HOMOTOPY MEASURES FOR REPRESENTATIVE TRAJECTORIES

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TRAJECTORIES
Let $P$ be $n$ points in the plane
TRAJECTORIES

- Let $P$ be $n$ points in the plane
- Now suppose your points run away
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Now suppose your points run away
$P$ traces a set of $n$ trajectories: curves in $\mathbb{R}^2$
• Trajectories are ubiquitous
TRAJECTORIES

- Trajectories are ubiquitous
  - GPS technology
  - Cyclists
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• Trajectories are interesting
  • Many different analysis tasks
REPRESENTATIVE TRAJECTORY

- Problem
  - Suppose we have lots of trajectories
• Problem
  • Suppose we have lots of trajectories
  • Suppose we want to extract significant patterns
Representative Trajectory

- Problem
  - Suppose we have lots of trajectories
  - Suppose we want to extract significant patterns
- Solution
REPRESENTATIVE TRAJECTORY

- Problem
  - Suppose we have lots of trajectories
  - Suppose we want to extract significant patterns

- Solution
  - Cluster the trajectories
REPRESENTATIVE TRAJECTORY

• Problem
  • Suppose we have lots of trajectories
  • Suppose we want to extract significant patterns

• Solution
  • Cluster the trajectories
  • Pick a good representative for each cluster
Suppose we have lots of trajectories
Suppose we want to extract significant patterns

Solution
Cluster the trajectories
Pick a good representative for each cluster
Keep only the representatives
REPRESENTATIVE TRAJECTORY

- Problem
  - Suppose we have lots of trajectories
  - Suppose we want to extract significant patterns

- Solution
  - Cluster the trajectories
  - Pick a good representative for each cluster
  - Keep only the representatives

- But what is a good representative?
REPRESENTATIVE TRAJECTORY

- Input: a set of ‘similar’ trajectories
  - Sort of the same shape
REPRESENTATIVE TRAJECTORY

- Input: a set of ‘similar’ trajectories
  - Sort of the same shape
- Output: a representative trajectory
Input: a set of ‘similar’ trajectories
   • Sort of the same shape

Output: a representative trajectory
   • Should also have sort of the same shape
**REPRESENTATIVE TRAJECTORY**

- **Input:** a set of ‘similar’ trajectories
  - Sort of the same shape

- **Output:** a representative trajectory
  - Should also have sort of the same shape
  - Shape should represent the whole set of input trajectories
REPRESENTATIVE TRAJECTORY

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  - Shape should represent the whole set of input trajectories
OBVIOUS REPRESENTATIVES
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- Use one of the input trajectories
OBVIOUS REPRESENTATIVES

- Use one of the input trajectories
- There may not be any single good representative!
OBVIOUS REPRESENTATIVES

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- Pick the mean trajectory
OBVIOUS REPRESENTATIVES

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- Pick the mean trajectory
  - May interfere with environment!
OBVIOUS REPRESENTATIVES

- Use one of the input trajectories
  - There may not be any single good representative!

- Pick the mean trajectory
  - May interfere with environment!

- Use pieces of different trajectories
MEDIAN TRAJECTORIES

- Buchin et.al. [ESA,2010] present two such representatives:
MEDIAN TRAJECTORIES

- Buchin et.al. [ESA,2010] present two such representatives:
  - Start in the middle, switch at every intersection
MEDIAN TRAJECTORIES

- Buchin et.al. [ESA,2010] present two such representatives:
  - Start in the middle, switch at every intersection
  - Mark important faces, pick the median that passes on "the right side" of each face.
OUR APPROACH
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- Trajectories are just curves
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  - Arrangement of curves forms a graph
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  - Edges are directed
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- Output $r$ is a path in this graph
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- Output \( r \) is a path in this graph

- Define the quality of a path?
**OUR APPROACH**

- Trajectories are just curves
  - Arrangement of curves forms a graph
  - Edges are directed

- Output $r$ is a path in this graph

- Define the quality of a path?
  - We define a distance measure between $r$ and all trajectories.
OUR APPROACH

- Let $D$ be a distance measure between two curves
OUR APPROACH

- Let $D$ be a distance measure between two curves

\[ D(r) = \sum_{T \in \mathcal{T}} D(r, T) \]

\[ M(r) = \max_{T \in \mathcal{T}} D(r, T) \]
OUR APPROACH

• Let $D$ be a distance measure between two curves
  • We use Homotopy Area

• $D(r) = \sum_{T \in \mathcal{T}} D(r, T)$

• $M(r) = \max_{T \in \mathcal{T}} D(r, T)$
HOMOTOPY AREA??????
\[ D(A, B) = \inf_{H \in \mathcal{H}(A, B)} \int_{u \in [0, 1]} \int_{w \in [0, 1]} \left| \frac{dH}{du} \times \frac{dH}{dw} \right| du \, dw, \]

where \( \mathcal{H}(A, B) = \ldots \)
**HOMOTOPY AREA? ? ? ? ?**

- \( D(A, B) = \) the minimum area that we have to sweep curve \( A \) over to transform it into \( B \).
HOMOTOPY AREA??????

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HOMOTOPY AREA

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- Why homotopy area?
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  - robust against outliers
HOMOTOPY AREA

- $D(A, B) =$ the minimum area that we have to sweep curve $A$ over to transform it into $B$.
- Why homotopy area?
  - it does not need a parametrization of the curves.
  - robust against outliers
  - tries to capture important faces automatically
HOMOTOPY AREA

- We assume that our trajectories:
  - start in $s$ and end in $t$
We assume that our trajectories:
- start in $s$ and end in $t$
- are simple
RESULTS

- Finding $r^*$ that minimizes
  - $\mathcal{M}(r) = \max_{T \in \mathcal{T}} D(r, T)$
  - $D(r) = \sum_{T \in \mathcal{T}} D(r, T)$
RESULTS

- Finding $r^*$ that minimizes
  - $\mathcal{M}(r) = \max_{T \in \mathcal{T}} D(r, T)$ is NP-hard
  - $\mathcal{D}(r) = \sum_{T \in \mathcal{T}} D(r, T)$
RESULTS

- Finding $r^*$ that minimizes
  - $M(r) = \max_{T \in \mathcal{T}} D(r, T)$
    is NP-hard, even for 2 $x$-monotone trajectories
  - $D(r) = \sum_{T \in \mathcal{T}} D(r, T)$
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  - $\mathcal{D}(r) = \sum_{T \in \mathcal{T}} D(r, T)$
    is NP-hard, even for 3 trajectories
RESULTS

- Finding $r^*$ that minimizes
  \[ M(r) = \max_{T \in T} D(r, T) \]
  is NP-hard, even for 2 $x$-monotone trajectories

- $D(r) = \sum_{T \in T} D(r, T)$
  is NP-hard, even for 3 trajectories

  Solvable efficiently when the trajectories from a DAG
MINIMIZING $D$

- Suppose that
MINIMIZING $D$

- Suppose that
- the trajectories are $x$-monotone
MINIMIZING $\mathcal{D}$

- Suppose that
  - the trajectories are $x$-monotone

- We can rewrite $\mathcal{D}(r)$ to
  $$\mathcal{D}(r) \simeq \int \sum_{T \in \mathcal{T}} |r(x) - T(x)| \, dx$$
MINIMIZING $\mathcal{D}$

- Suppose that
  - the trajectories are $x$-monotone

- We can rewrite $\mathcal{D}(r)$ to
  
  $$
  \mathcal{D}(r) \equiv \int_x \sum_{T \in \mathcal{T}} |r(x) - T(x)| \, dx
  $$
MINIMIZING $\mathcal{D}$

- Suppose that
  - the trajectories are $x$-monotone

- We can rewrite $\mathcal{D}(r)$ to

$$
\mathcal{D}(r) \simeq \int_x \sum_{T \in \mathcal{T}} |r(x) - T(x)| \, dx
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MINIMIZING $\mathcal{D}$

- Suppose that
  - the trajectories are $x$-monotone

- We can rewrite $\mathcal{D}(r)$ to
  $$\mathcal{D}(r) \leq \int \sum_{T \in T} |r(x) - T(x)| \, dx$$

- Let $r^*$ be the the $n/2$ level.
MINIMIZING $\mathcal{D}$

- Suppose that
  - the trajectories are $x$-monotone

- We can rewrite $\mathcal{D}(r)$ to
  \[ \mathcal{D}(r) \preceq \int_x \sum_{T \in \mathcal{T}} |r(x) - T(x)| \, dx \]

- Let $r^*$ be the $n/2$ level.
  - $r^*$ minimizes $\mathcal{D}$
MINIMIZING $\mathcal{D}$

- Suppose that
  - the trajectories are $x$-monotone

- We can rewrite $\mathcal{D}(r)$ to
  \[ \mathcal{D}(r) \leq \int \sum_{T \in \mathcal{T}} |r(x) - T(x)| \, dx \]

- Let $r^*$ be the the $n/2$ level.
  - $r^*$ minimizes $\mathcal{D}$
  - $r^*$ is the simple median
Suppose that
• the trajectories form a DAG
• \( s \) and \( t \), lie in the outer face
MINIMIZING $\mathcal{D}$

- Suppose that
  - the trajectories form a DAG
  - $s$ and $t$, lie in the outer face
- We can rewrite $\mathcal{D}(r)$ to

\[
\mathcal{D}(r) \subseteq \int_{\lambda} \sum_{T \in T} \text{curvelength}(r, T, \lambda) \, d\lambda
\]
MINIMIZING $\mathcal{D}$

- Suppose that
  - the trajectories form a DAG

- Transform the space s.t. $s$ and $t$ lie on the outer face.
FUTURE WORK

• Done?
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• Done?
• No
  • How to handle larger class of graphs?
FUTURE WORK

- Done?
- No
  - How to handle larger class of graphs?
FUTURE WORK

- Done?
- No
  - How to handle larger class of graphs?
  - Lift to space in which graph is a DAG
FUTURE WORK

• Done?
• No
  • How to handle larger class of graphs?
  • Lift to space in which graph is a DAG
  • How to define “corridor”? 
FUTURE WORK

- Done?
- No
  - How to handle larger class of graphs?
  - Lift to space in which graph is a DAG
  - How to define “corridor”?

Thank You!
Reduction from PARTITION:

Partition a set of integers $S = \{a_1, a_2, \ldots, a_n\}$ into two subsets $S_1$ and $S_2$ with equal total sums:

$$\sum_{a \in S_1} a = \sum_{a \in S_2} a$$