Algorithms for Hotspot Computation on Trajectory Data

Joachim Gudmundsson
Marc van Kreveld
Frank Staals

University of Sydney
Utrecht University
Problem Statement

Given a trajectory $T$ of a moving entity.

Find a small region where the entity spends a large amount of time:
Problem Statement

Given a trajectory $\mathcal{T}$ of a moving entity.

Find a small region where the entity spends a large amount of time: a hotspot $\mathcal{H}$.  

Problem Statement

Given a trajectory $\mathcal{T}$ of a moving entity.

Find a small square where the entity spends a large amount of time: a hotspot $\mathcal{H}$. 
Problem Statement

Given a trajectory $\mathcal{T}$ of a moving entity.

Find a small square where the entity spends a large amount of time: a hotspot $\mathcal{H}$.

$\text{time} \approx \text{length}$
Problem Statement

Given a trajectory $\mathcal{T}$ of a moving entity.

Find a small square containing a lot of trajectory length: a hotspot $\mathcal{H}$.

\[ \text{time} \approx \text{length} \]
Problem Statement 1

Given a trajectory $\mathcal{T}$ of a moving entity and a square $\mathcal{H}$.

Find a placement of $\mathcal{H}$ maximizing

$$\text{length}(\mathcal{T} \cap \mathcal{H})$$
Problem Statement 2

Given a trajectory $\mathcal{T}$ of a moving entity and a length $L$

Find a placement of $\mathcal{H}$ minimizing $\text{size}(\mathcal{H})$, s.t.

$$\text{length}(\mathcal{T} \cap \mathcal{H}) \geq L$$
Given a trajectory $\mathcal{T}$ of a moving entity and a length $L$

Find a placement of $\mathcal{H}$ minimizing $\text{size}(\mathcal{H})$, s.t. $\text{length}(\mathcal{T} \cap \mathcal{H}) \geq L$
Problem Statement 3

Given a trajectory $\mathcal{T}$ of a moving entity and a length $L$

Find a placement of $\mathcal{H}$ minimizing $\text{size}(\mathcal{H})$, s.t.

$$\text{cont.length}(\mathcal{T} \cap \mathcal{H}) \geq L$$
Problem Statement 4

Given a trajectory $\mathcal{T}$ of a moving entity and a square $\mathcal{H}$.

Find a placement of $\mathcal{H}$ maximizing

$$\text{cont.length}(\mathcal{T} \cap \mathcal{H})$$
Problem Statement 5

Given a trajectory $\mathcal{T}$ of a moving entity.

Find a placement of $\mathcal{H}$ maximizing

$$\frac{\text{length}(\mathcal{T} \cap \mathcal{H})}{\text{sidelength}(\mathcal{H})}$$
Problem Statement 6

Given a trajectory $\mathcal{T}$ of a moving entity.

Find a placement of $\mathcal{H}$ maximizing

$$\frac{cont.\ length(\mathcal{T} \cap \mathcal{H})}{\text{sidelength}(\mathcal{H})}$$
## Results

<table>
<thead>
<tr>
<th></th>
<th>Fixed Size</th>
<th>Fixed Length</th>
<th>Relative</th>
</tr>
</thead>
<tbody>
<tr>
<td>length</td>
<td>$O(n^2)$</td>
<td>$O(n^2 \log^2 n)$</td>
<td>$O(n^3)$</td>
</tr>
<tr>
<td>cont.length</td>
<td>$O(n \log n)$</td>
<td>$O(n \log n)$</td>
<td>-</td>
</tr>
</tbody>
</table>
### Results

<table>
<thead>
<tr>
<th></th>
<th>Fixed Size</th>
<th>Fixed Length</th>
<th>Relative</th>
</tr>
</thead>
<tbody>
<tr>
<td>length</td>
<td>$O(n^2)$</td>
<td>$O(n^2 \log^2 n)$</td>
<td>$O(n^3)$</td>
</tr>
<tr>
<td>cont. length</td>
<td>$O(n \log n)$</td>
<td>$O(n \log n)$</td>
<td>-</td>
</tr>
</tbody>
</table>

Our algorithms also work for multiple trajectories,
Results

<table>
<thead>
<tr>
<th></th>
<th>Fixed Size</th>
<th>Fixed Length</th>
<th>Relative</th>
</tr>
</thead>
<tbody>
<tr>
<td>length</td>
<td>$O(n^2)$</td>
<td>$O(n^2 \log^2 n)$</td>
<td>$O(n^3)$</td>
</tr>
<tr>
<td>cont.length</td>
<td>$O(n \log n)$</td>
<td>$O(n \log n)$</td>
<td>-</td>
</tr>
</tbody>
</table>

Our algorithms also work for multiple trajectories, and for weighted edges.
Applications

- Finding places
Applications

- Finding places
- Segmentation
Applications
  • Finding places
  • Segmentation
  • Clustering
Applications

- Finding places
- Segmentation
- Clustering
- Visualization
Applications

- Finding places
- Segmentation
- Clustering
- Visualization
## Results

<table>
<thead>
<tr>
<th></th>
<th>Fixed Size</th>
<th>Fixed Length</th>
<th>Relative</th>
</tr>
</thead>
<tbody>
<tr>
<td>length</td>
<td>$\mathcal{O}(n^2)$</td>
<td>$\mathcal{O}(n^2 \log^2 n)$</td>
<td>$\mathcal{O}(n^3)$</td>
</tr>
<tr>
<td>maxlen</td>
<td>$\mathcal{O}(n \log n)$</td>
<td>$\mathcal{O}(n \log n)$</td>
<td>-</td>
</tr>
</tbody>
</table>
Total Length, Fixed Size

Parameterize $\Upsilon(c) = \text{length}(\mathcal{T} \cap \mathcal{H})$ by the center $c$ of $\mathcal{H}$. 
Lemma 1. \( \Upsilon \) is piecewise linear.
Consider the subdivision $A$ of the parameter space of $\Upsilon$.

Parameterize $\Upsilon(c) = \text{length}(\mathcal{T} \cap \mathcal{H})$ by the center $c$ of $\mathcal{H}$.

**Lemma 1.** $\Upsilon$ is piecewise linear.

Consider the subdivision $A$ of the parameter space of $\Upsilon$. 

**Total Length, Fixed Size**

Parameterize $\Upsilon(c) = \text{length}(\mathcal{T} \cap \mathcal{H})$ by the center $c$ of $\mathcal{H}$.

**Lemma 1.** $\Upsilon$ is piecewise linear.

Consider the subdivision $A$ of the parameter space of $\Upsilon$. 

Consider the subdivision $A$ of the parameter space of $\Upsilon$. Parameterize $\Upsilon(c) = \text{length}(T \cap H)$ by the center $c$ of $H$. 

**Lemma 1.** $\Upsilon$ is piecewise linear.

Consider the subdivision $A$ of the parameter space of $\Upsilon$. $\max \Upsilon$ occurs at a vertex of $A$. So, compute $\Upsilon$ at each vertex of $A$. 

**Total Length, Fixed Size**

Parameterize $\Upsilon(c) = \text{length}(T \cap H)$ by the center $c$ of $H$. 

**Lemma 1.** $\Upsilon$ is piecewise linear.
Consider the subdivision $\mathcal{A}$ of the parameter space of $\Upsilon$. Parameterize $\Upsilon(c) = \text{length}(T \cap H)$ by the center $c$ of $H$.

$max \ Upsilon$ occurs at a vertex of $\mathcal{A}$. So, compute $\Upsilon$ at each vertex of $\mathcal{A}$.

**Lemma 1.** $\Upsilon$ is piecewise linear.

Total Length, Fixed Size

Parameterize $\Upsilon(c) = \text{length}(T \cap H)$ by the center $c$ of $H$.

**Lemma 1.** $\Upsilon$ is piecewise linear.

Consider the subdivision $\mathcal{A}$ of the parameter space of $\Upsilon$.

$max \ Upsilon$ occurs at a vertex of $\mathcal{A}$. So, compute $\Upsilon$ at each vertex of $\mathcal{A}$.

**Complexity $\mathcal{A}$:** $O(n^2)$

**Find max $\Upsilon$:** $O(n^2)$

**Total:** $O(n^2)$
Contiguous Length, Fixed Size

Find $\mathcal{H}$ by finding $\mathcal{T}[p, q]$. 

Contiguous Length, Fixed Size

Find $\mathcal{H}$ by finding $\mathcal{T}[p, q]$.

**Lemma 2.**  There is an optimal hotspot $\mathcal{H}$ s.t. a vertex $v$ of $\mathcal{T}$ lies on $\partial \mathcal{H}$. 
Contiguous Length, Fixed Size

Find $\mathcal{H}$ by finding $T[p, q]$.

**Lemma 2.** There is an optimal hotspot $\mathcal{H}$ s.t. a vertex $v$ of $T[p, q]$ lies on $\partial \mathcal{H}$. 
Contiguous Length, Fixed Size

Find $\mathcal{H}$ by finding $T[p, q]$.

**Lemma 2.** There is an optimal hotspot $\mathcal{H}$ s.t. a vertex $v$ of $T[p, q]$ lies on $\partial \mathcal{H}$.

**Corollary 3.** Starting point $p$ on one of the horizontal or vertical lines.
Contiguous Length, Fixed Size

Find $\mathcal{H}$ by finding $\mathcal{T}[p, q]$.

**Lemma 2.** There is an optimal hotspot $\mathcal{H}$ s.t. a vertex $v$ of $\mathcal{T}[p, q]$ lies on $\partial \mathcal{H}$.

**Corollary 3.** Starting point $p$ on one of the horizontal or vertical lines.

How to find $p$ and $q$?
Contiguous Length, Fixed Size

Find $\mathcal{H}$ by finding $\mathcal{T}[p, q]$.

**Lemma 2.** There is an optimal hotspot $\mathcal{H}$ s.t. a vertex $v$ of $\mathcal{T}[p, q]$ lies on $\partial \mathcal{H}$.

**Corollary 3.** Starting point $p$ on one of the horizontal or vertical lines.

How to find $p$ and $q$?
Consider $\mathcal{T}_x$ and $\mathcal{T}_y$ separately.
Lemma 2. There is an optimal hotspot $H$ s.t. a vertex $v$ of $T[p, q]$ lies on $\partial H$.

Corollary 3. Starting point $p$ on one of the horizontal or vertical lines.

How to find $p$ and $q$?

Consider $T_x$ and $T_y$ separately.

Try to find $[t_p, t_q]$.
Lemma 2. There is an optimal hotspot $\mathcal{H}$ s.t. a vertex $v$ of $\mathcal{T}_{[p,q]}$ lies on $\partial \mathcal{H}$.

Corollary 3. Starting point $p$ on one of the horizontal or vertical lines.

How to find $p$ and $q$?
Consider $\mathcal{T}_x$ and $\mathcal{T}_y$ separately.
Try to find $[t_p, t_q]$.
Use ray shooting queries.
There is an optimal hotspot $H$ s.t. a vertex $v$ of $T[p, q]$ lies on $\partial H$.

Starting point $p$ on one of the horizontal or vertical lines.

How to find $p$ and $q$?

Consider $T_x$ and $T_y$ separately.

Try to find $[t_p, t_q]$.

Use ray shooting queries.

Running time: $O(n \log n)$
Relative Length
Lemma 4. There is an optimal $\mathcal{H}$ bounded by 3 objects. 

3 vertices on $\partial \mathcal{H}$. 

Relative Length
Lemma 4. There is an optimal $\mathcal{H}$ bounded by 3 objects.

3 vertices on $\partial \mathcal{H}$.
2 vertices on $\partial \mathcal{H} + 1$ edge through a corner of $\mathcal{H}$. 
Lemma 4. There is an optimal $\mathcal{H}$ bounded by 3 objects.

- 3 vertices on $\partial \mathcal{H}$.
- 2 vertices on $\partial \mathcal{H} + 1$ edge through a corner of $\mathcal{H}$.
- ...
- 3 edges through corners of $\mathcal{H}$. 
Relative Length

Lemma 4. There is an optimal $\mathcal{H}$ bounded by 3 objects.

3 vertices on $\partial \mathcal{H}$.
2 vertices on $\partial \mathcal{H} + 1$ edge through a corner of $\mathcal{H}$.
...
3 edges through corners of $\mathcal{H}$.
Relative Length

Lemma 4. There is an optimal $\mathcal{H}$ bounded by 3 objects.

Fix 2 bounding objects, consider $\text{relative length}(\mathcal{T} \cap \mathcal{H})$ as a function $\Psi$ of the remaining degree of freedom $\alpha$. 
Lemma 4. There is an optimal $\mathcal{H}$ bounded by 3 objects.

Fix 2 bounding objects, consider $\text{relative length}(\mathcal{T} \cap \mathcal{H})$ as a function $\Psi$ of the remaining degree of freedom $a$.

Case: 3 vertices on $\partial \mathcal{H}$. 

Relative Length
Relative Length

**Lemma 4.** There is an optimal $H$ bounded by 3 objects.

Fix 2 bounding objects, consider $\text{relative length}(T \cap H)$ as a function $\Psi$ of the remaining degree of freedom $a$.

Case: 3 vertices on $\partial H$. 
Lemma 4. There is an optimal $\mathcal{H}$ bounded by 3 objects.

Fix 2 bounding objects, consider $\text{relative length}(\mathcal{T} \cap \mathcal{H})$ as a function $\Psi$ of the remaining degree of freedom $a$.

Case: 3 vertices on $\partial \mathcal{H}$. 

Relative Length
**Lemma 4.** There is an optimal $\mathcal{H}$ bounded by 3 objects.

Fix 2 bounding objects, consider $\text{relative length}(T \cap \mathcal{H})$ as a function $\Psi$ of the remaining degree of freedom $a$.

Case: 3 vertices on $\partial \mathcal{H}$.
Lemma 4. There is an optimal $\mathcal{H}$ bounded by 3 objects.

Fix 2 bounding objects, consider $\text{relativelength}(T \cap \mathcal{H})$ as a function $\Psi$ of the remaining degree of freedom $a$.

Case: 3 vertices on $\partial \mathcal{H}$.
Lemma 4. There is an optimal $\mathcal{H}$ bounded by 3 objects.

Fix 2 bounding objects, consider $\text{relativelength}(\mathcal{T} \cap \mathcal{H})$ as a function $\Psi$ of the remaining degree of freedom $a$.

Case: 3 vertices on $\partial \mathcal{H}$. 
Lemma 4. There is an optimal $\mathcal{H}$ bounded by 3 objects.

Fix 2 bounding objects, consider \( \text{relative length}(T \cap \mathcal{H}) \) as a function $\Psi$ of the remaining degree of freedom $a$.

Case: 3 vertices on $\partial \mathcal{H}$.

\( O(n) \) breakpoints/events: \( O(n) \) time.
Lemma 4. There is an optimal $H$ bounded by 3 objects.

Fix 2 bounding objects, consider $\text{relativelength}(T \cap H)$ as a function $\Psi$ of the remaining degree of freedom $a$.

Case: 3 vertices on $\partial H$.

- $O(n)$ breakpoints/events: $O(n)$ time.
- $O(n^2)$ pairs

Total: $O(n^3)$ time.
Lemma 4. There is an optimal $\mathcal{H}$ bounded by 3 objects.

Fix 2 bounding objects, consider $\text{relativelength}(\mathcal{T} \cap \mathcal{H})$ as a function $\Psi$ of the remaining degree of freedom $a$.

Case: 3 vertices on $\partial \mathcal{H}$.

- $O(n)$ breakpoints/events: $O(n)$ time.
- $O(n^2)$ pairs

Total: $O(n^3)$ time.

Same for the other cases.

So $O(n^3)$ time to find a $\mathcal{H}$ that maximizes $\text{relativelength}(\mathcal{T} \cap \mathcal{H})$. 
Hotspot Shapes

Can we handle hotspots of a different shape?
Hotspot Shapes

Can we handle hotspots of a different shape?

$\mathcal{H}$ is a convex polygon of given shape: yes
Hotspot Shapes

Can we handle hotspots of a different shape?

\( \mathcal{H} \) is a convex polygon of given shape: yes
\( \mathcal{H} \) is a polygon of given shape: no
Hotspot Shapes

Can we handle hotspots of a different shape?

\( \mathcal{H} \) is a convex polygon of given shape: yes
\( \mathcal{H} \) is a polygon of given shape: no
\( \mathcal{H} \) has fixed but curved boundaries: no
<table>
<thead>
<tr>
<th>Question</th>
<th>Yes</th>
<th>No</th>
</tr>
</thead>
<tbody>
<tr>
<td>Can we handle hotspots of a different shape?</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\mathcal{H}$ is a convex polygon of given shape:</td>
<td>yes</td>
<td></td>
</tr>
<tr>
<td>$\mathcal{H}$ is a polygon of given shape:</td>
<td>no</td>
<td></td>
</tr>
<tr>
<td>$\mathcal{H}$ has fixed but curved boundaries:</td>
<td>no</td>
<td></td>
</tr>
<tr>
<td>The shape of $\mathcal{H}$ is not predefined:</td>
<td>no</td>
<td></td>
</tr>
</tbody>
</table>
Can we handle hotspots of a different shape?

\( \mathcal{H} \) is a polygon of given shape: no
\( \mathcal{H} \) has fixed but curved boundaries: no
The shape of \( \mathcal{H} \) is not predefined: no
Can we handle hotspots of a different shape?

\(\mathcal{H}\) is a polygon of given shape: no
\(\mathcal{H}\) has fixed but curved boundaries: no
The shape of \(\mathcal{H}\) is not predefined: no
Can we handle hotspots of a different shape?

$\mathcal{H}$ is a polygon of given shape: no
$\mathcal{H}$ has fixed but curved boundaries: no
The shape of $\mathcal{H}$ is not predefined: no

More variations for multiple entities:

Find a smallest hotspot s.t. all entities spend at least $L$ time in $\mathcal{H}$. 
Can we handle hotspots of a different shape?

\( \mathcal{H} \) is a polygon of given shape: no
\( \mathcal{H} \) has fixed but curved boundaries: no
The shape of \( \mathcal{H} \) is not predefined: no

More variations for multiple entities:

Find a smallest hotspot s.t. all entities spend at least \( L \) time in \( \mathcal{H} \).
Total Length, Fixed Length

Goal: minimize the side length of $\mathcal{H}$ for a fixed trajectory length $L$.

Use parametric search, using the Fixed-Size algorithm as a decision algorithm.

Running time: $O(n^2 \log^2 n)$. 
Contiguous Length, Fixed Length

Find $\mathcal{H}$ by finding $\mathcal{T}[p, q]$. 
Contiguous Length, Fixed Length

Find $\mathcal{H}$ by finding $\mathcal{T}[p, q]$. 
Contiguous Length, Fixed Length

Find $\mathcal{H}$ by finding $\mathcal{T}[p, q]$. 
Contiguous Length, Fixed Length

Find $\mathcal{H}$ by finding $T[p, q]$.
Contiguous Length, Fixed Length

Find $\mathcal{H}$ by finding $\mathcal{T}[p, q]$.

Consider $\maxlength(\mathcal{T} \cap \mathcal{H})$ as a function $\psi$ depending on $t_p$. 
Contiguous Length, Fixed Length

Find $\mathcal{H}$ by finding $T[p, q]$.

Consider $\text{maxlength}(T \cap \mathcal{H})$ as a function $\psi$ depending on $t_p$.

**Lemma 5.** $\phi$ is piecewise linear, its break points corresponding to hotspots $\mathcal{H}$ s.t.

...
Contiguous Length, Fixed Length

Find \( \mathcal{H} \) by finding \( T[p, q] \).

Consider \( \text{maxlength}(T \cap \mathcal{H}) \) as a function \( \psi \) depending on \( t_p \).

**Lemma 5.** \( \phi \) is piecewise linear, its break points corresponding to hotspots \( \mathcal{H} \) s.t.

... 

Compute \( \phi \) at all break points and select the maximum.

Running time: \( O(n \log n) \)