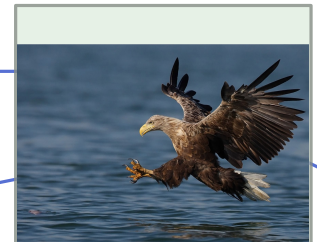
Our algorithms also work for multiple trajectories,

and for weighted edges.



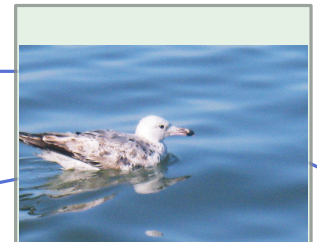
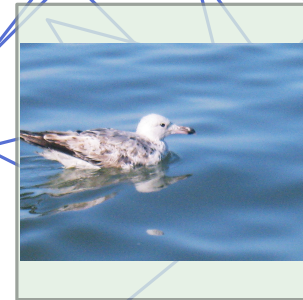
Applications

- Finding places



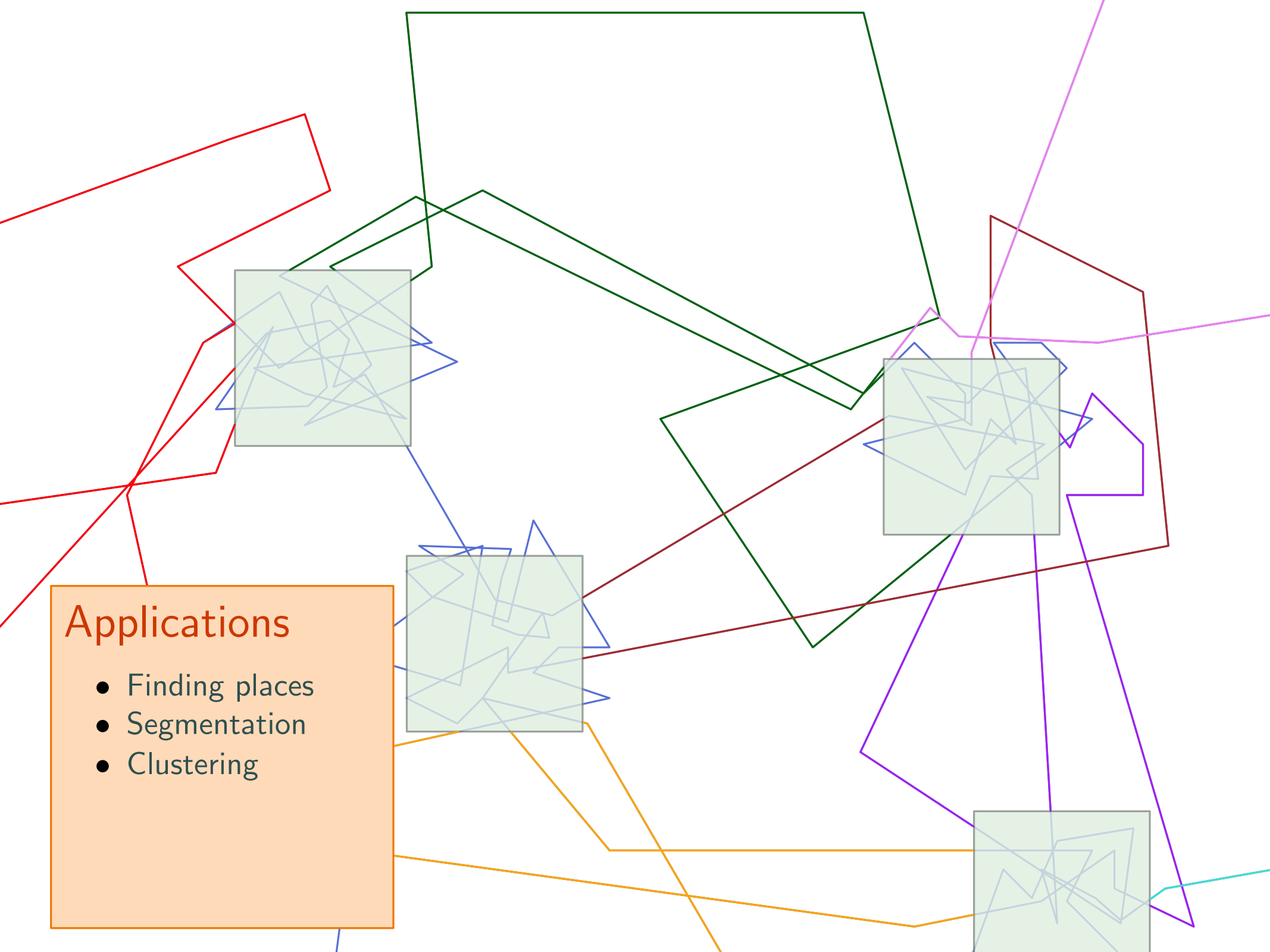
Applications

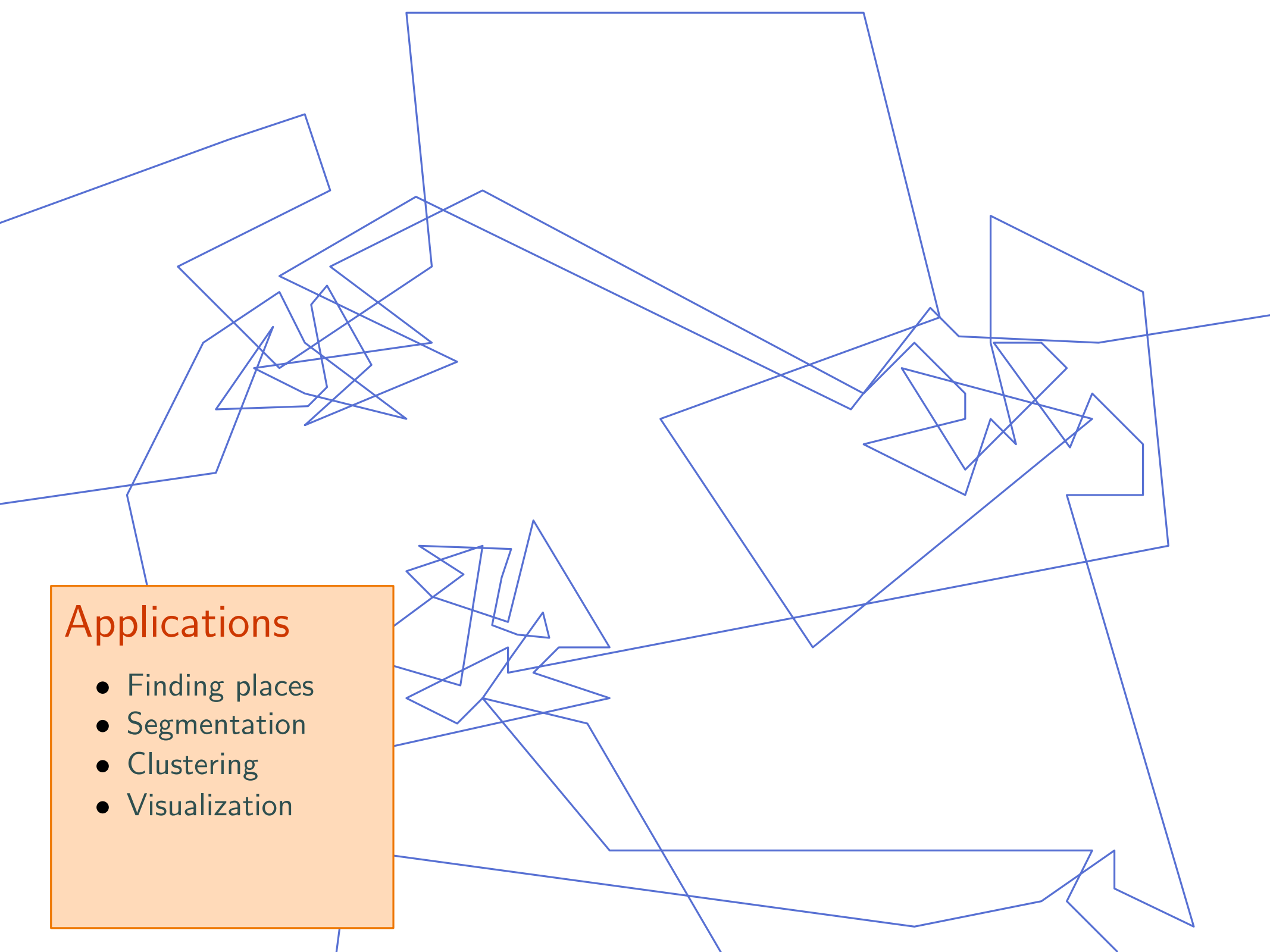
- Finding places
- Segmentation



Applications

- Finding places
- Segmentation
- Clustering



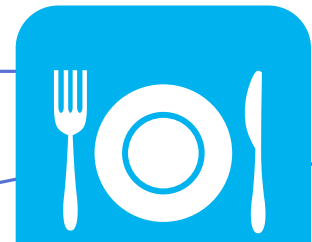
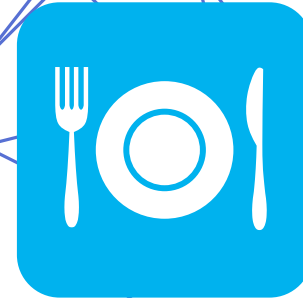
The background of the slide is a complex, abstract pattern of thin blue lines and overlapping polygons of various shapes and sizes, creating a sense of movement and depth.

Applications

- Finding places
- Segmentation
- Clustering
- Visualization

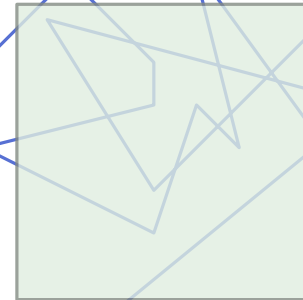
Applications

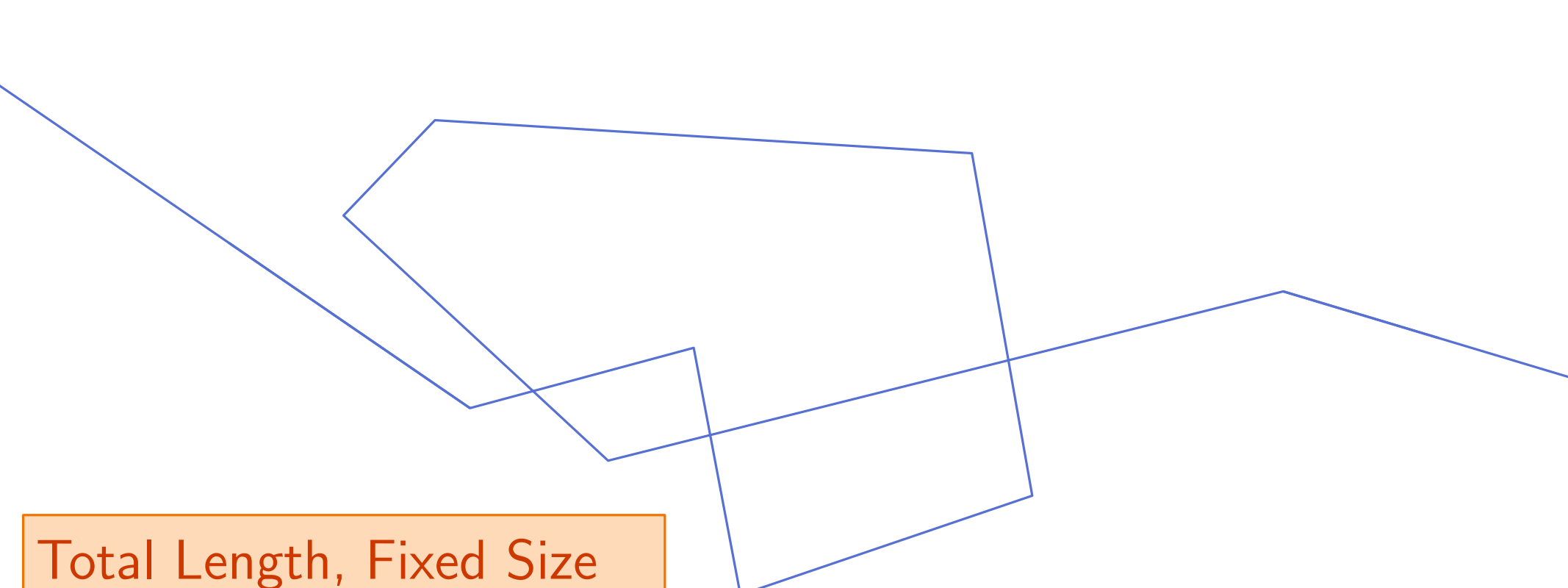
- Finding places
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Results

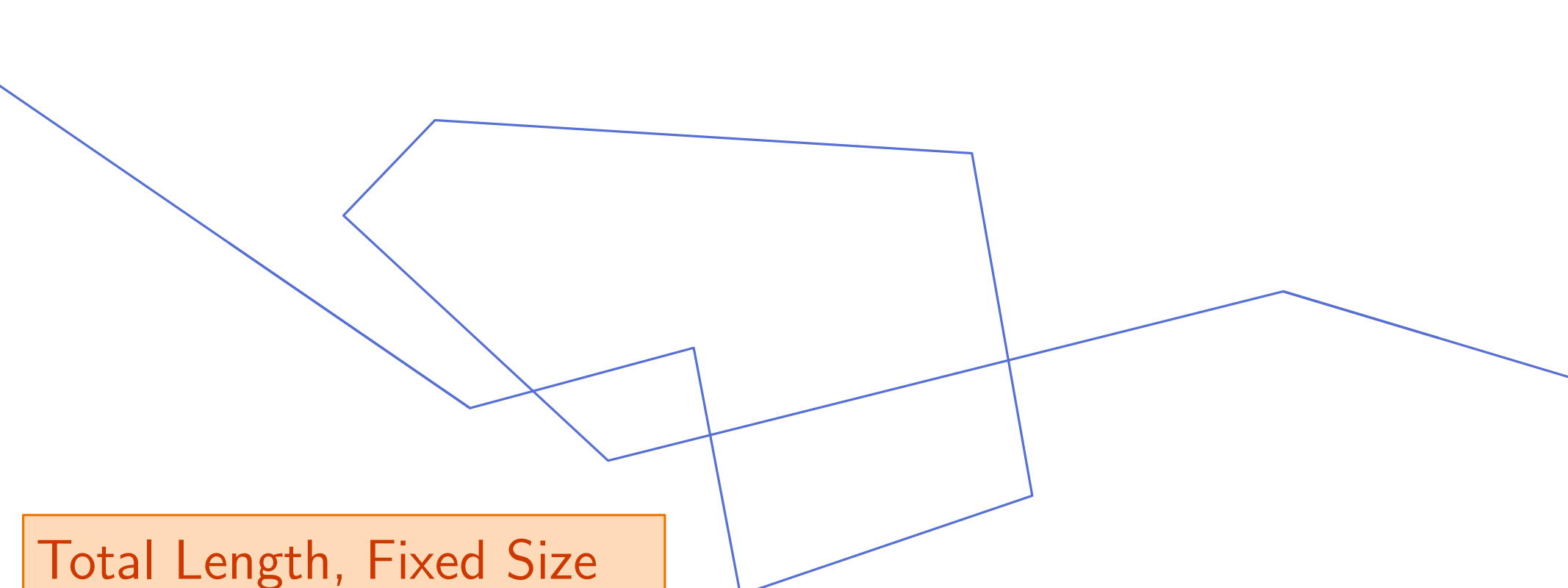
	Fixed Size	Fixed Length	Relative
<i>length</i> :	$O(n^2)$	$O(n^2 \log^2 n)$	$O(n^3)$
<i>maxlength</i> :	$O(n \log n)$	$O(n \log n)$	-





Total Length, Fixed Size

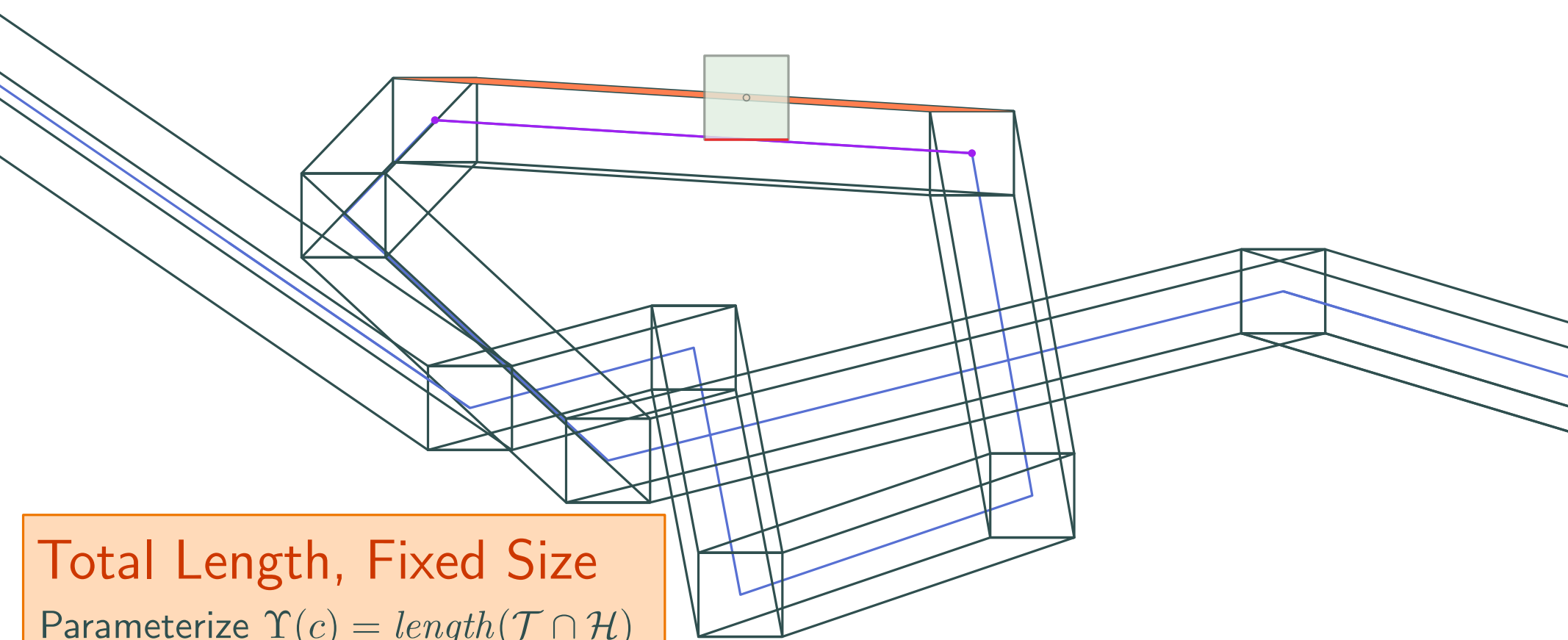
Parameterize $\Upsilon(c) = \text{length}(\mathcal{T} \cap \mathcal{H})$
by the center c of $\mathcal{H}$.



Total Length, Fixed Size

Parameterize $\Upsilon(c) = \text{length}(\mathcal{T} \cap \mathcal{H})$
by the center c of $\mathcal{H}$.

Lemma 1. Υ is piecewise linear.

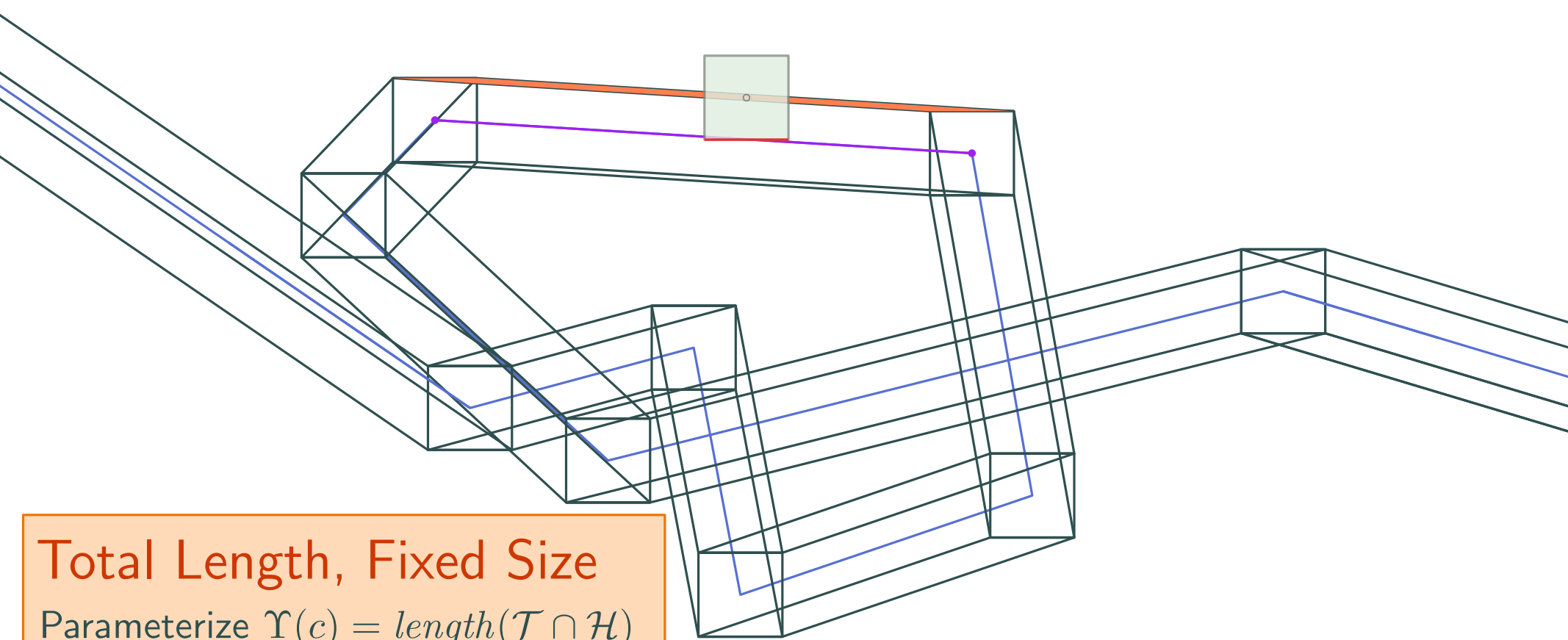


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by the center c of $\mathcal{H}$.

Lemma 1. Υ is piecewise linear.

Consider the subdivision $\mathcal{A}$ of the
parameter space of Υ .



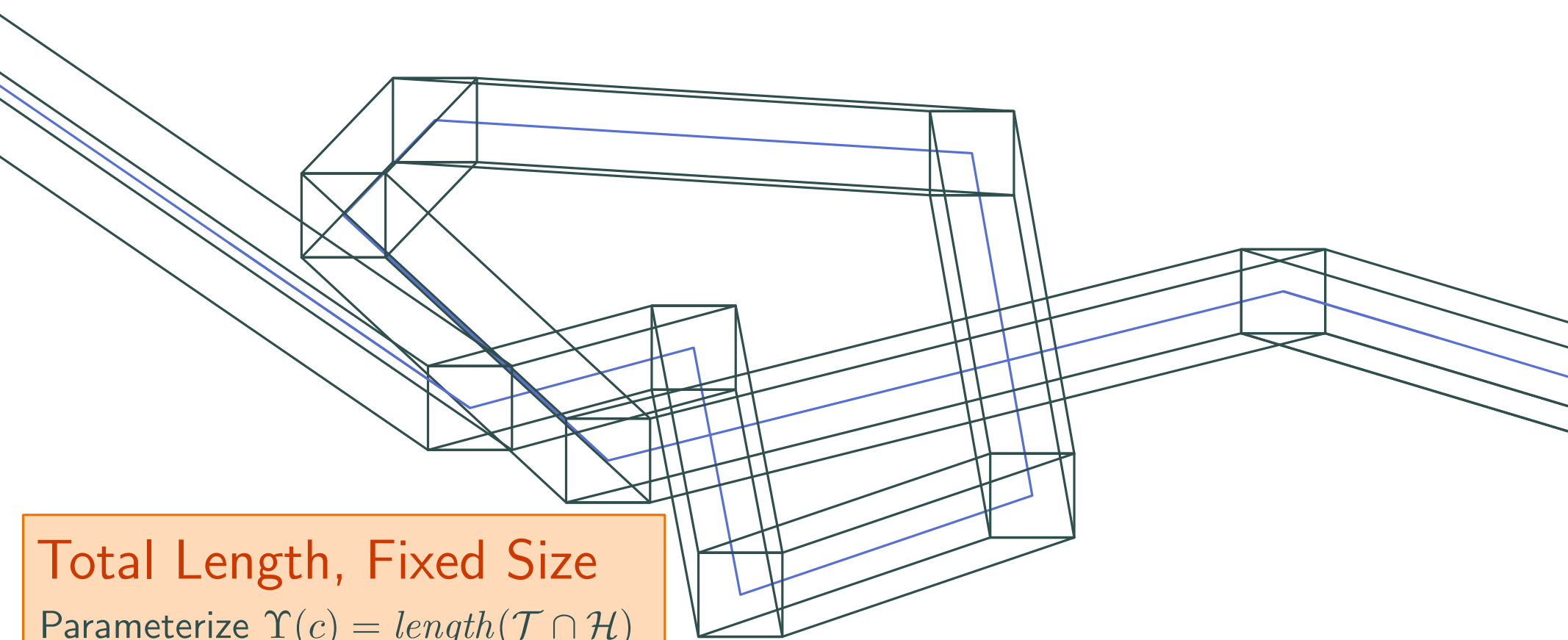
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$\max \Upsilon$ occurs at a vertex of $\mathcal{A}$. So, compute Υ at each vertex of $\mathcal{A}$.



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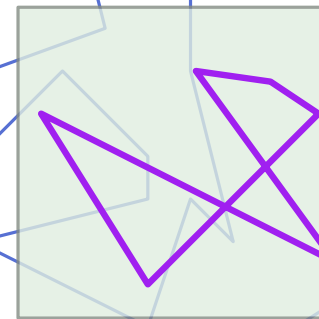
Complexity $\mathcal{A}$: $O(n^2)$

Find $\max \Upsilon$: $O(n^2)$

Total: $O(n^2)$

Contiguous Length, Fixed Size

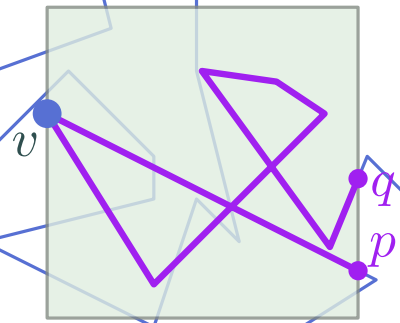
Find $\mathcal{H}$ by finding $\mathcal{T}[p, q]$.



Contiguous Length, Fixed Size

Find $\mathcal{H}$ by finding $\mathcal{T}[p, q]$.

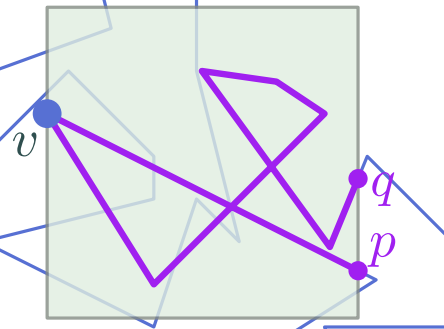
Lemma 2. *There is an optimal hotspot $\mathcal{H}$ s.t. a vertex v of $\mathcal{T}$ lies on $\partial\mathcal{H}$.*



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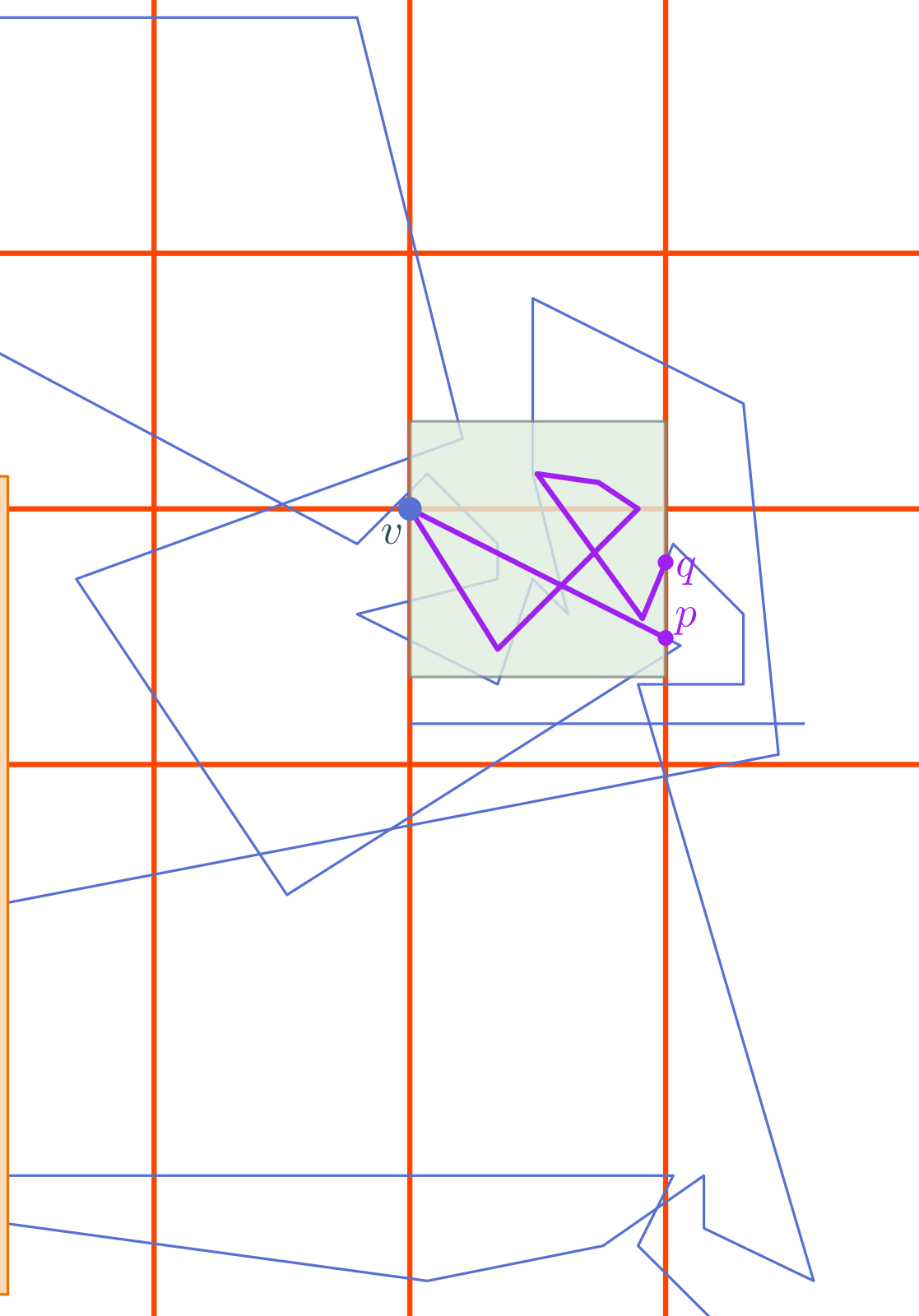


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Lemma 2. *There is an optimal hotspot $\mathcal{H}$ s.t. a vertex v of $\mathcal{T}[p, q]$ lies on $\partial\mathcal{H}$.*

Corollary 3. *Starting point p on one of the horizontal or vertical lines.*



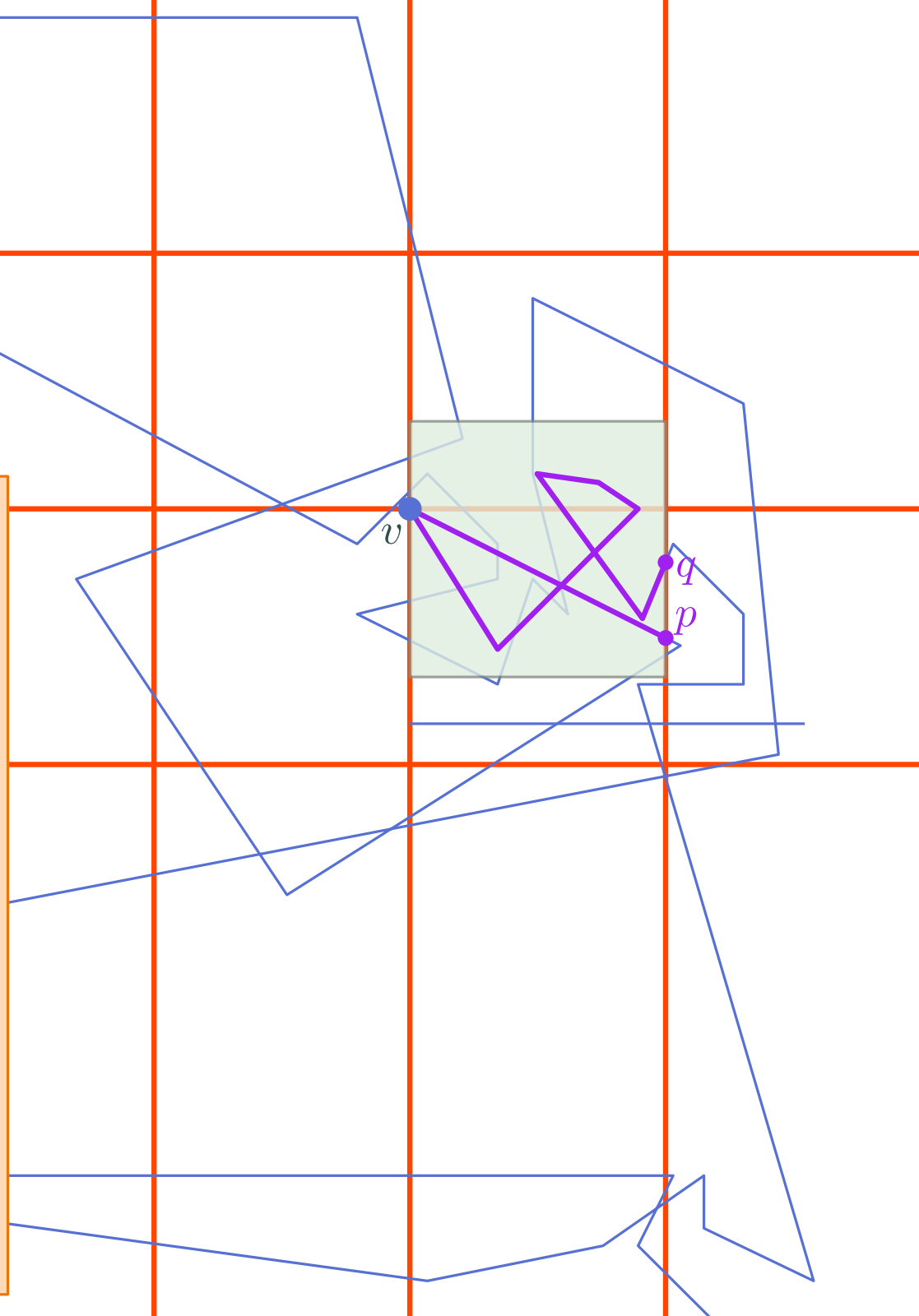
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How to find p and q ?



Contiguous Length, Fixed Size

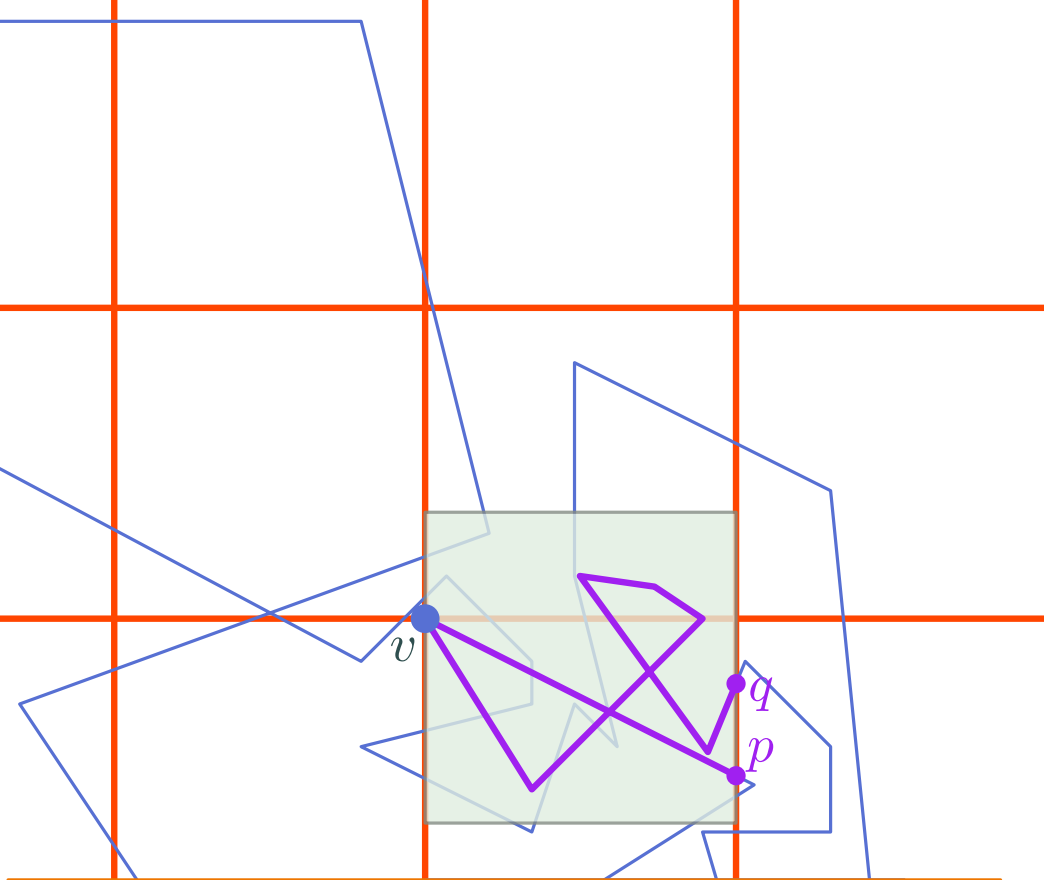
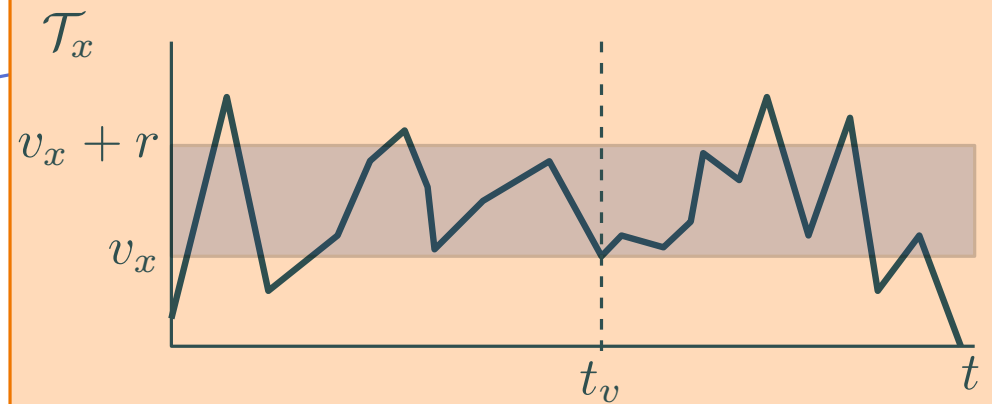
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How to find p and q ?

Consider $\mathcal{T}_x$ and $\mathcal{T}_y$ separately.



Contiguous Length, Fixed Size

Find $\mathcal{H}$ by finding $\mathcal{T}[p, q]$.

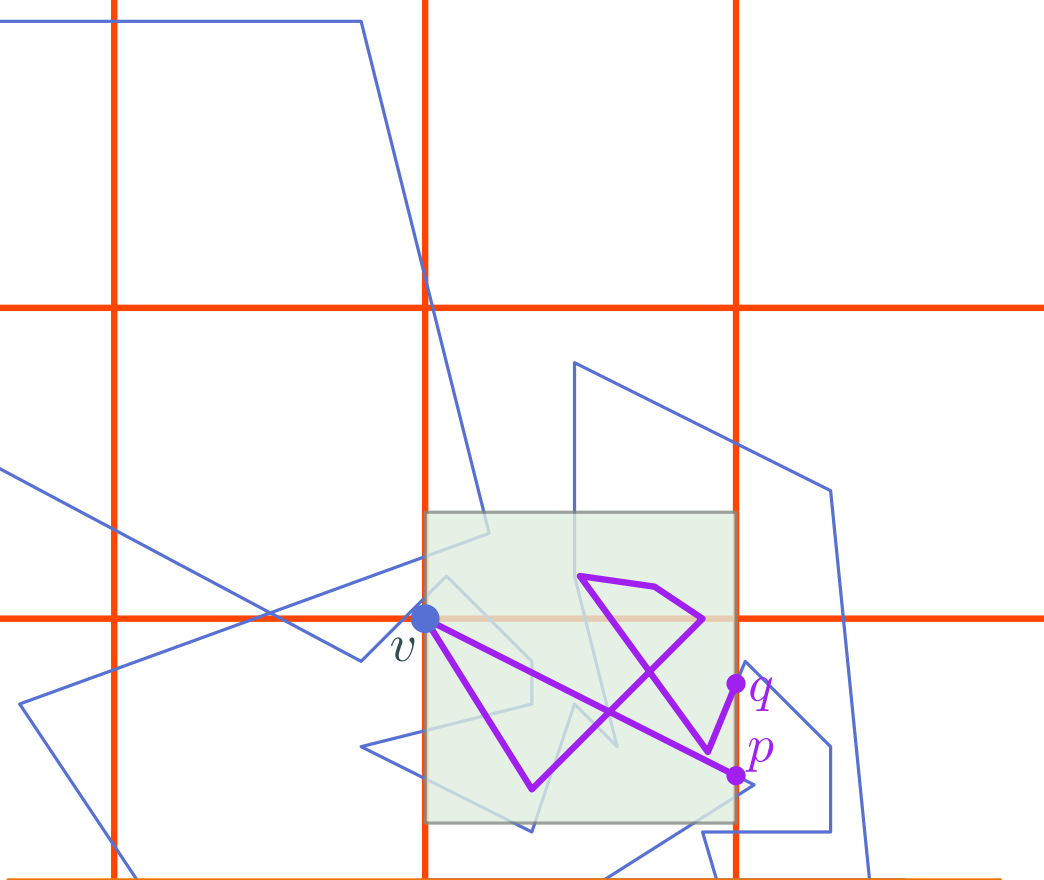
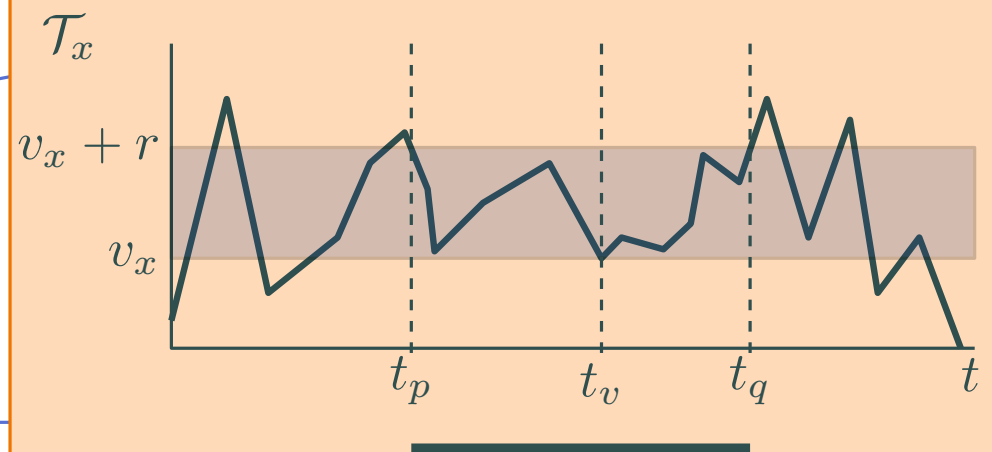
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Consider $\mathcal{T}_x$ and $\mathcal{T}_y$ separately.

Try to find $[t_p, t_q]$.



Contiguous Length, Fixed Size

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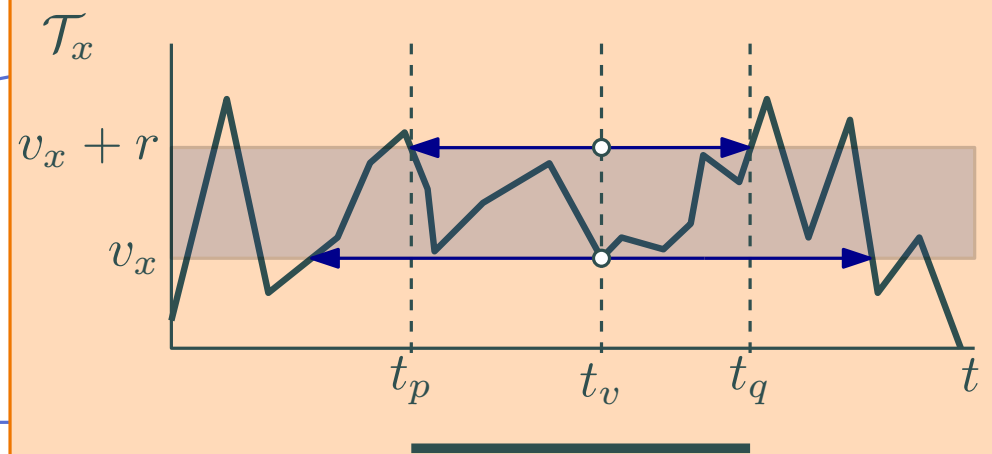
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Use ray shooting queries.



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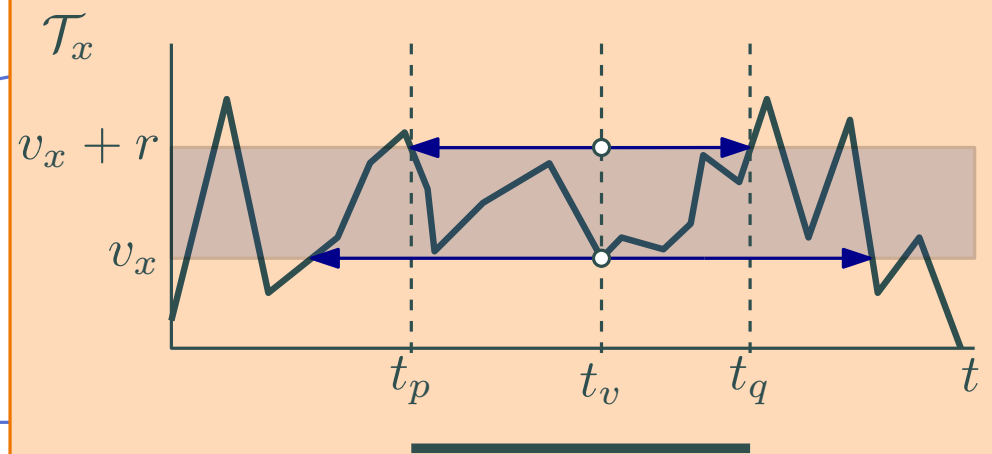
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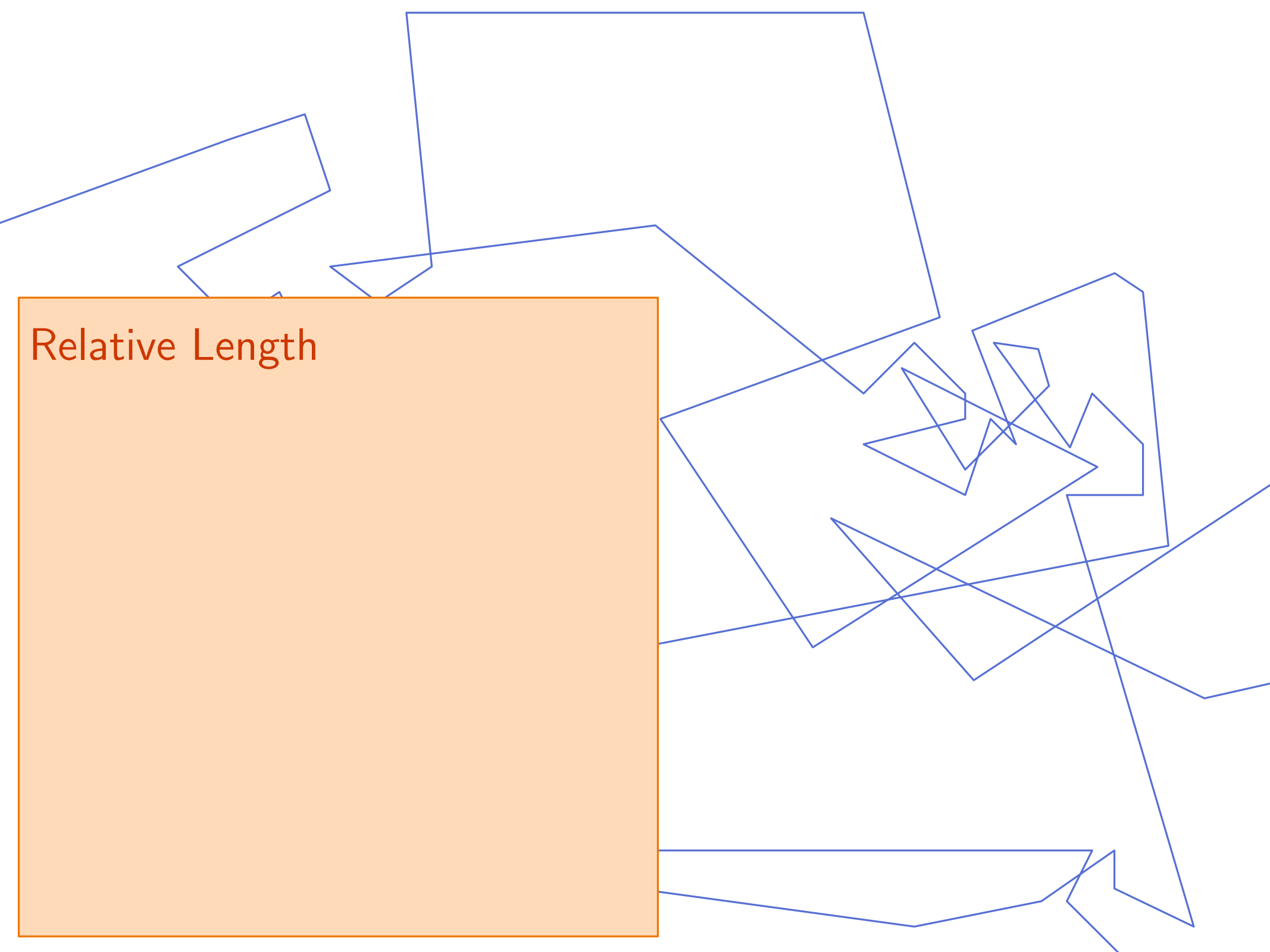
Try to find $[t_p, t_q]$.

Use ray shooting queries.

Running time: $O(n \log n)$



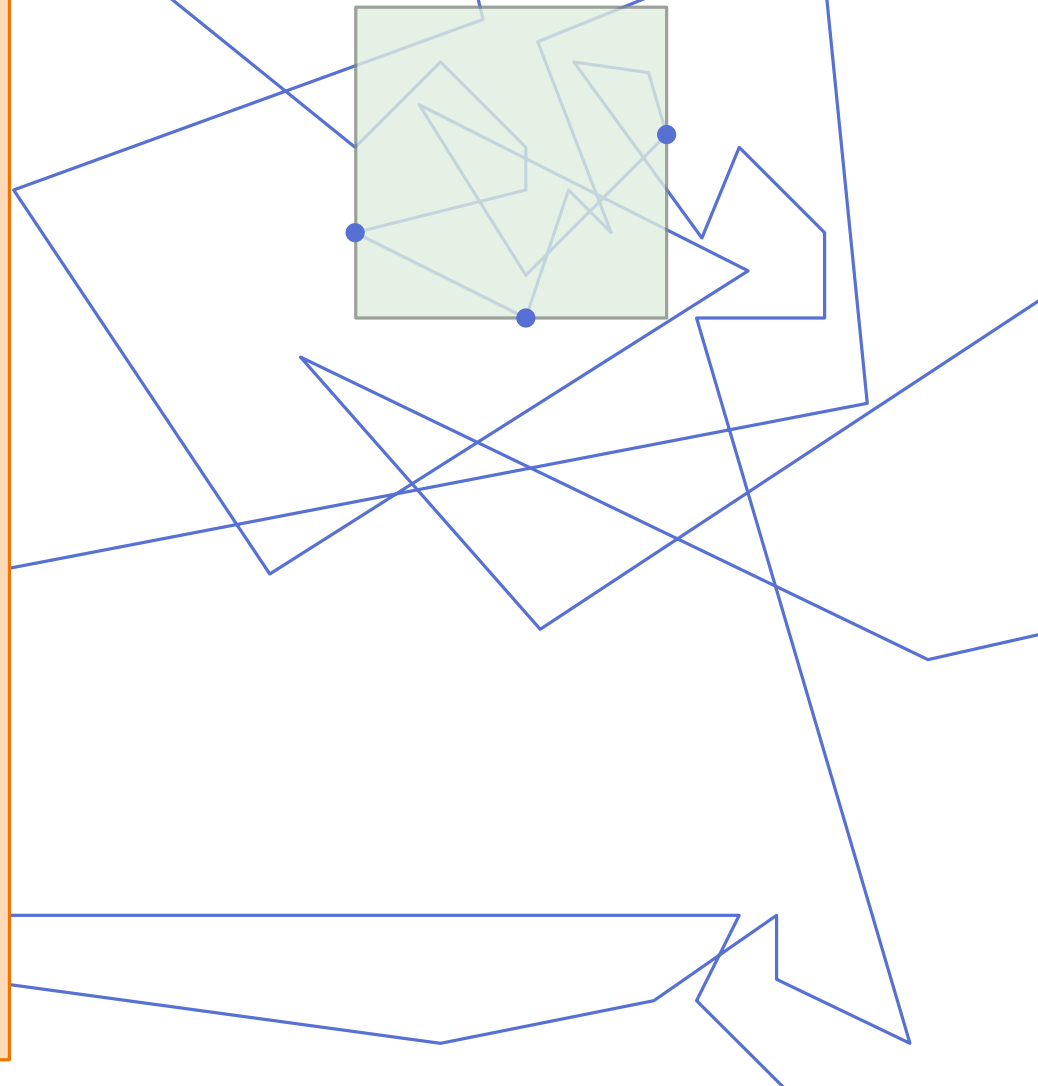
Relative Length



Relative Length

Lemma 4. *There is an optimal $\mathcal{H}$ bounded by 3 objects.*

3 vertices on $\partial\mathcal{H}$.

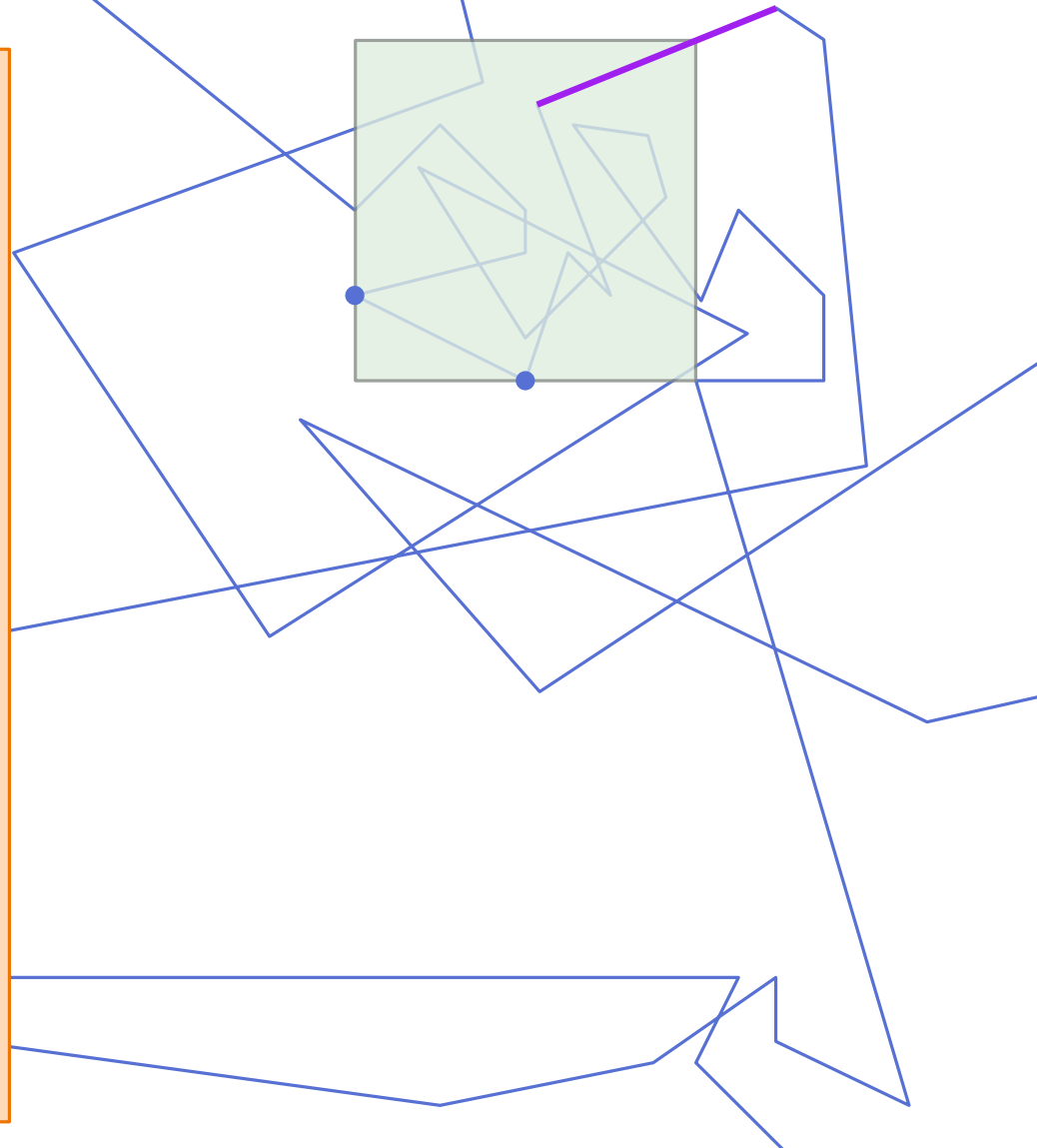


Relative Length

Lemma 4. *There is an optimal $\mathcal{H}$ bounded by 3 objects.*

3 vertices on $\partial\mathcal{H}$.

2 vertices on $\partial\mathcal{H}$ + 1 edge through a corner of $\mathcal{H}$.



Relative Length

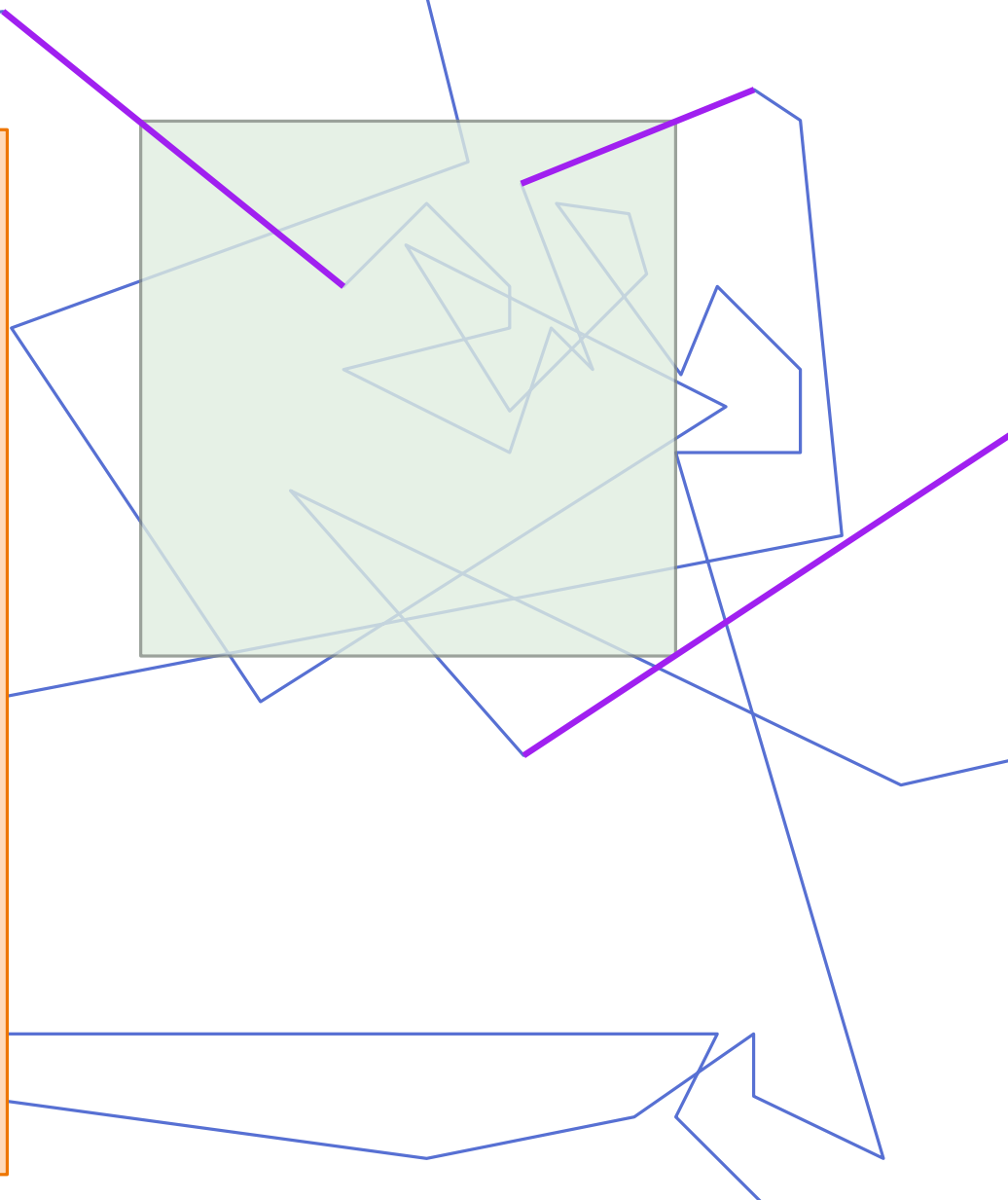
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...

3 edges through corners of $\mathcal{H}$.



Relative Length

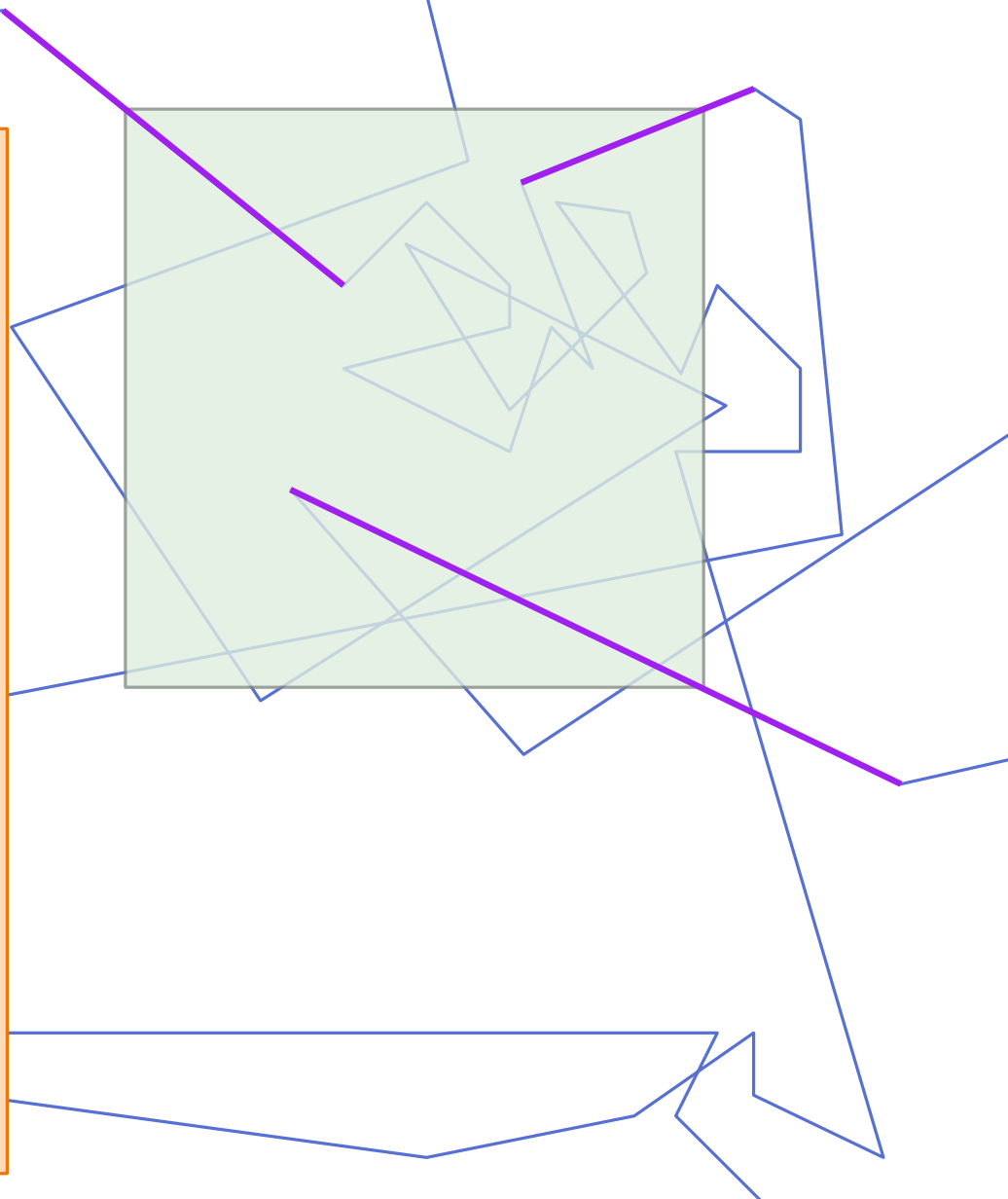
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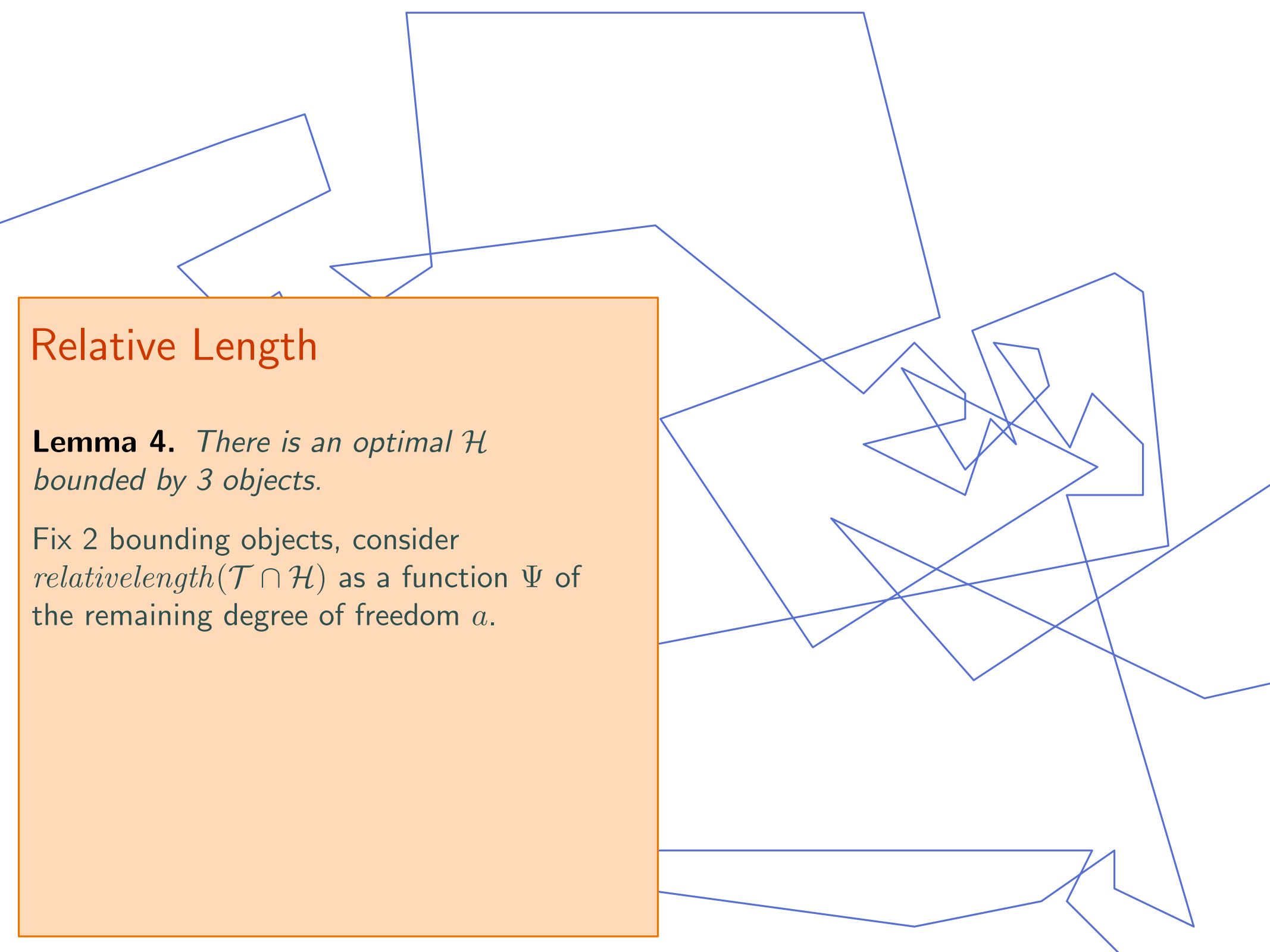
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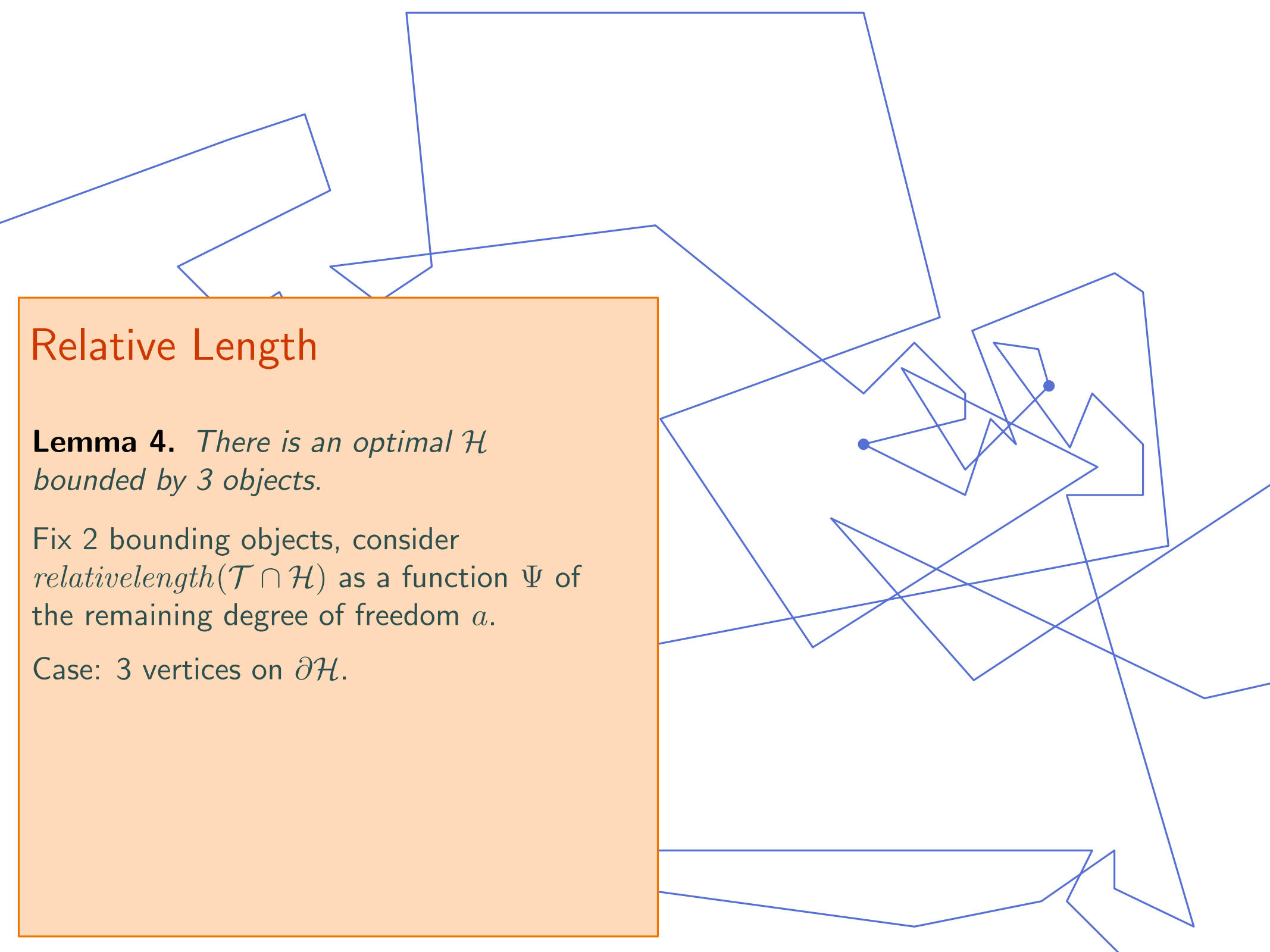


The background of the slide features a complex, abstract geometric pattern composed of numerous thin, blue, irregular lines. These lines form a variety of shapes, including triangles, quadrilaterals, and larger, more complex polygons, some of which are nested or overlapping. The lines are scattered across the white background, creating a sense of dynamic movement and geometric complexity. In the lower-left quadrant, there is a prominent orange rectangular box with a thin orange border. This box contains the title 'Relative Length' in a bold, red, sans-serif font, followed by a lemma statement in a black, italicized, serif font, and a paragraph of text in a black, serif font. The text is left-aligned within the box.

Relative Length

Lemma 4. *There is an optimal $\mathcal{H}$ bounded by 3 objects.*

Fix 2 bounding objects, consider $relativelength(\mathcal{T} \cap \mathcal{H})$ as a function Ψ of the remaining degree of freedom a .

The background of the slide features a complex geometric diagram composed of several blue line segments. These segments form various polygons and intersect to create a web-like structure. Two specific points are highlighted with solid blue dots: one is located in the upper-middle section, and the other is in the lower-right section. The lines vary in length and orientation, creating a non-regular, abstract shape.

Relative Length

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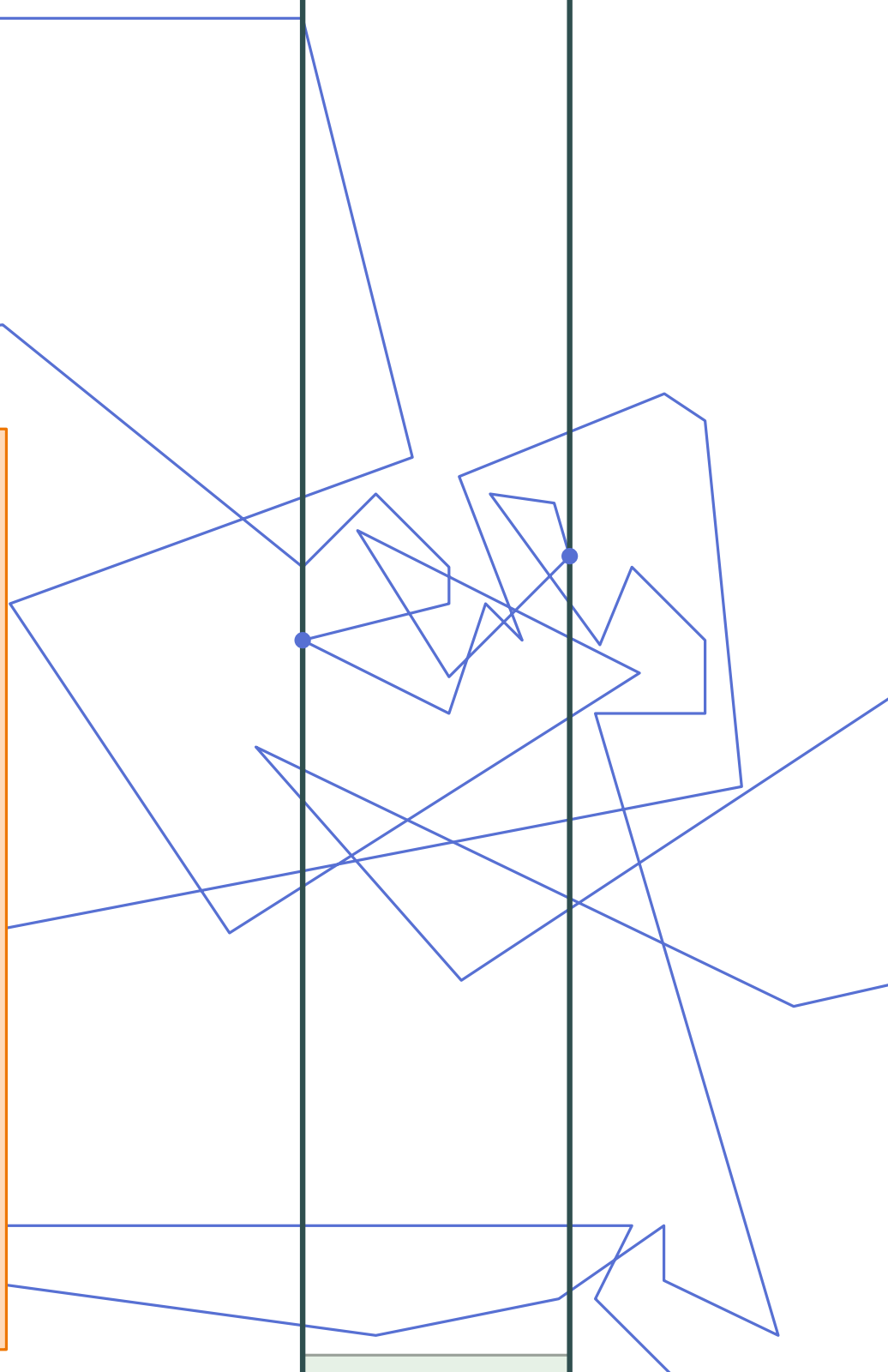
Case: 3 vertices on $\partial\mathcal{H}$.

Relative Length

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Case: 3 vertices on $\partial\mathcal{H}$.

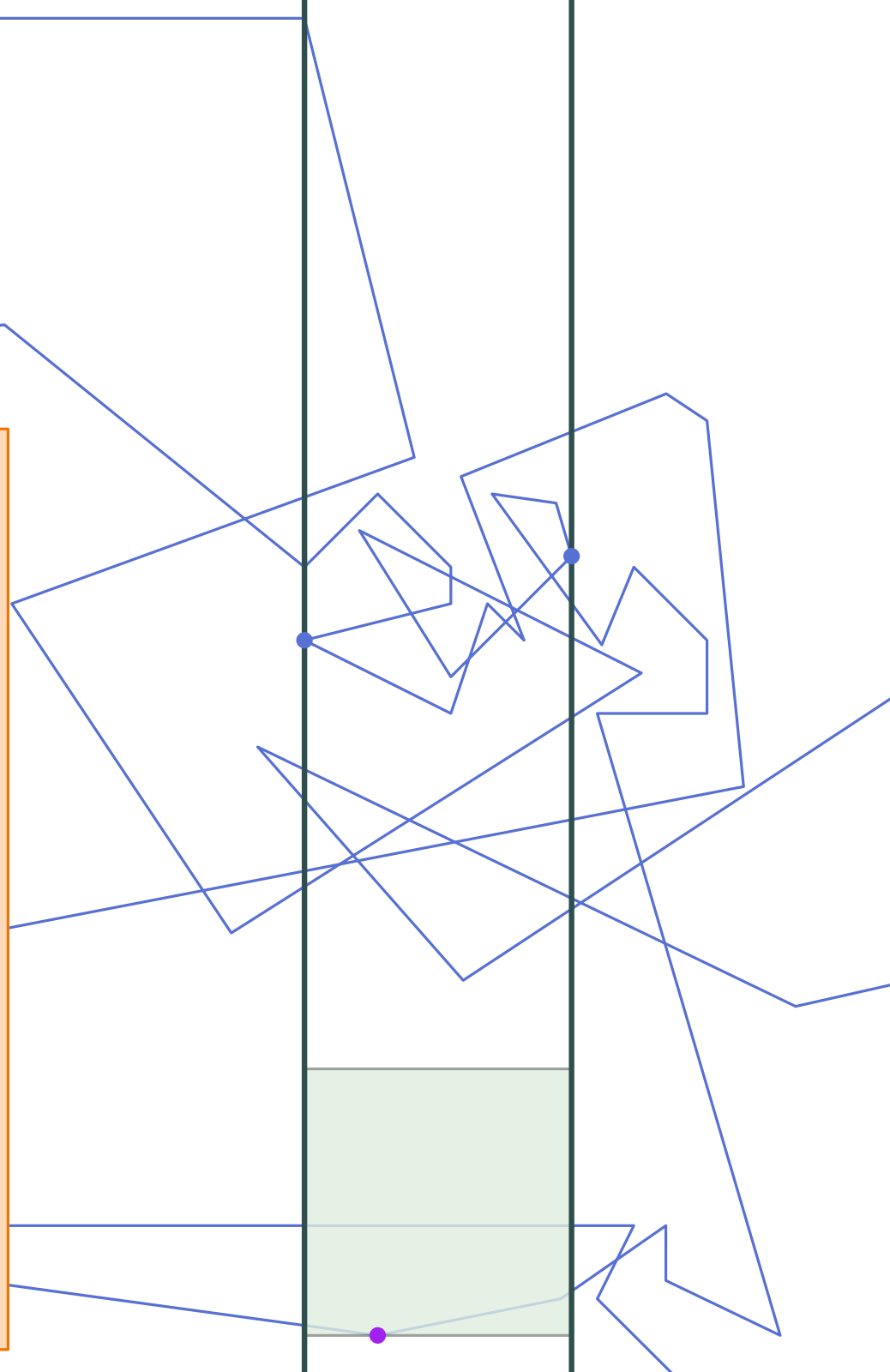


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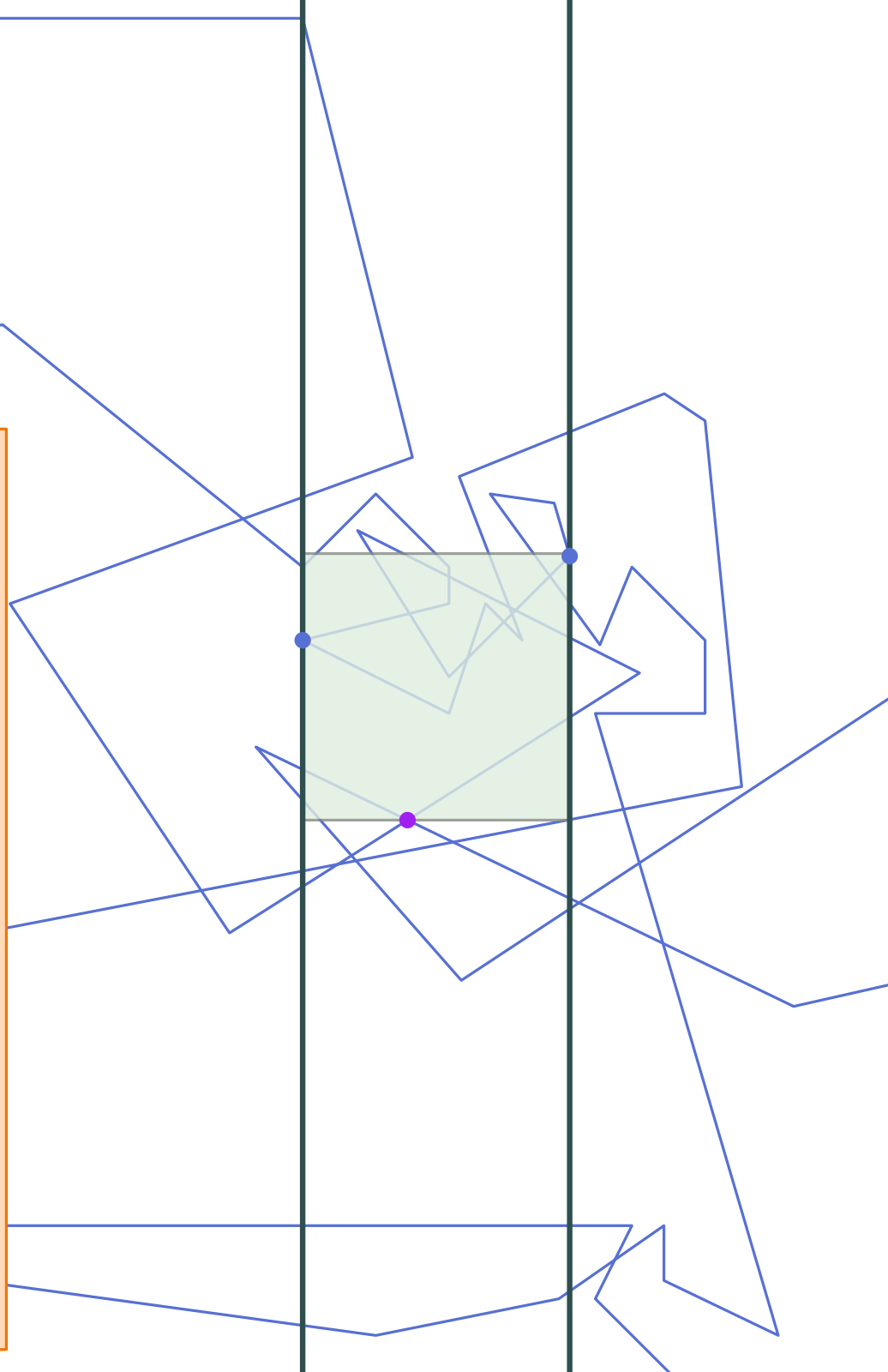


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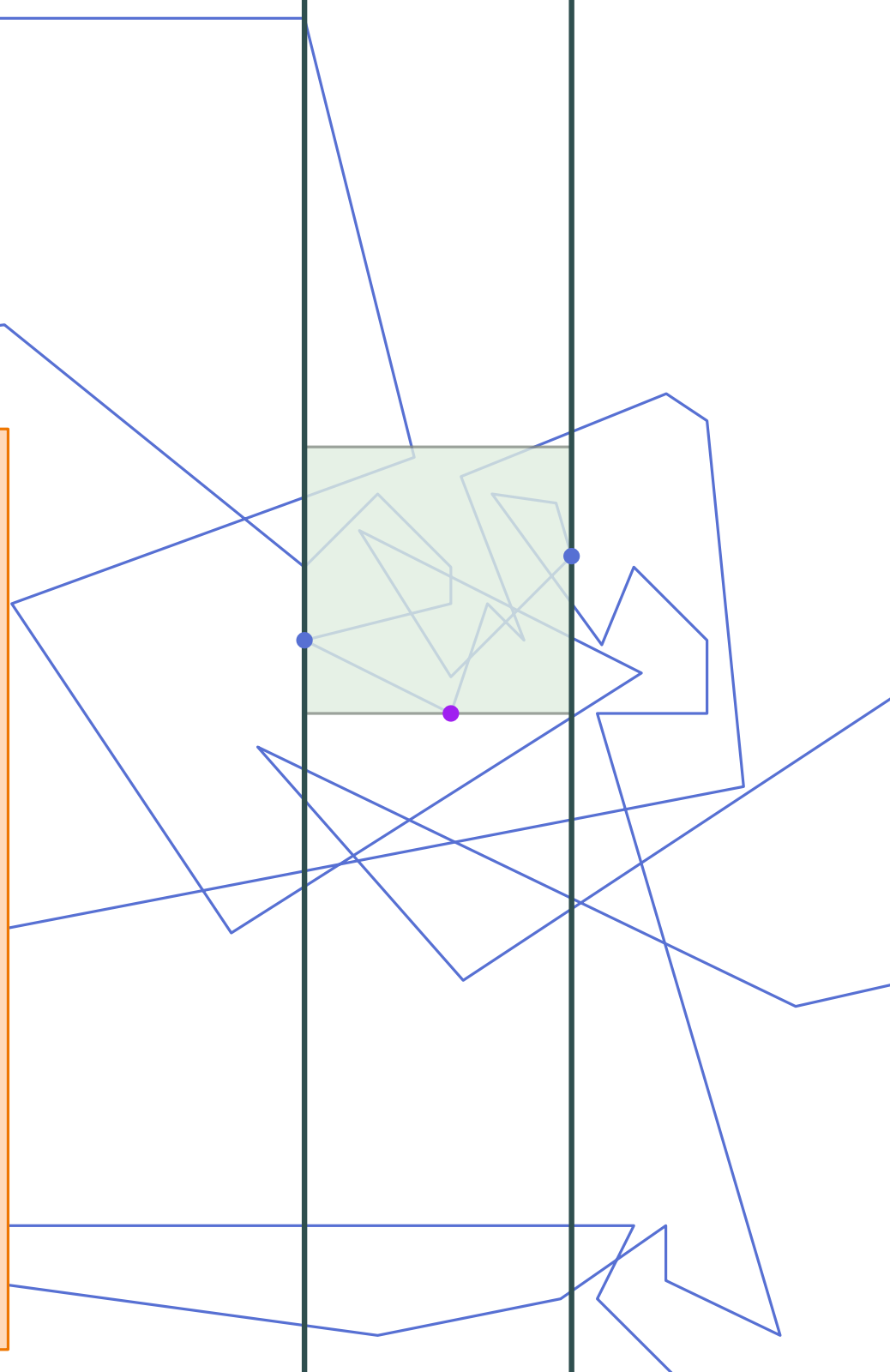


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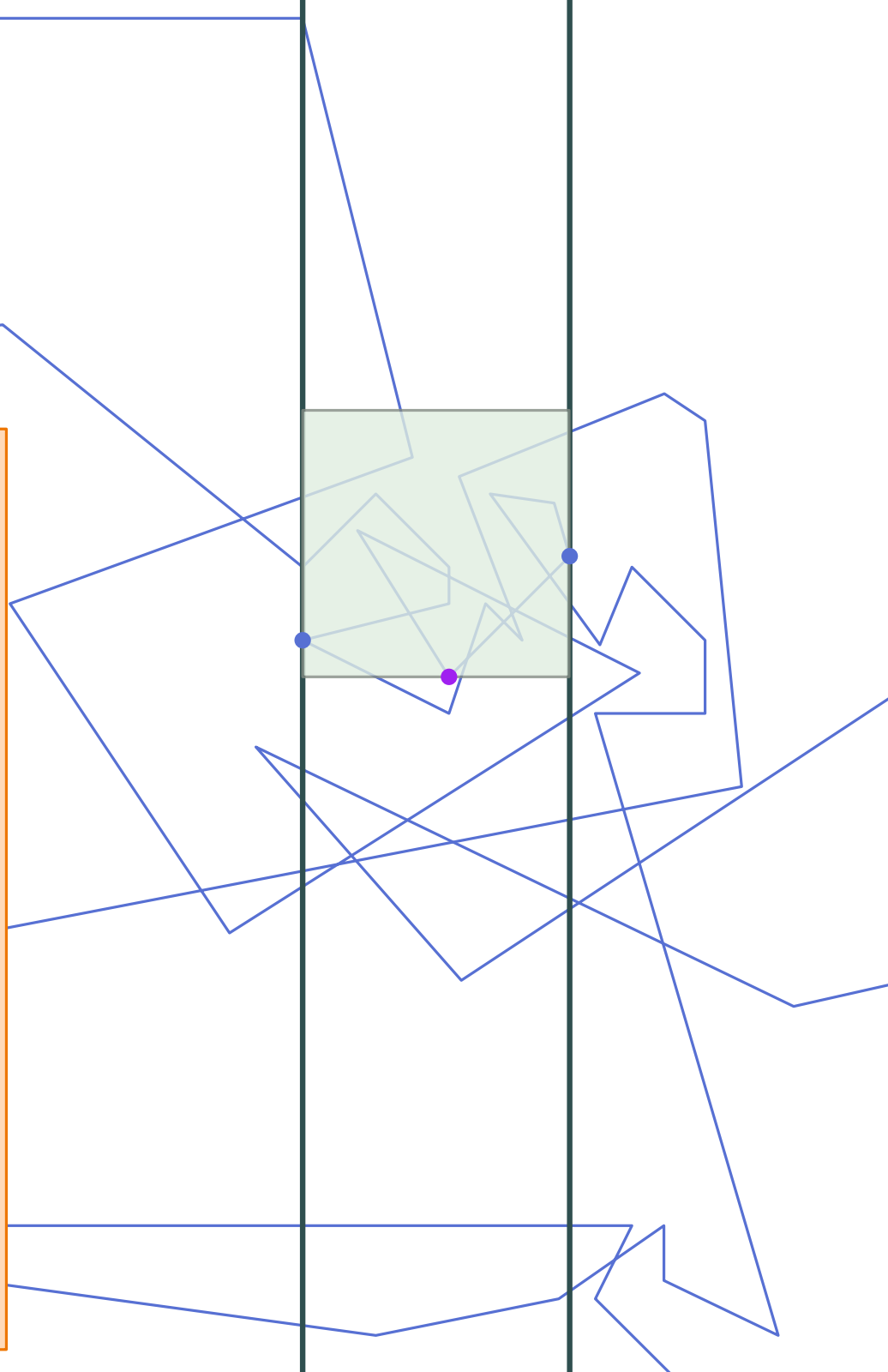


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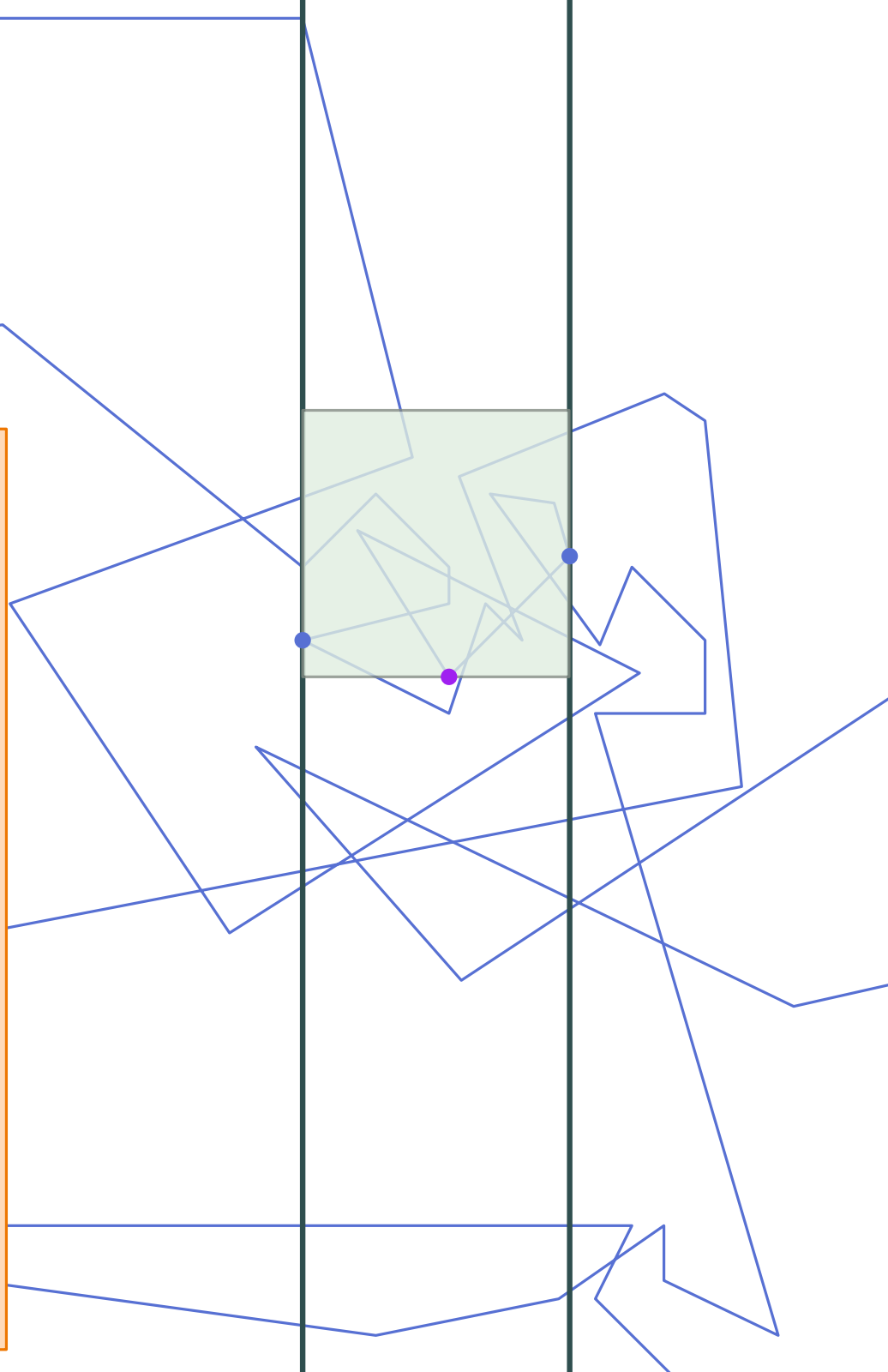
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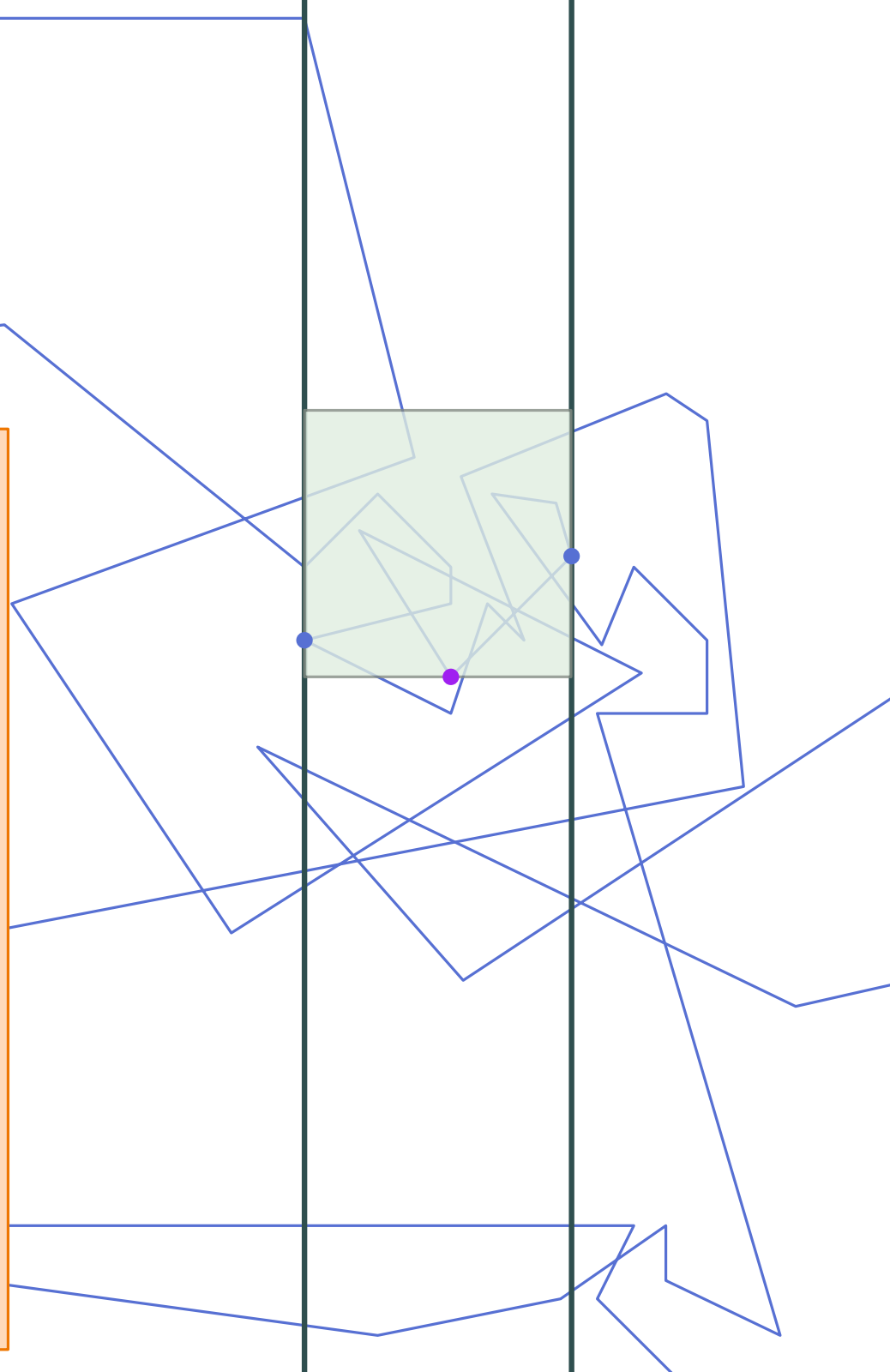
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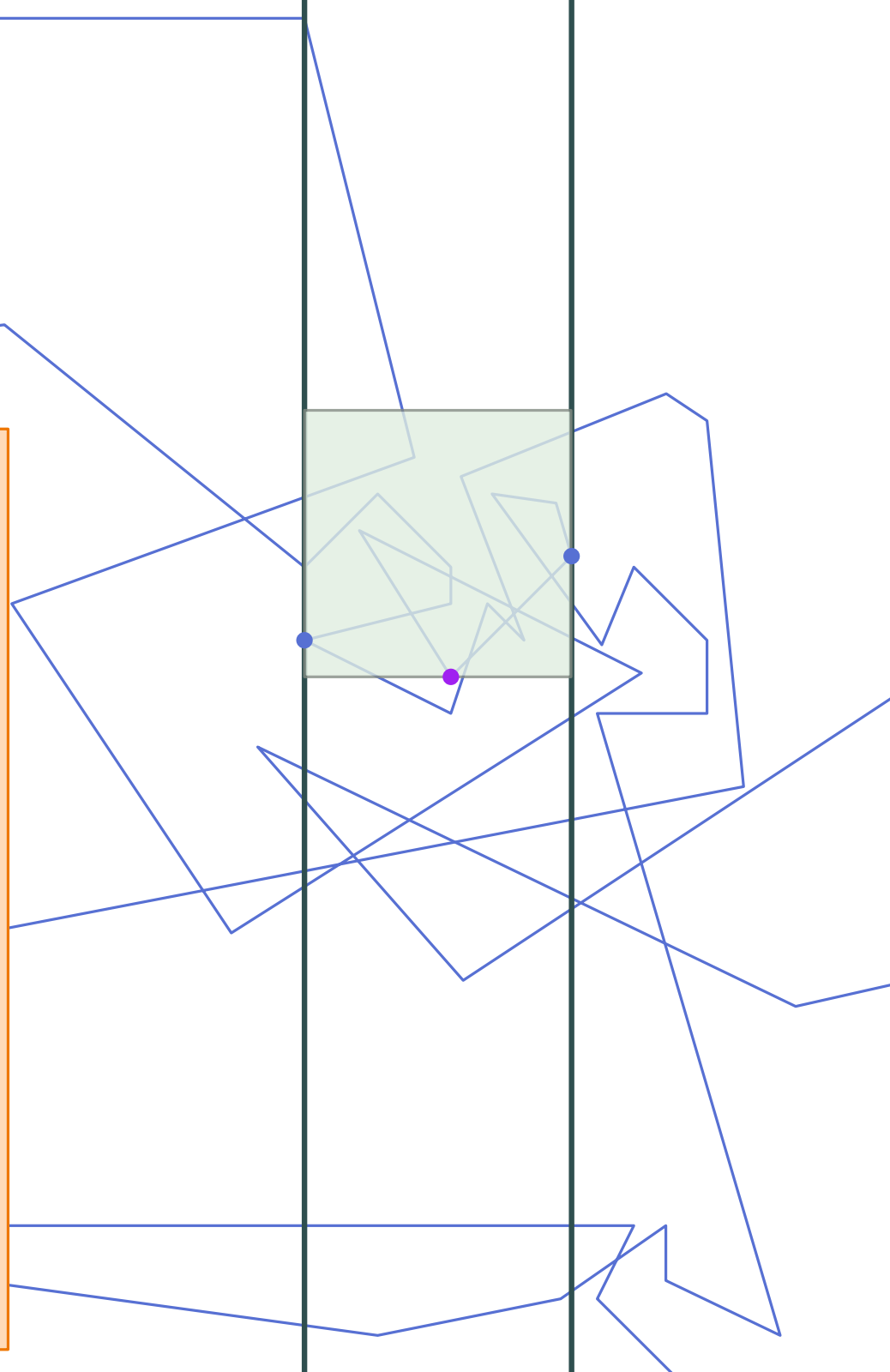
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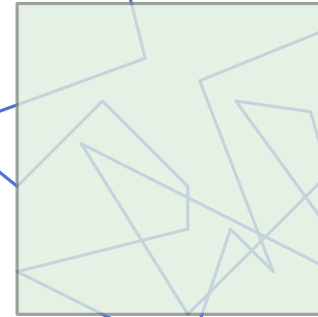
Same for the other cases.

So $O(n^3)$ time to find a $\mathcal{H}$ that maximizes $relativelength(\mathcal{T} \cap \mathcal{H})$.



Hotspot Shapes

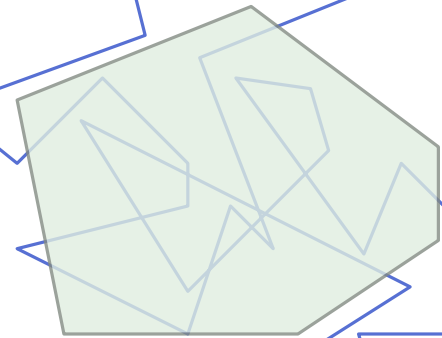
Can we handle hotspots of a different shape?



Hotspot Shapes

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$\mathcal{H}$ is a convex polygon of given shape: yes

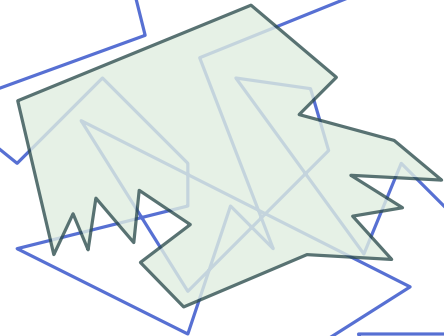


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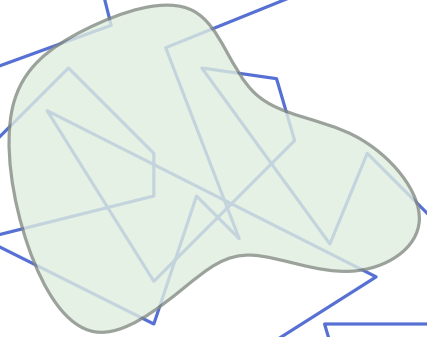
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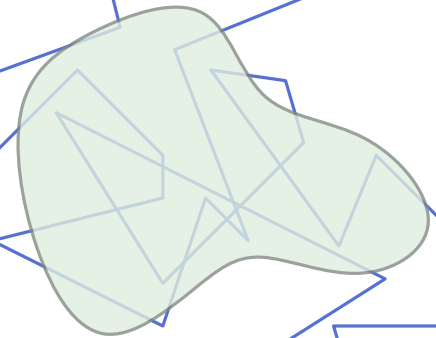
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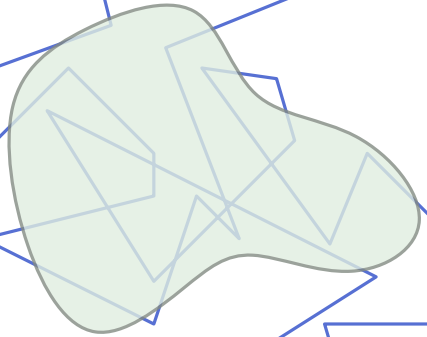
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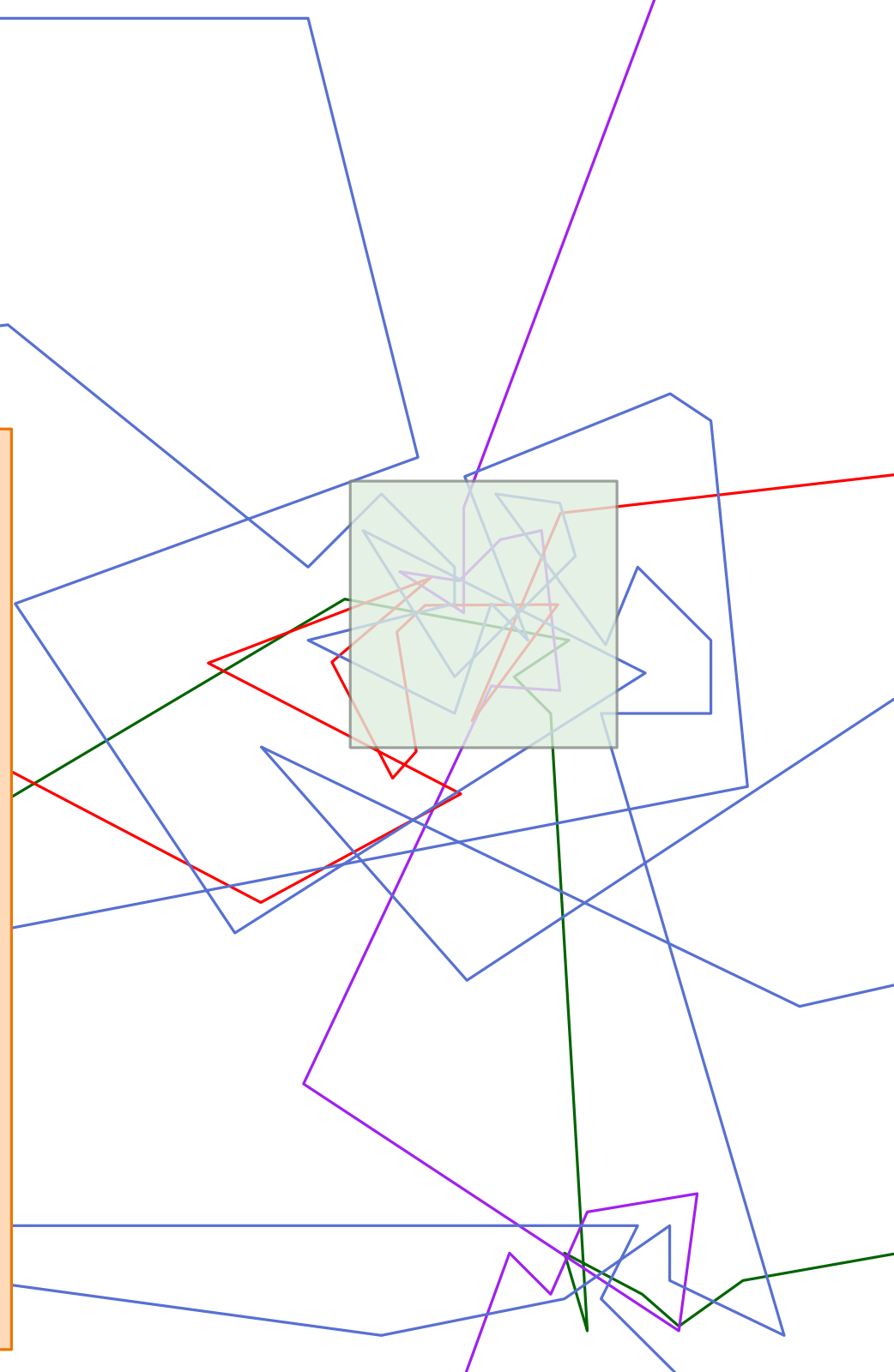
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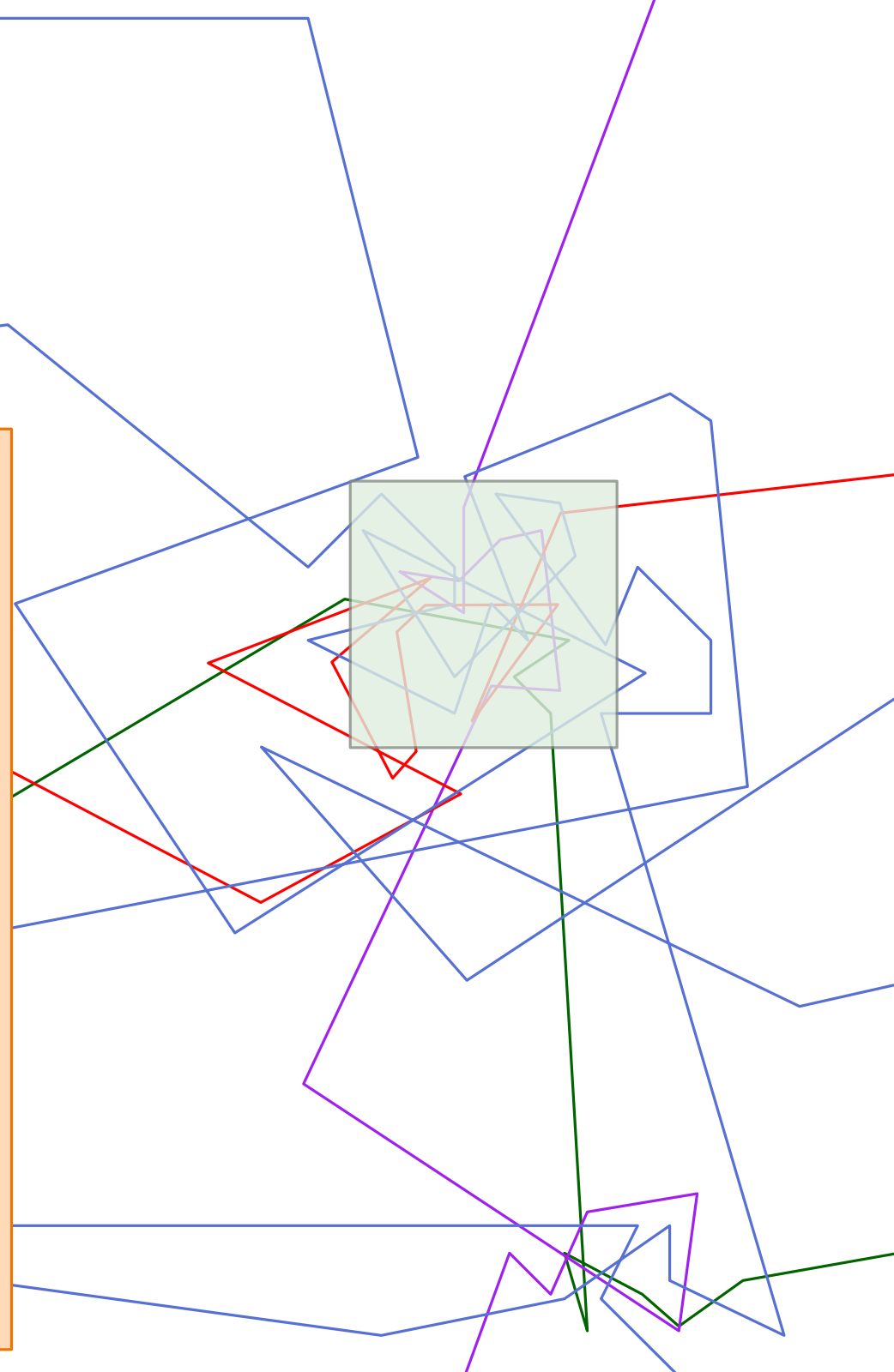
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More variations for multiple entities:

Find a smallest hotspot s.t. **all** entities spend at least L time in $\mathcal{H}$.



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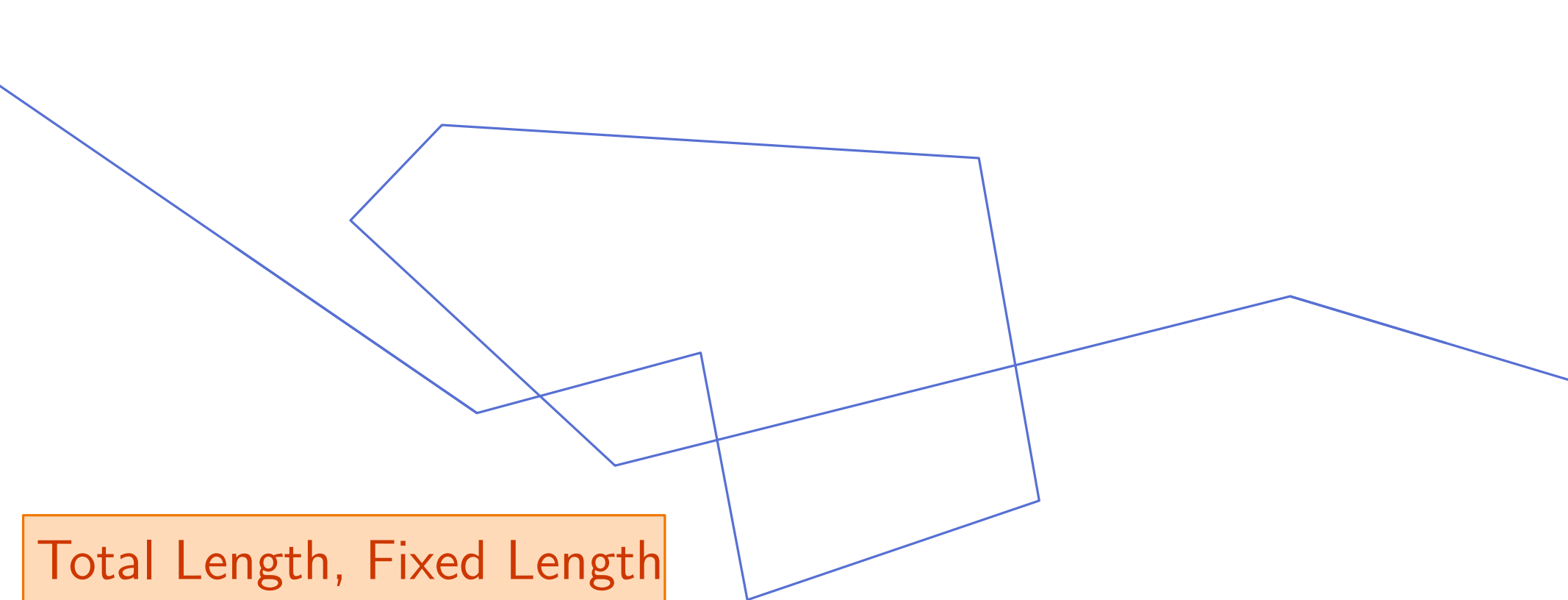
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Thank you!



Total Length, Fixed Length

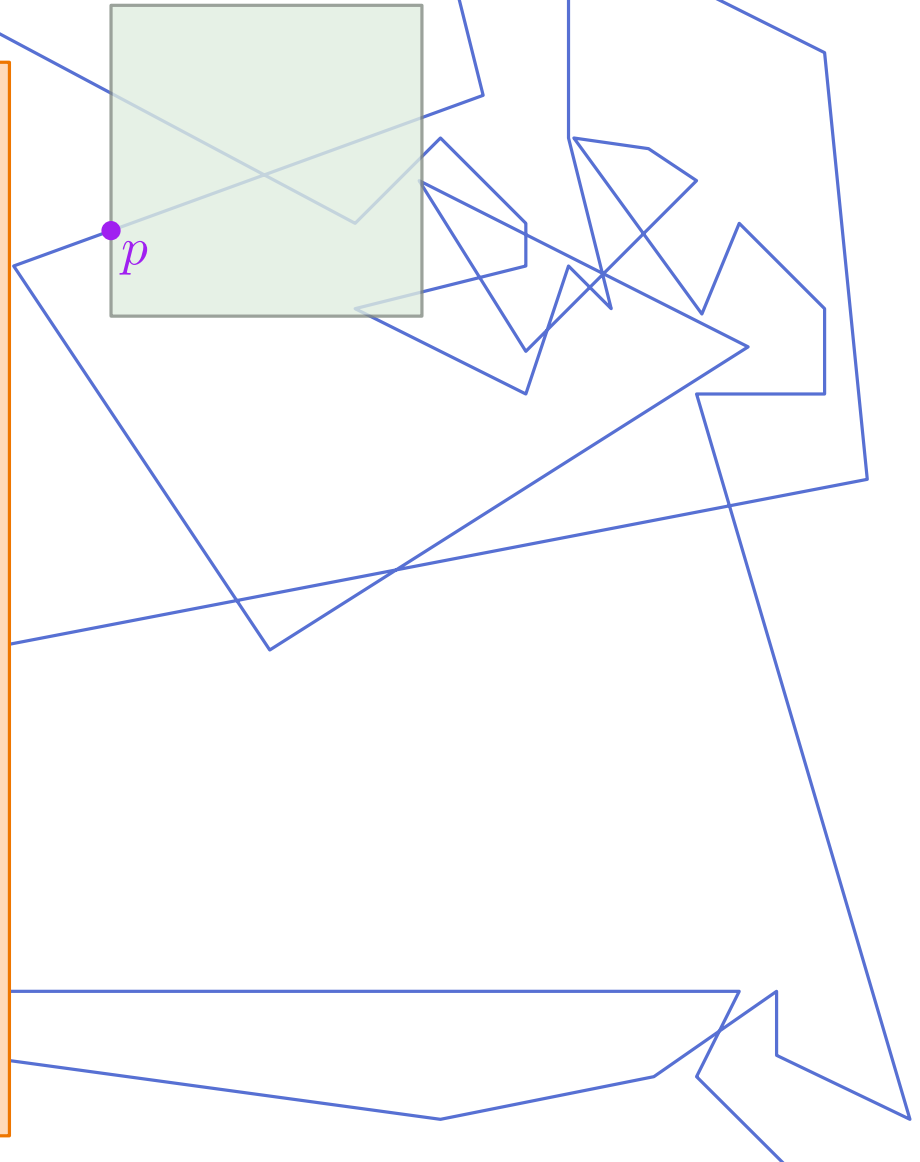
Goal: minimize the side length of $\mathcal{H}$ for a fixed trajectory length L .

Use parametric search, using the Fixed-Size algorithm as a decision algorithm.

Running time: $O(n^2 \log^2 n)$.

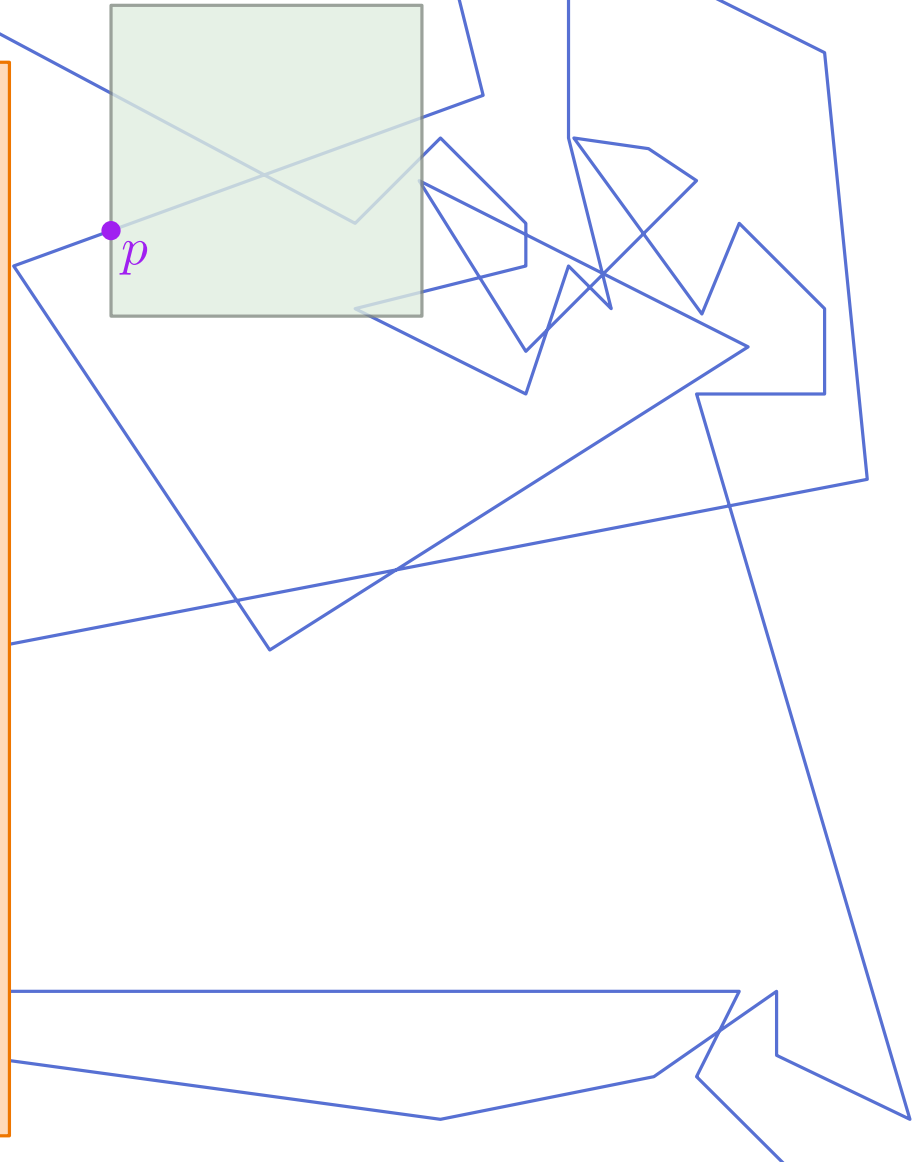
Contiguous Length, Fixed Length

Find $\mathcal{H}$ by finding $\mathcal{T}[p, q]$.



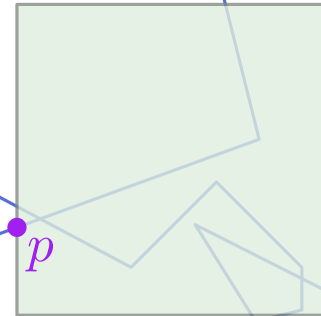
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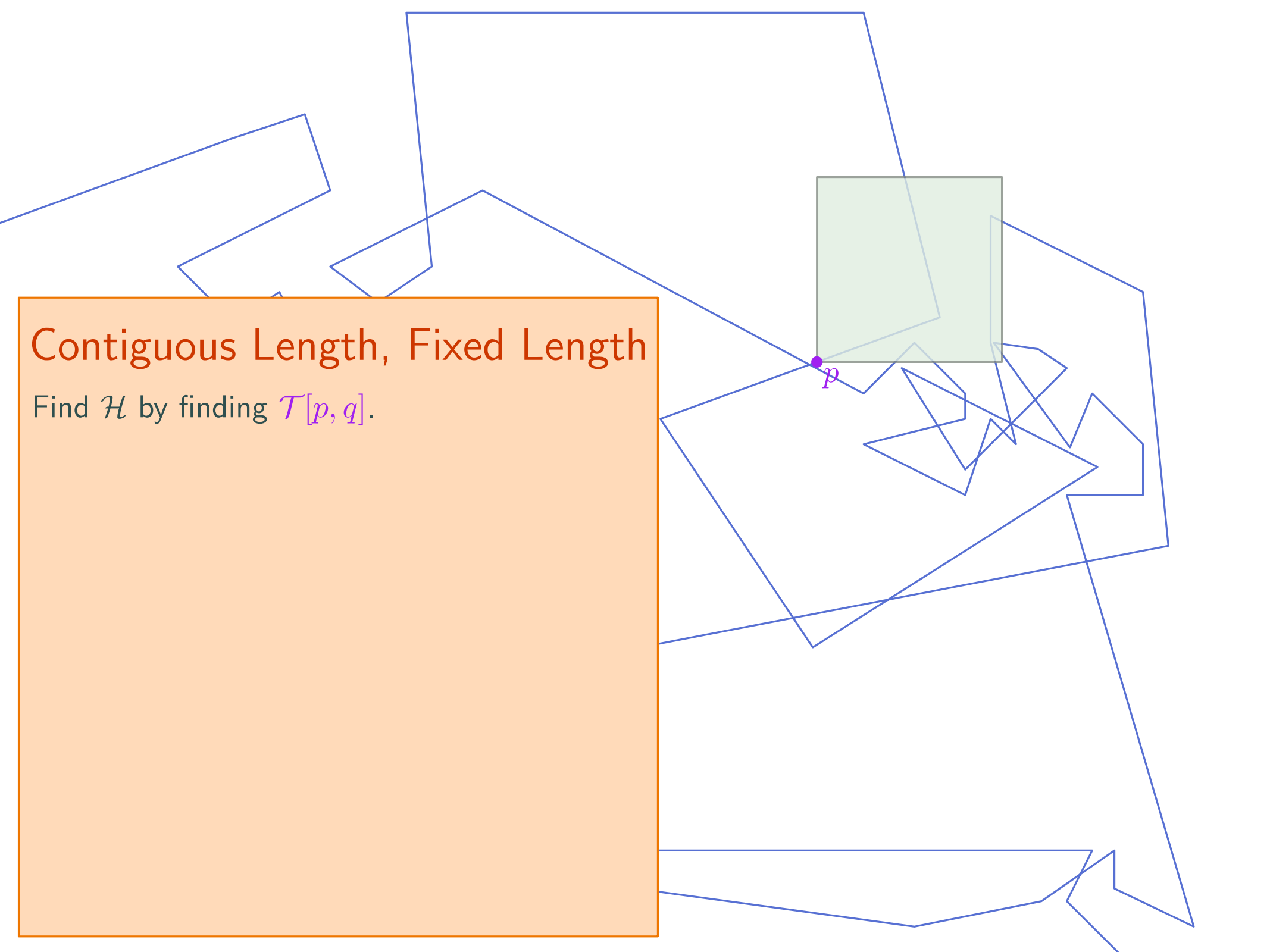
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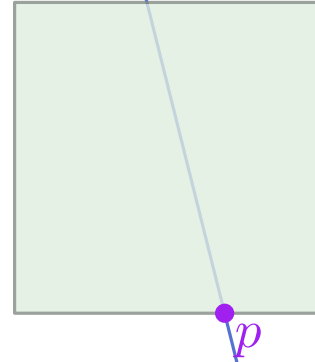


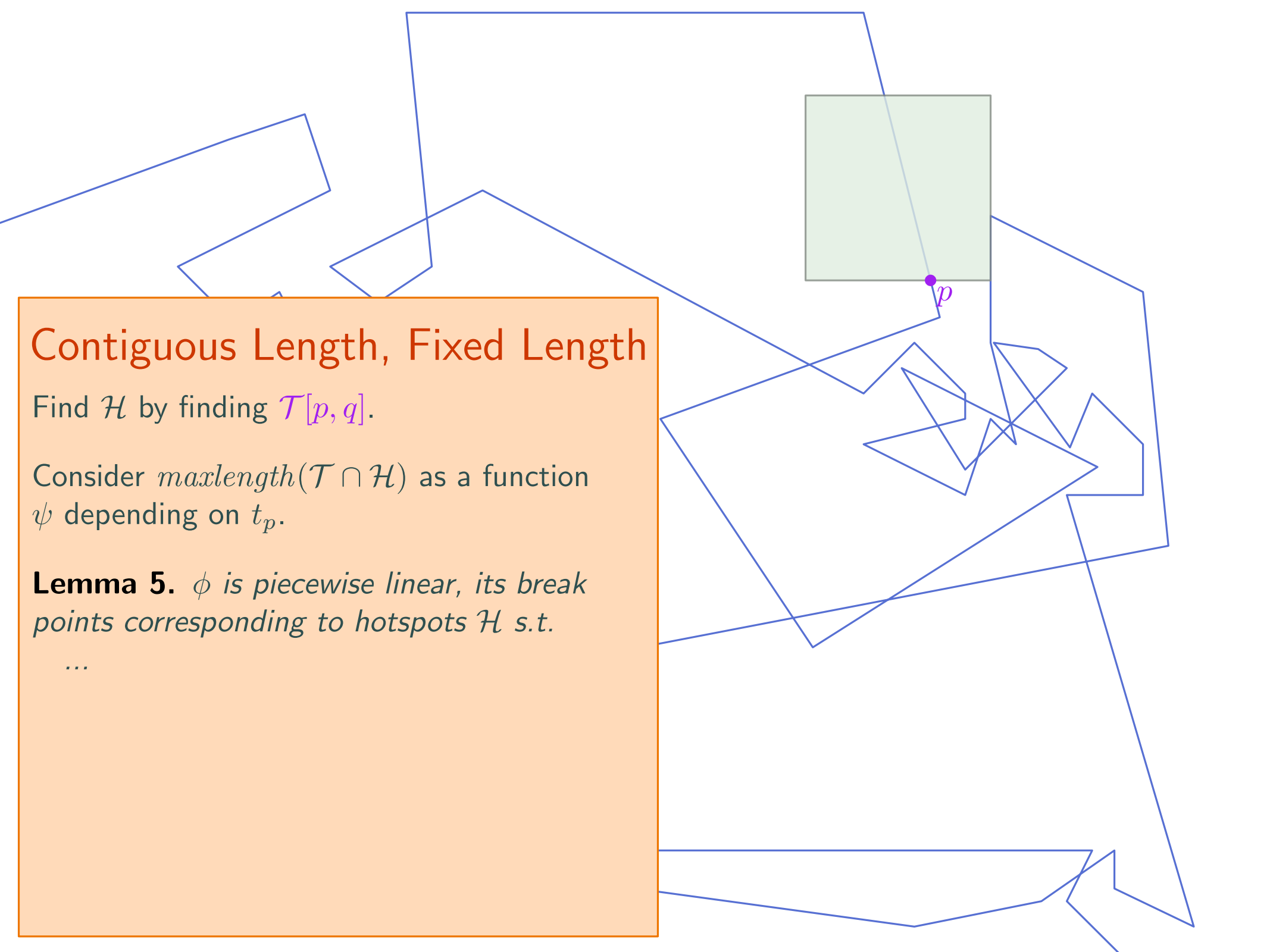
A diagram illustrating a geometric problem. It features a complex, self-intersecting polygon outlined in blue. A specific point, labeled p in purple, is located on one of the polygon's edges. A light green square is positioned such that its bottom-left corner is exactly at point p . The square is oriented horizontally and vertically. The background is white, and the overall image appears to be a slide from a presentation.

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Consider $\text{maxlength}(\mathcal{T} \cap \mathcal{H})$ as a function ψ depending on t_p .





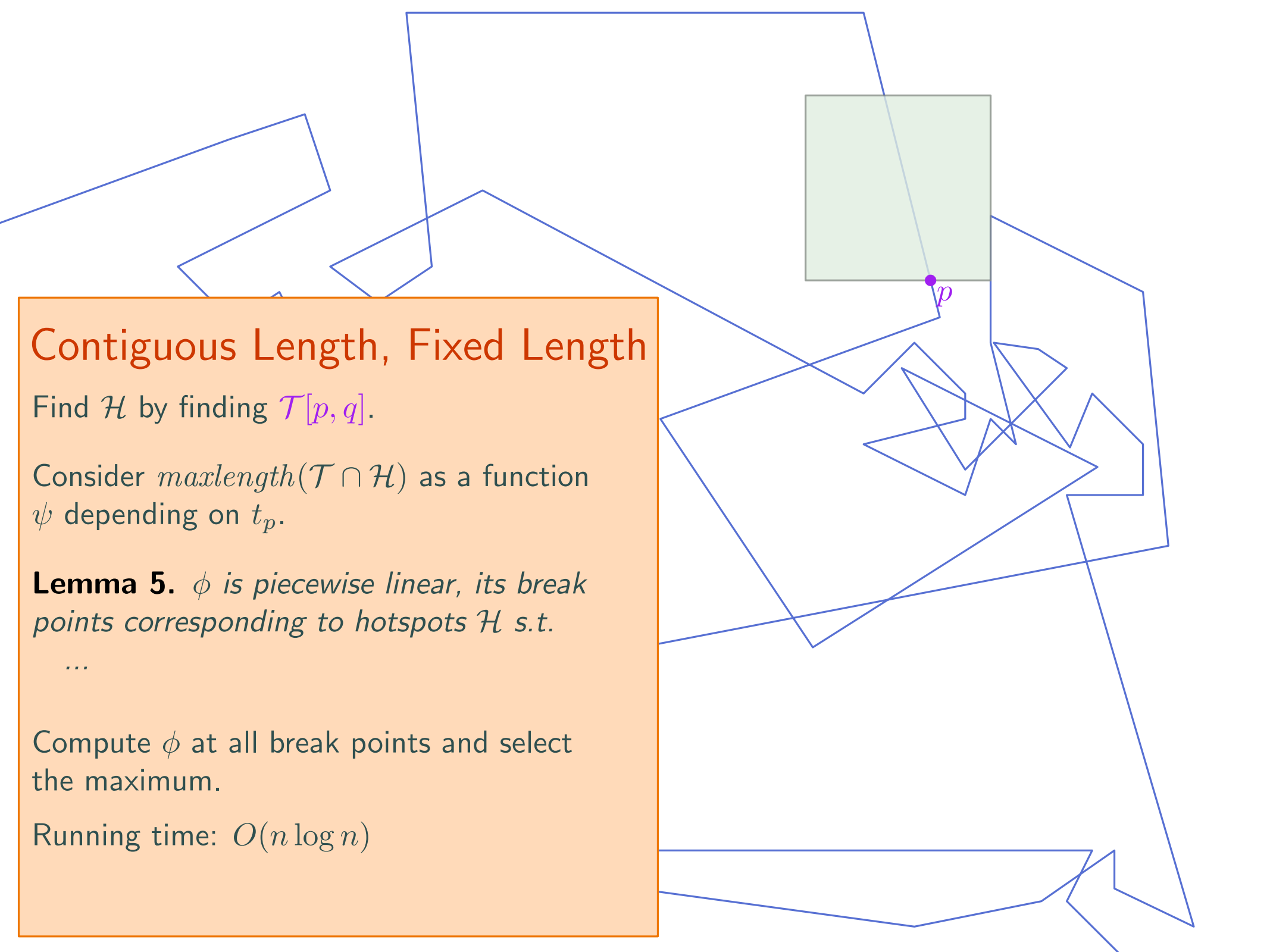
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Compute ϕ at all break points and select the maximum.

Running time: $O(n \log n)$