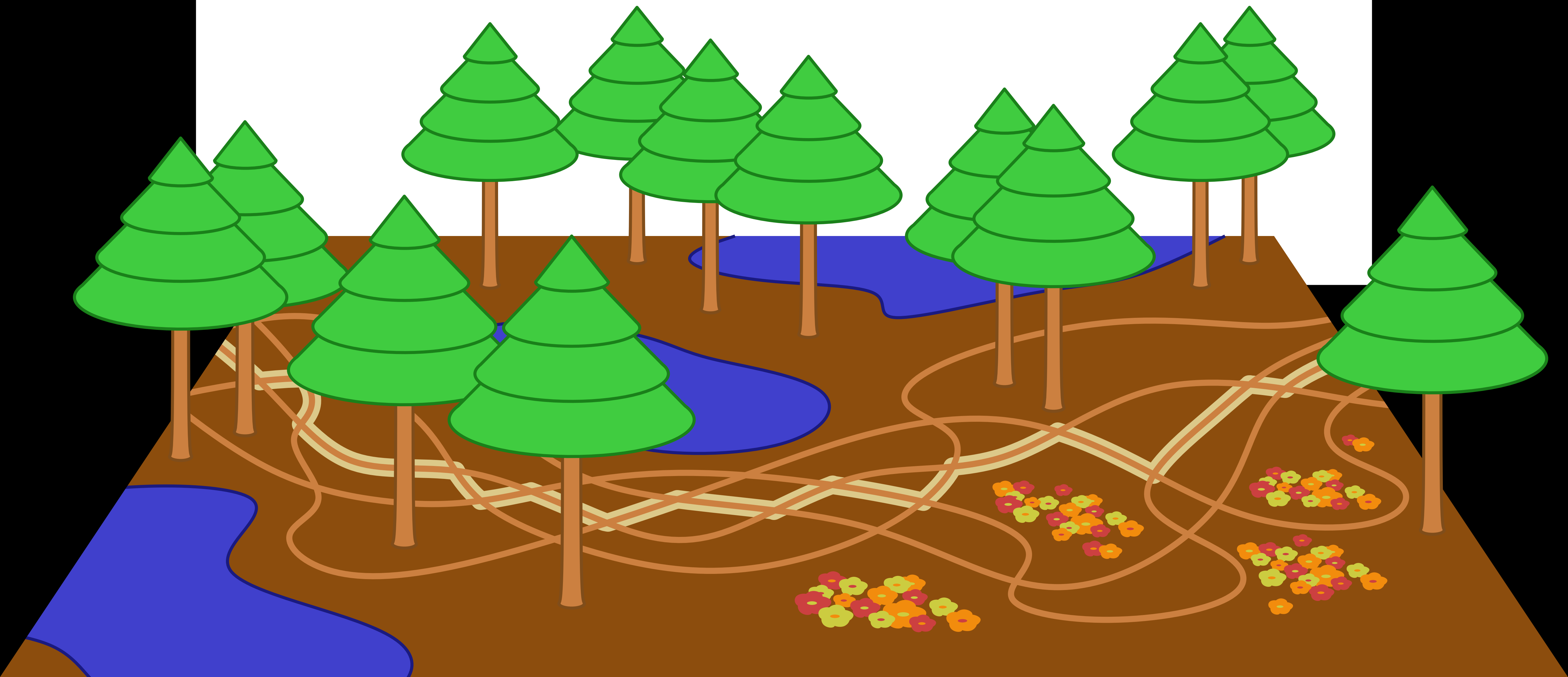
HOMOTOPY MEASURES FOR REPRESENTATIVE TRAJECTORIES

Erin Chambers *Maarten Löffler*

Irina Kostitsyna *Frank Staals*



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- Let P be n points in the plane

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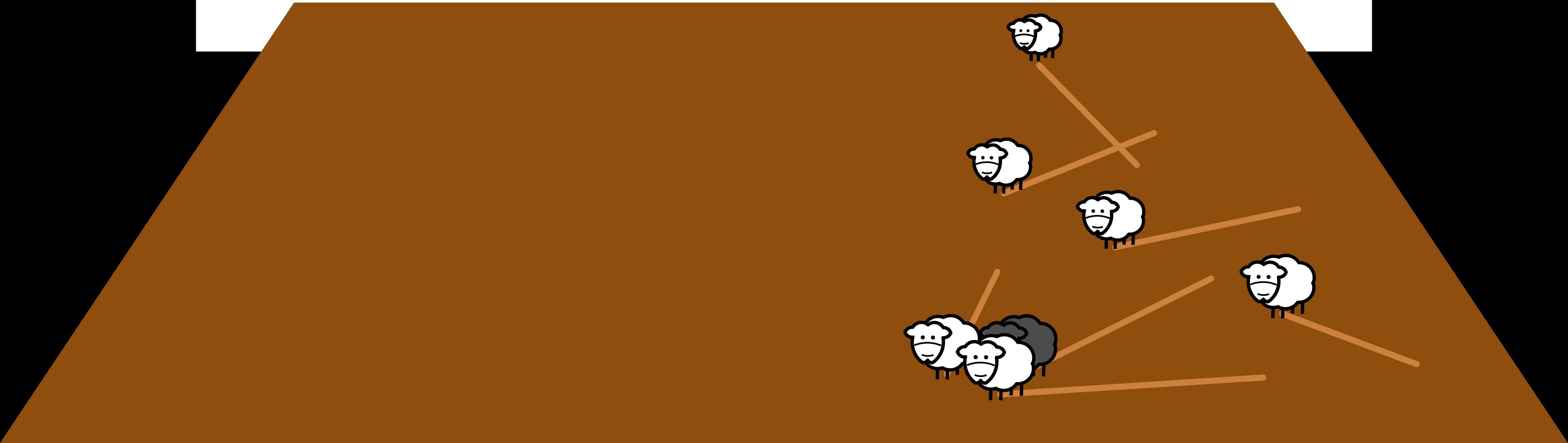

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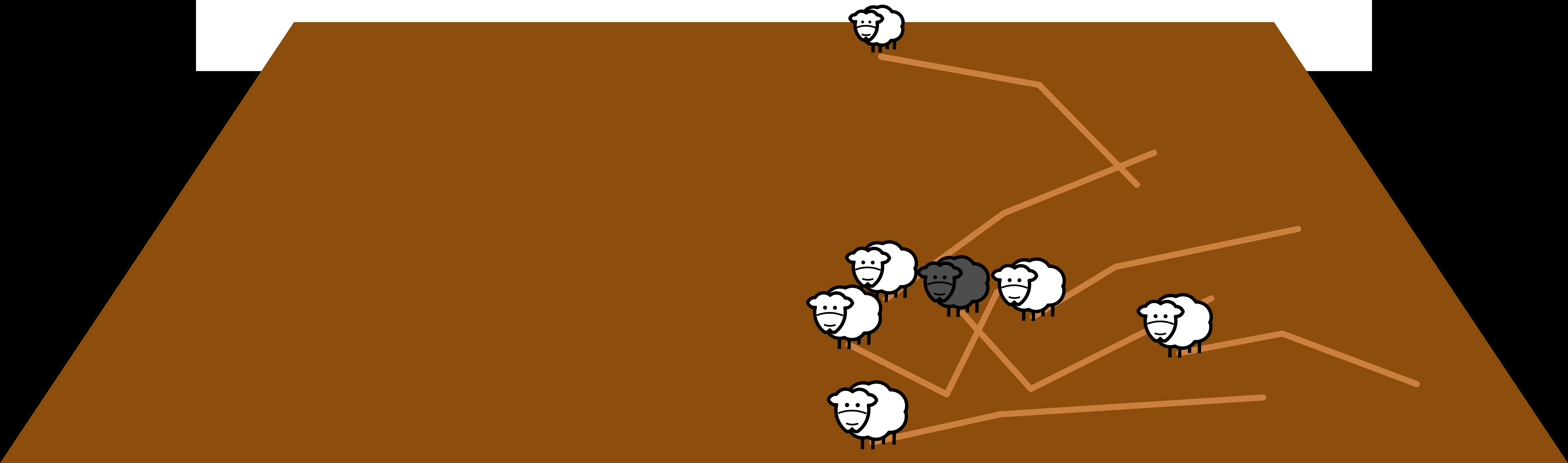

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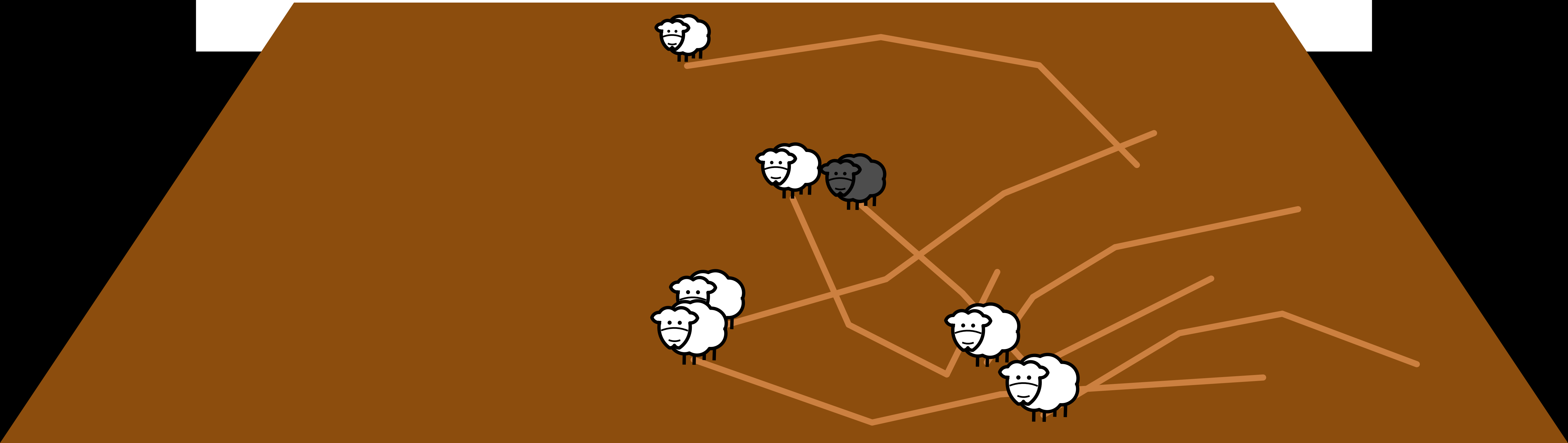

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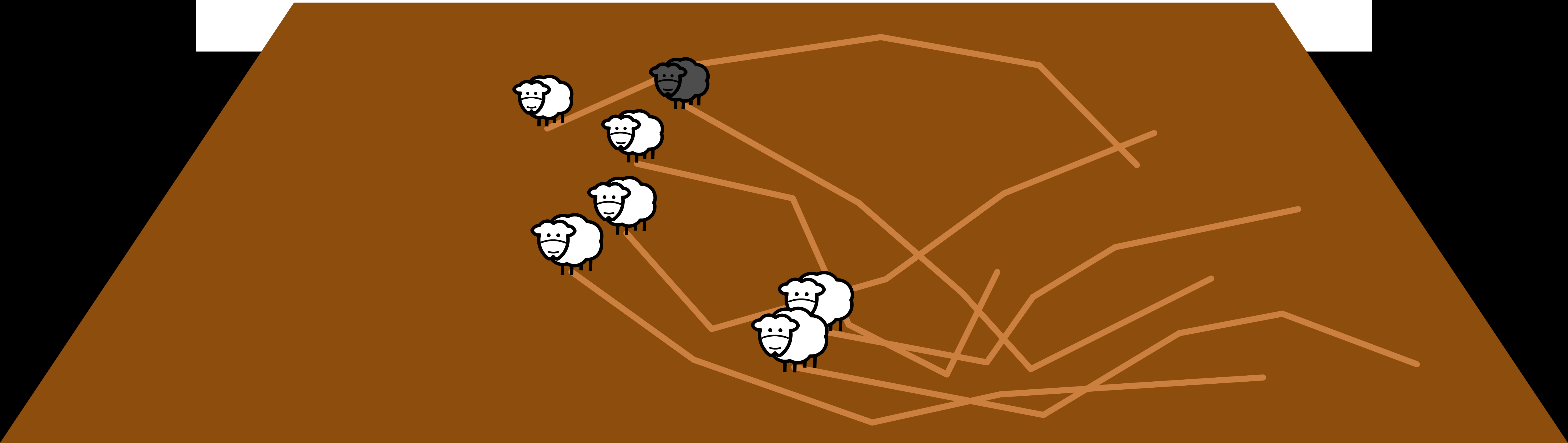

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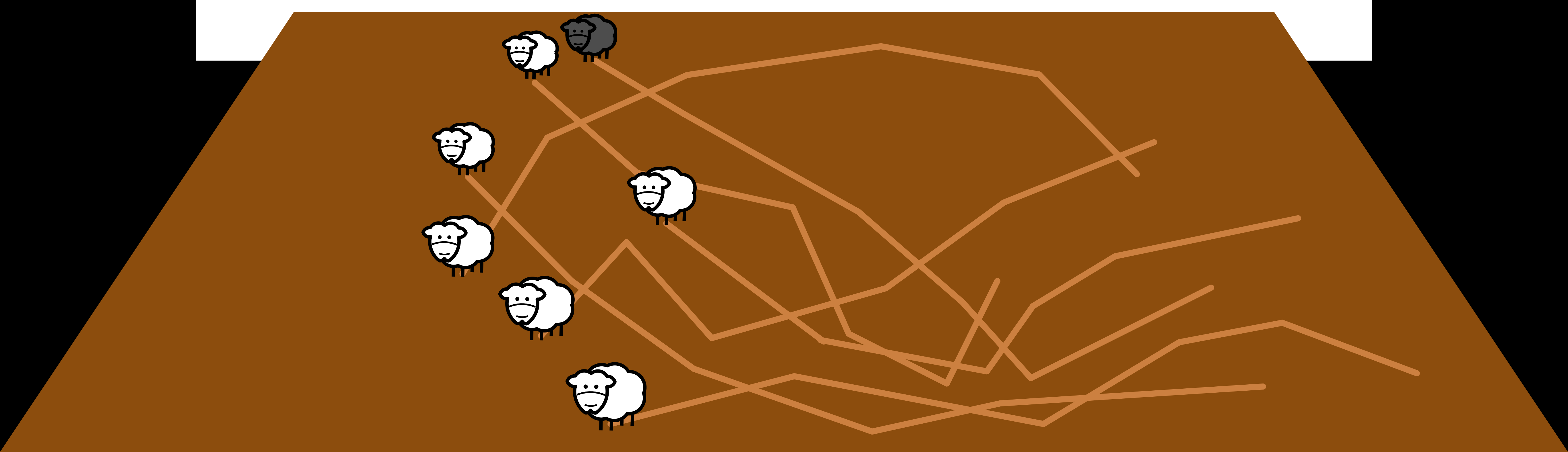
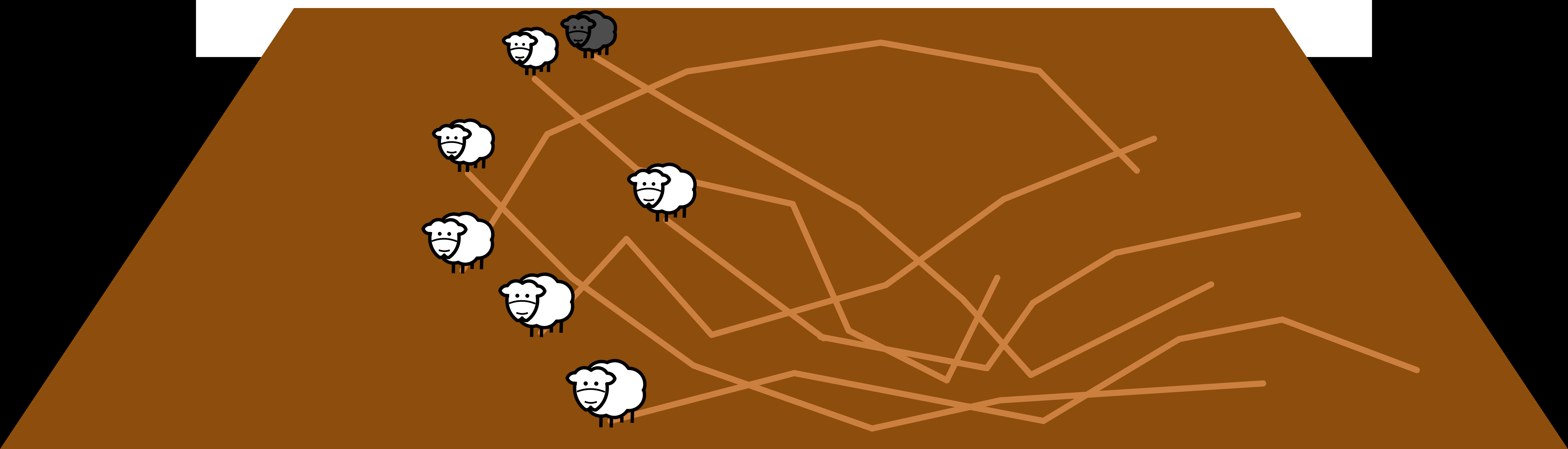


TRAJECTORIES

- Let P be n points in the plane
- Now suppose your points run away
- P traces a set of n *trajectories*: curves in $\mathbb{R}^2$



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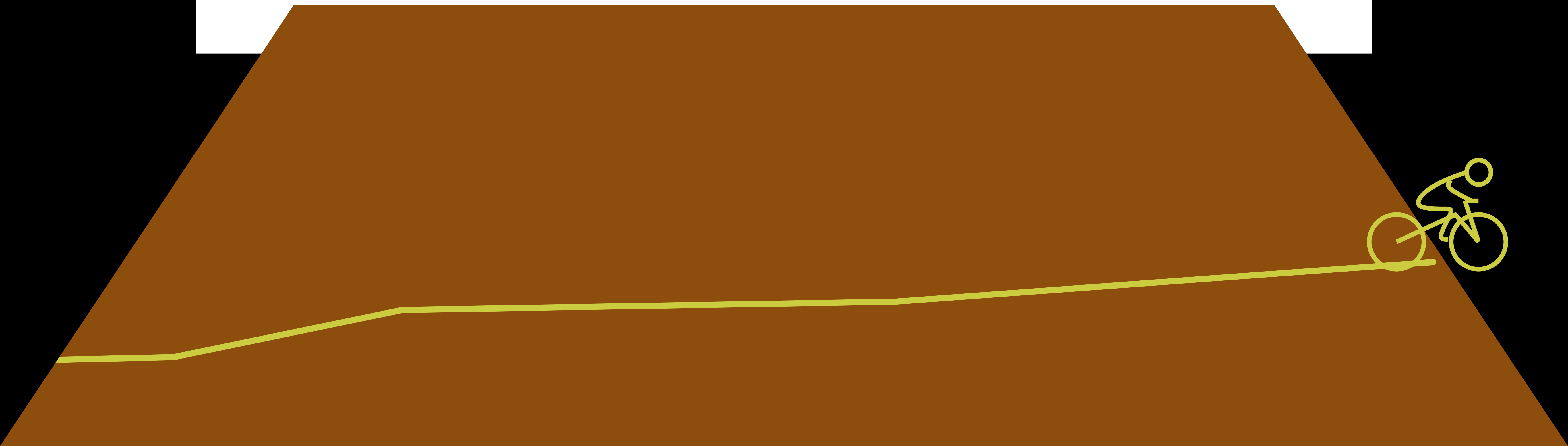

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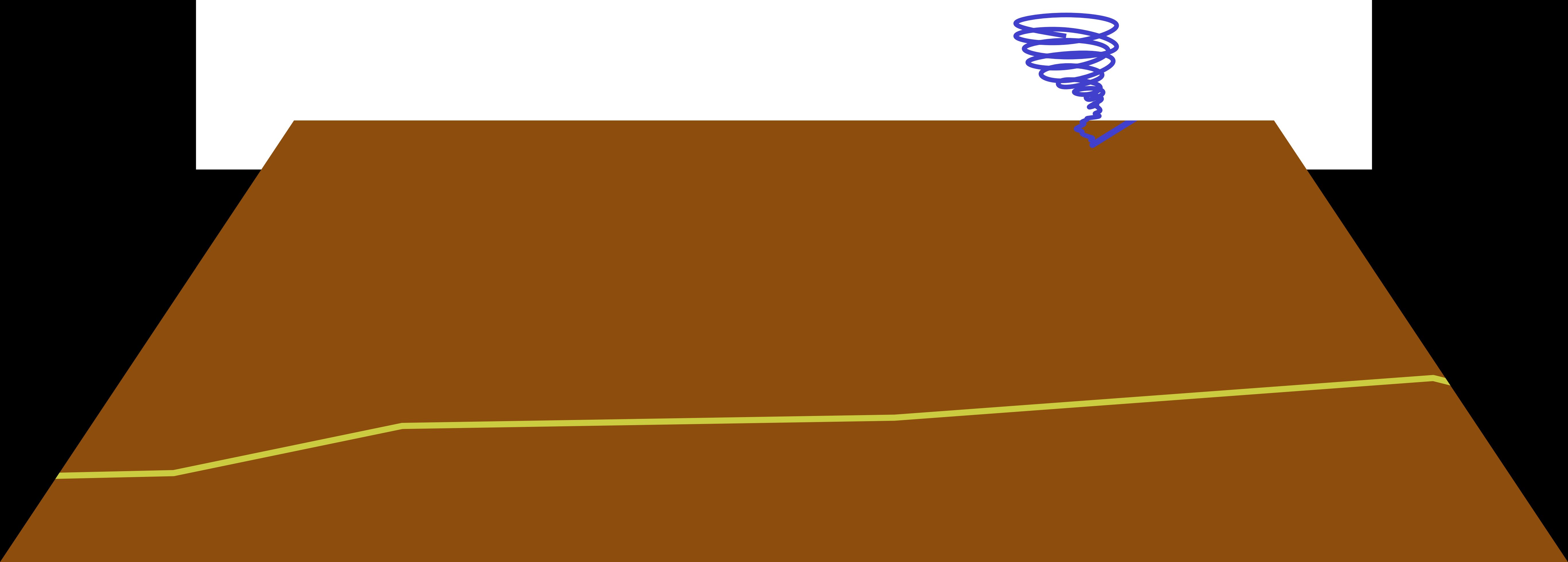

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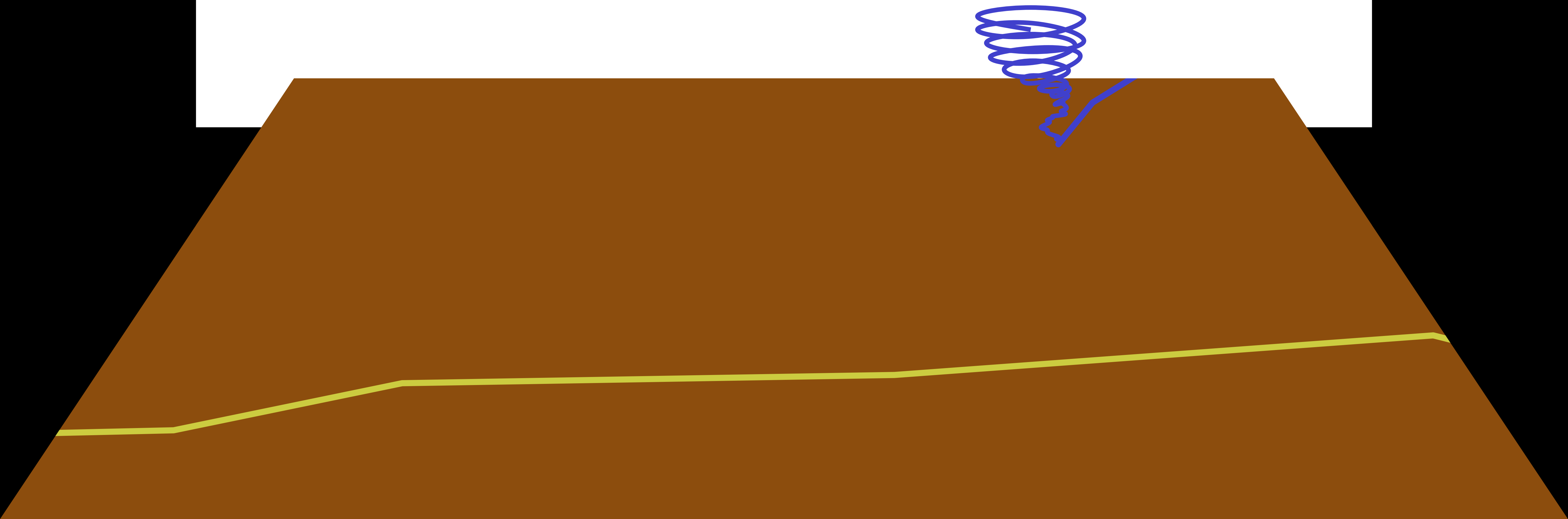

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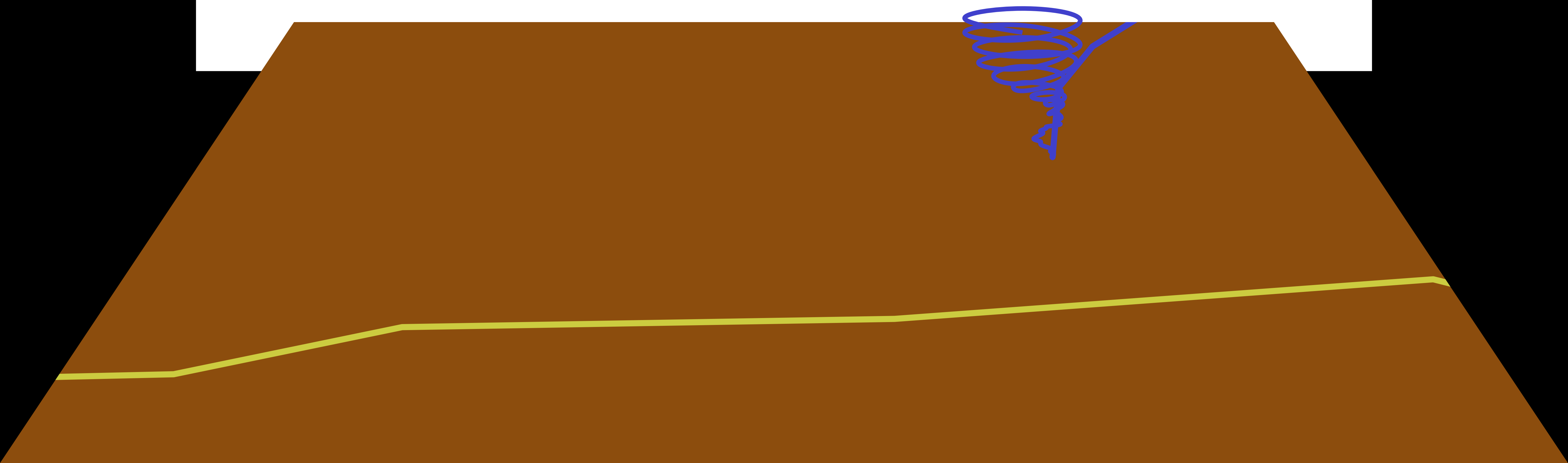

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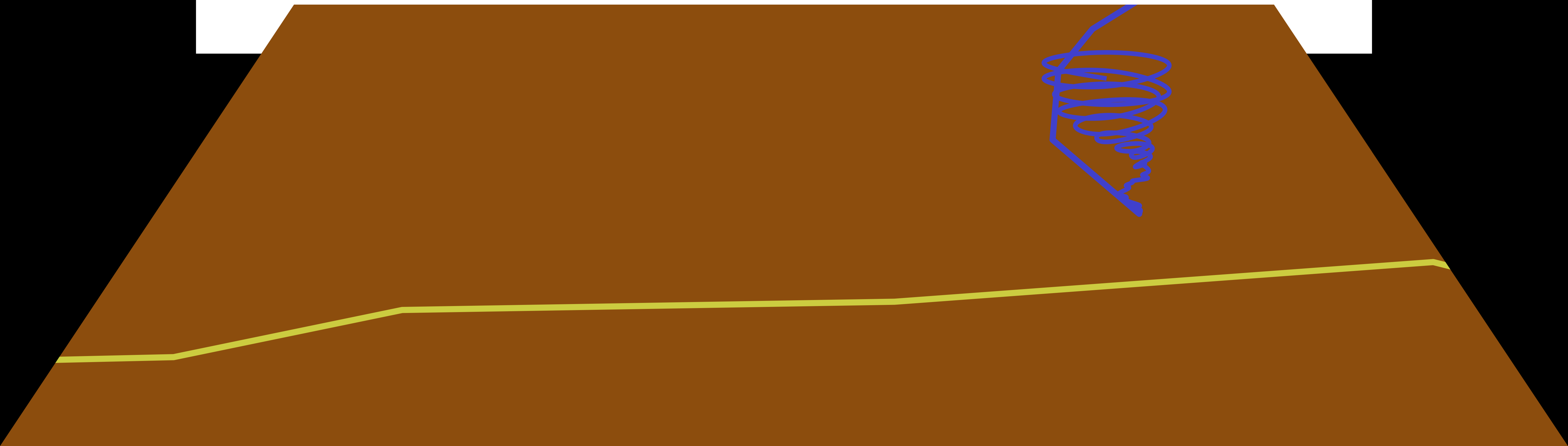

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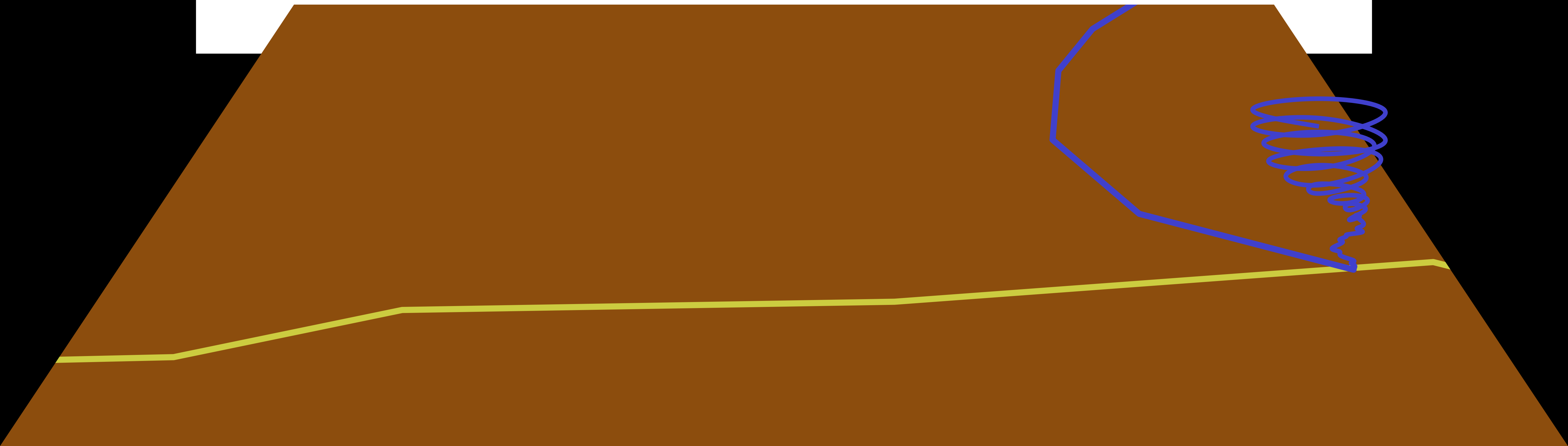

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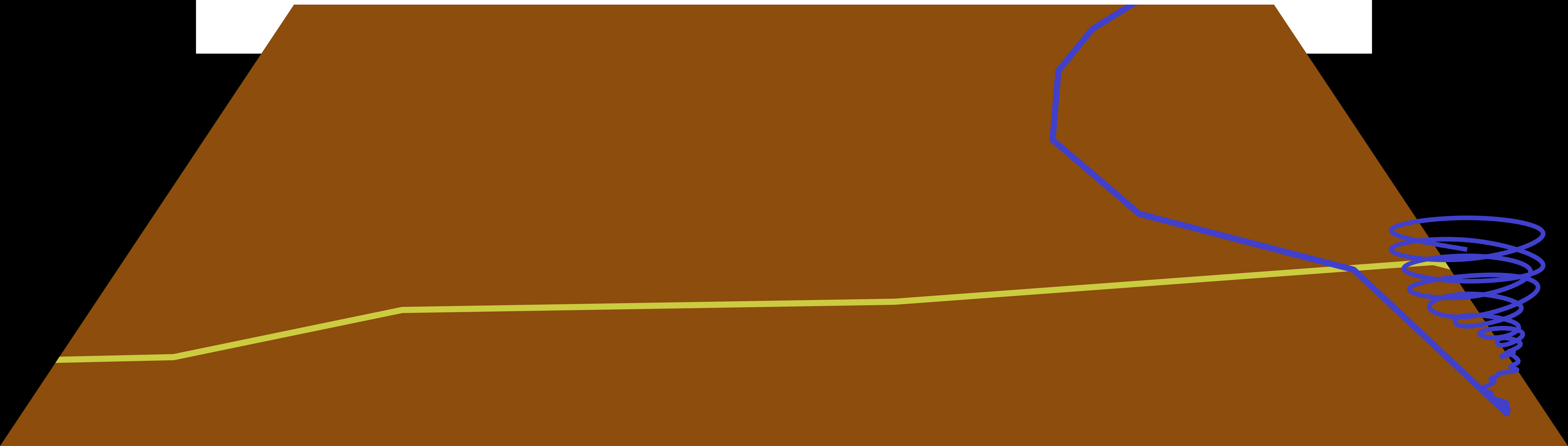

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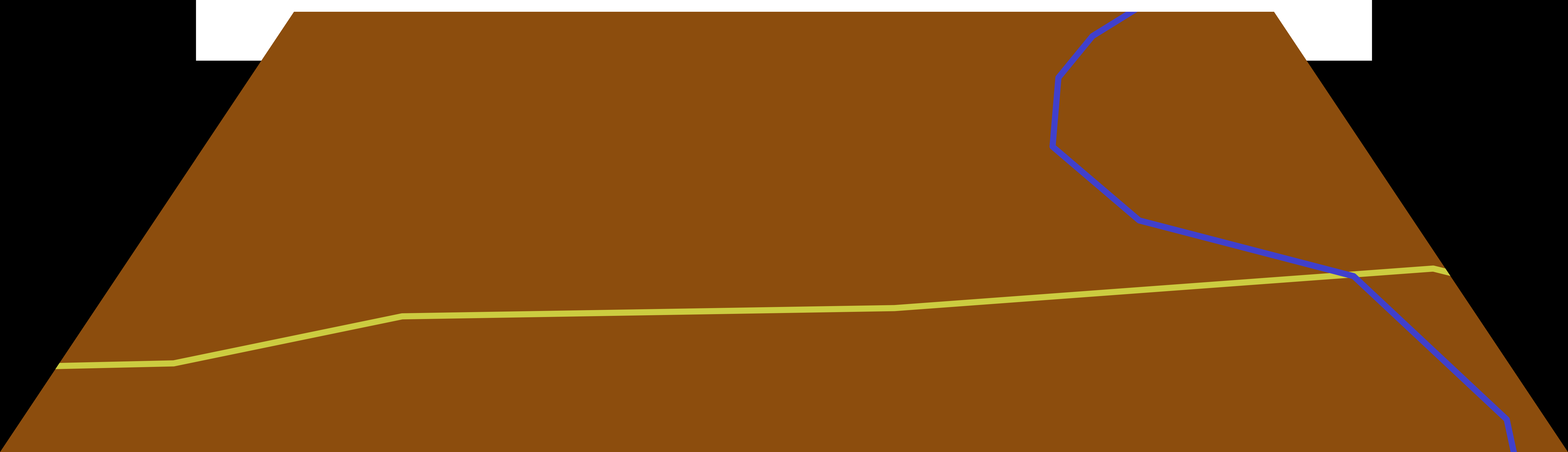

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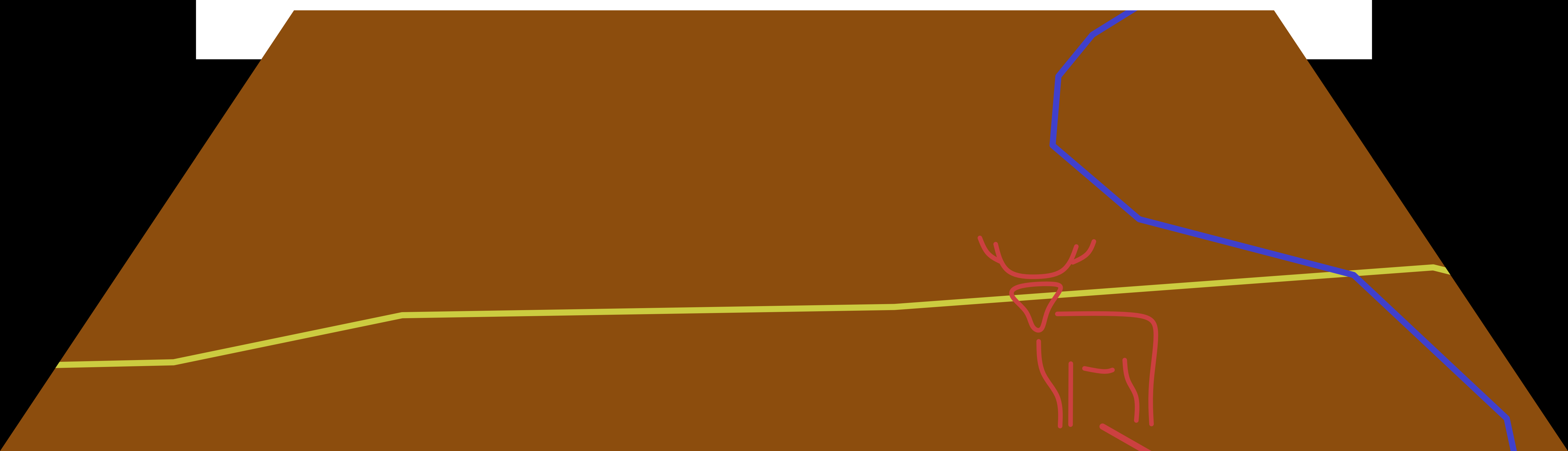

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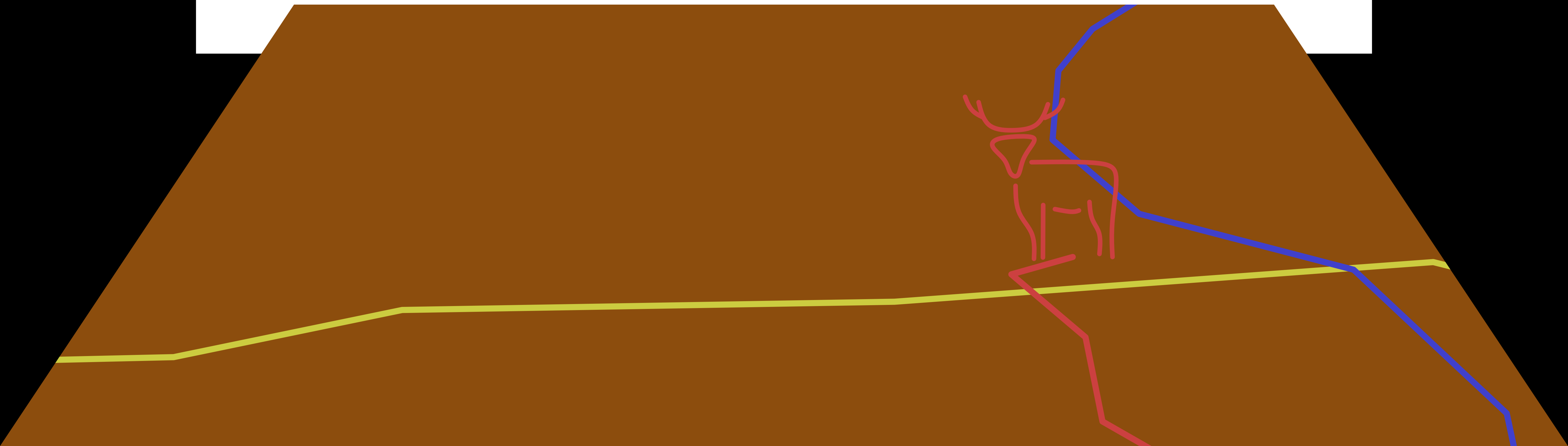

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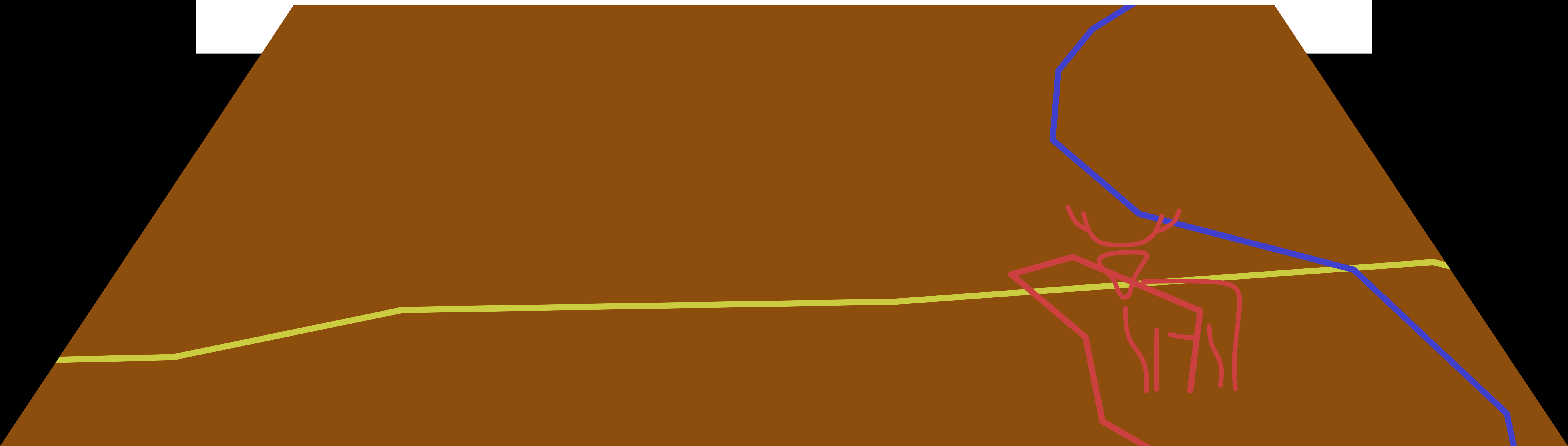

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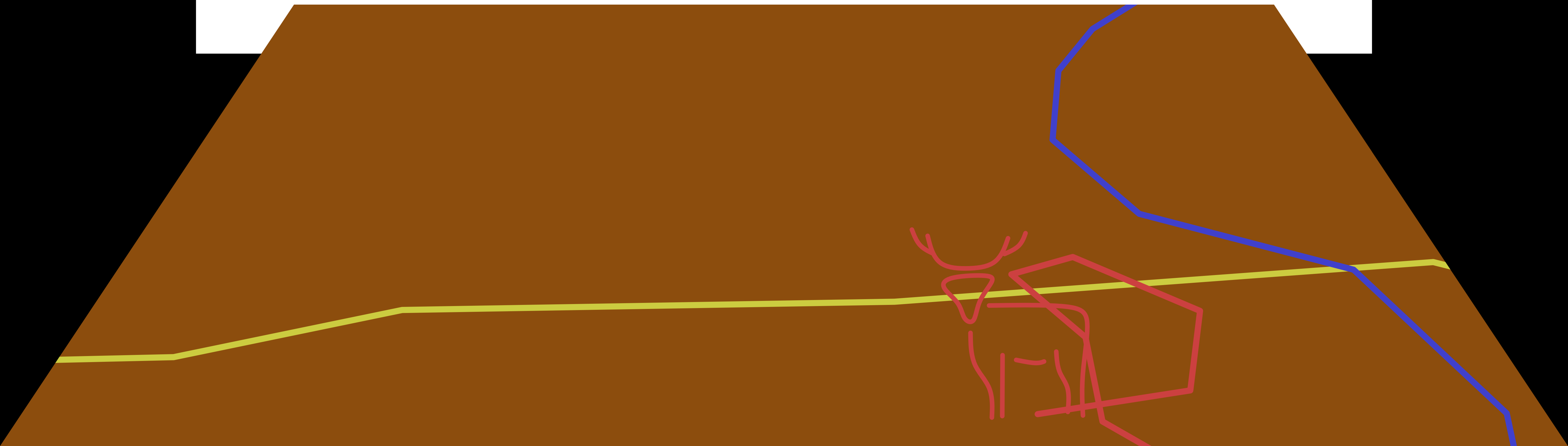

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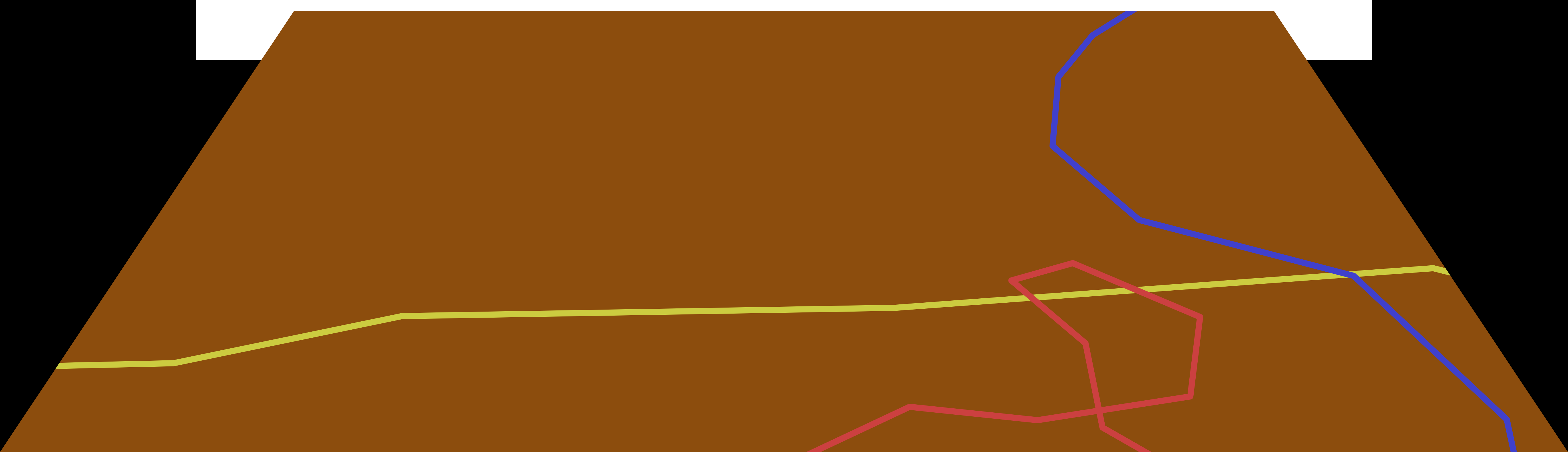
- Trajectories are interesting
  - Many different analysis tasks



REPRESENTATIVE TRAJECTORY

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  - Suppose we have lots of trajectories

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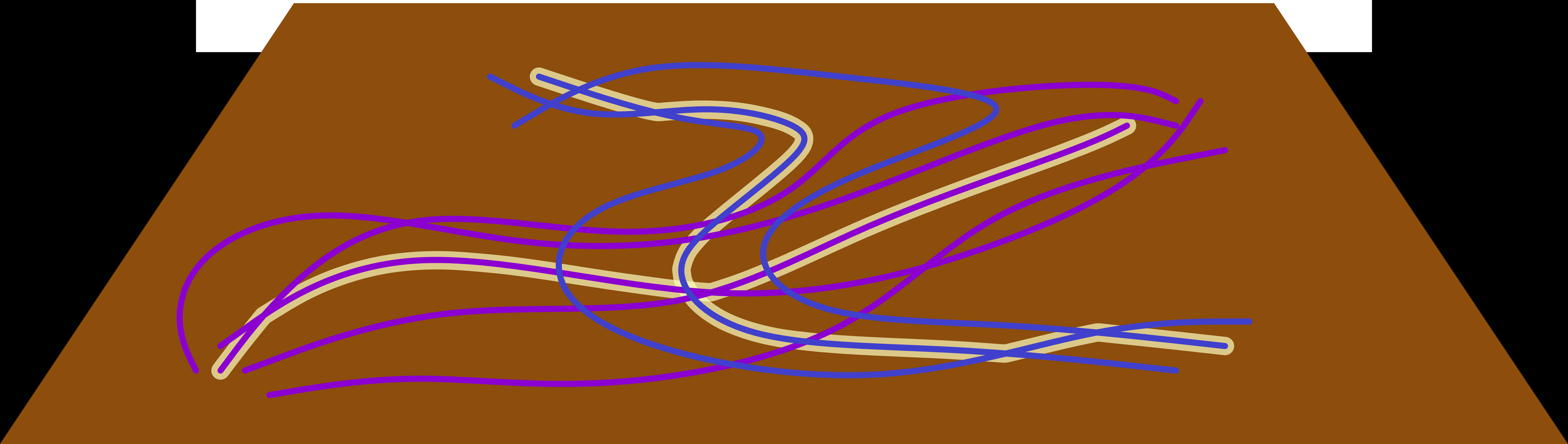

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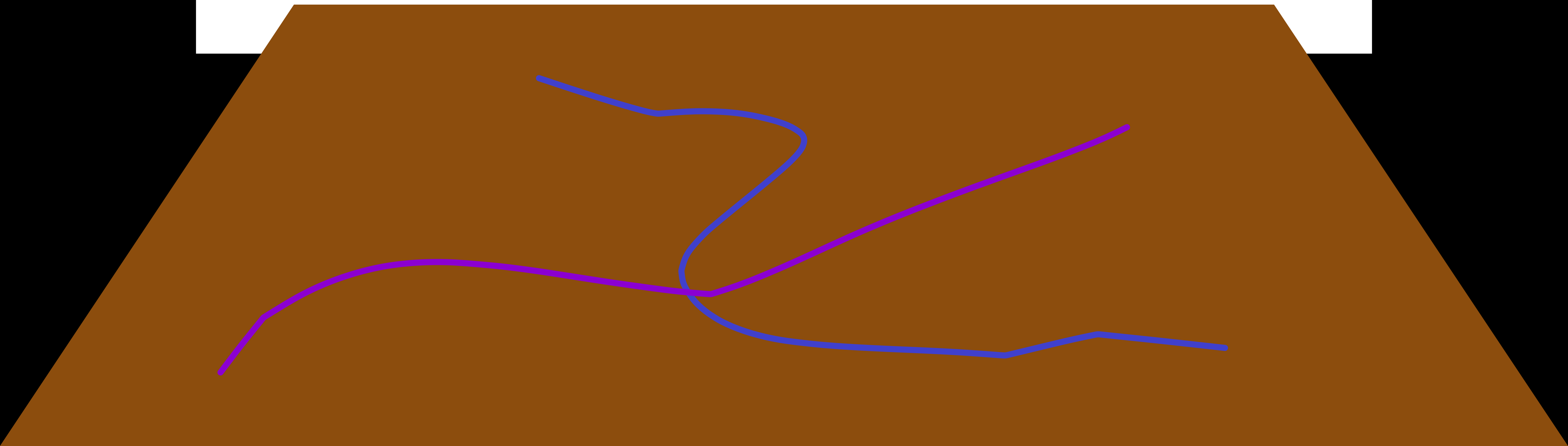
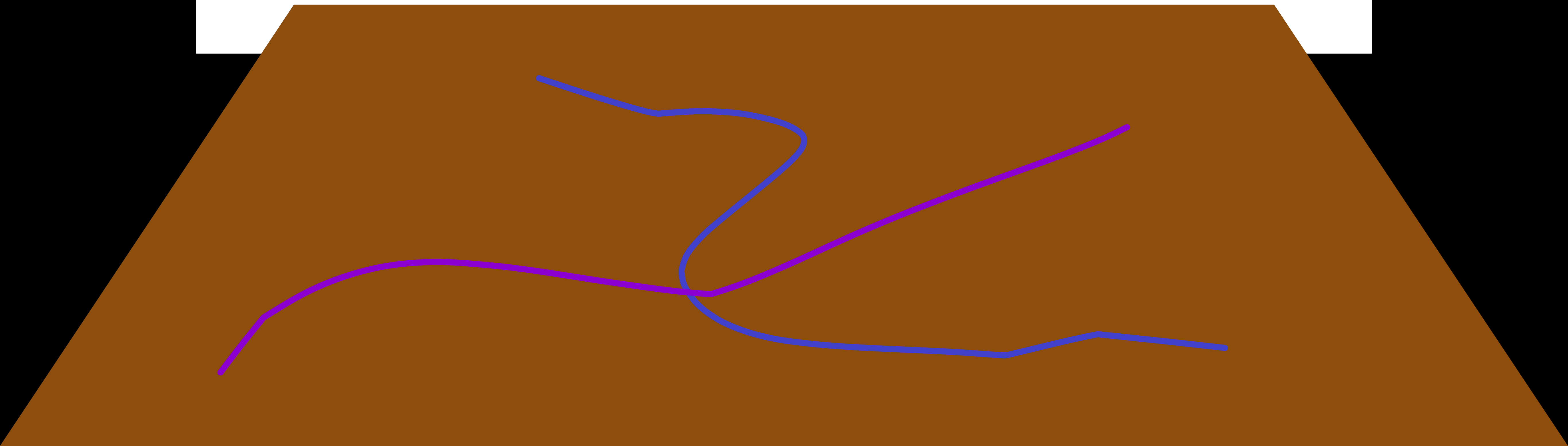


REPRESENTATIVE TRAJECTORY

- Problem
  - Suppose we have lots of trajectories
  - Suppose we want to extract significant patterns
- Solution
  - Cluster the trajectories
  - Pick a good representative for each cluster
  - Keep only the representatives
- But what is a good representative?



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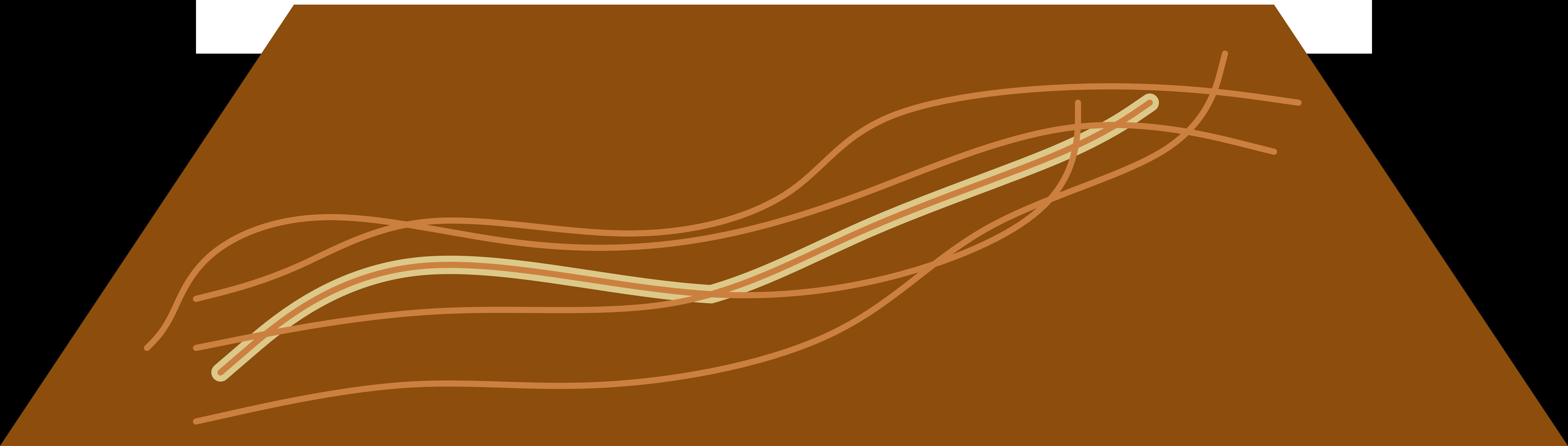

OBVIOUS REPRESENTATIVES

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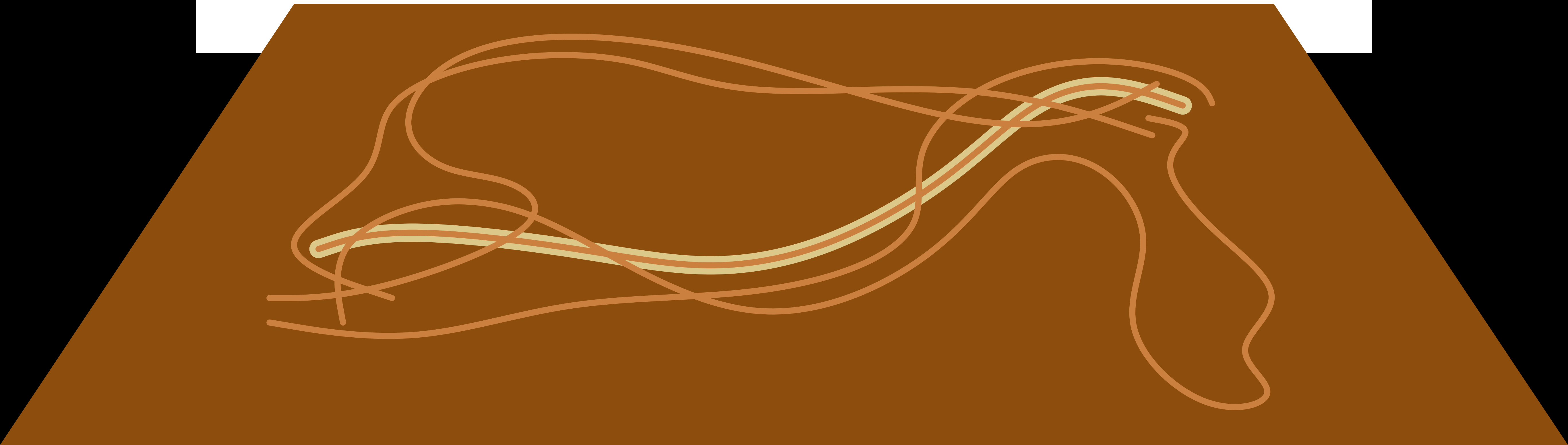
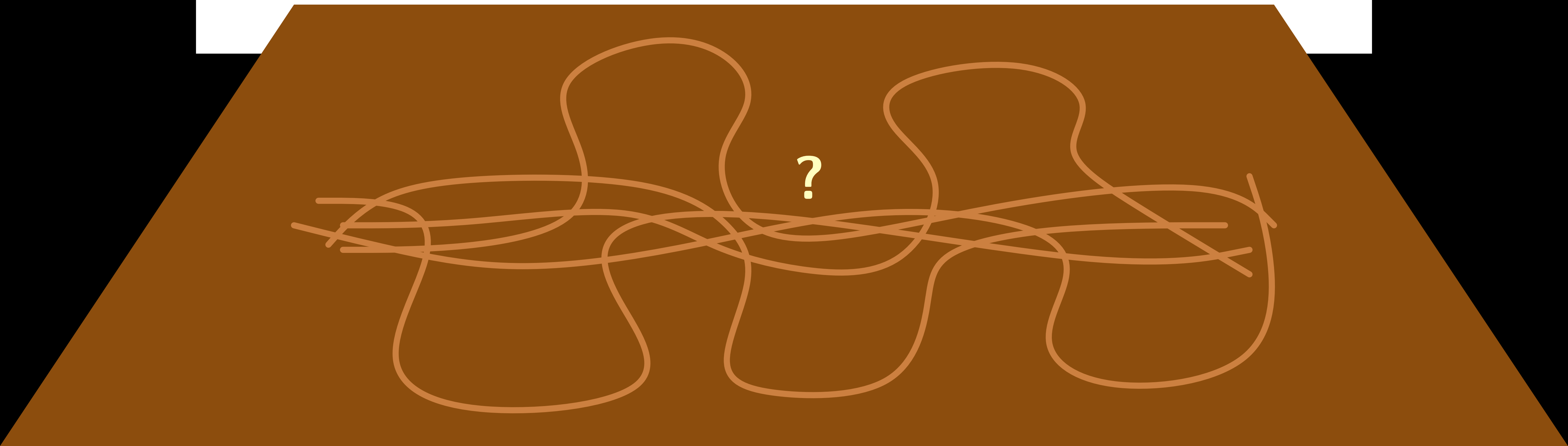
- Use one of the input trajectories



OBVIOUS REPRESENTATIVES

- Use one of the input trajectories
  - There may not be any single good representative!



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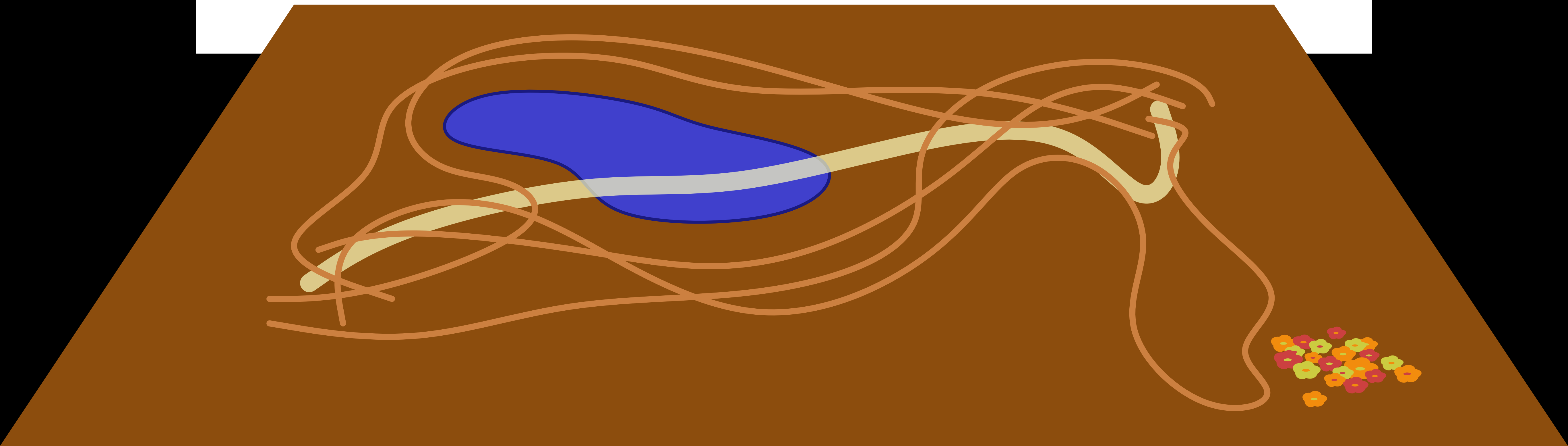
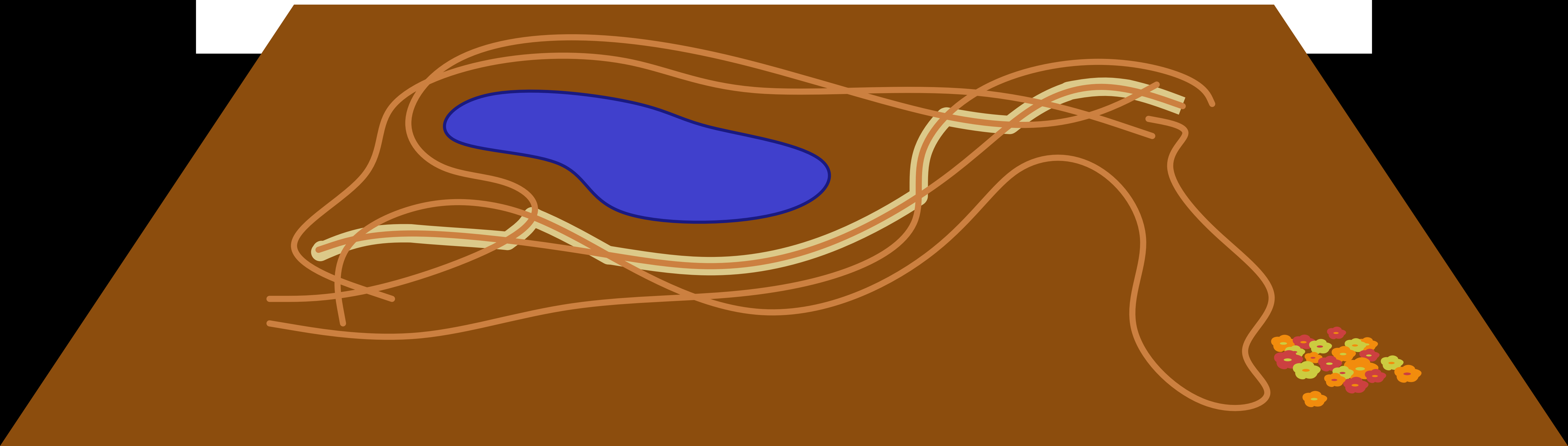


OBVIOUS REPRESENTATIVES

- Use one of the input trajectories
  - There may not be any single good representative!
- Pick the mean trajectory
  - May interfere with environment!
- Use pieces of different trajectories



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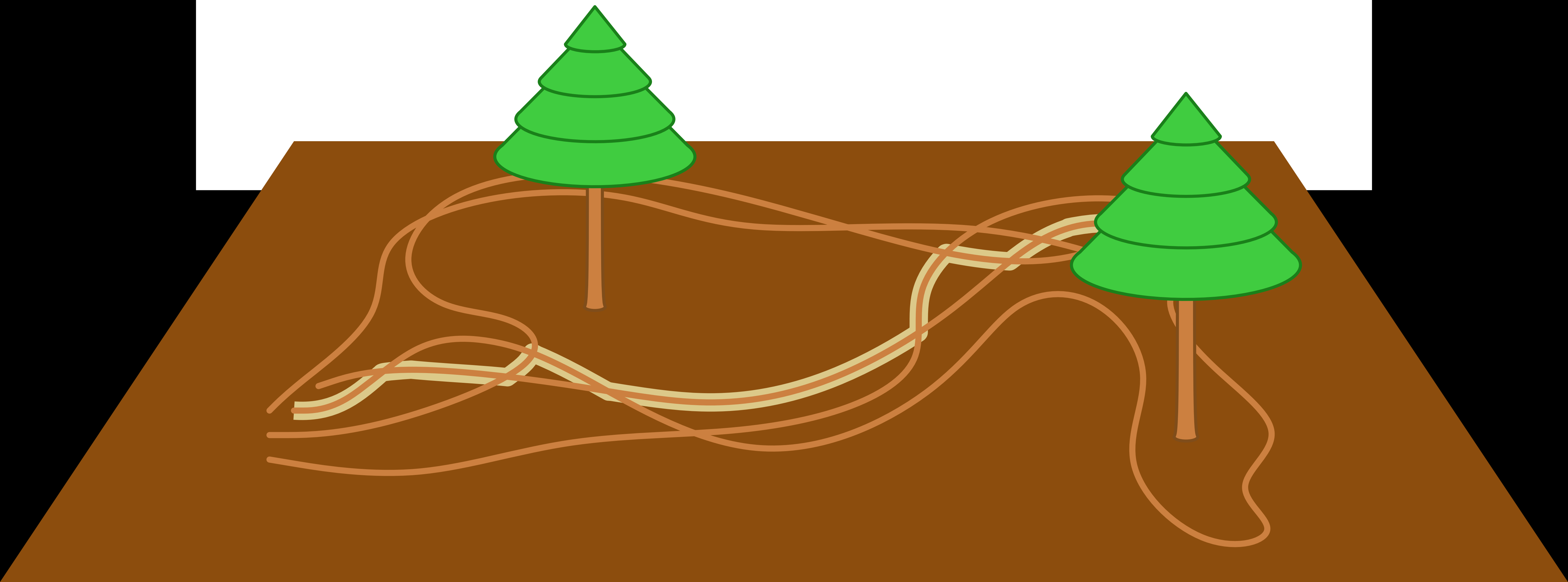
MEDIAN TRAJECTORIES

- Buchin et.al. [ESA,2010] present two such representatives:
  - Start in the middle, switch at every intersection
  - Mark important faces, pick the median that passes on "the right side" of each face.



OUR APPROACH

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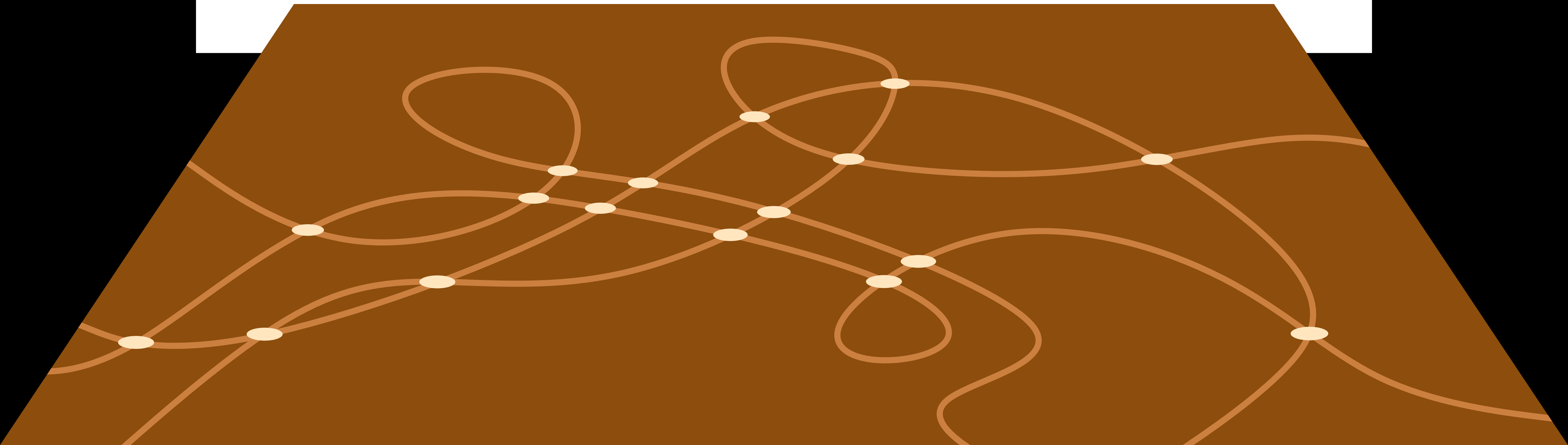
  - Arrangement of curves forms a graph



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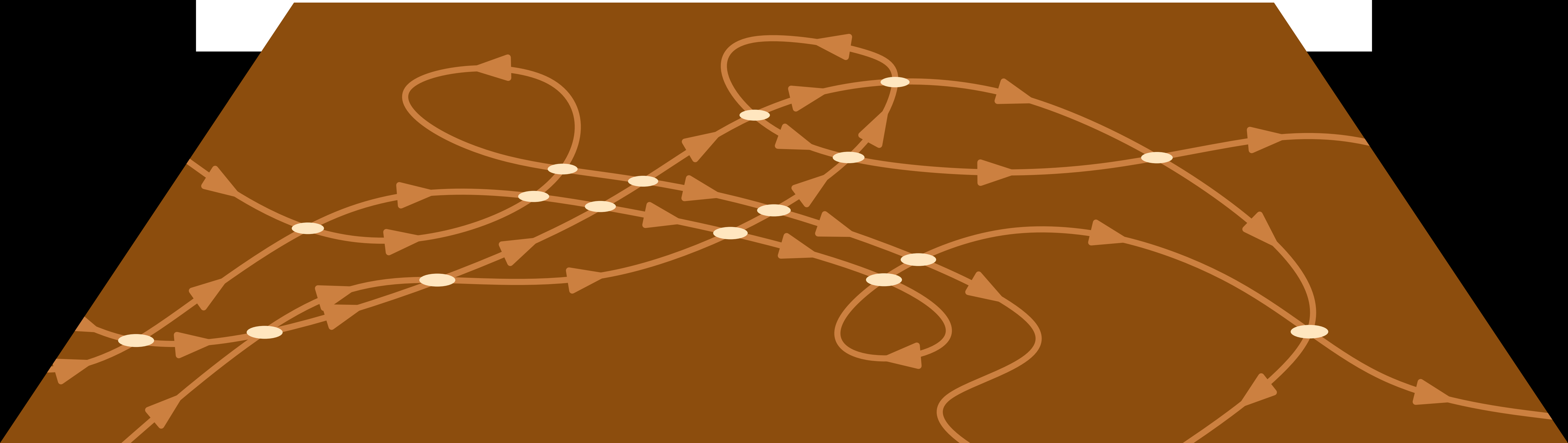

OUR APPROACH

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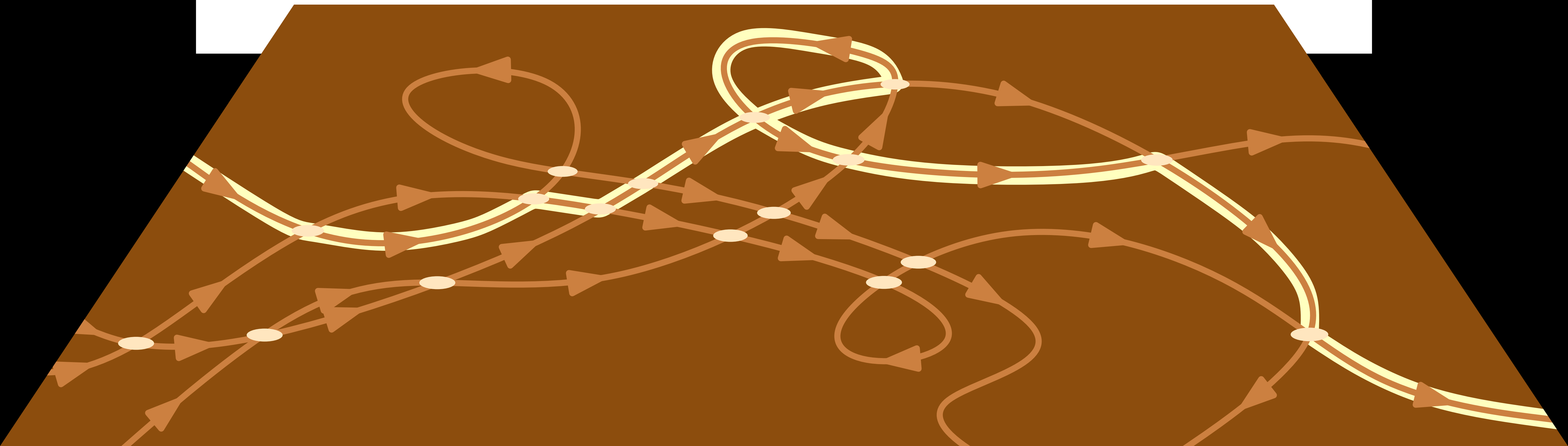
  - Arrangement of curves forms a graph
  - Edges are directed
- Output r is a path in this graph



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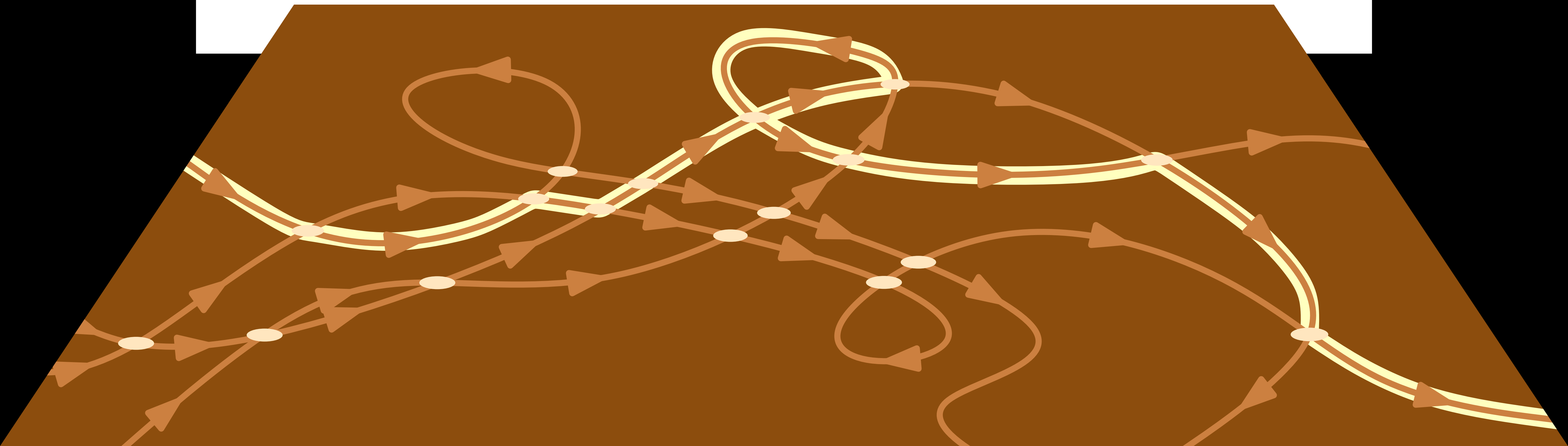
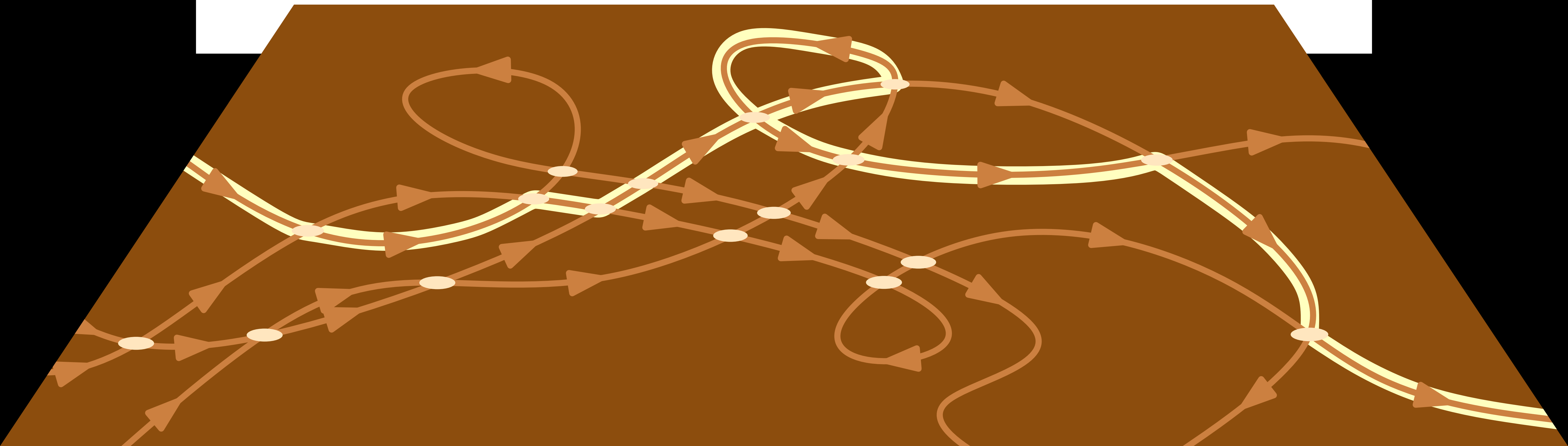
- Define the quality of a path?



OUR APPROACH

- Trajectories are just curves
  - Arrangement of curves forms a graph
  - Edges are directed
- Output r is a path in this graph
- Define the quality of a path?
  - We define a distance measure between r and all trajectories.



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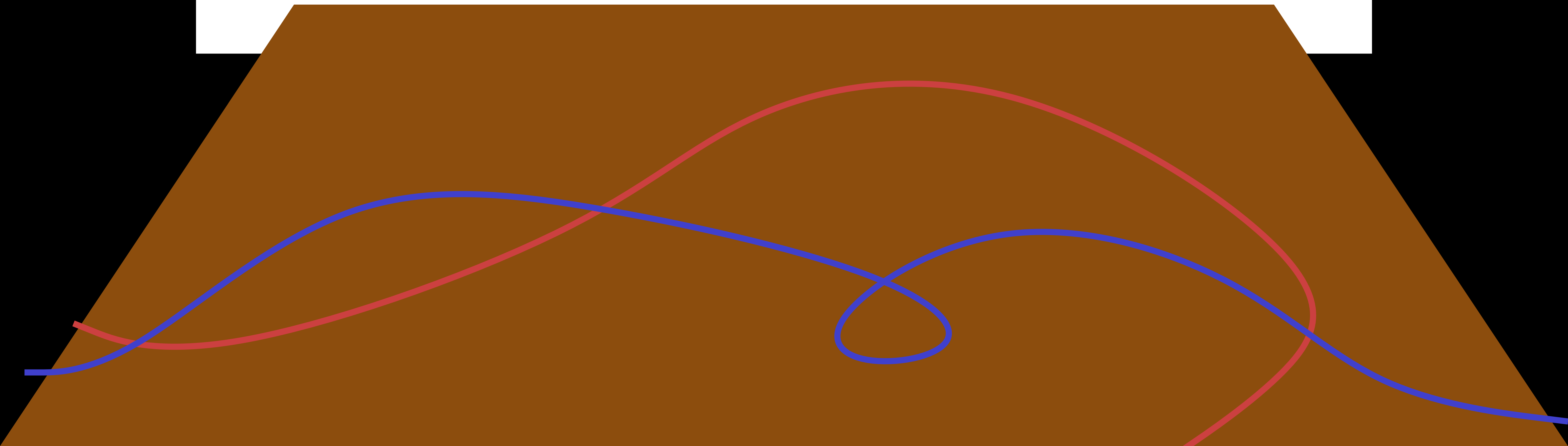
- Let D be a distance measure between two curves



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- Let D be a distance measure between two curves

- $\mathcal{D}(r) = \sum_{T \in \mathcal{T}} D(r, T)$

- $\mathcal{M}(r) = \max_{T \in \mathcal{T}} D(r, T)$

OUR APPROACH

- Let D be a distance measure between two curves
  - We use Homotopy Area

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HOMOTOPY AREA?????

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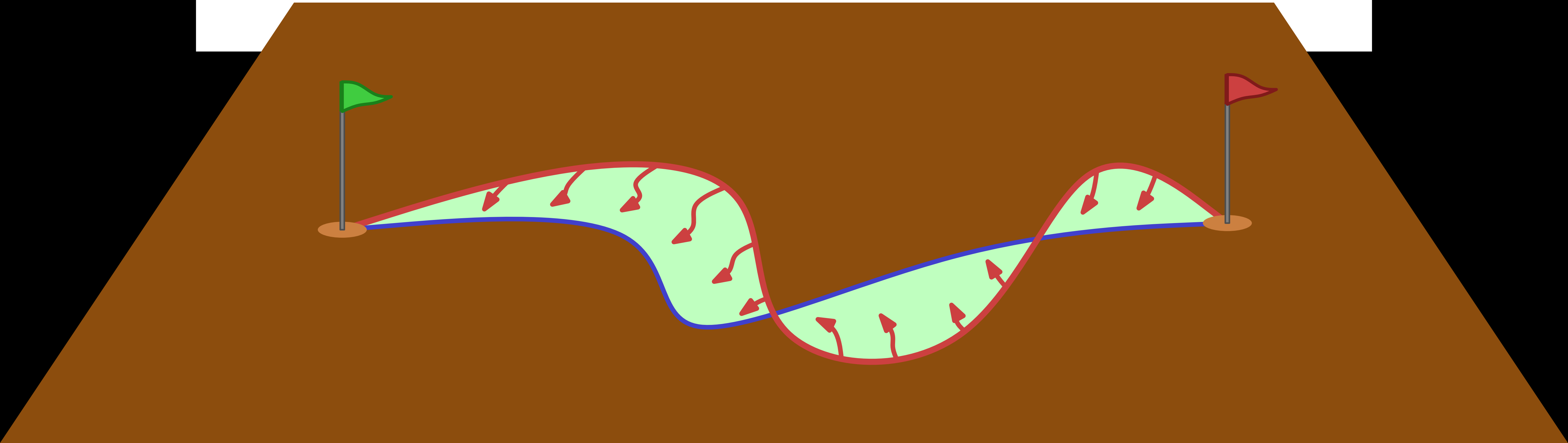
- $D(A, B) =$

$$\inf_{H \in \mathcal{H}(A,B)} \int_{u \in [0,1]} \int_{w \in [0,1]} \left| \frac{\mathrm{d}H}{\mathrm{d}u} \times \frac{\mathrm{d}H}{\mathrm{d}w} \right| \, \mathrm{d}u \, \mathrm{d}w \,,$$

where $\mathcal{H}(A, B) =$ .....

HOMOTOPY AREA?????

- $D(A, B) =$ the minimum area that we have to sweep curve A over to transform it into B.



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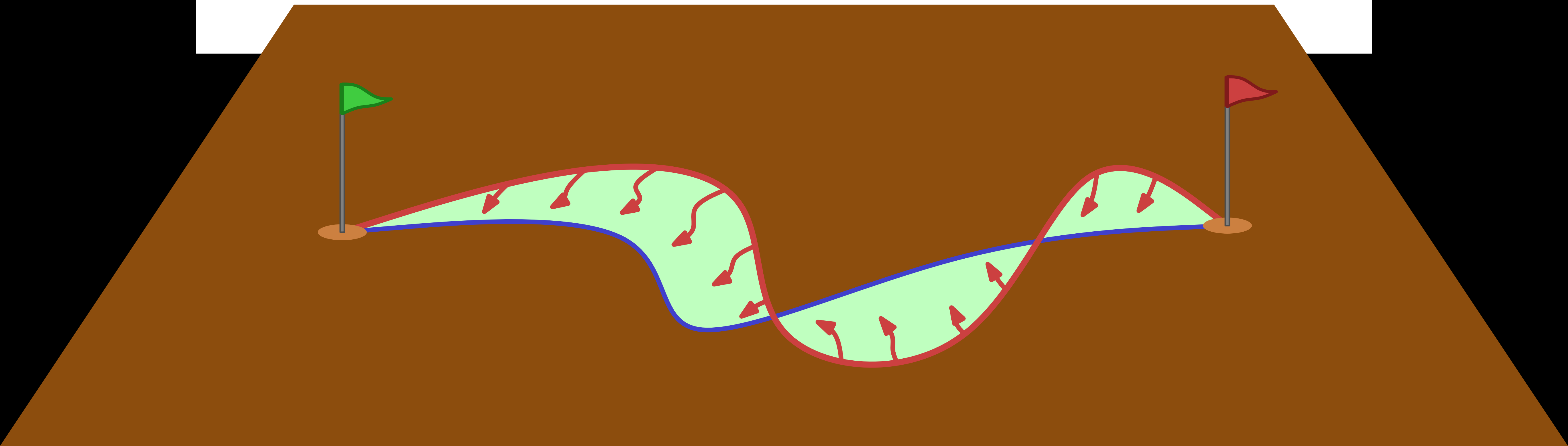

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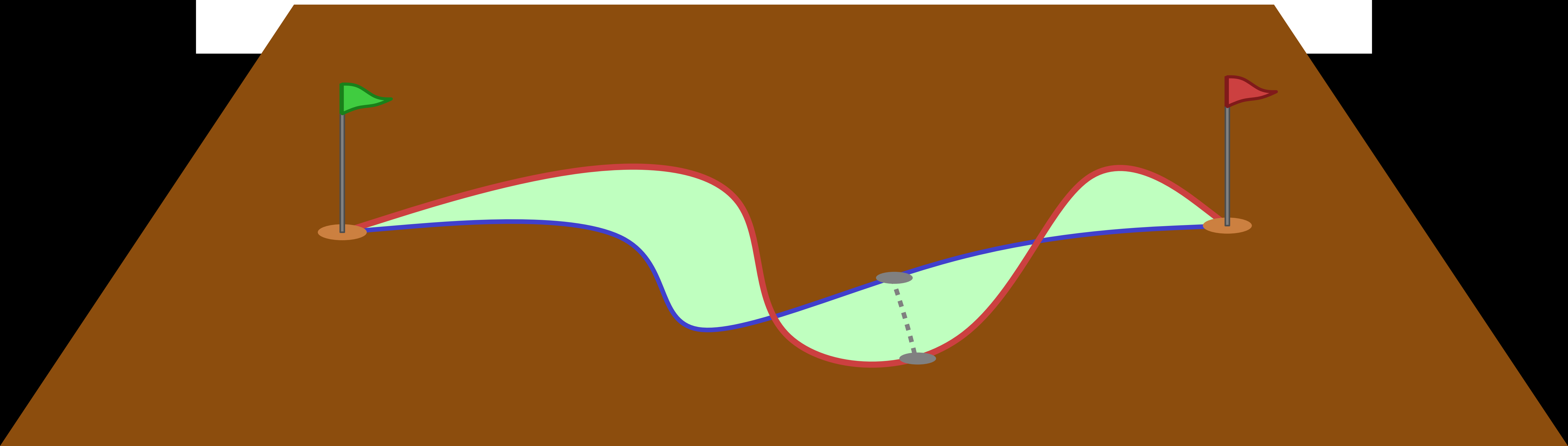
  - it does not need a parametrization of the curves.



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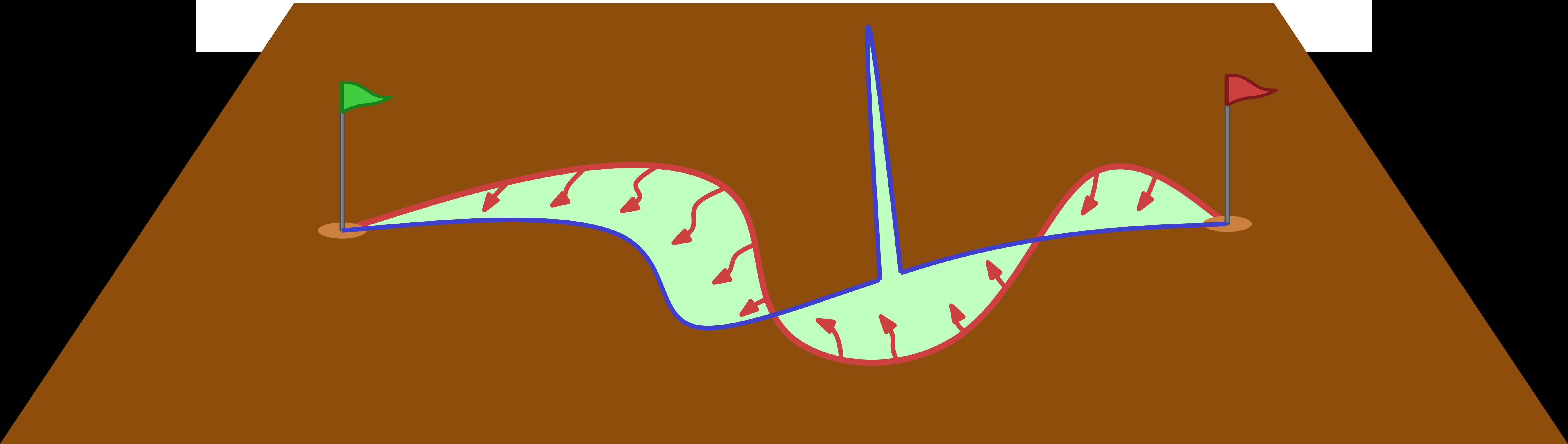

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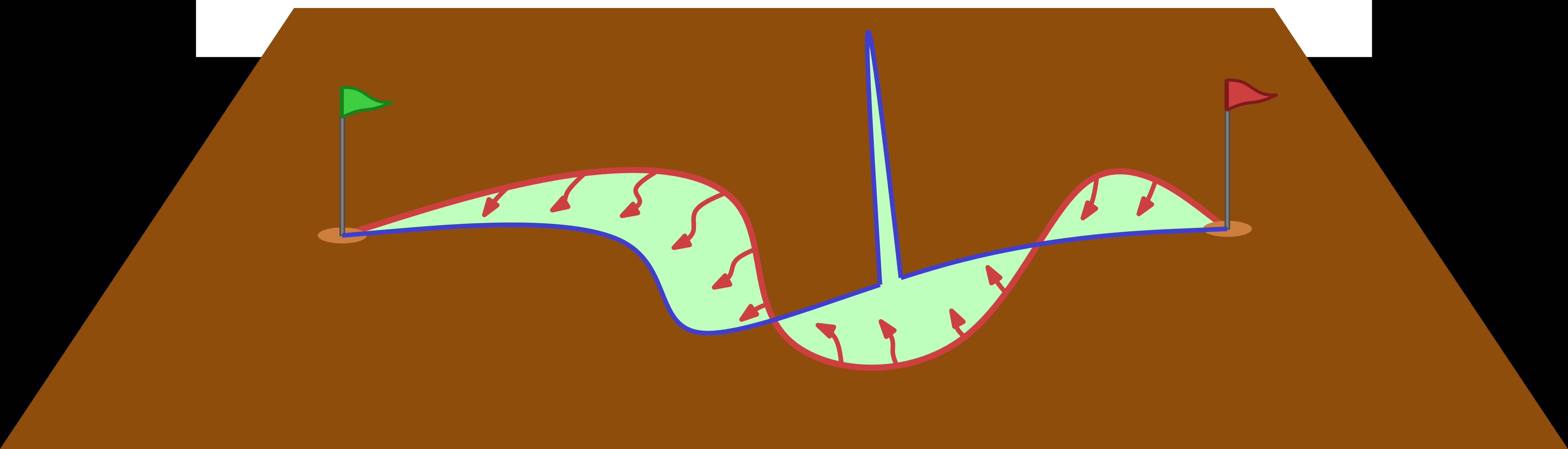
- $D(A, B) =$ the minimum area that we have to sweep curve A over to transform it into B.
- Why homotopy area?
  - it does not need a parametrization of the curves.
  - robust against outliers
  - tries to capture important faces automatically



HOMOTOPY AREA?????

- We assume that our trajectories:
  - start in s and end in t

HOMOTOPY AREA?????

- We assume that our trajectories:
  - start in s and end in t
  - are simple

RESULTS

- Finding r^* that minimizes
  - $\mathcal{M}(r) = \max_{T \in \mathcal{T}} D(r, T)$
  - $\mathcal{D}(r) = \sum_{T \in \mathcal{T}} D(r, T)$

RESULTS

- Finding r^* that minimizes
  - $\mathcal{M}(r) = \max_{T \in \mathcal{T}} D(r, T)$ is NP-hard
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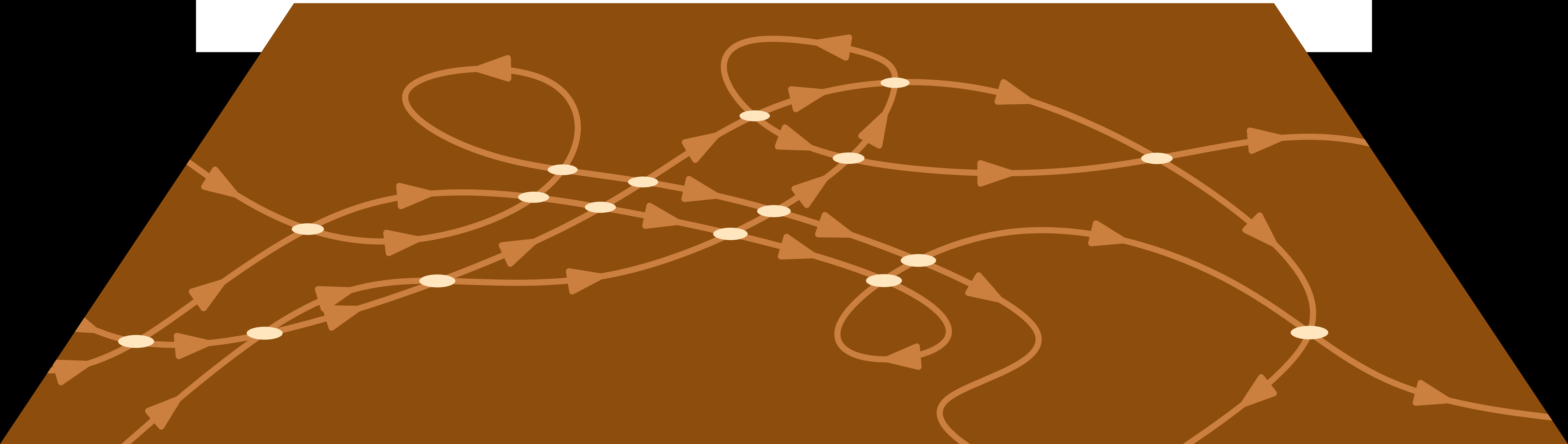
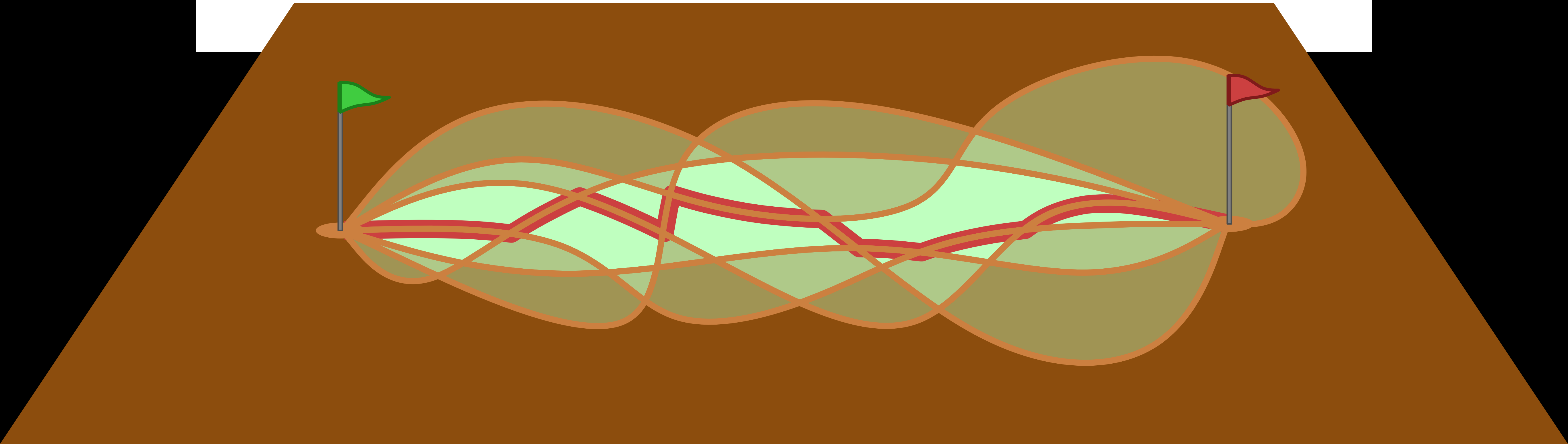
    Solvable efficiently when the trajectories from a DAG



MINIMIZING $\mathcal{D}$

- Suppose that



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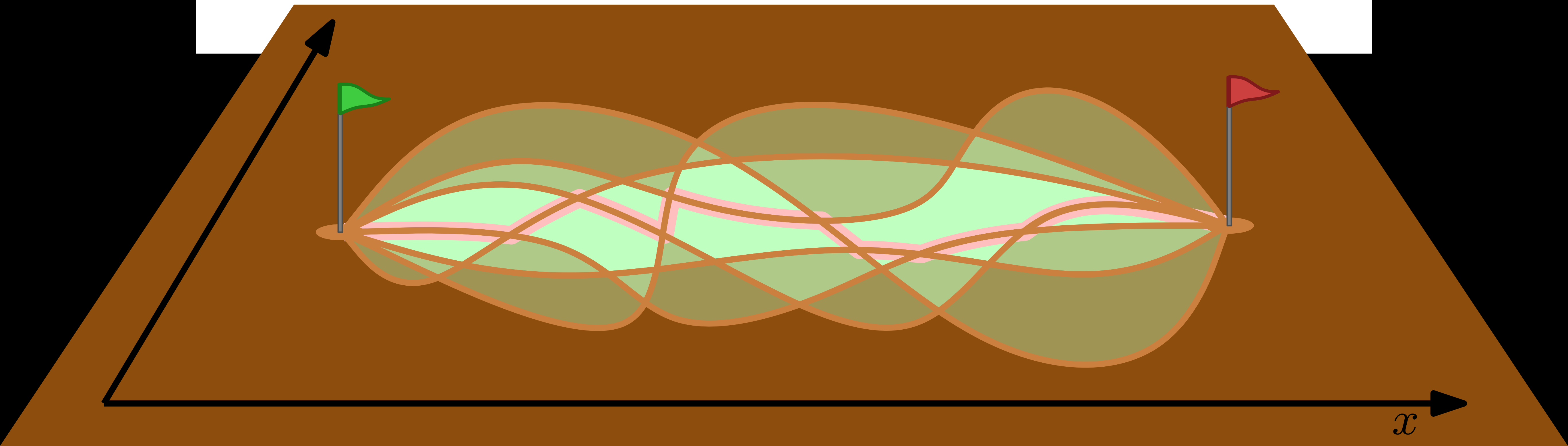

MINIMIZING $\mathcal{D}$

- Suppose that
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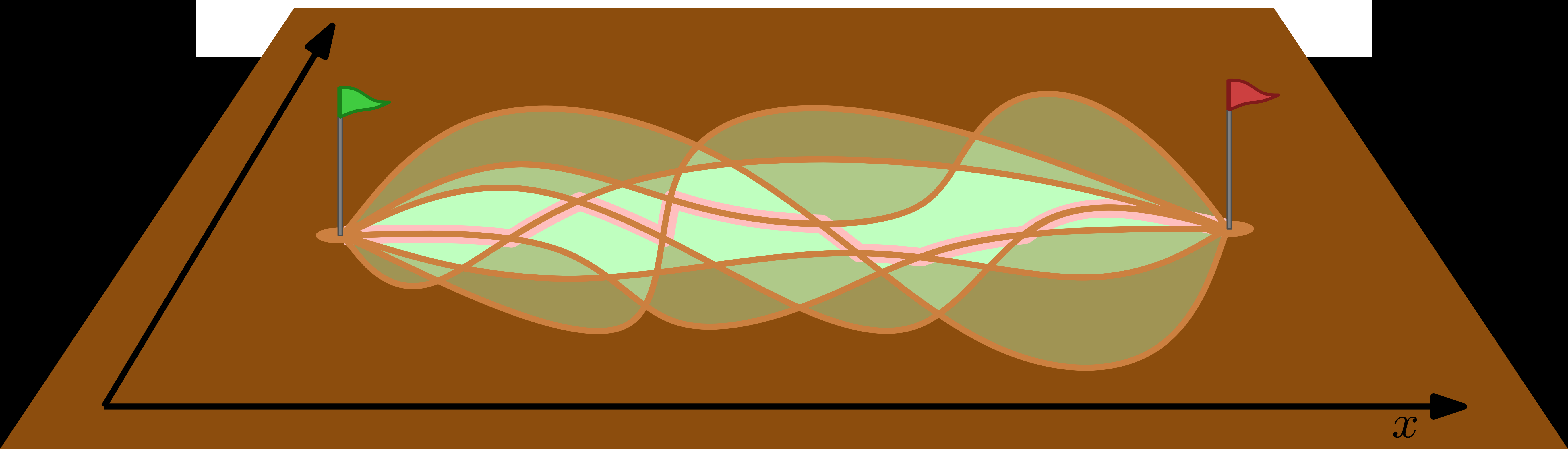
- We can rewrite $\mathcal{D}(r)$ to

$$\mathcal{D}(r) \backsimeq \int_x \sum_{T \in \mathcal{T}} |r(x) - T(x)| \, \mathrm{d}x$$



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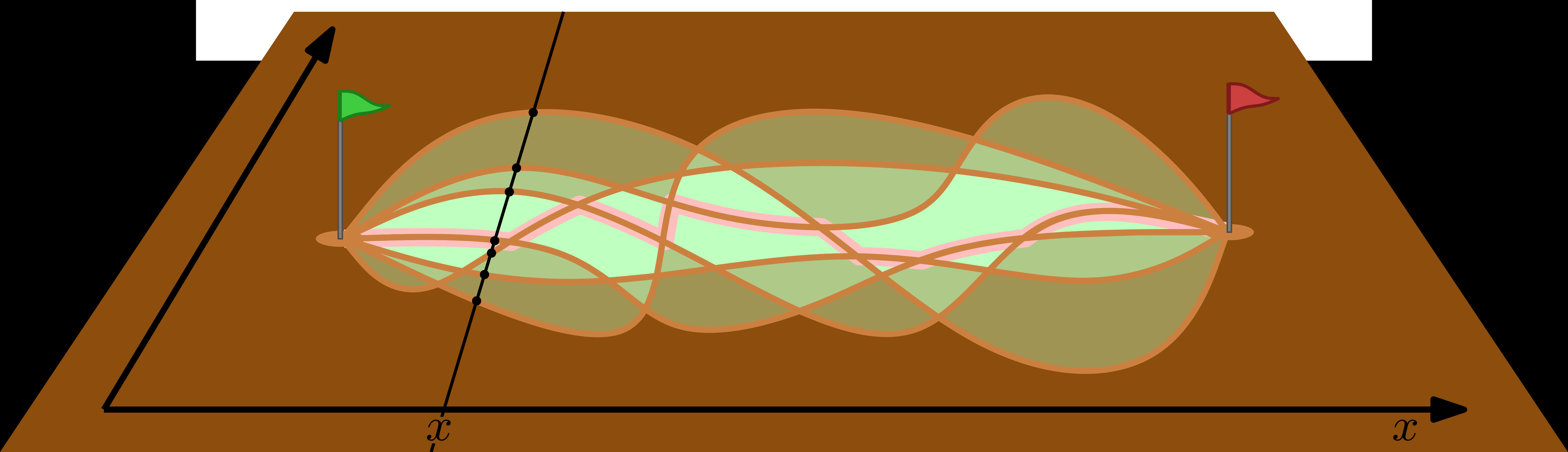
$$\mathcal{D}(r) \backsimeq \int_x \sum_{T \in \mathcal{T}} |r(x) - T(x)| \, \mathrm{d}x$$



MINIMIZING $\mathcal{D}$

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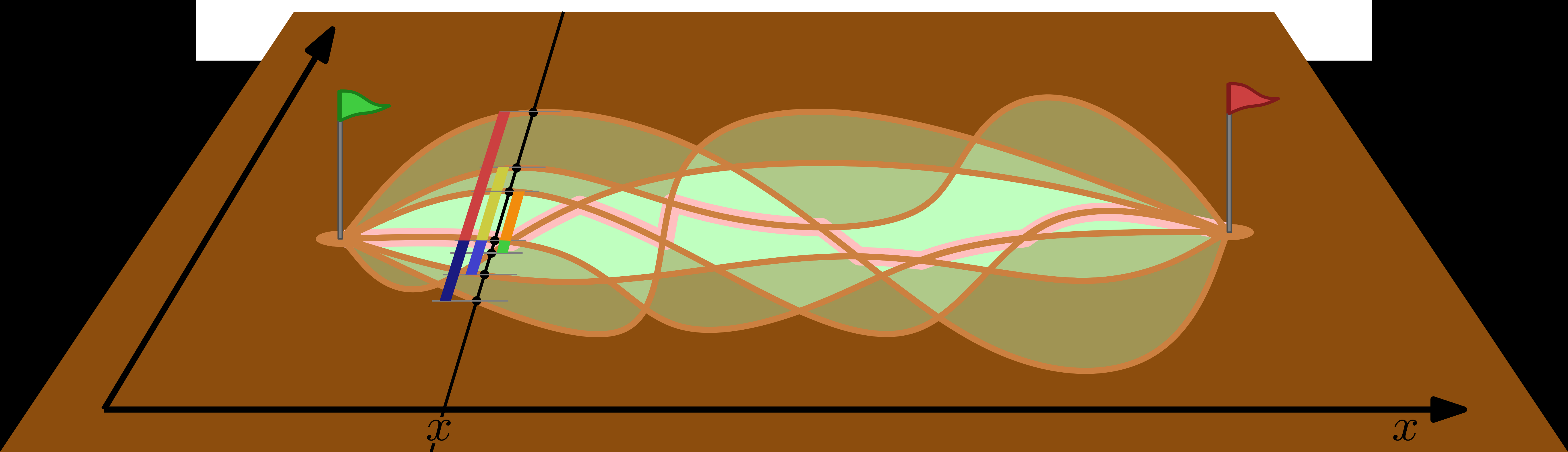

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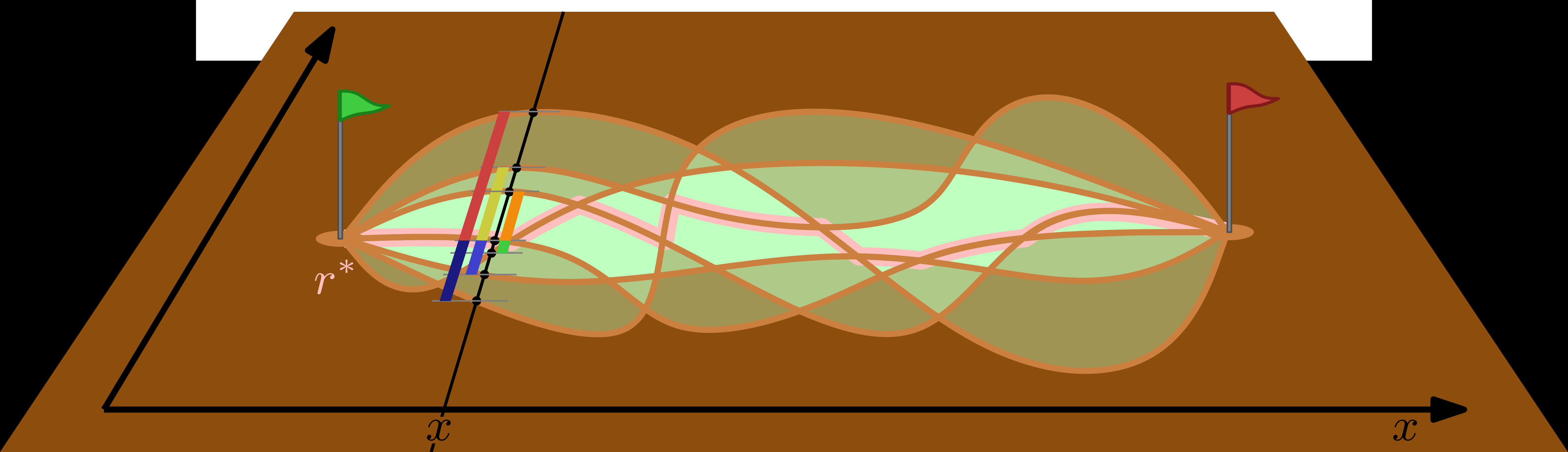

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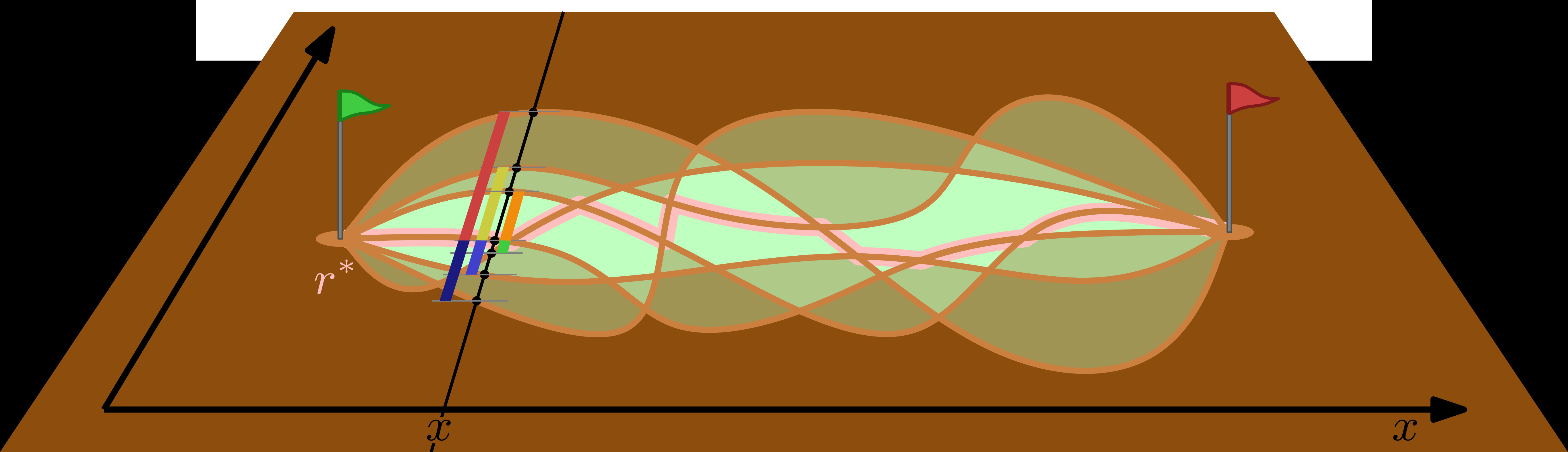

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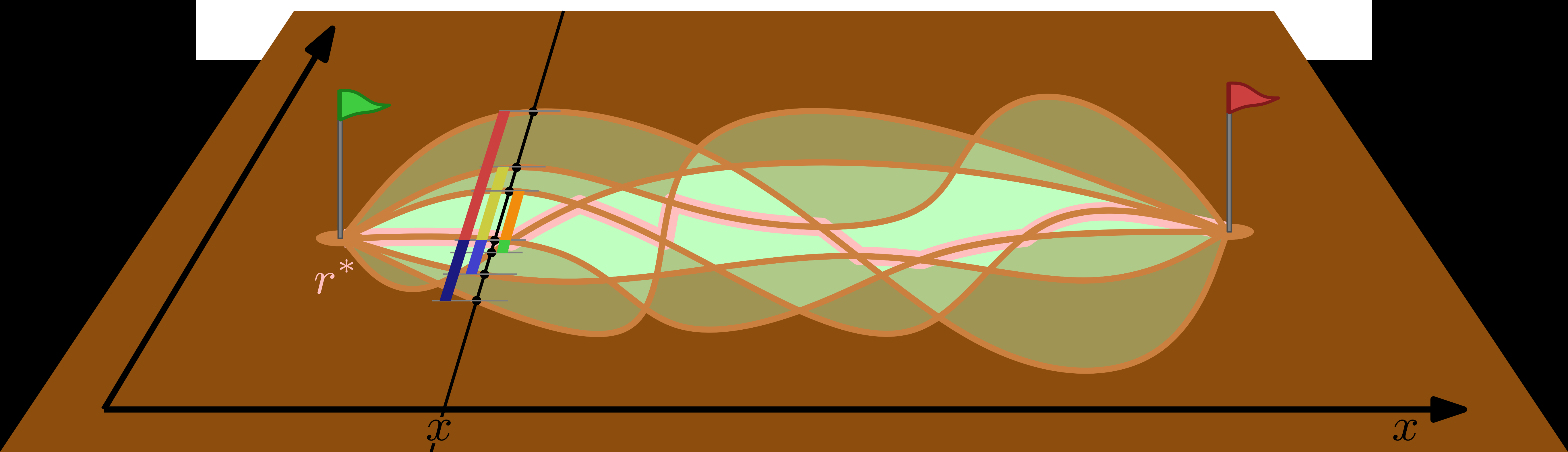

MINIMIZING $\mathcal{D}$

- Suppose that
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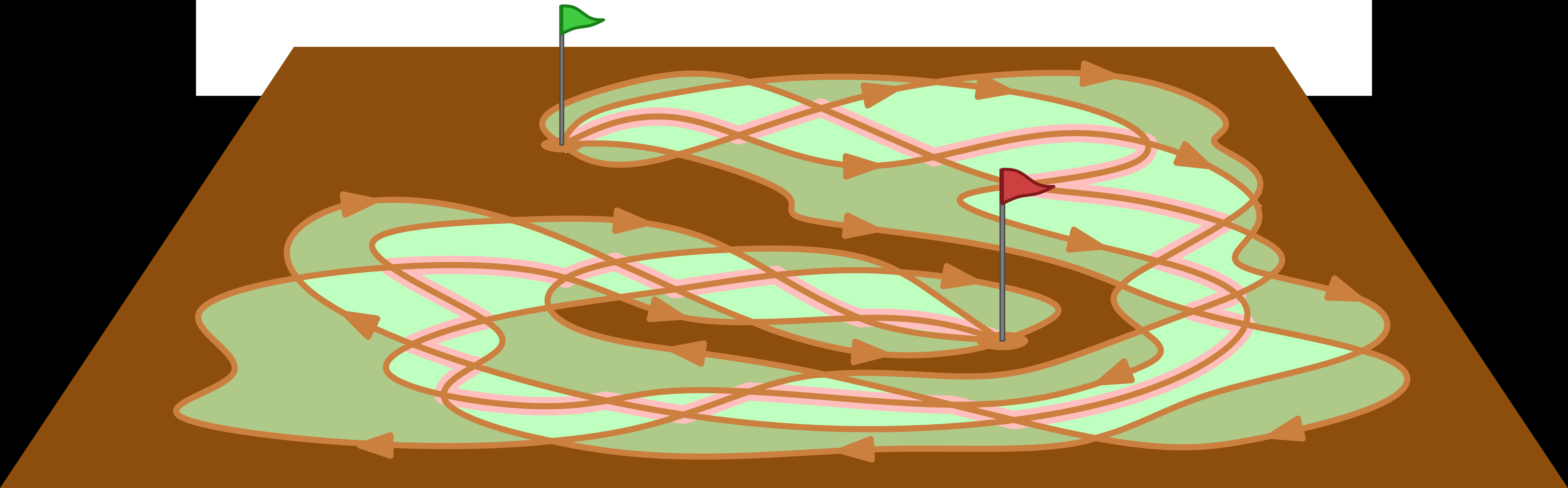
  - s and t, lie in the outer face



MINIMIZING $\mathcal{D}$

- Suppose that
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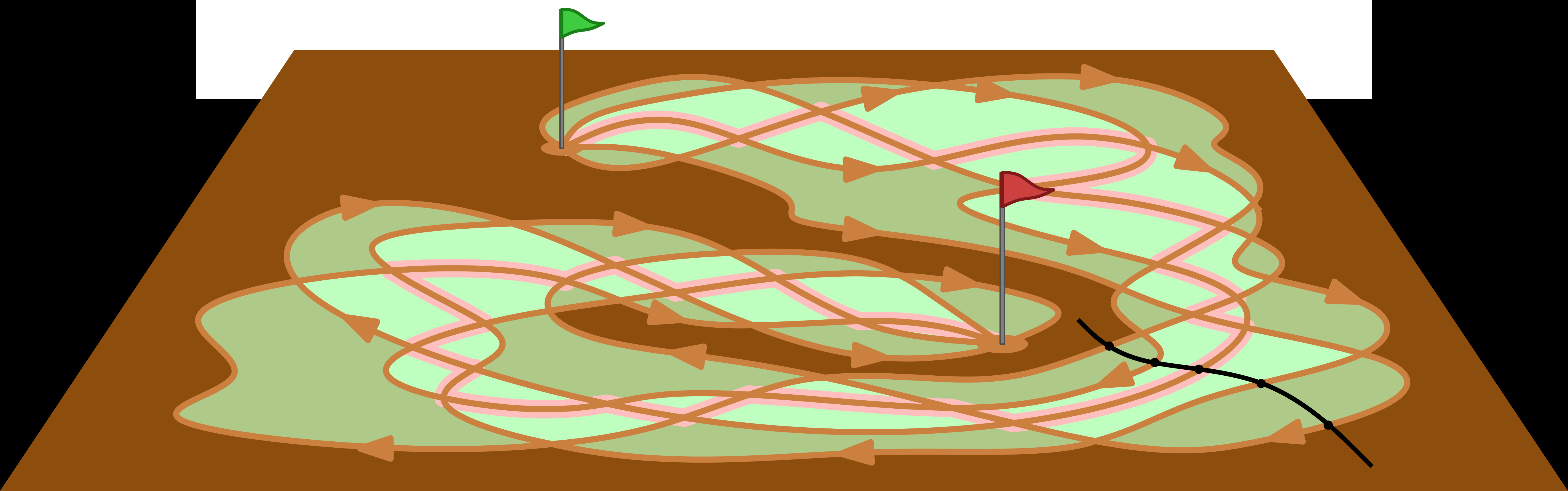
  - s and t, lie in the outer face
- We can rewrite $\mathcal{D}(r)$ to

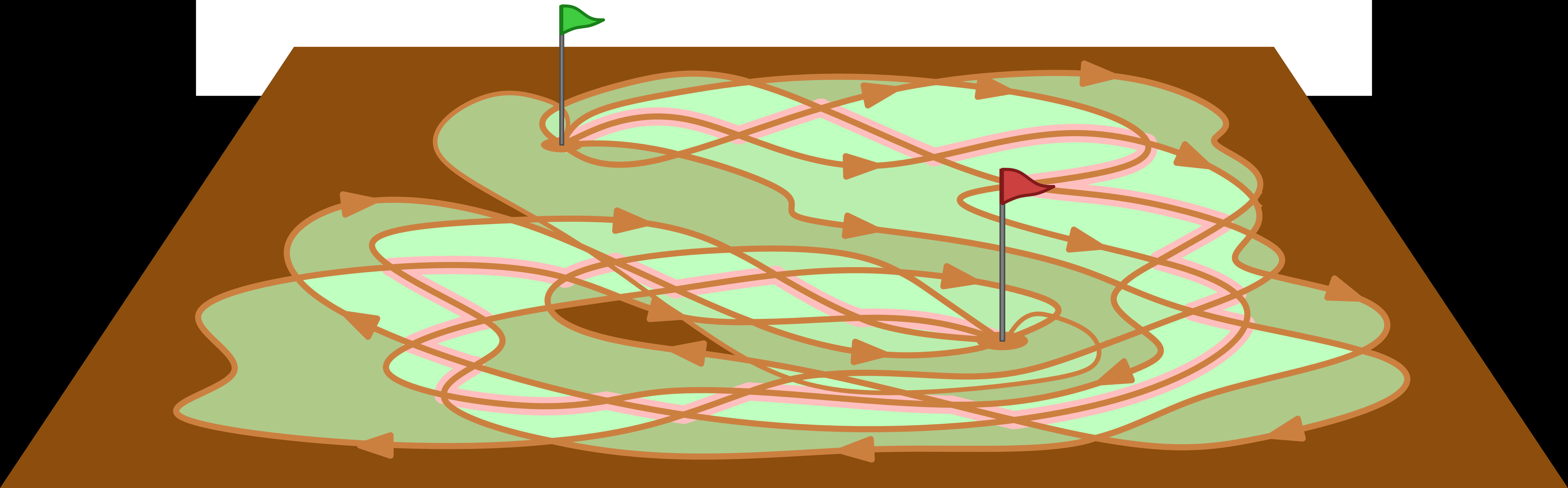
$$\mathcal{D}(r) \backsimeq \int_{\lambda} \sum_{T \in \mathcal{T}} curvelength(r, T, \lambda)\,\mathrm{d}\lambda$$



MINIMIZING $\mathcal{D}$

- Suppose that
  - the trajectories form a DAG

- Transform the space s.t. s and t lie on the outer face.



FUTURE WORK

- Done?

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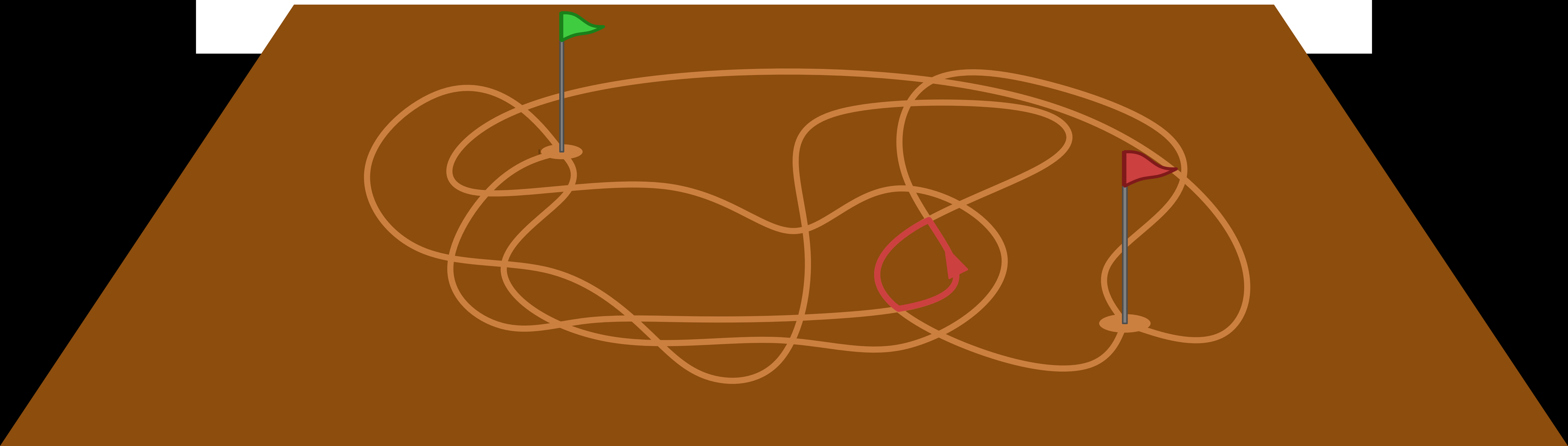
- No
  - How to handle larger class of graphs?



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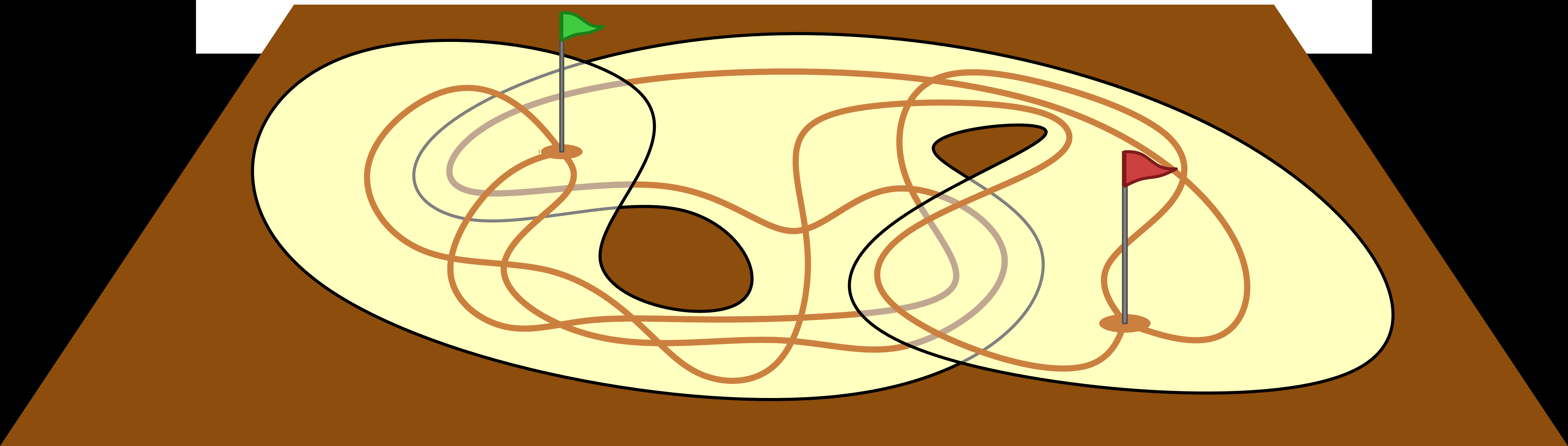

FUTURE WORK

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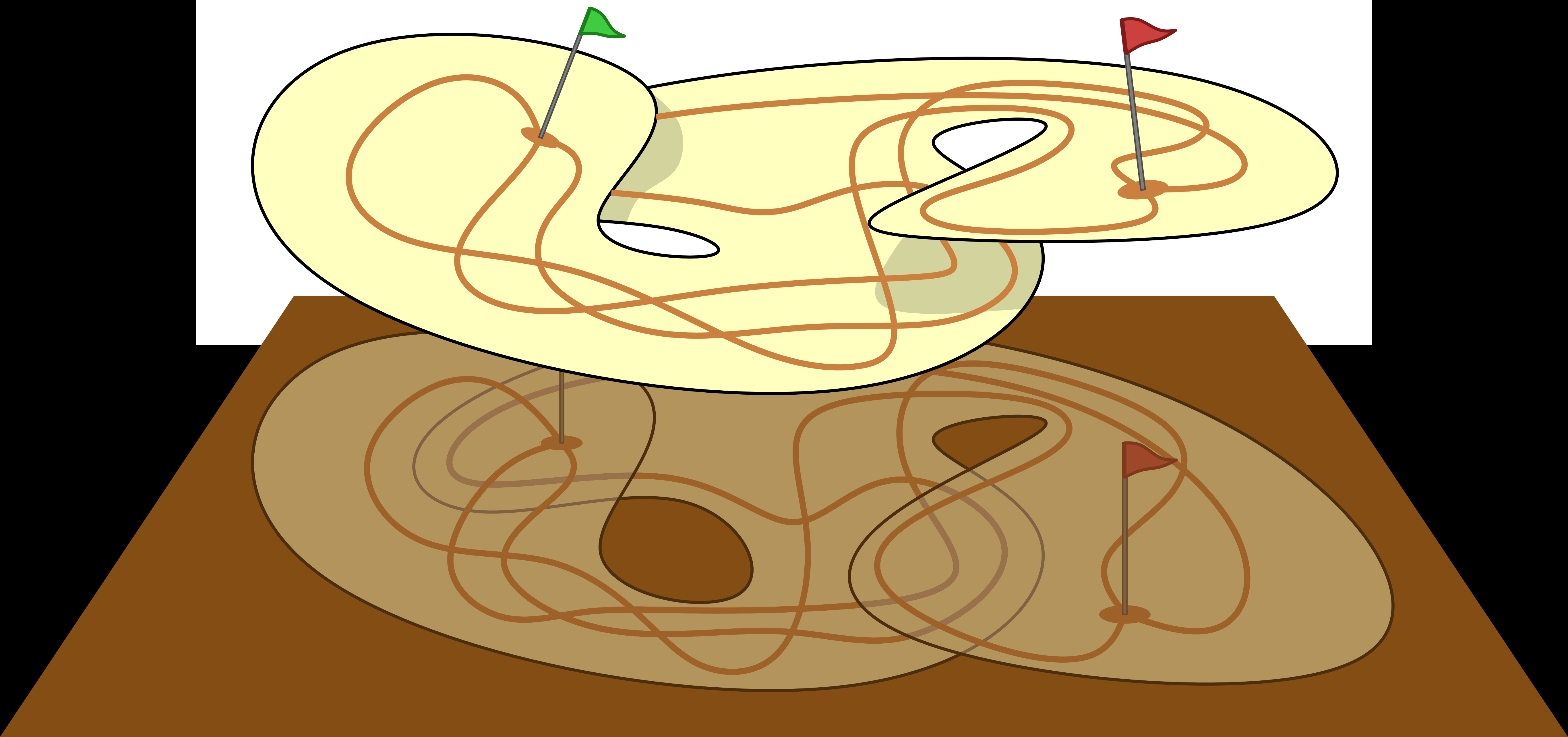
  - How to handle larger class of graphs?
  - Lift to space in which graph is a DAG



FUTURE WORK

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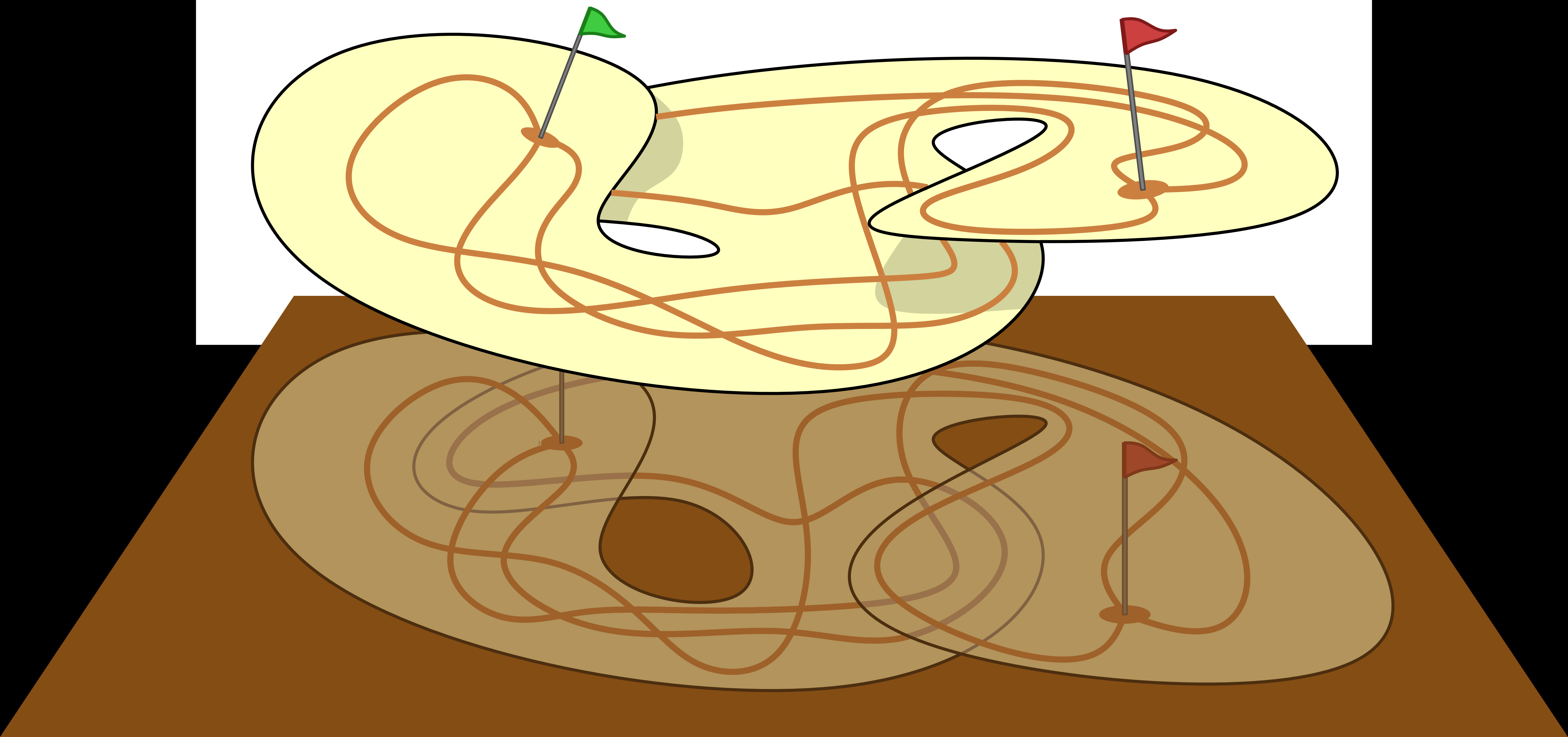
- No
  - How to handle larger class of graphs?
  - Lift to space in which graph is a DAG
  - How to define “corridor”?



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Thank You!

MIN MAX IS NP-HARD

Reduction from PARTITION:

Partition a set of integers $S = \{a_1, a_2, \ldots, a_n\}$ into two subsets S_1 and S_2 with equal total sums:

$$\sum_{a \in S_1} a = \sum_{a \in S_2} a$$