

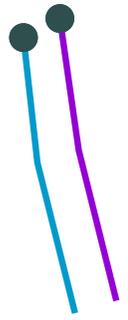
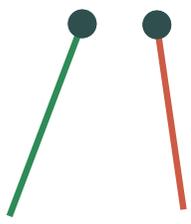
# Trajectory Grouping Structure

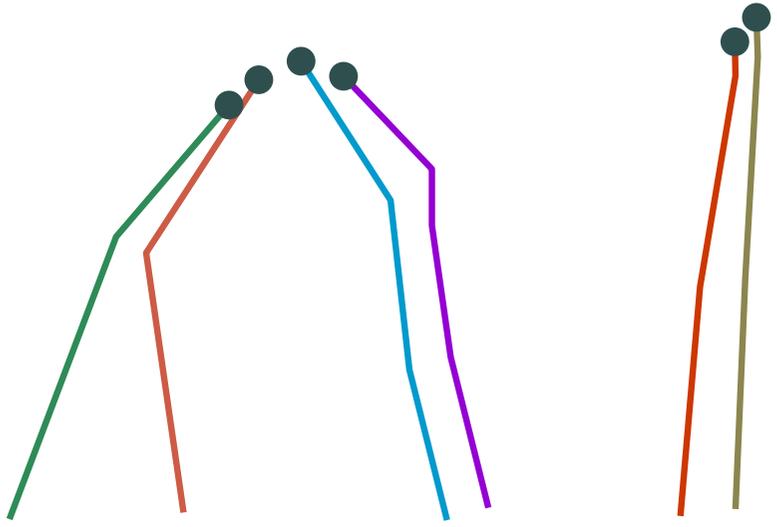
Kevin Buchin  
Maïke Buchin  
Marc van Kreveld  
Bettina Speckmann  
Frank Staals

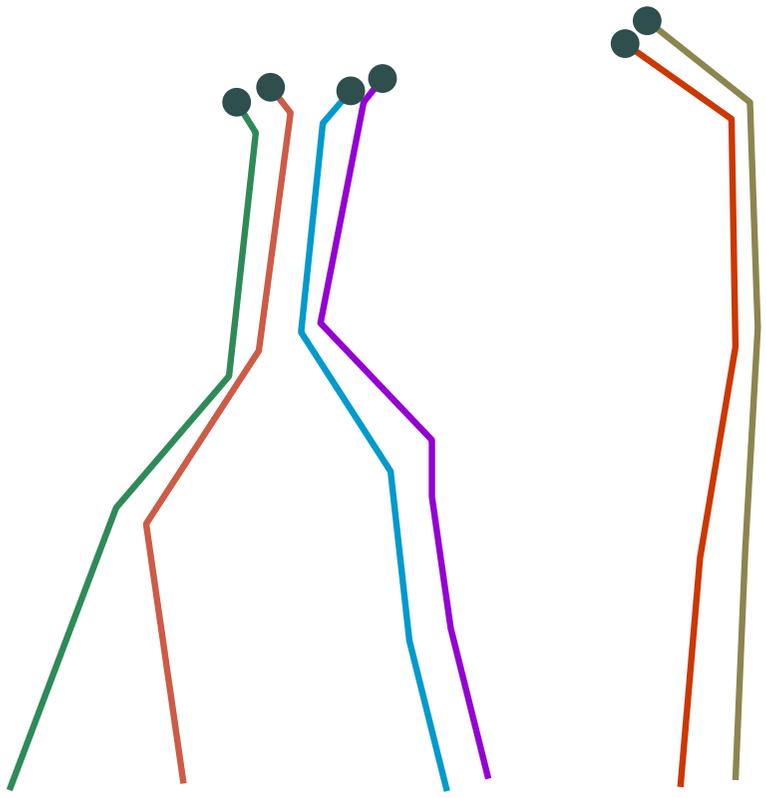
TU Eindhoven  
Utrecht University

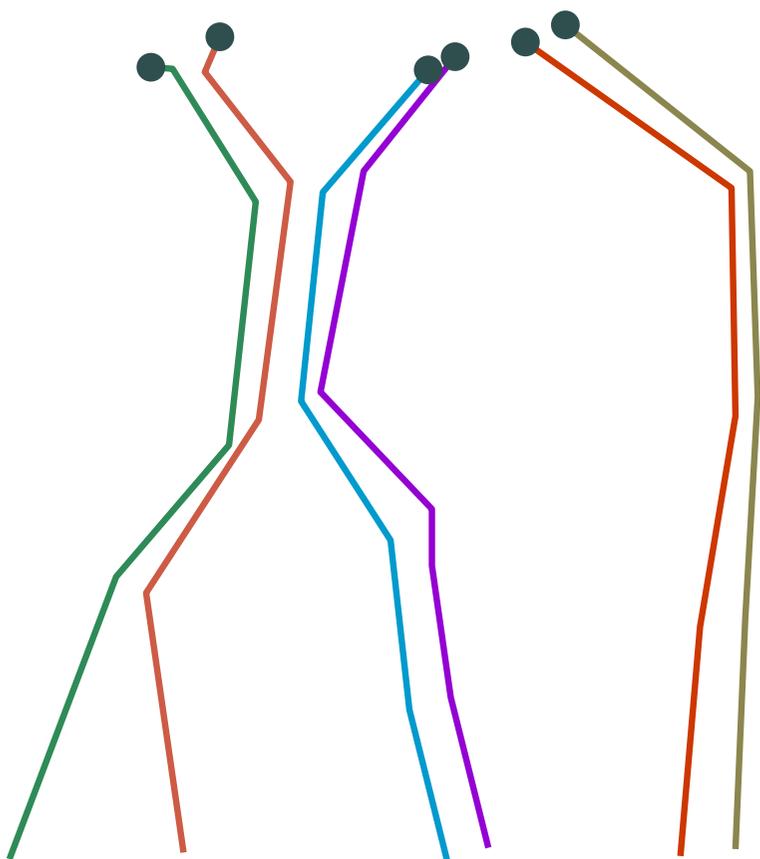


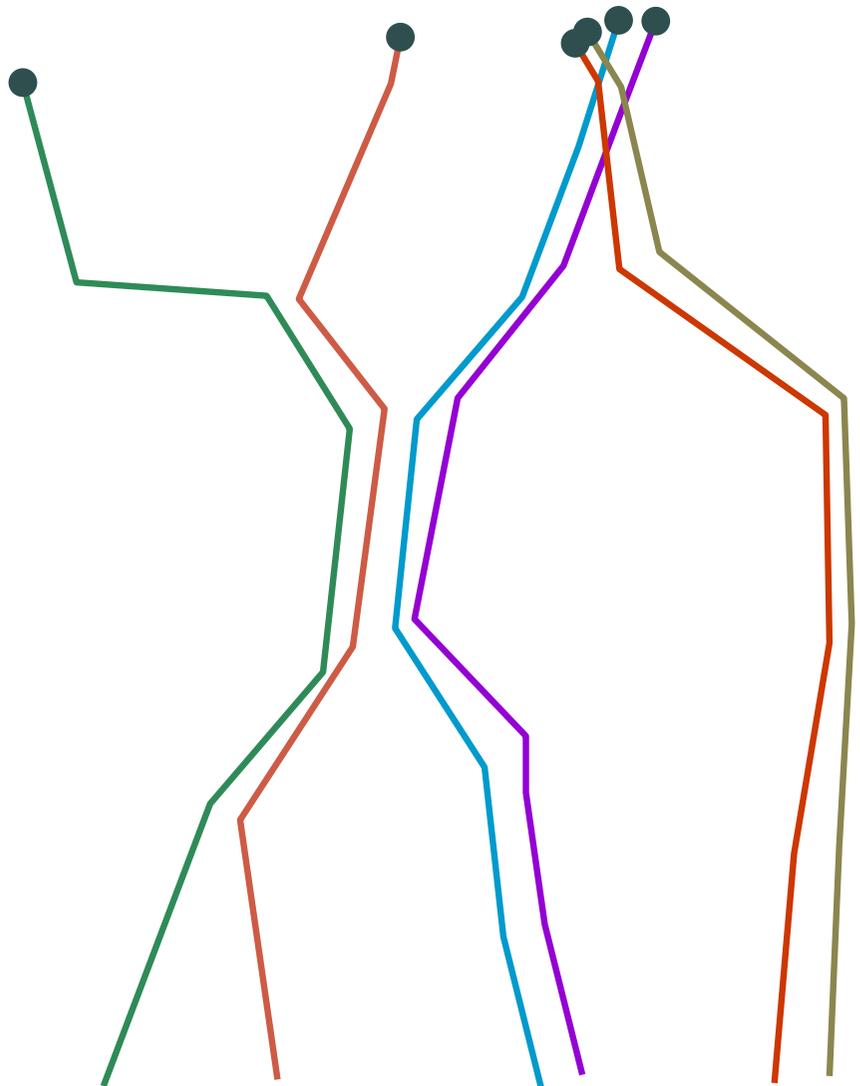


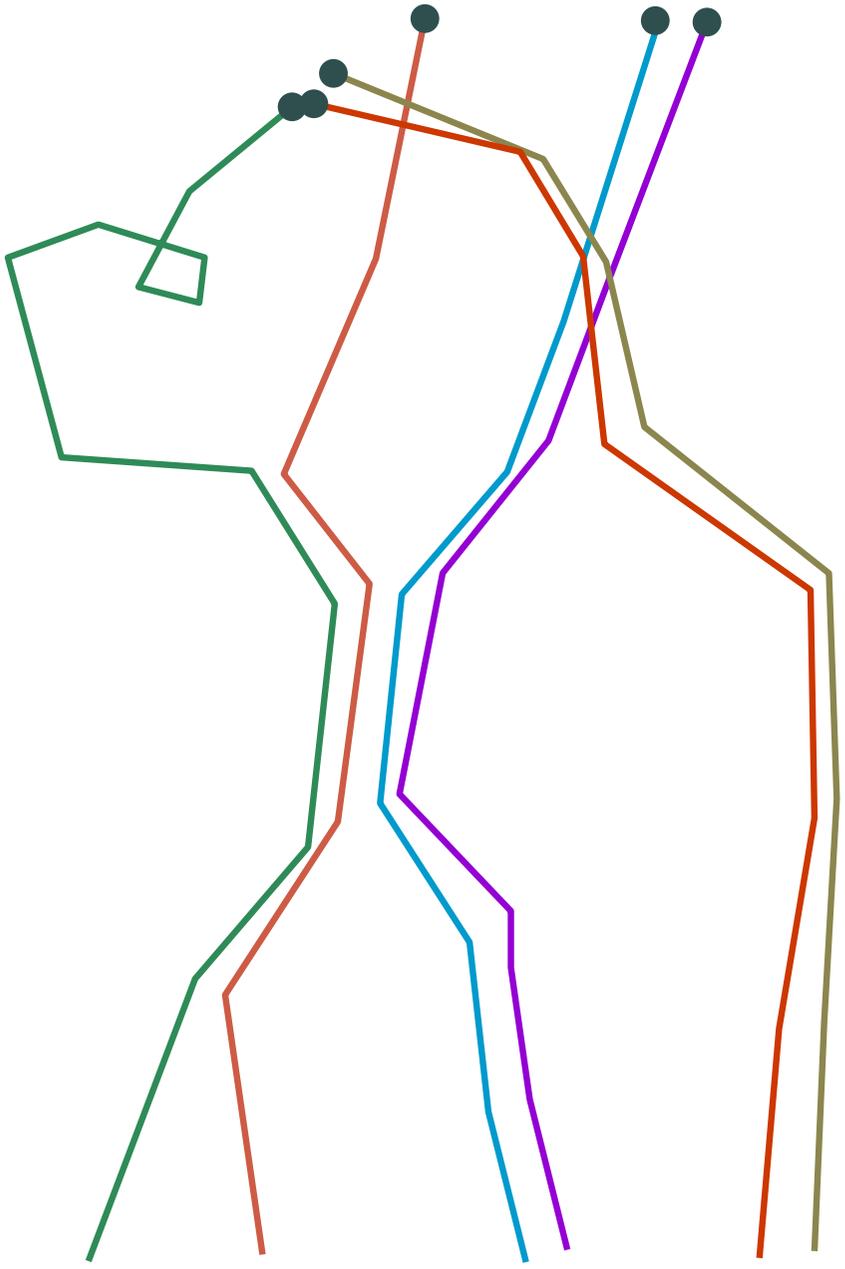


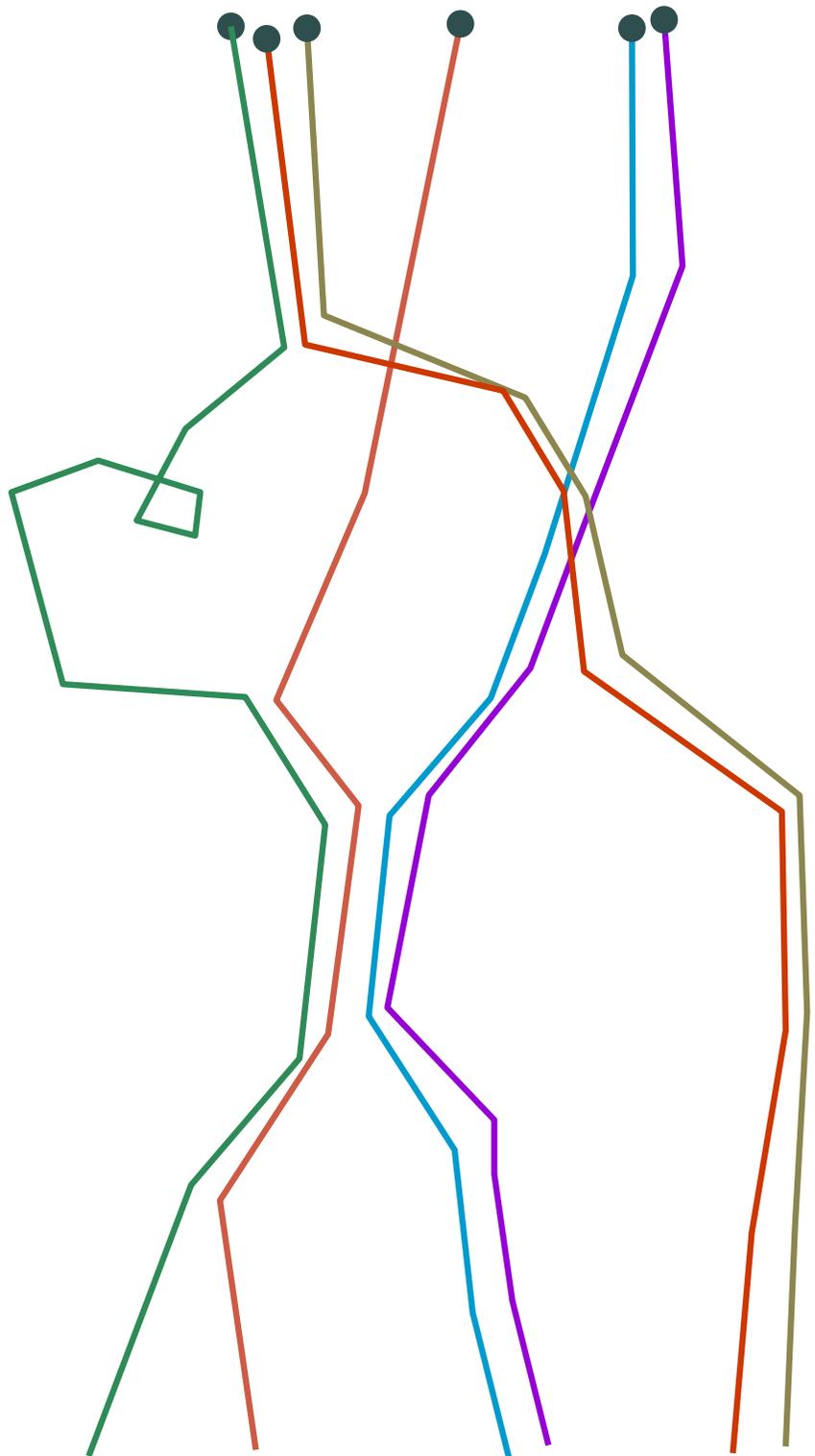


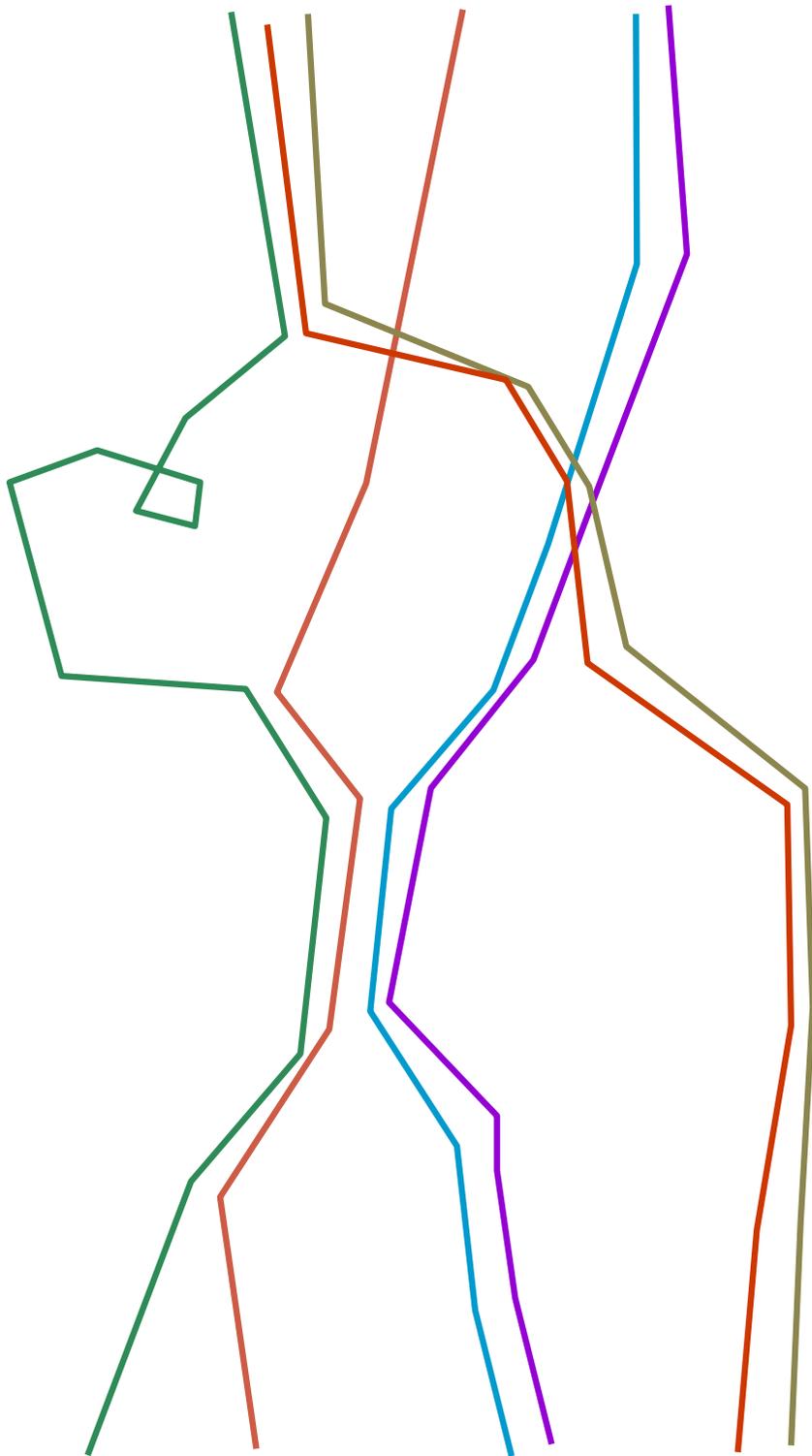








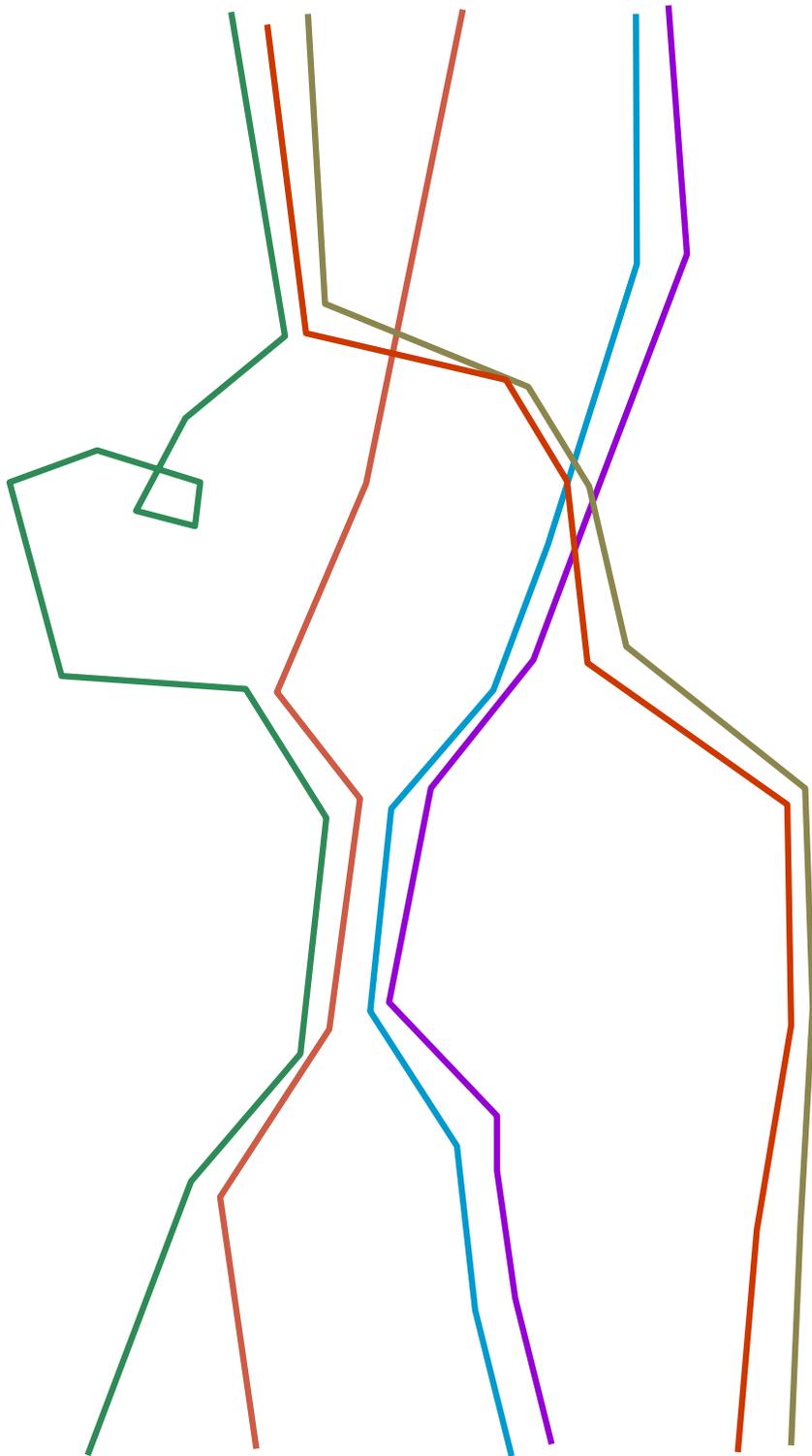




# Groups

We want to find **groups**,

and how the groups change over time.

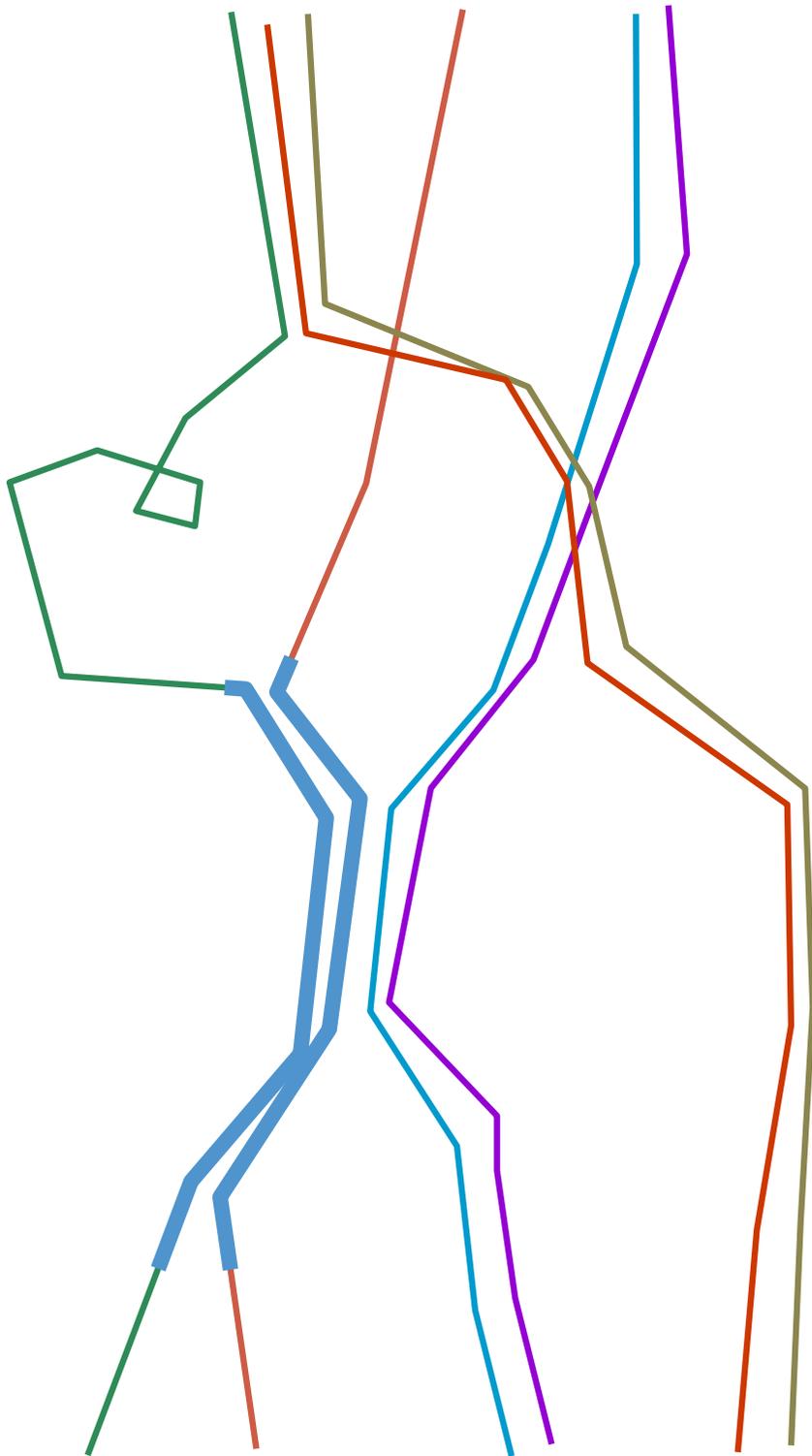


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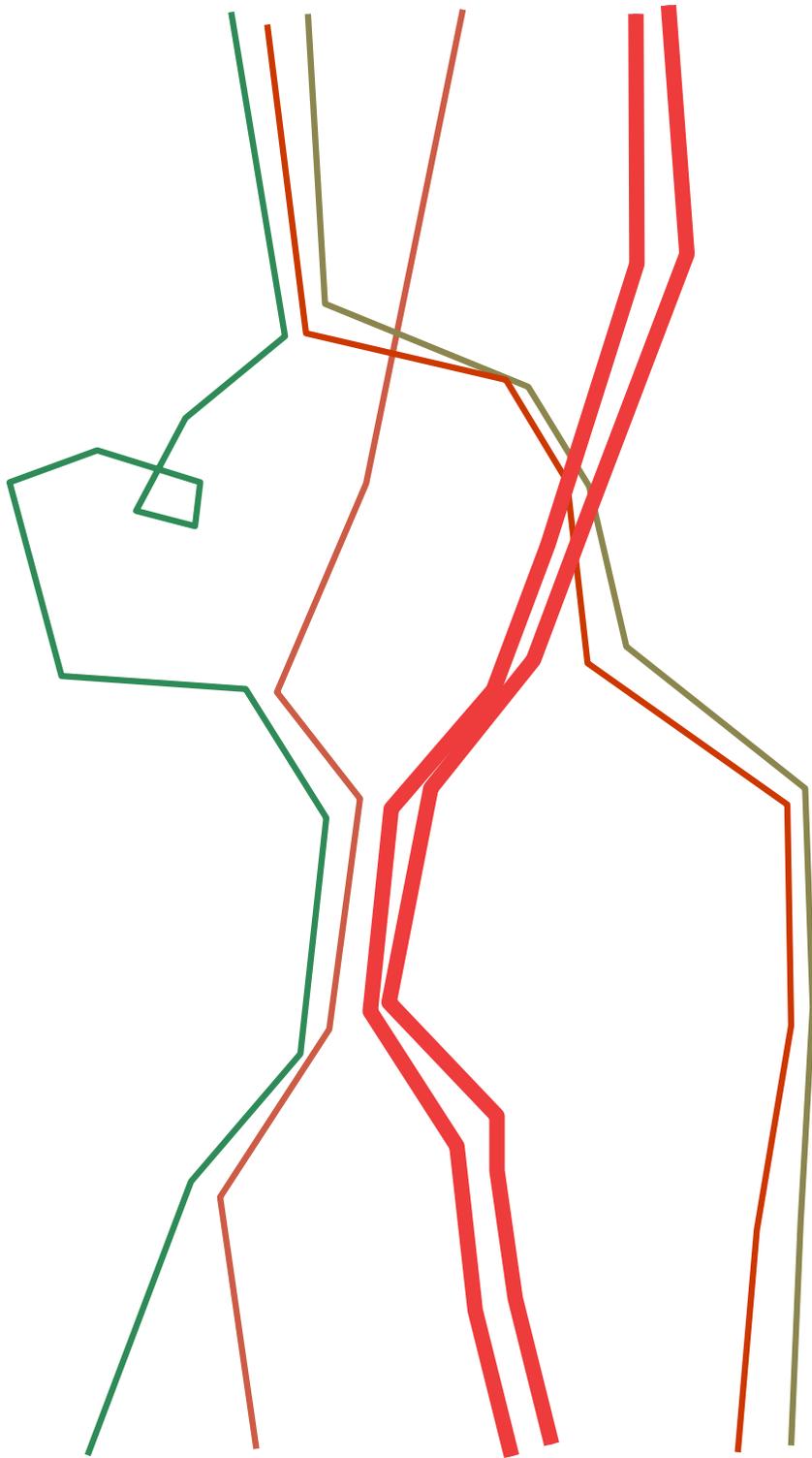


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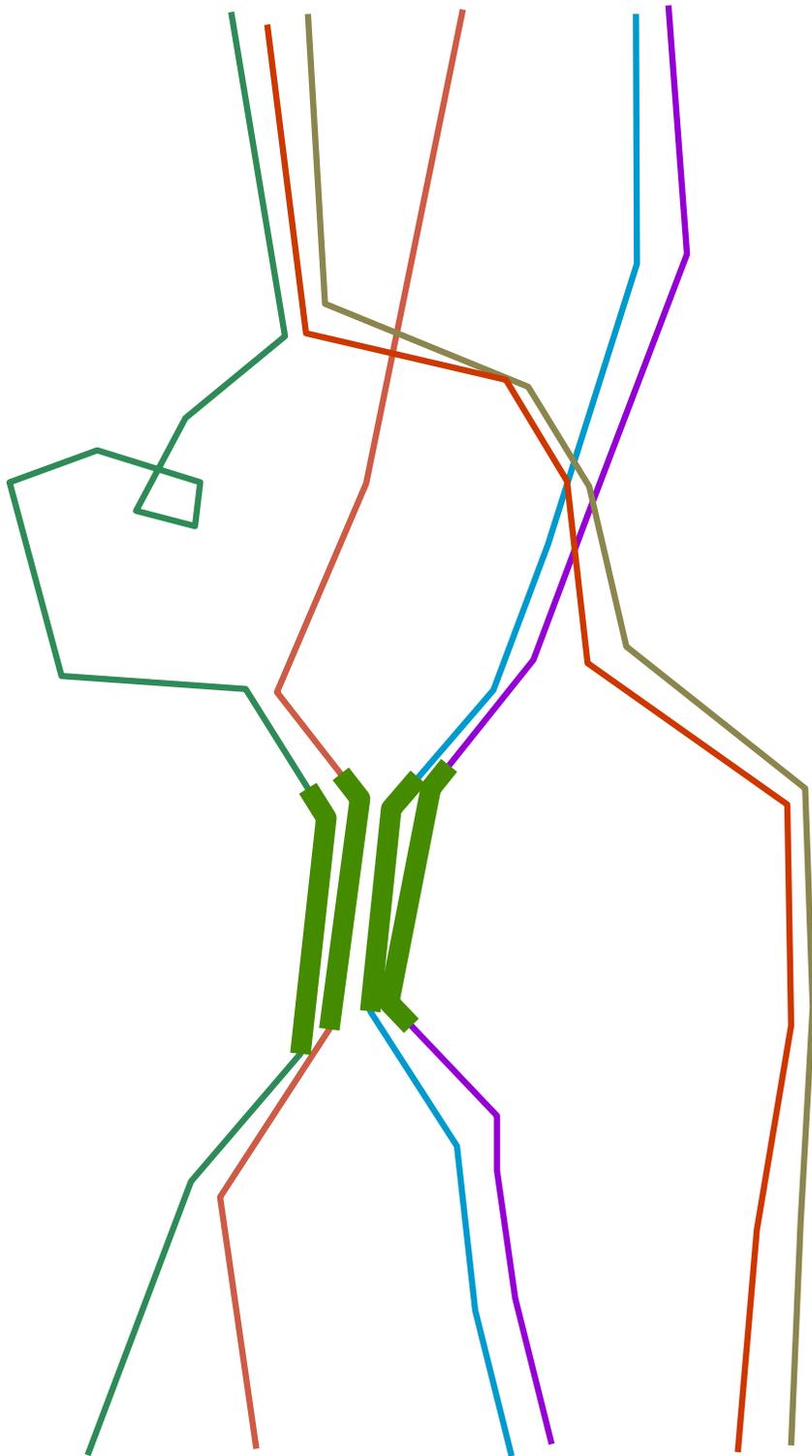


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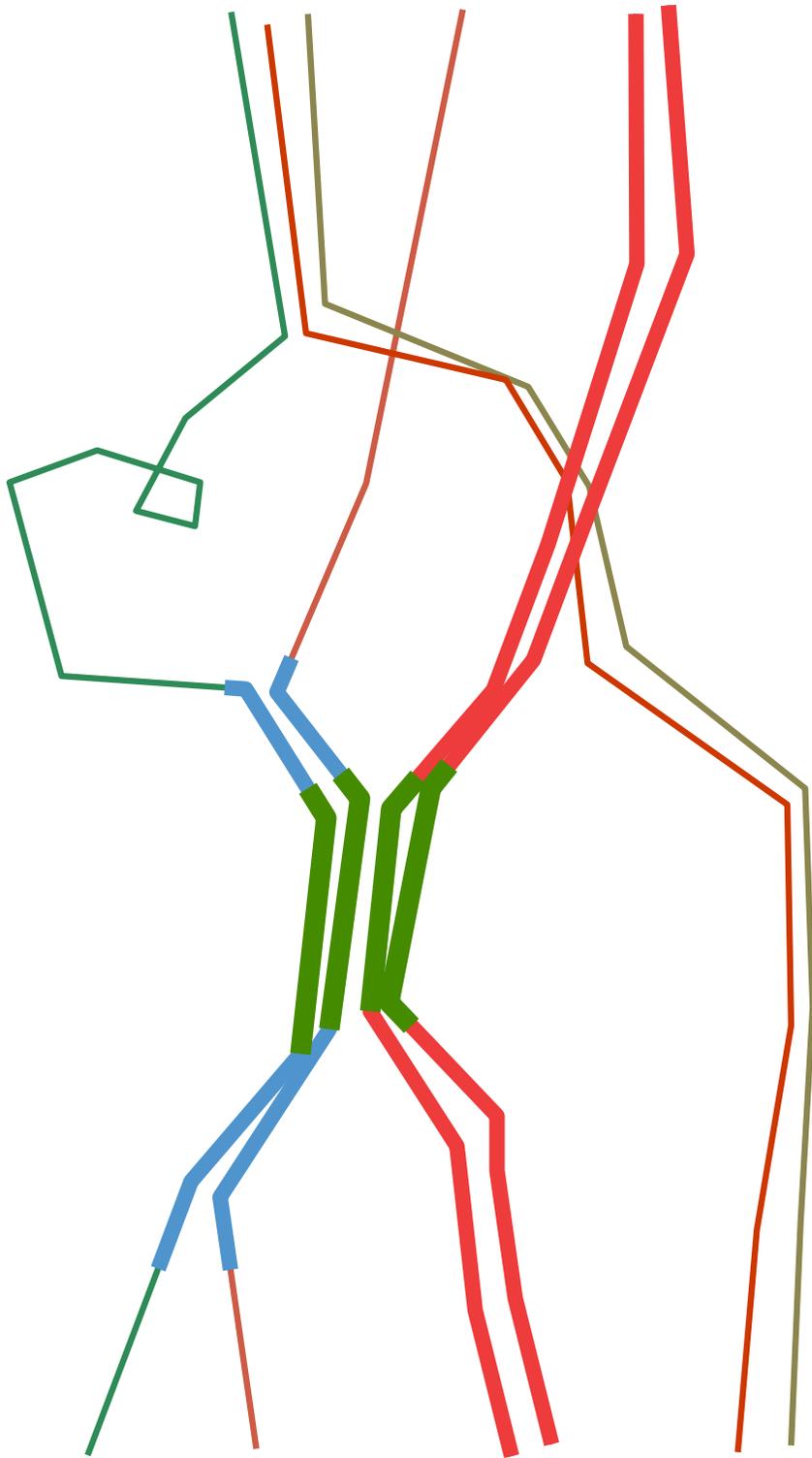


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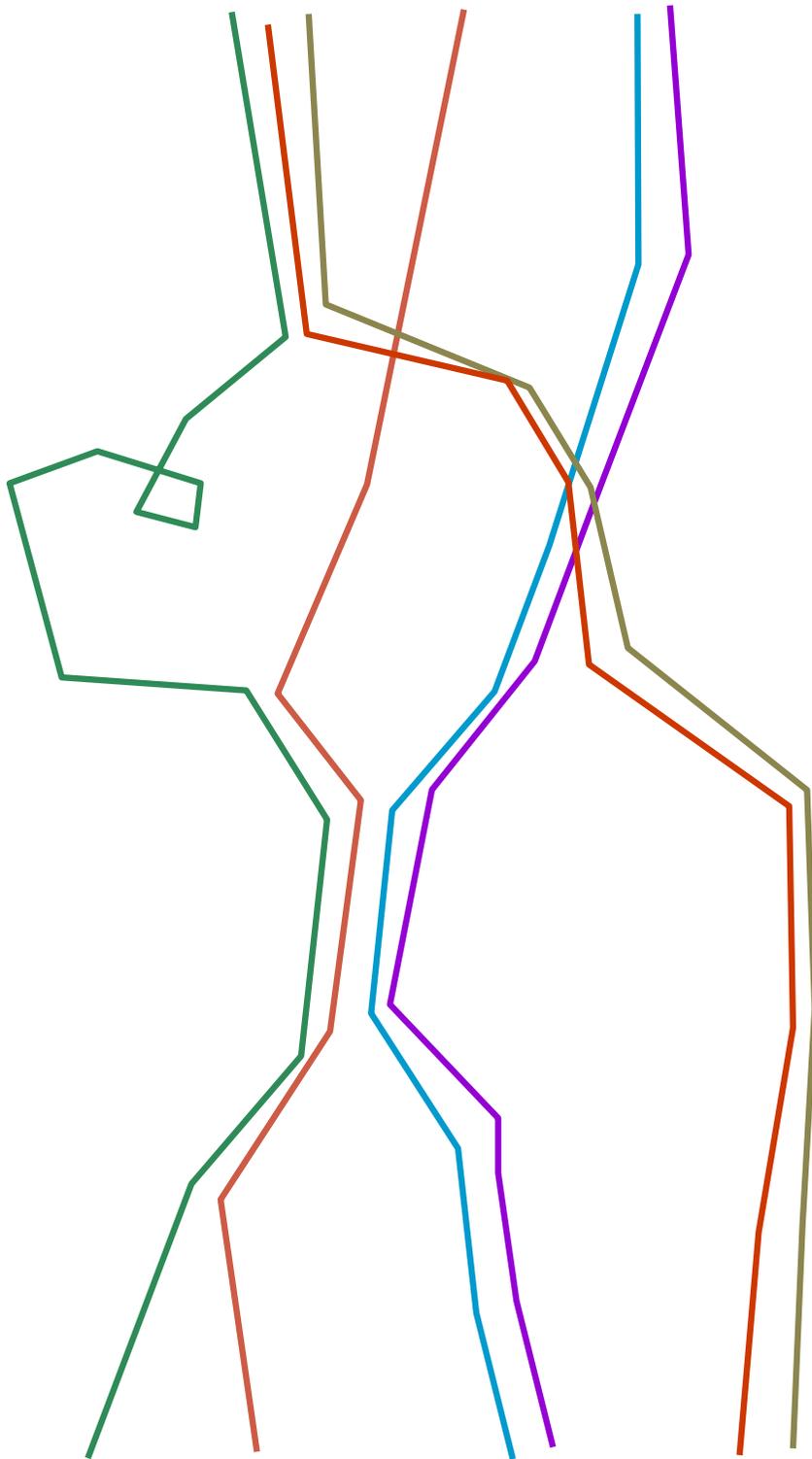
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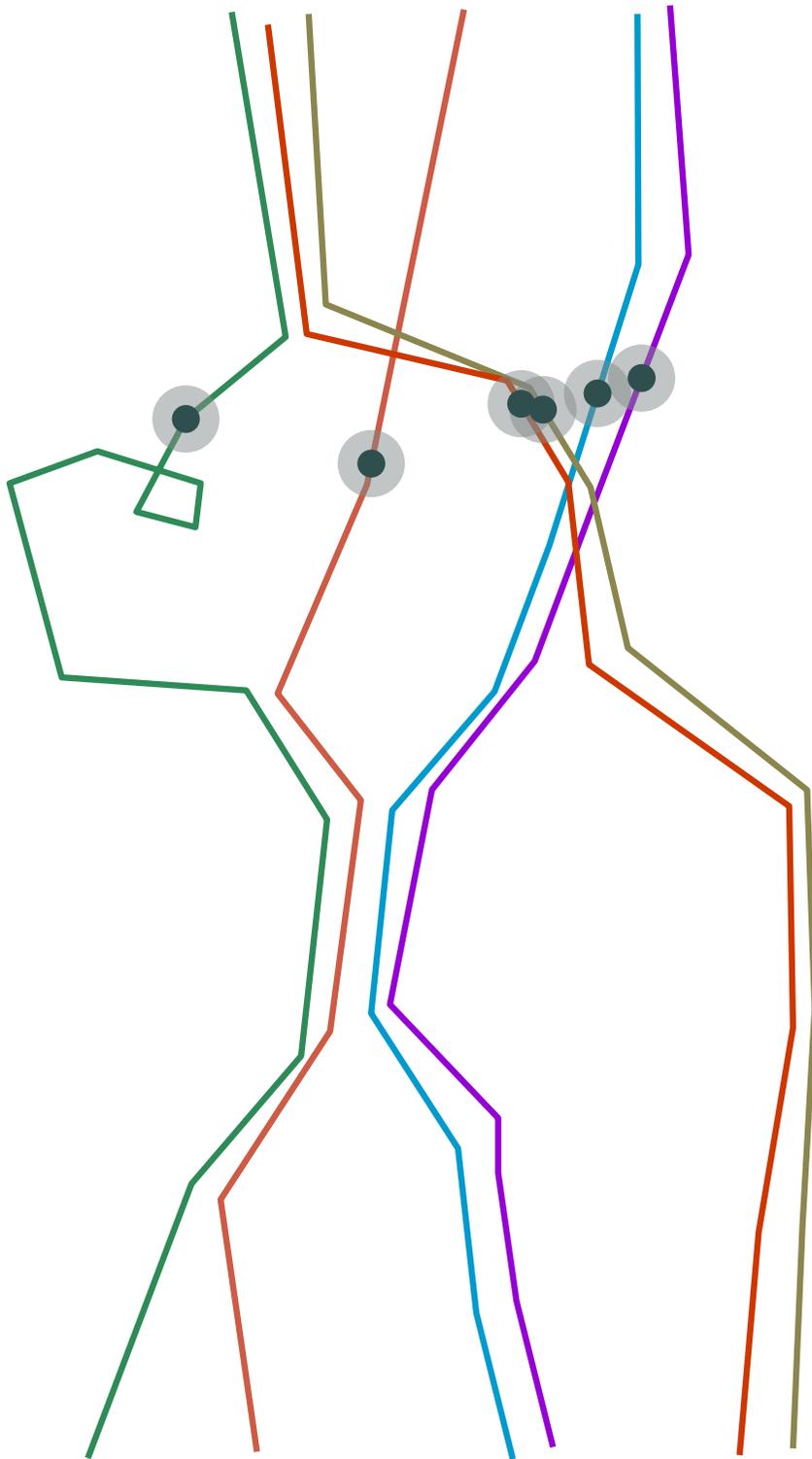
Trajectory Grouping Structure



# Groups

Group parameters:

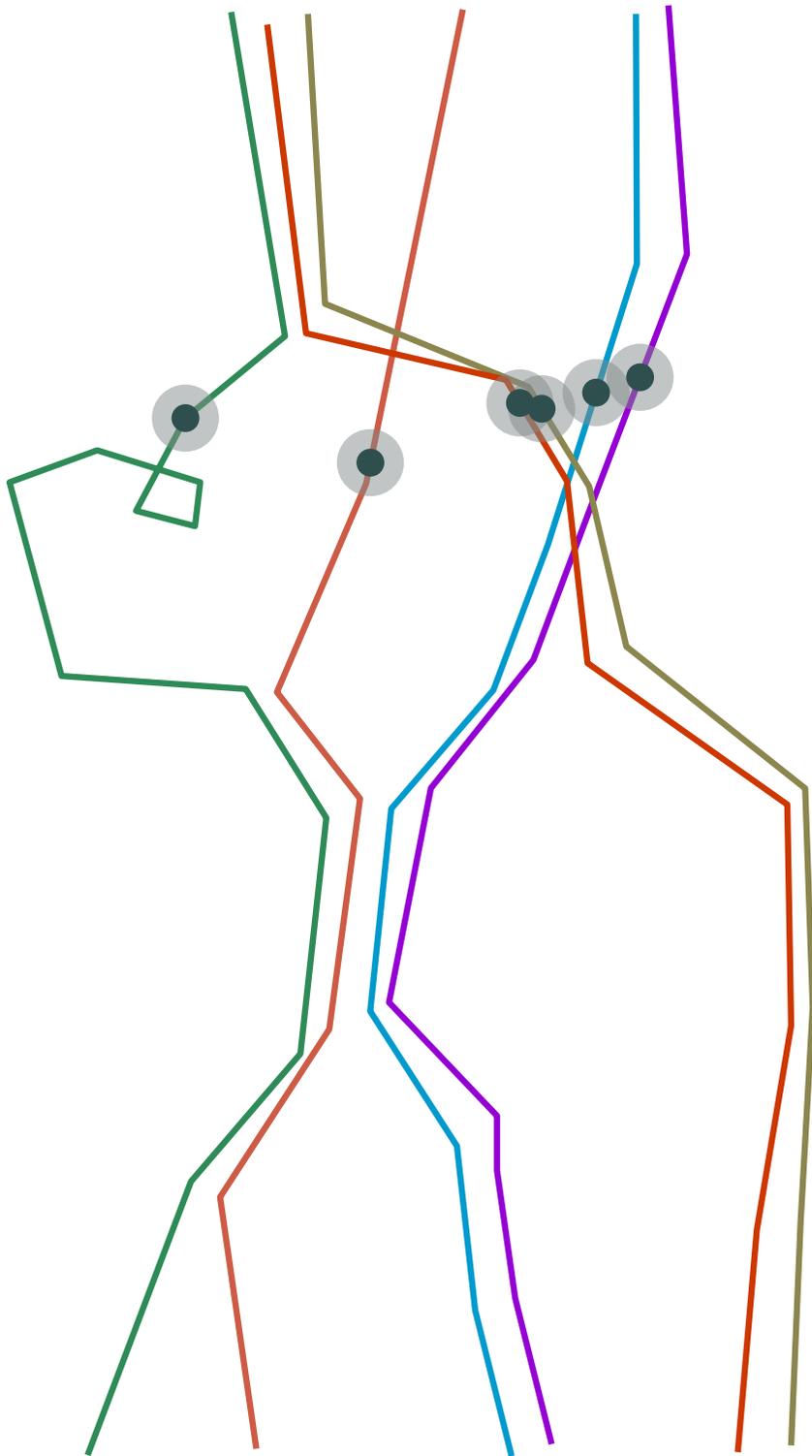
- min group size:  $m$
- min duration:  $\delta$
- max distance between entities:  $\varepsilon$



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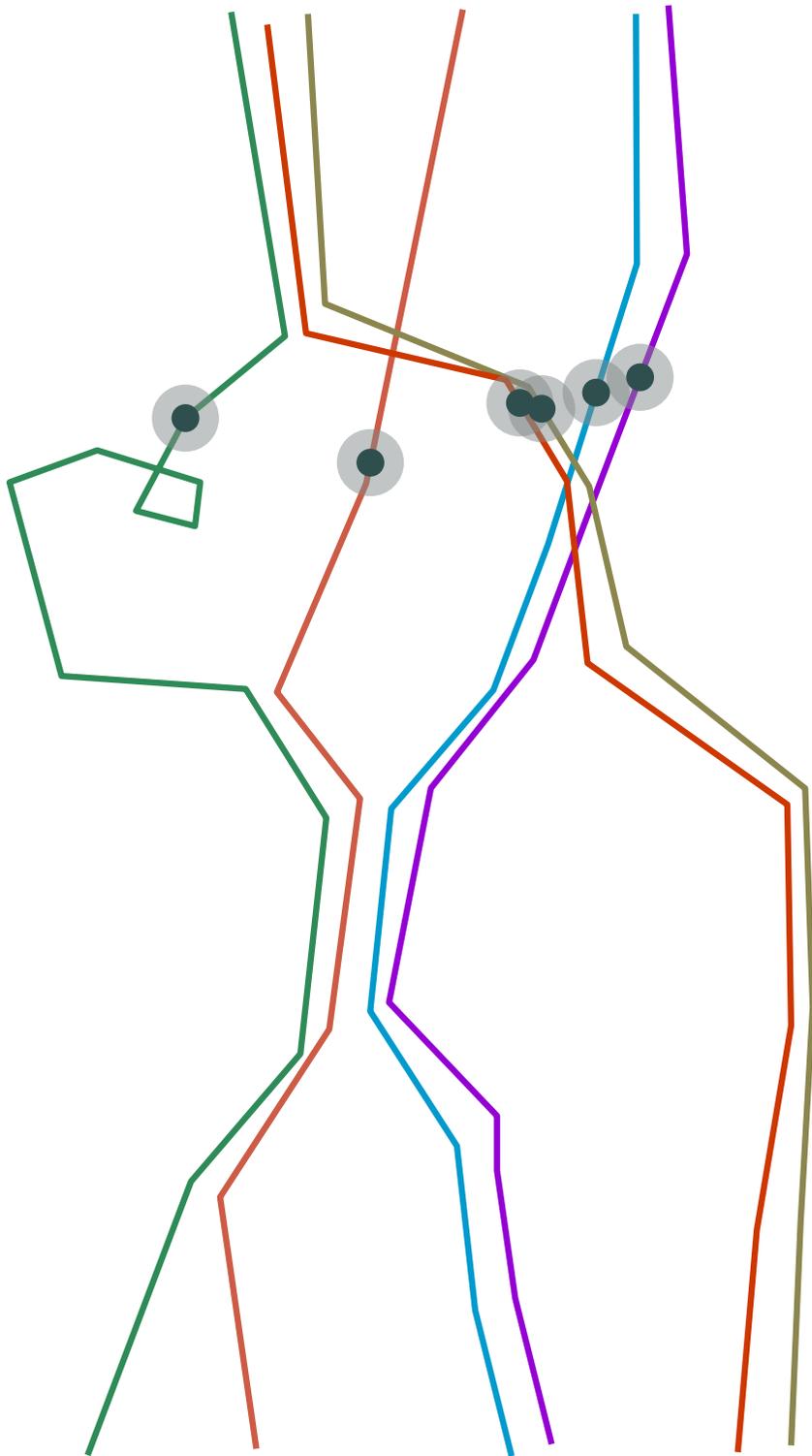


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The grouping structure should allow study of groups at different scales.



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Computational Topology

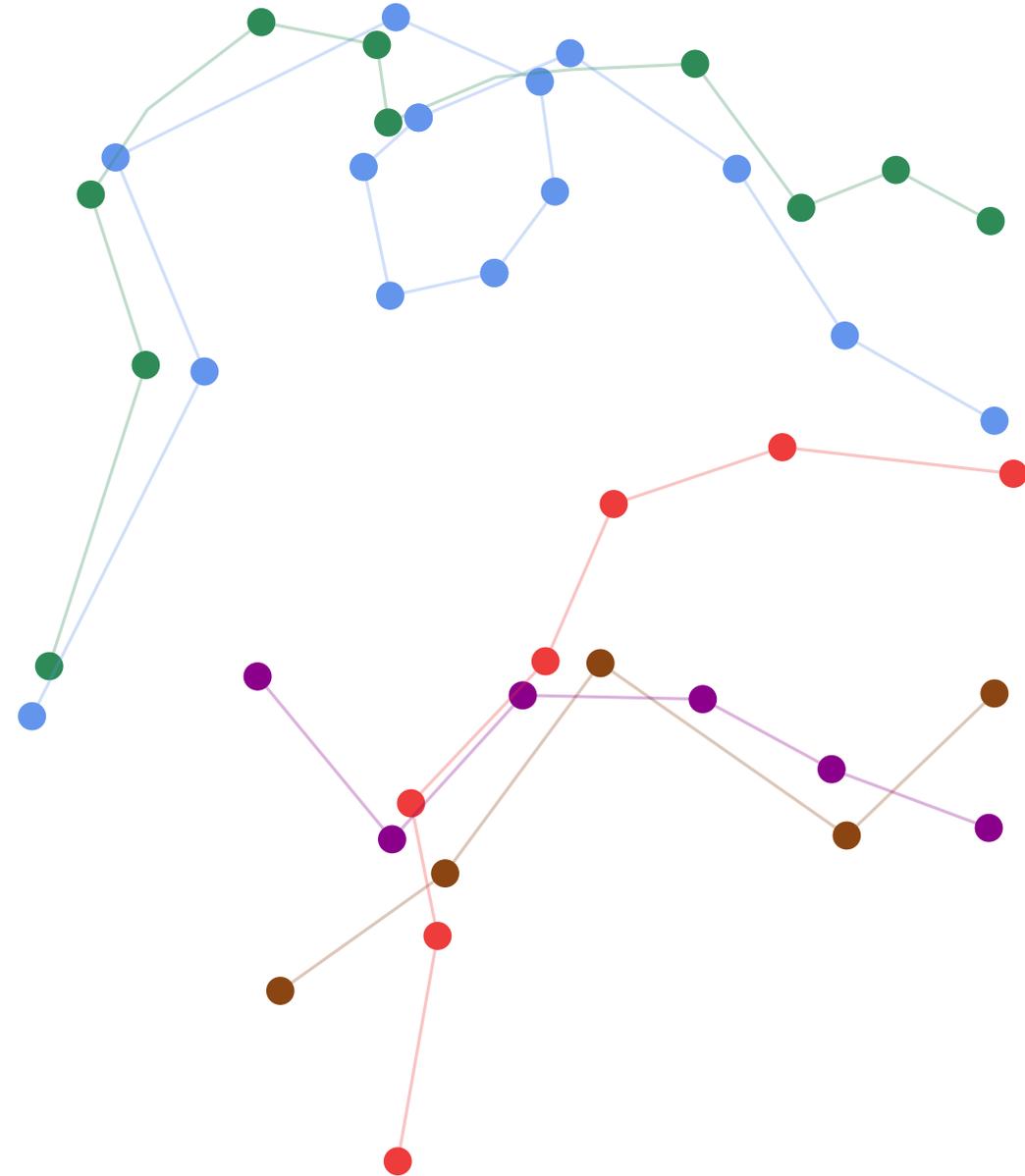
# Related Work

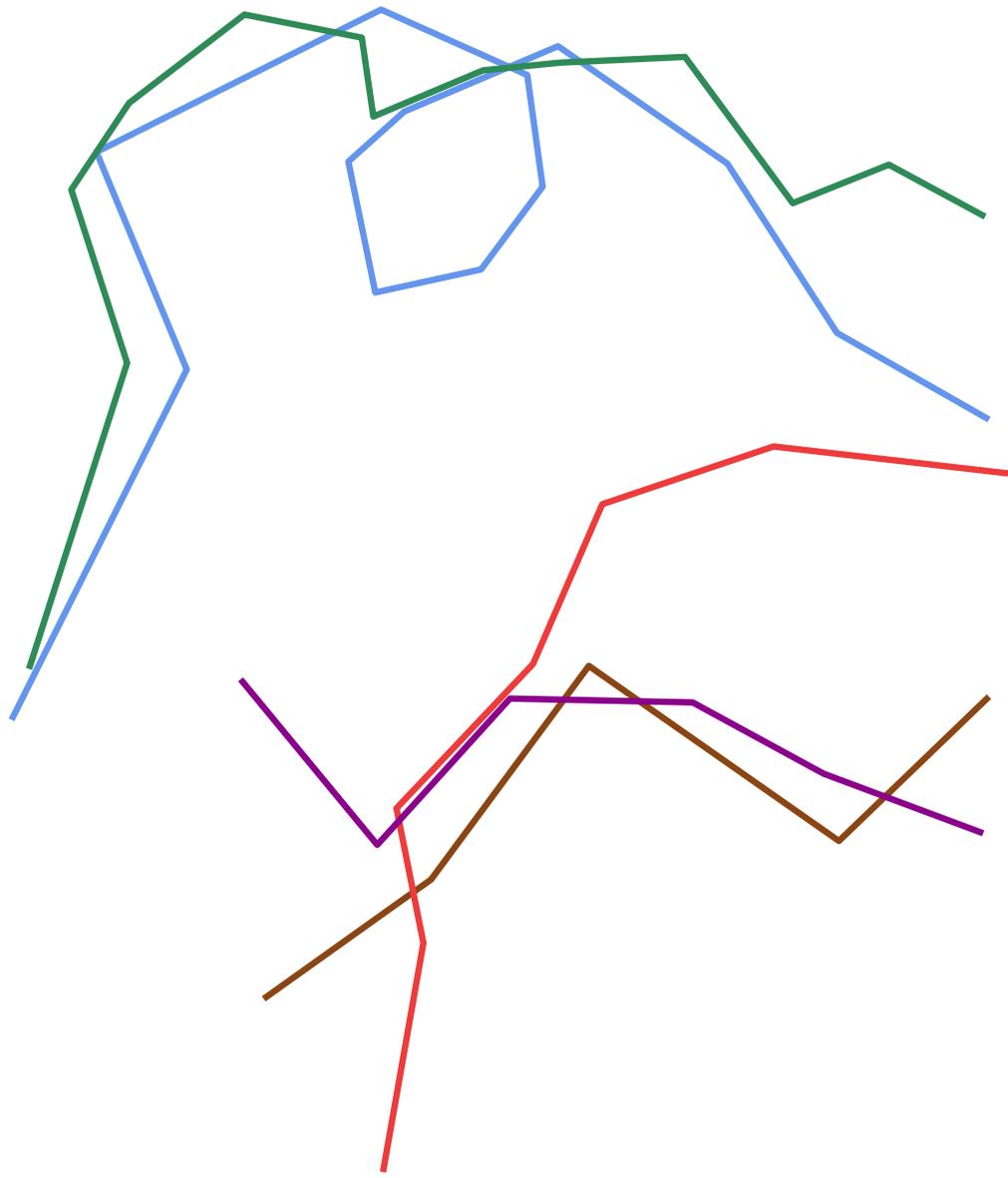
convoys, moving clusters,  
mobile groups, swarms, flocks,  
herds, ....

# Related Work

convoys, moving clusters,  
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herds, ....

Discrete trajectory data





## Related Work

convoys, moving clusters,  
mobile groups, swarms, flocks,  
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Continuous trajectory data

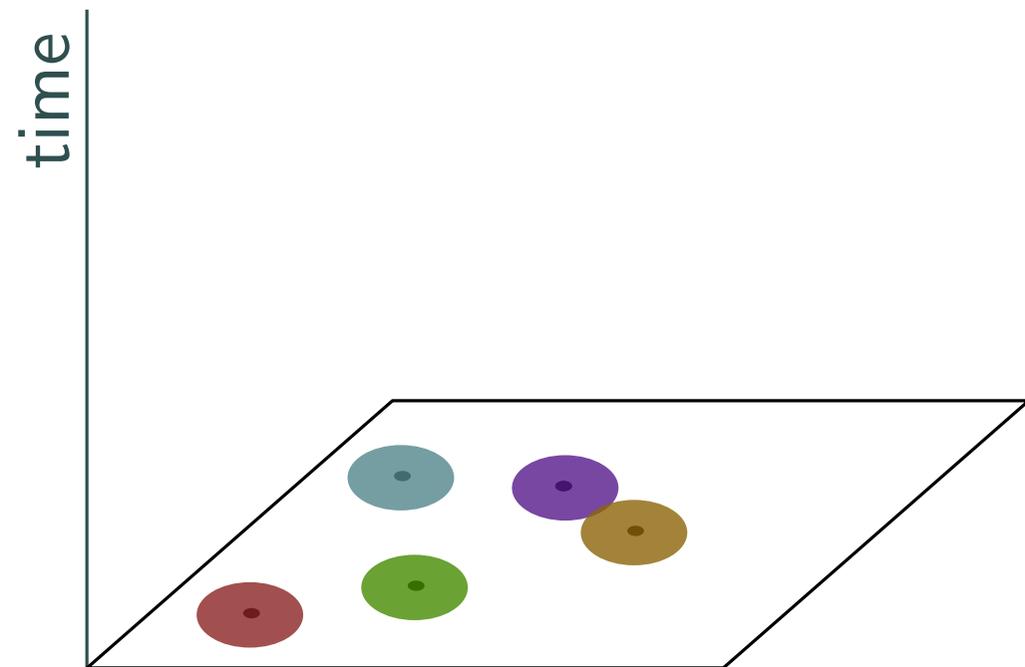


## Related Work

convoys, moving clusters,  
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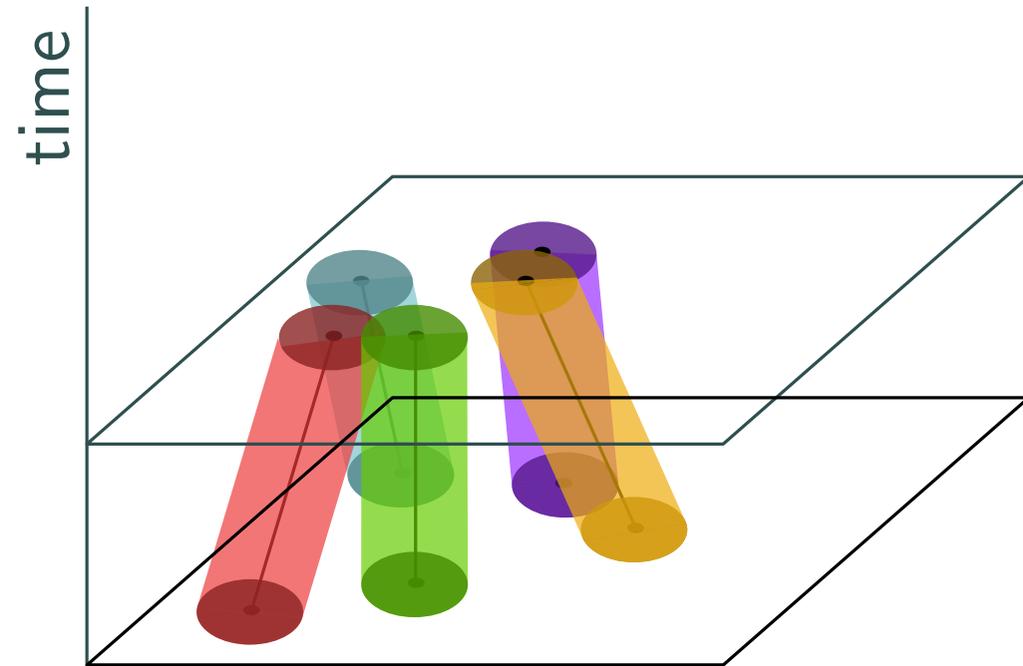
Continuous trajectory data

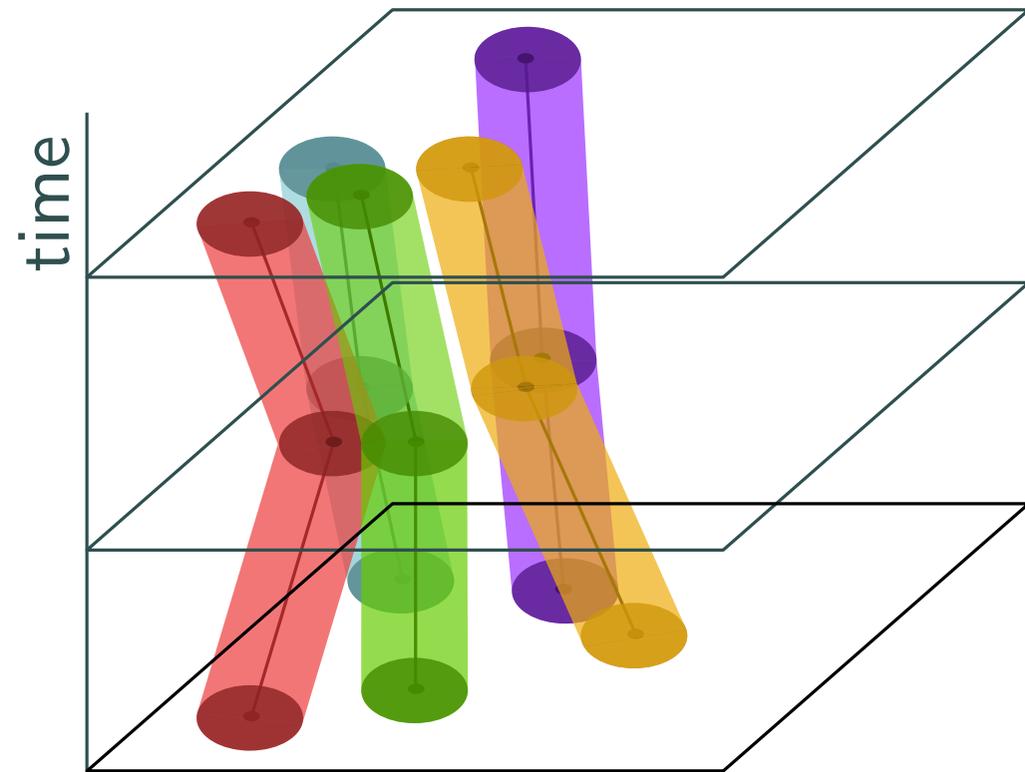
We provide an experimental  
evaluation.



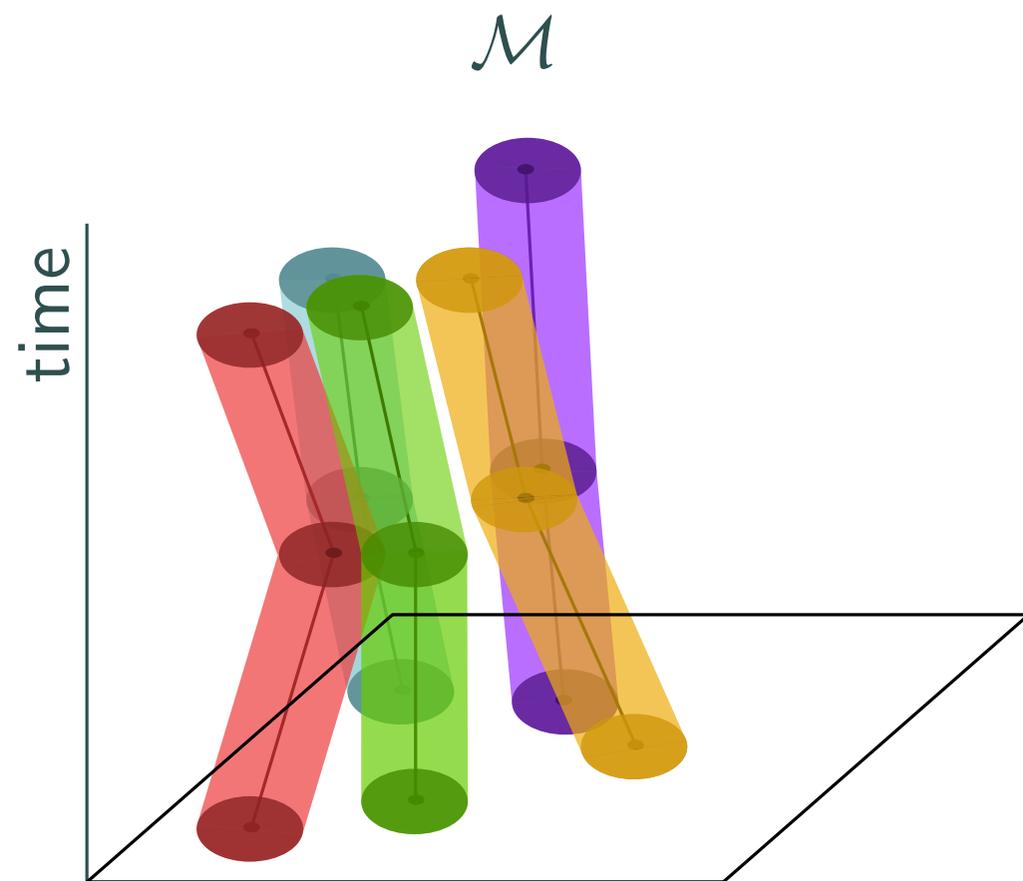
Consider tracing  $\varepsilon$ -discs over the trajectories.

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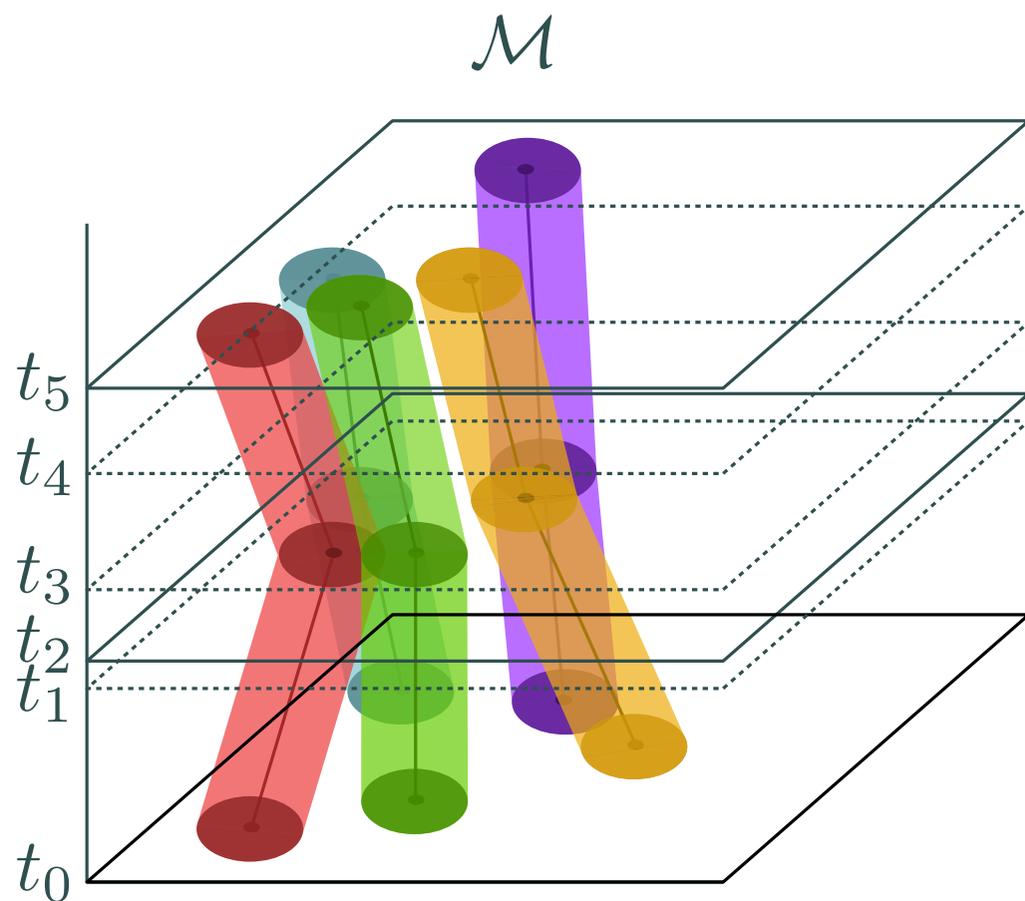


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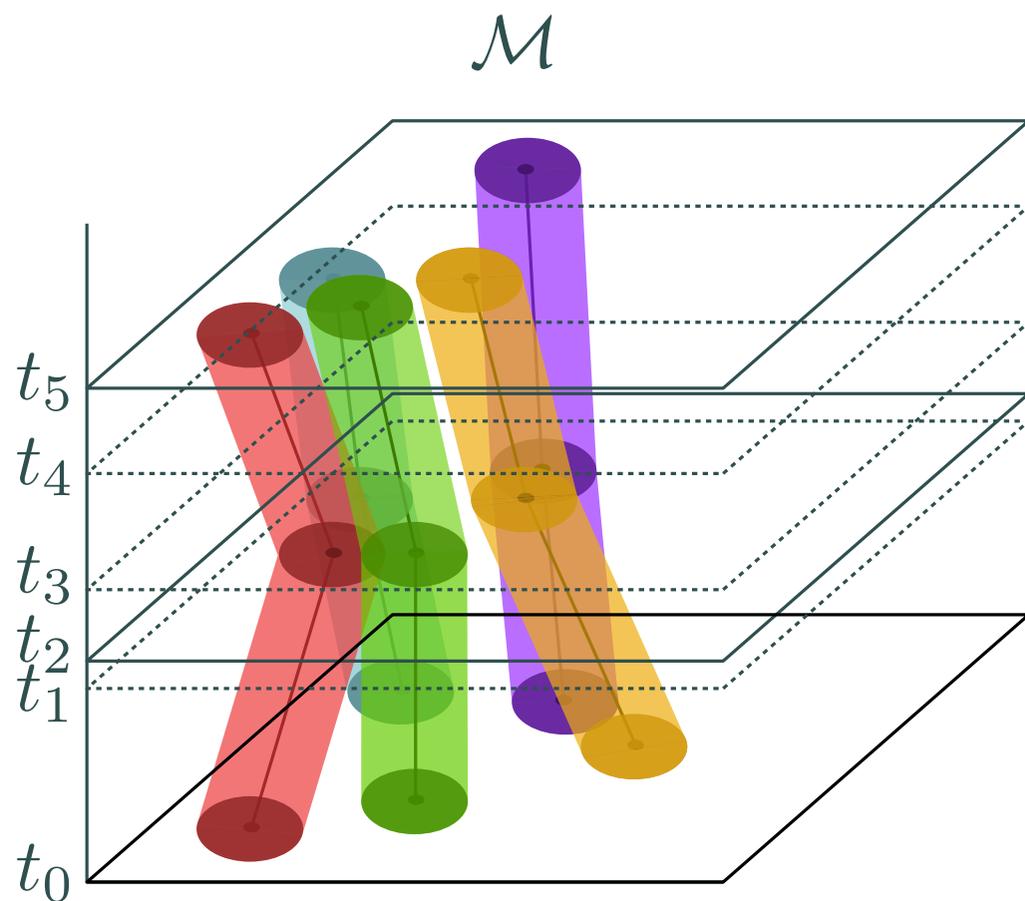
Each entity produces a “tube”. Let  $\mathcal{M}$  be the union of those  $n$  “tubes”.



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Groups change when connectivity in  $\mathcal{M}$  changes.



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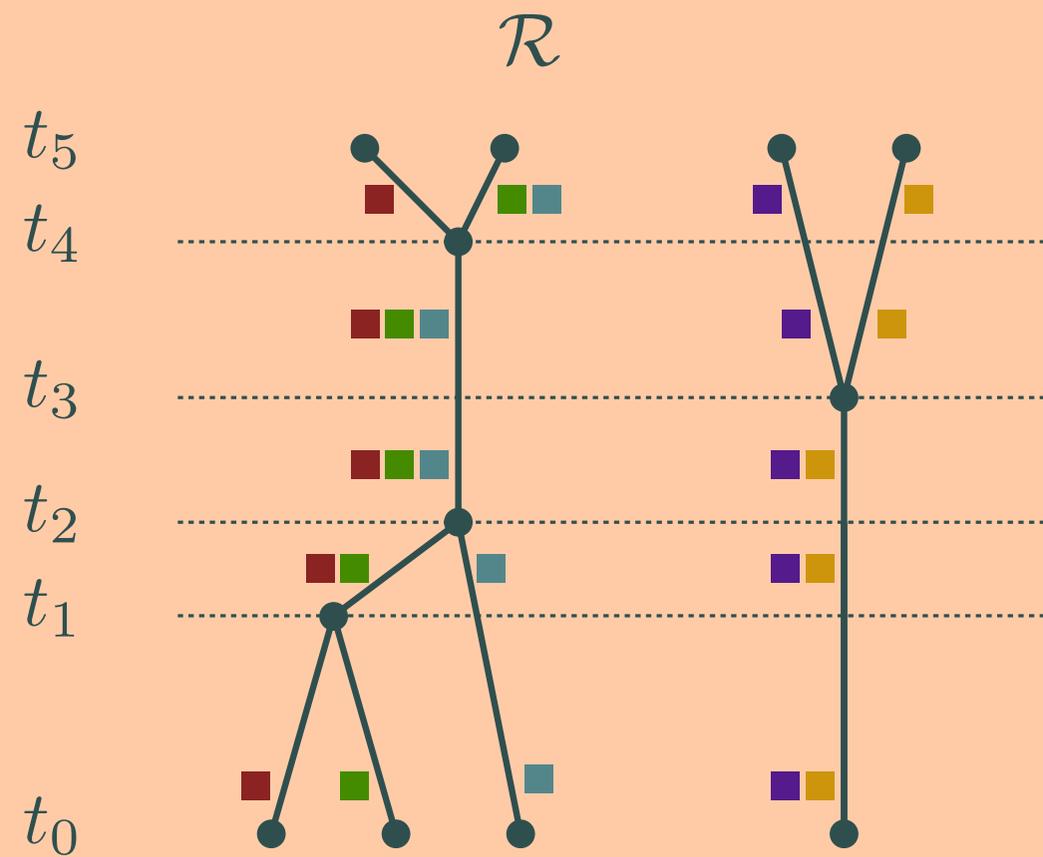
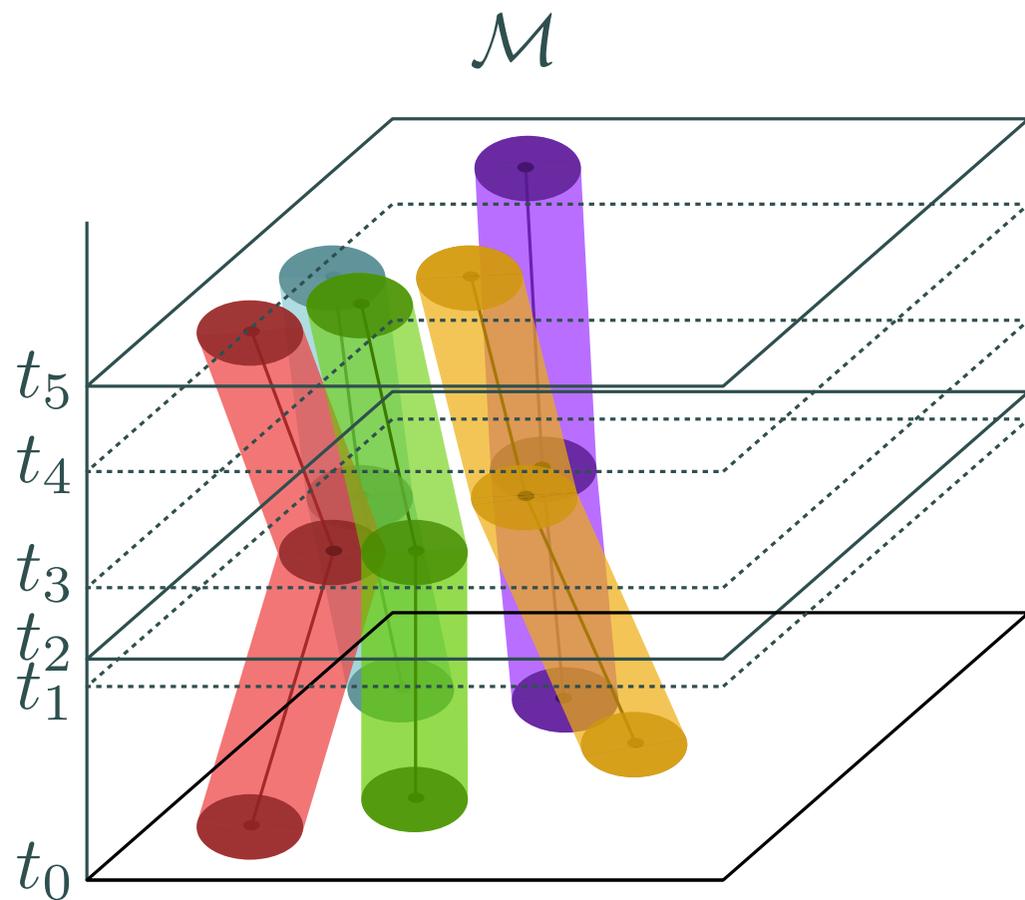
Groups change when connectivity in  $\mathcal{M}$  changes.

The **Reeb graph**  $\mathcal{R}$  of  $\mathcal{M}$  captures connectivity changes.



**Reeb graph** captures all group changes.

# The Reeb Graph

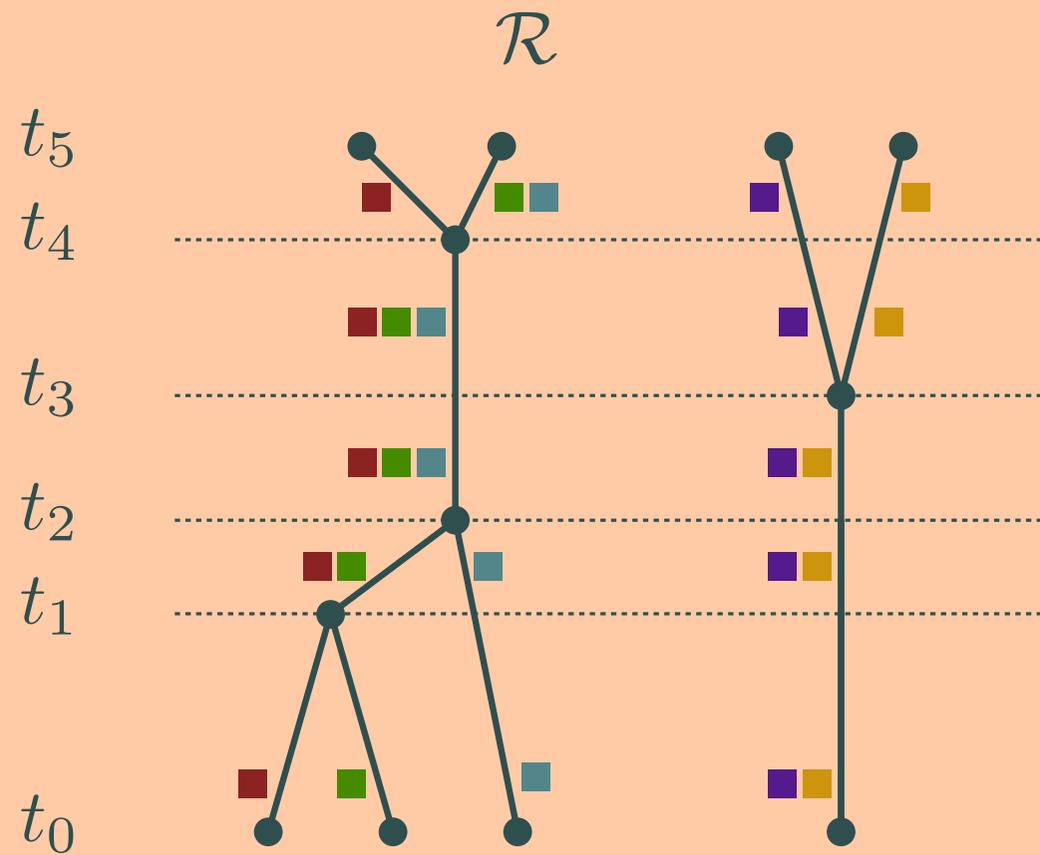
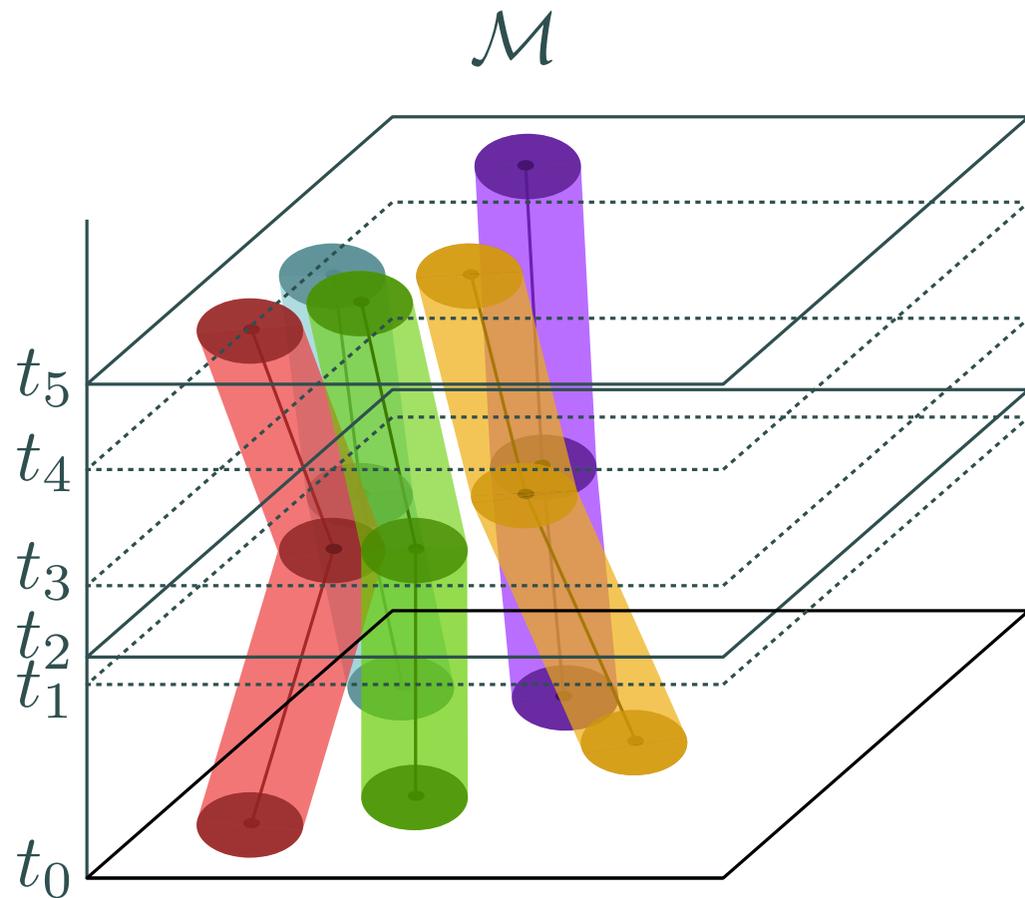


# The Reeb Graph

Annotate  $\mathcal{R}$  with all groups



Trajectory Grouping Structure



# The Reeb Graph

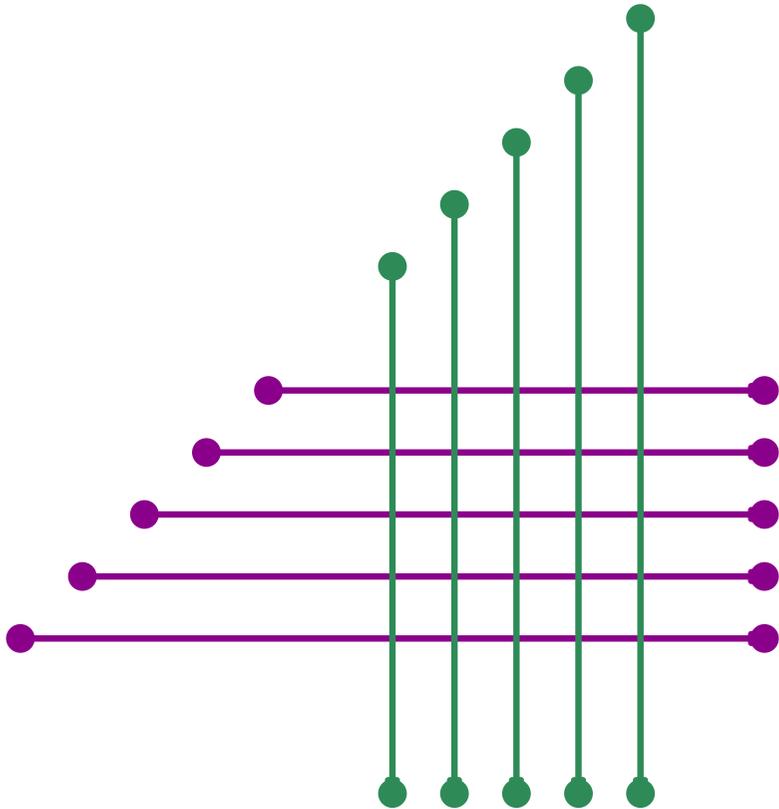
Complexity  $\mathcal{R}$ ?

# The Reeb Graph

Complexity  $\mathcal{R}$ ?

$$\Theta(\tau n^2)$$

$\tau =$  trajectory length

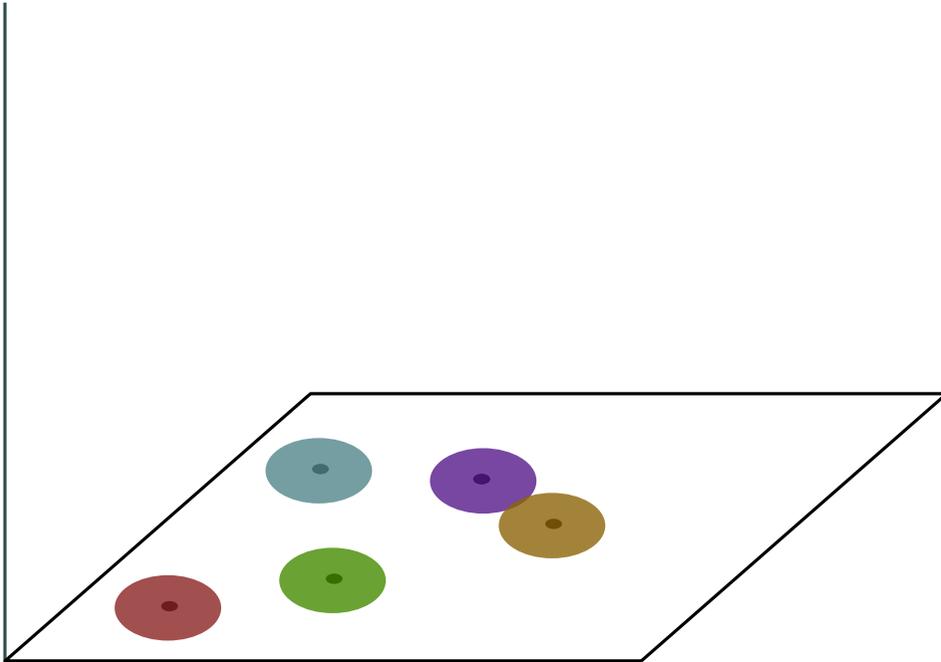


# The Reeb Graph

How to compute  $\mathcal{R}$ ?

Sweep  $\mathcal{M}$  while maintaining connected components.

$\mathcal{M}$



$\mathcal{R}$

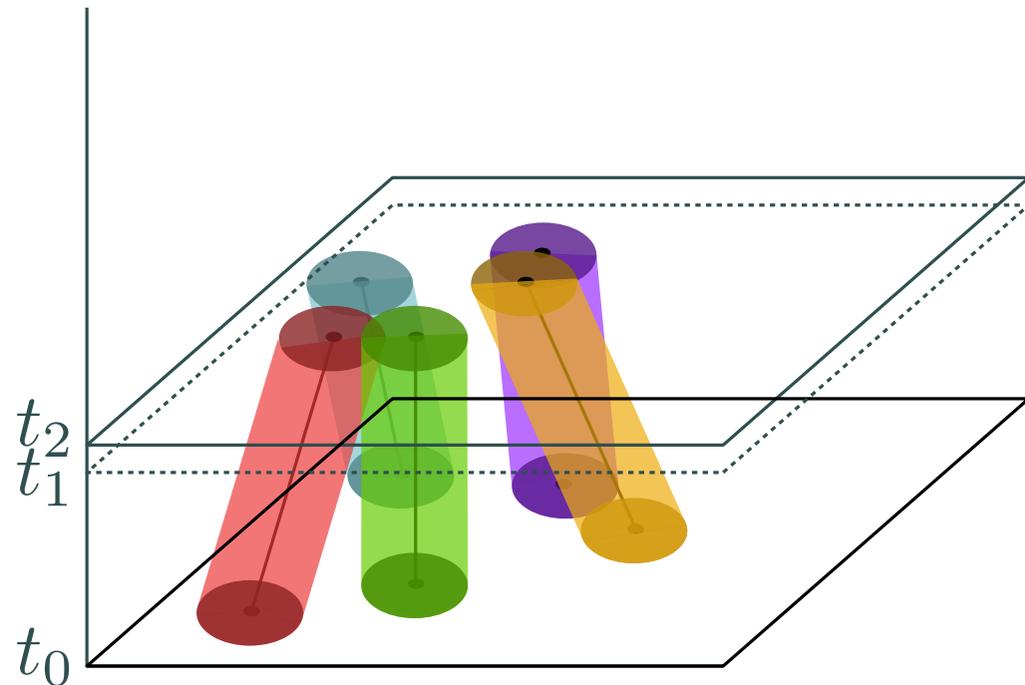


# The Reeb Graph

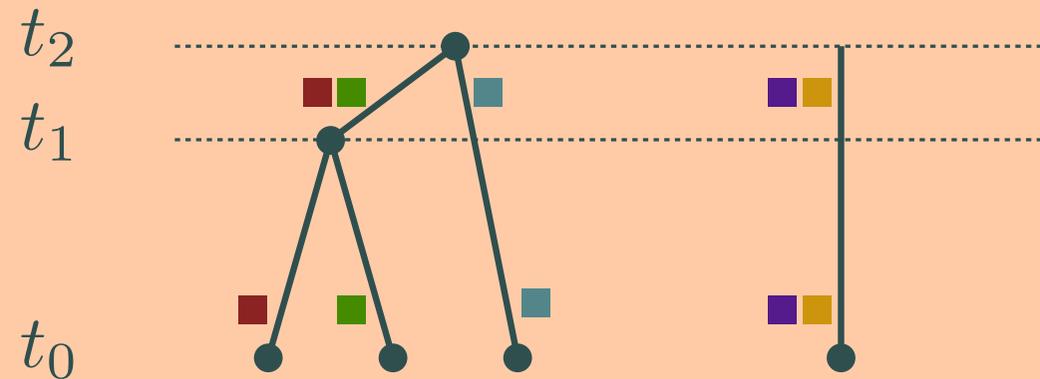
How to compute  $\mathcal{R}$ ?

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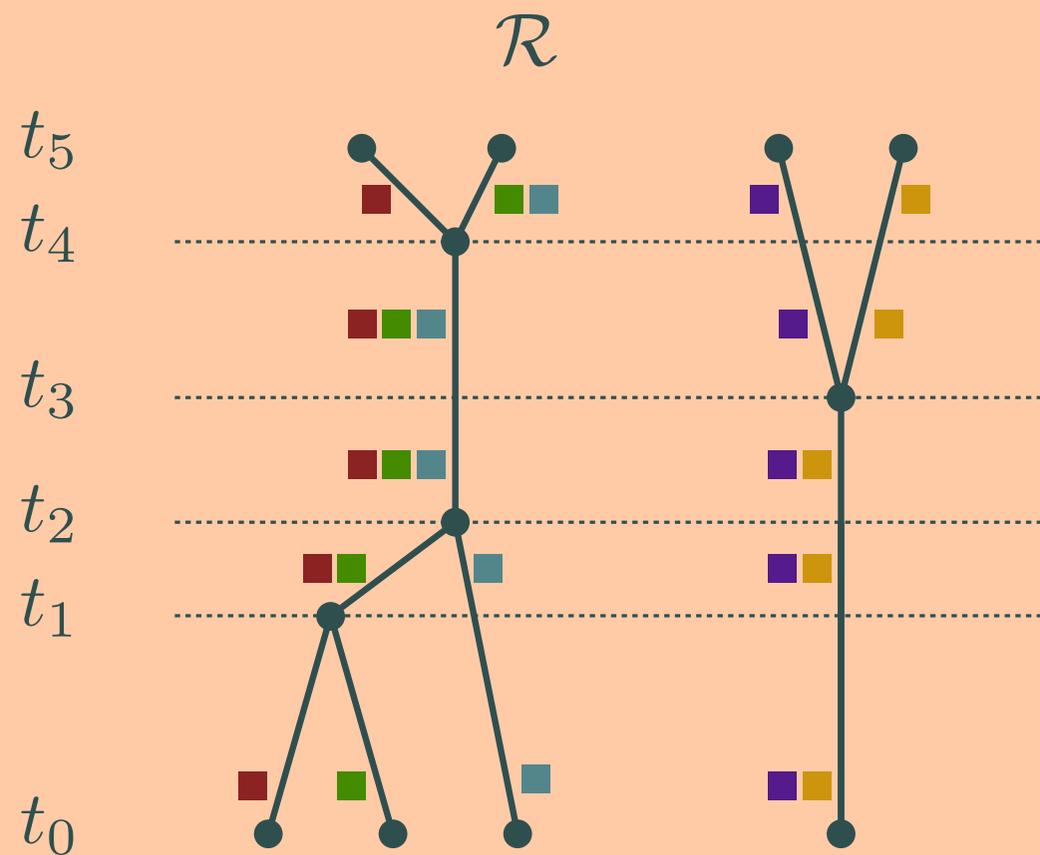
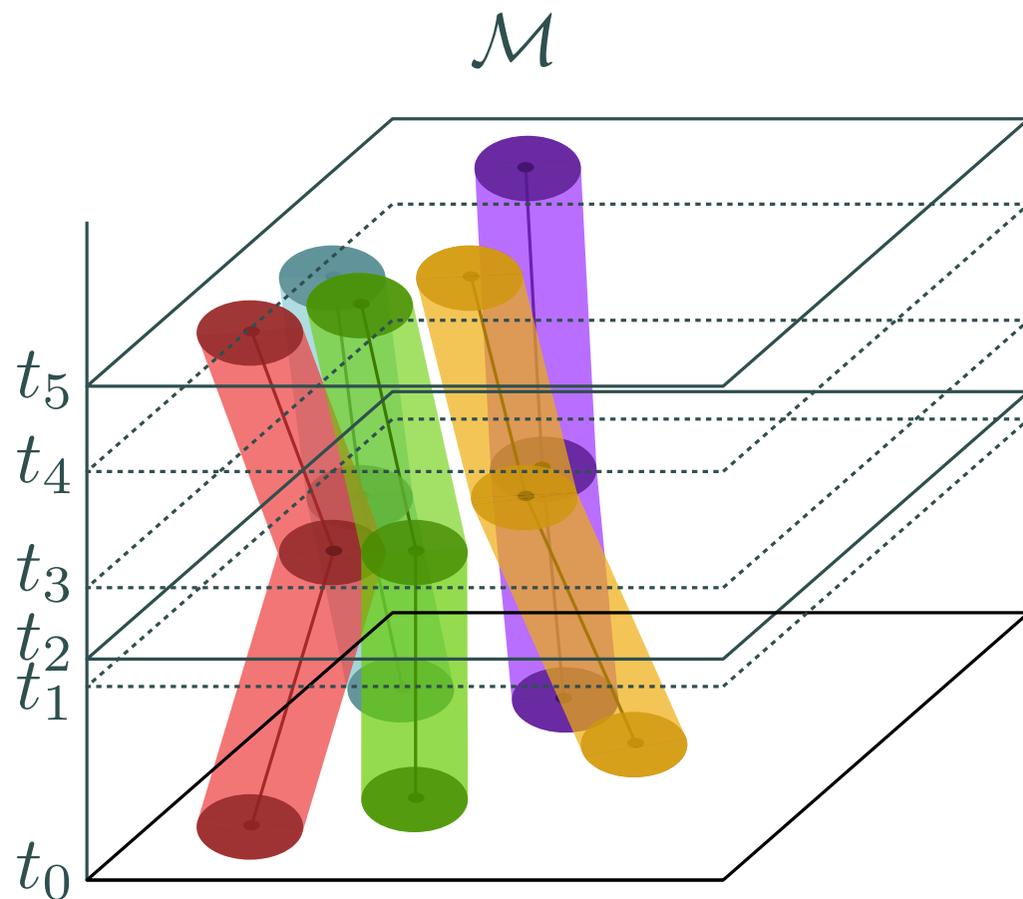
$\mathcal{R}$



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Sweep  $\mathcal{M}$  while maintaining connected components.



# The Reeb Graph

How to compute  $\mathcal{R}$ ?

Sweep  $\mathcal{M}$  while maintaining connected components.

Running time?

Compute & Sort all events:

$O(\tau n^2 \log n)$  time

Initialize graph:

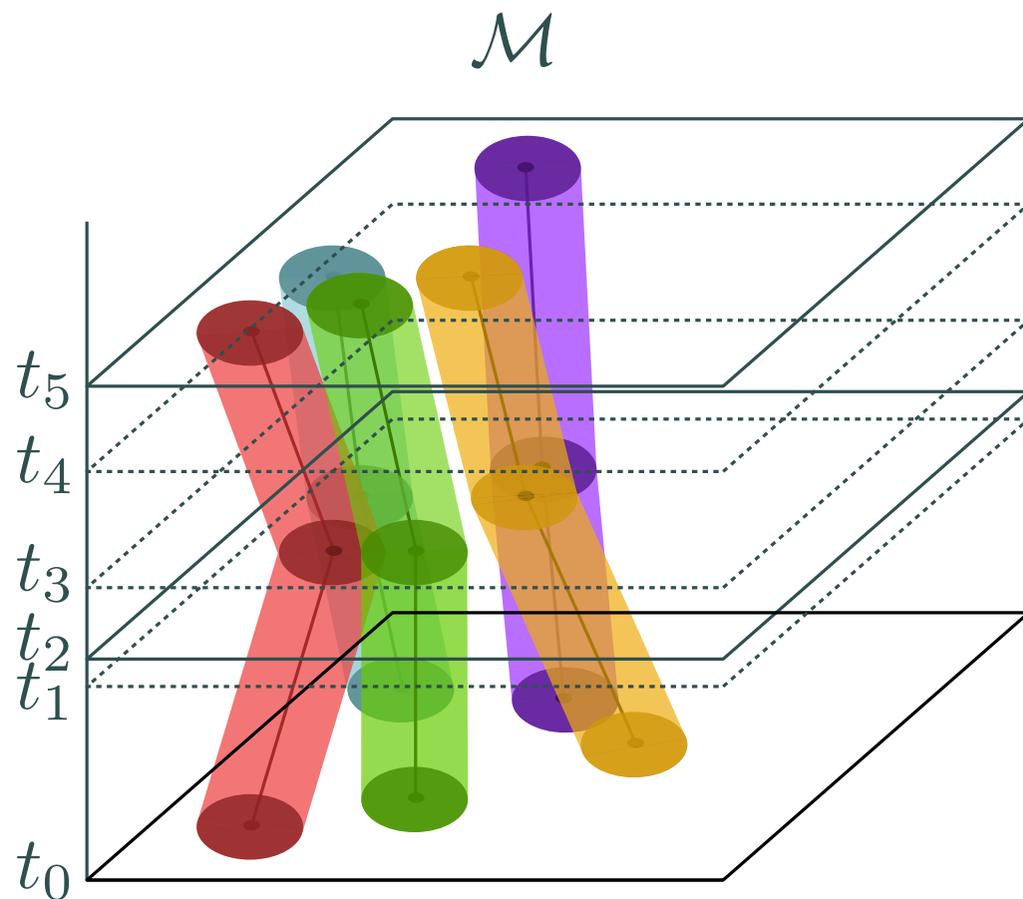
$O(n^2)$  time

Handle an event:

$O(\log n)$  time

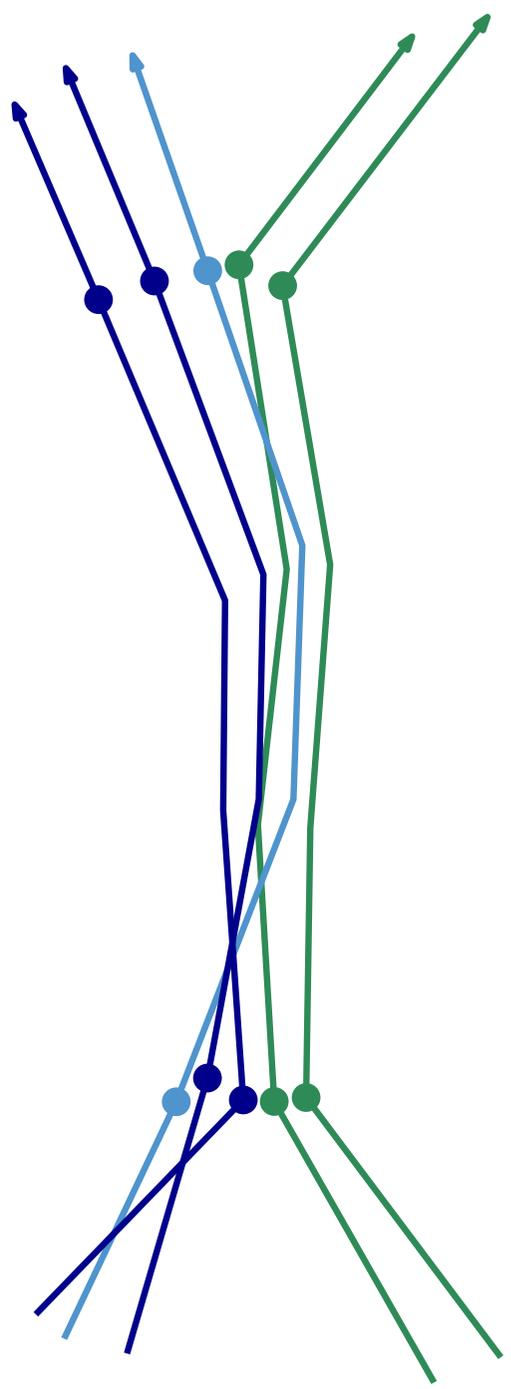
Total:

$O(\tau n^2 \log n)$  time



# Computing all Groups

How many groups?

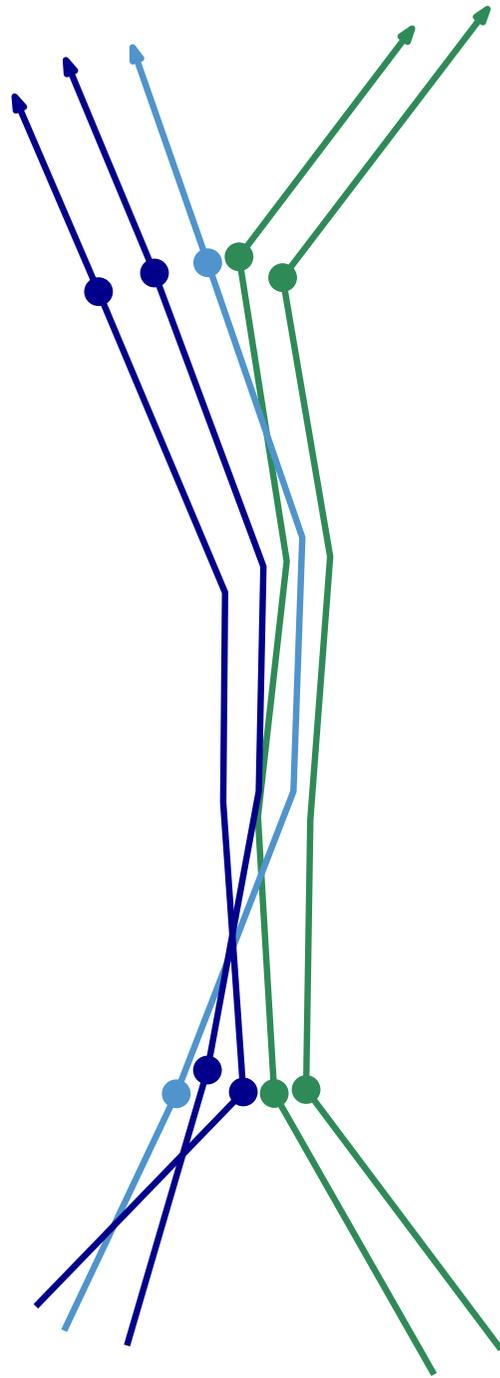


# Computing all Groups

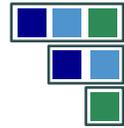
How many groups?

# Computing all Groups

How many maximal groups?



maximal:



not maximal:

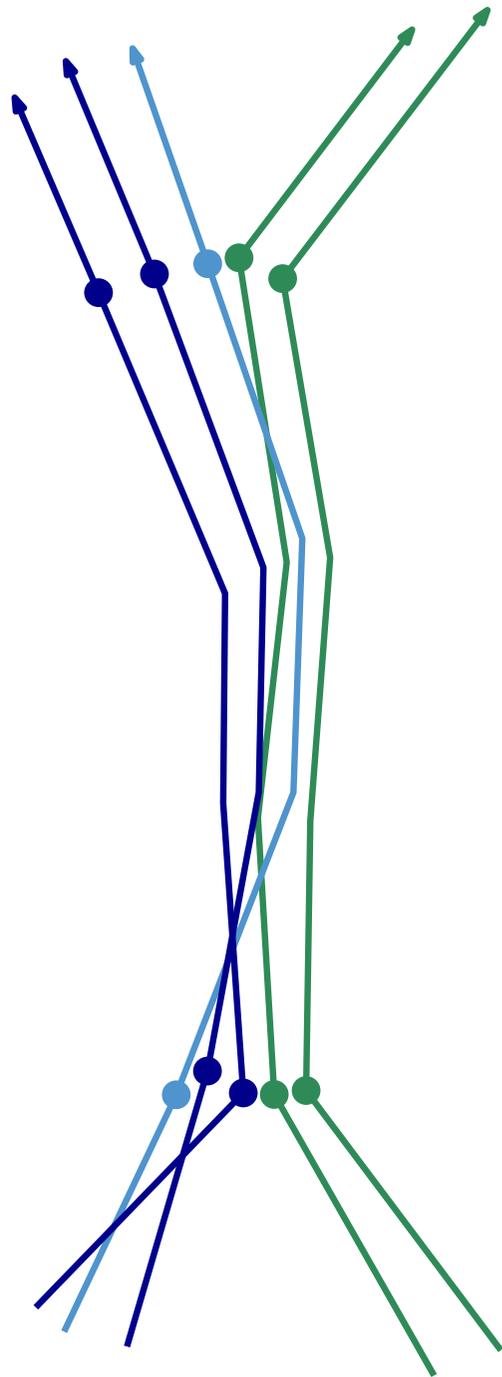


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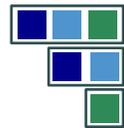
How many maximal groups?

At most  $n$  per vertex

$\implies \Theta(\tau n^3)$  in total.



maximal:



not maximal:



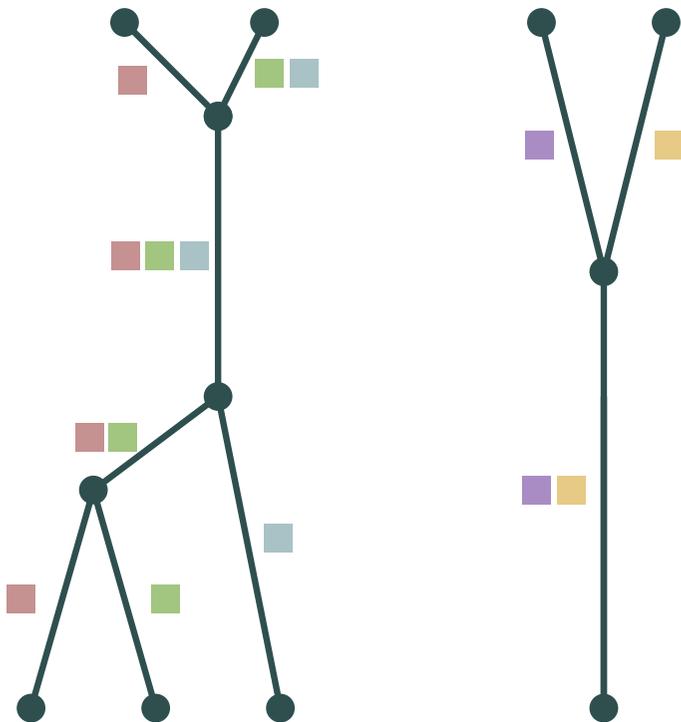
# Computing all Groups

How to compute all maximal groups?

with parameters  $\varepsilon$ ,  $m$ , and  $\delta$

1. Compute  $\mathcal{R}$  for the given  $\varepsilon$ .
2. Process the vertices in time order.
3. Label the outgoing edges with all known maximal groups, assuming  $m = 1$  and  $\delta = 0$ .
4. Filter the groups on  $\delta$  and  $m$ .

$\mathcal{R}$

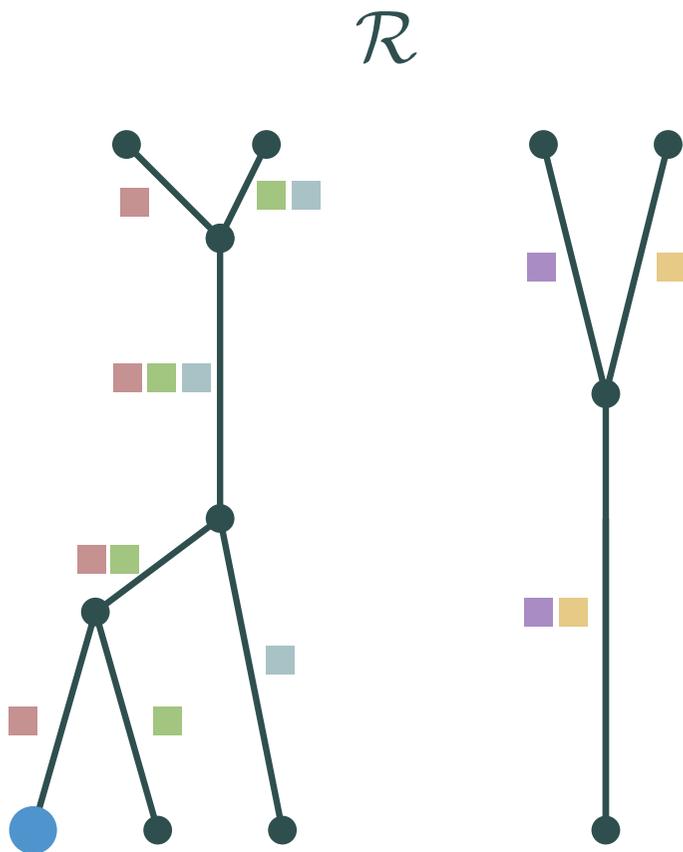


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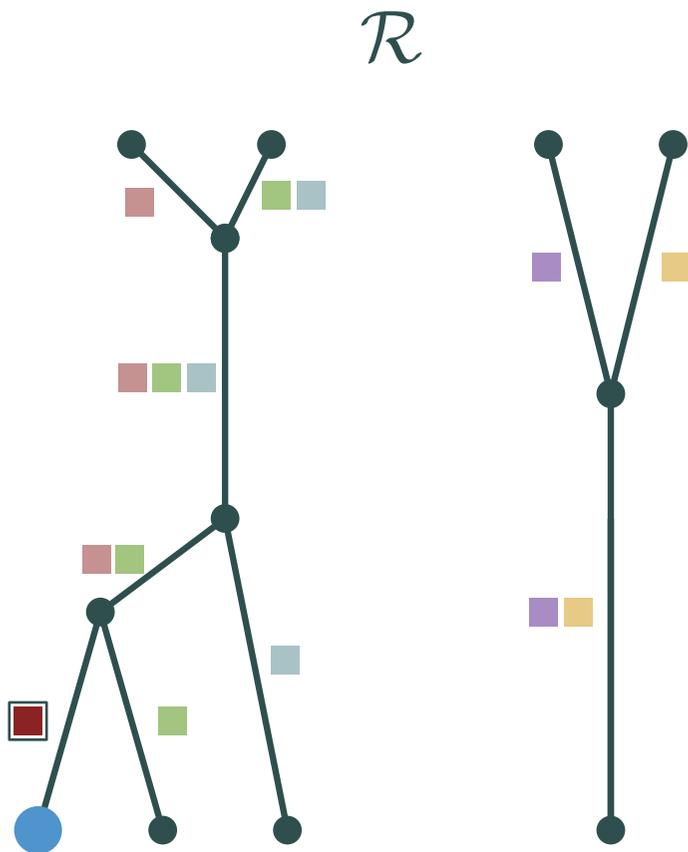


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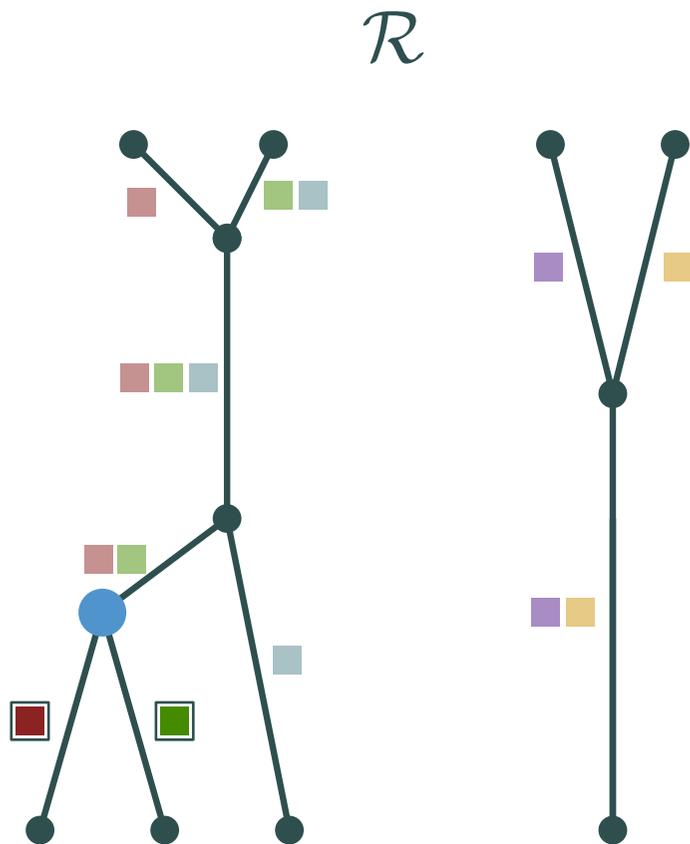


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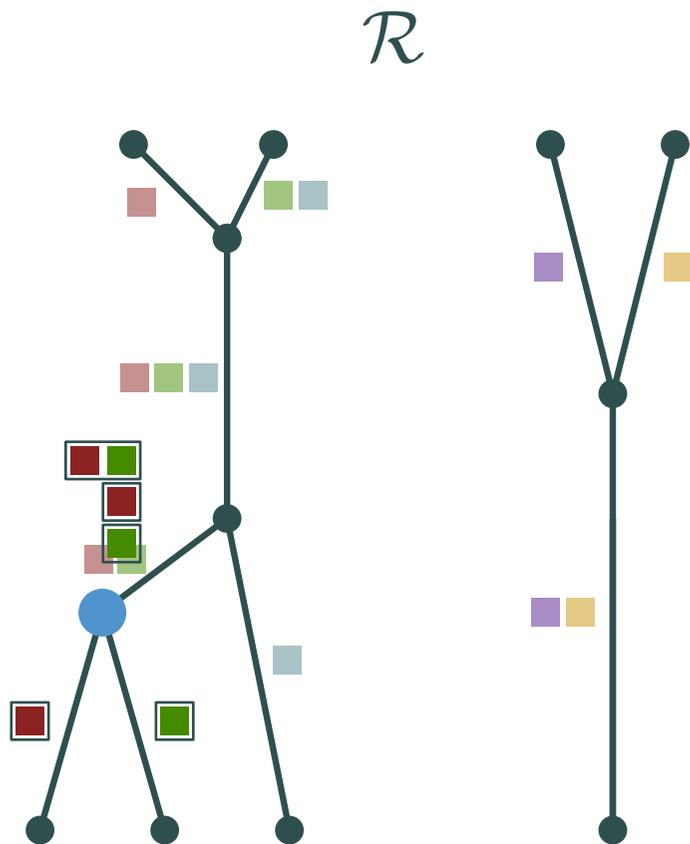


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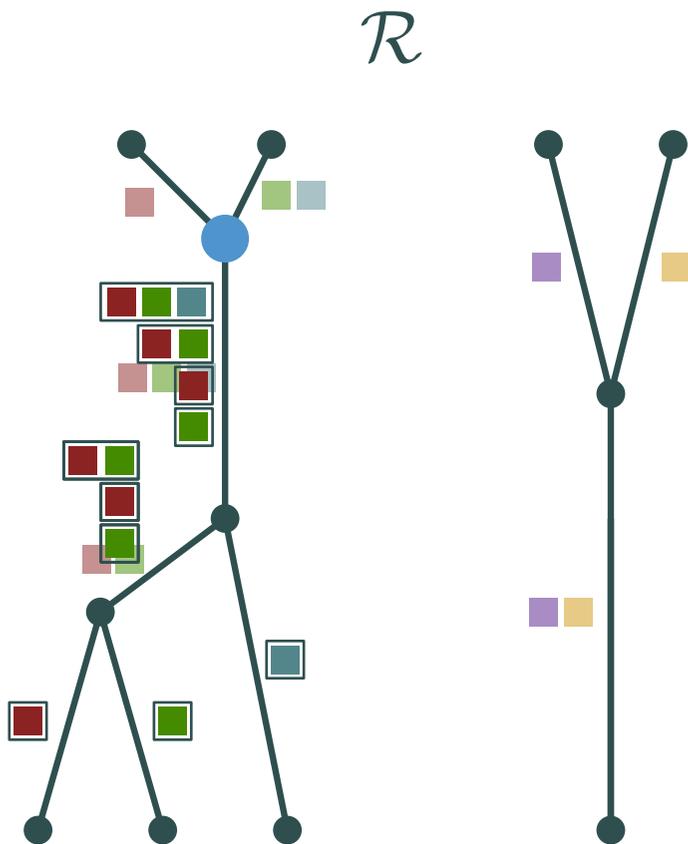


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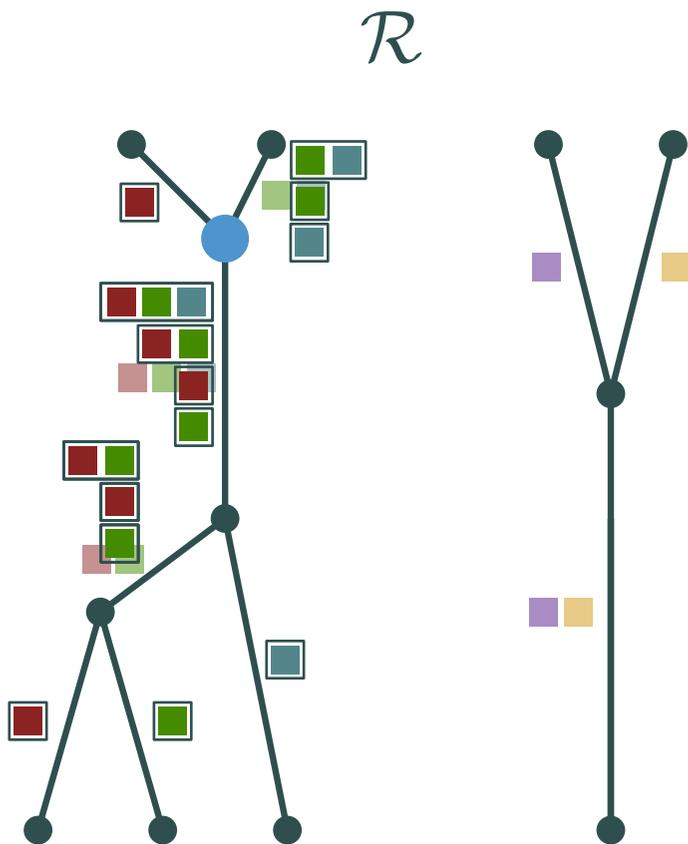


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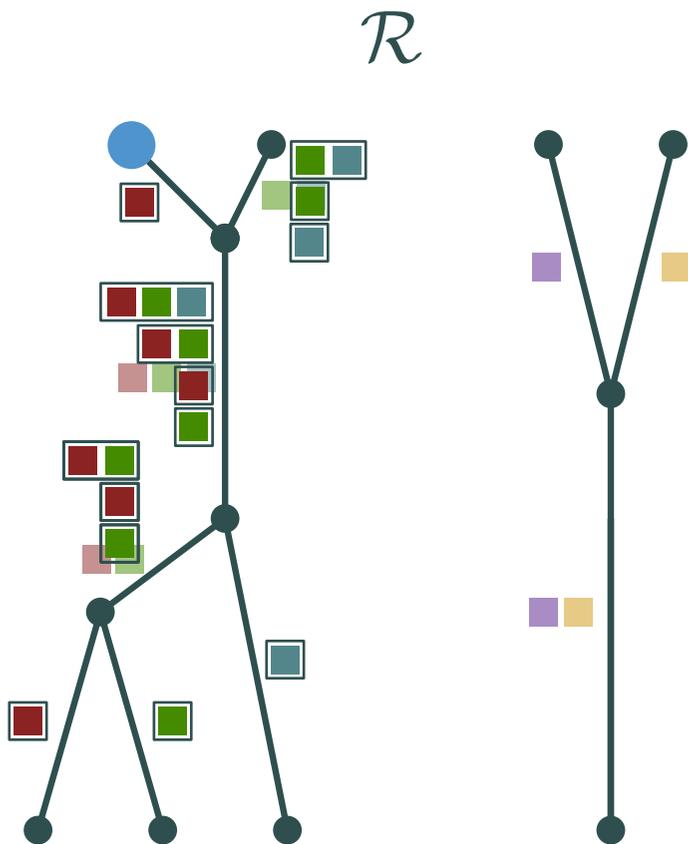


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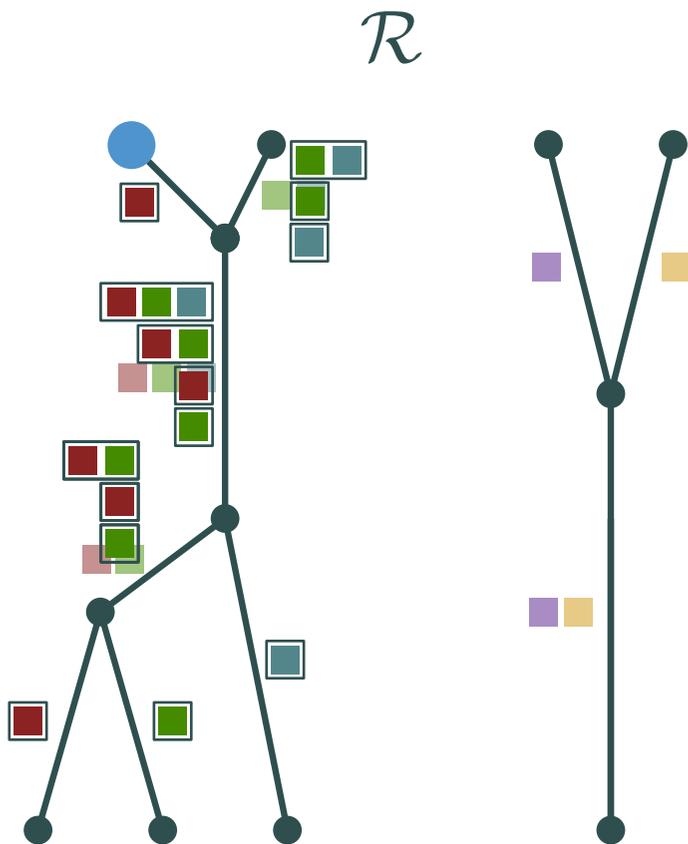
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How to represent the groups?





# Computing all Groups

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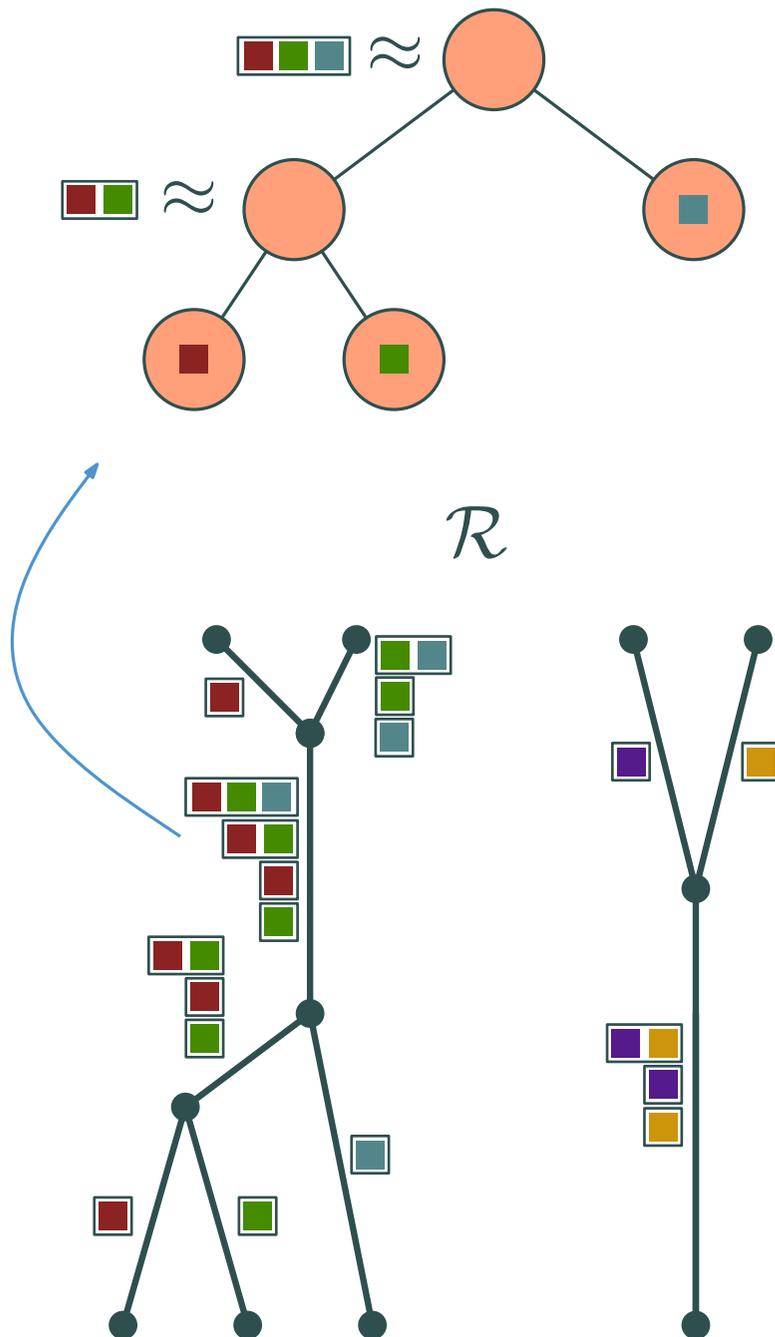
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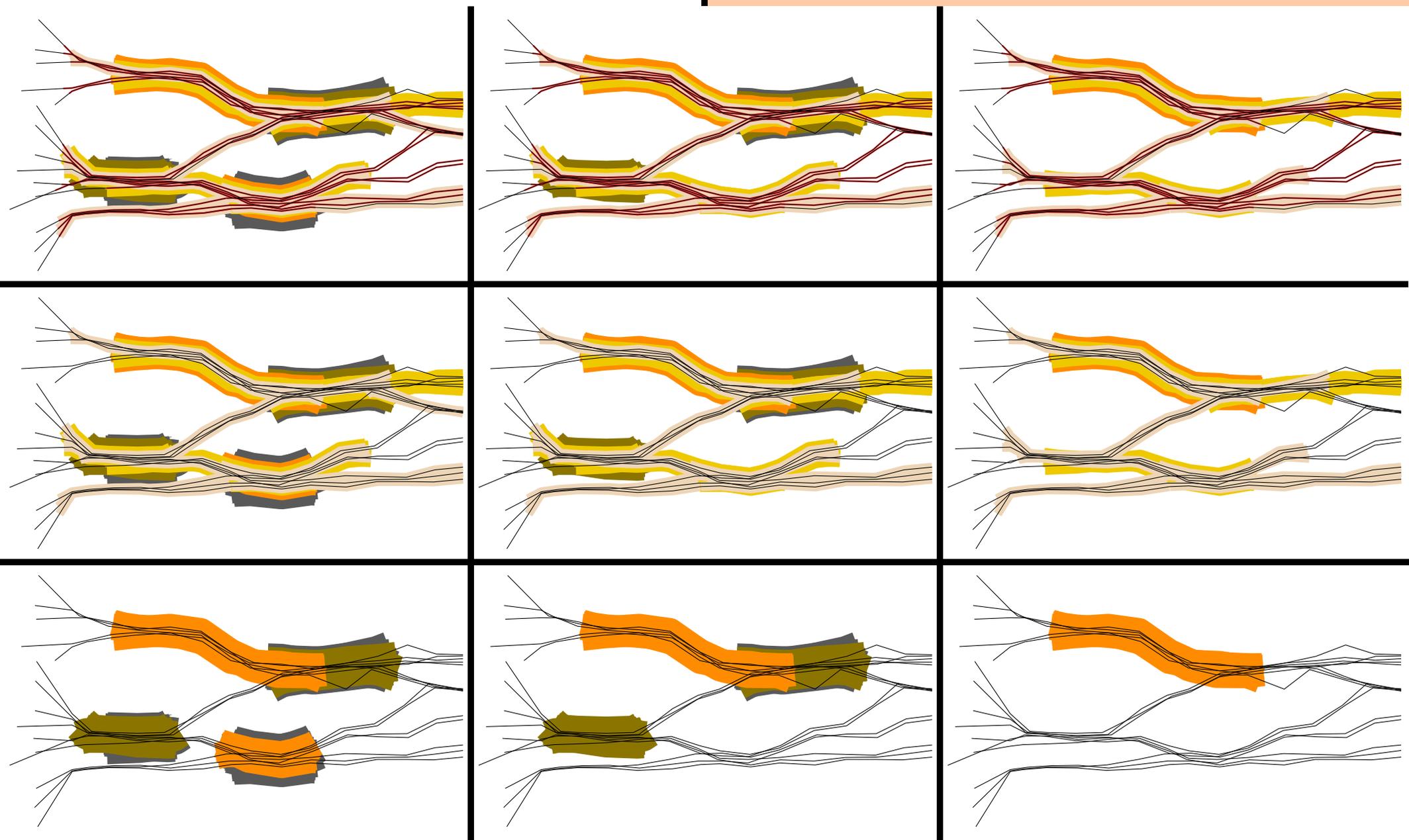
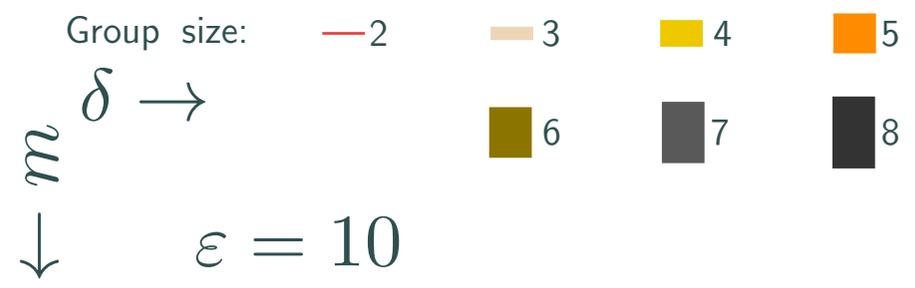
Use a binary tree.

Running time:  $O(\tau n^3 + N)$ .

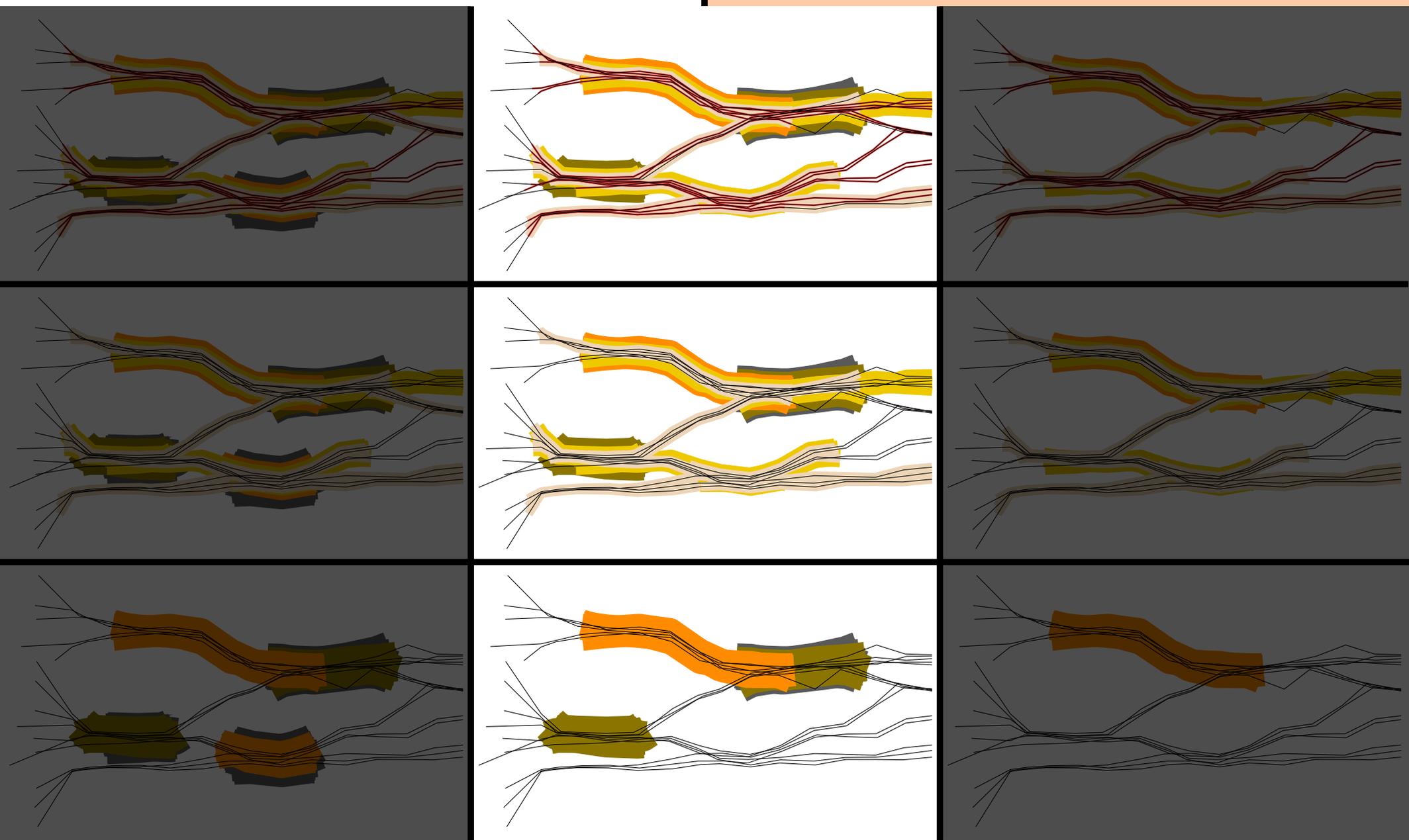
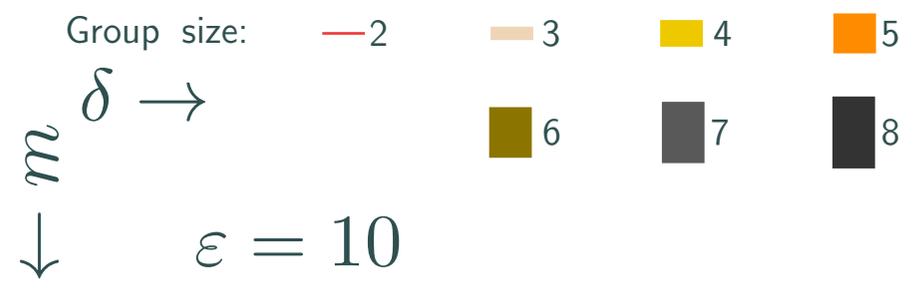
$N$  = total group size



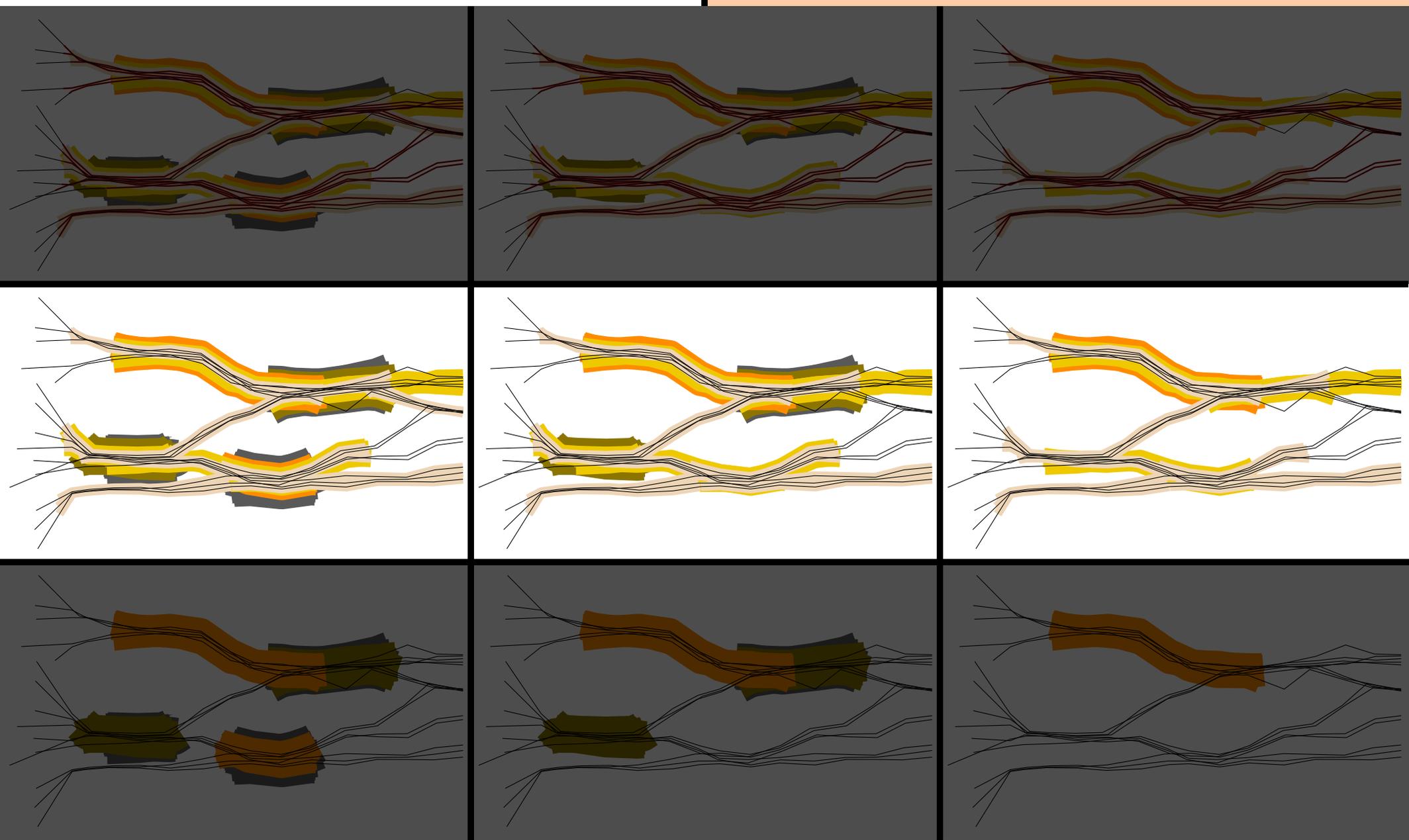
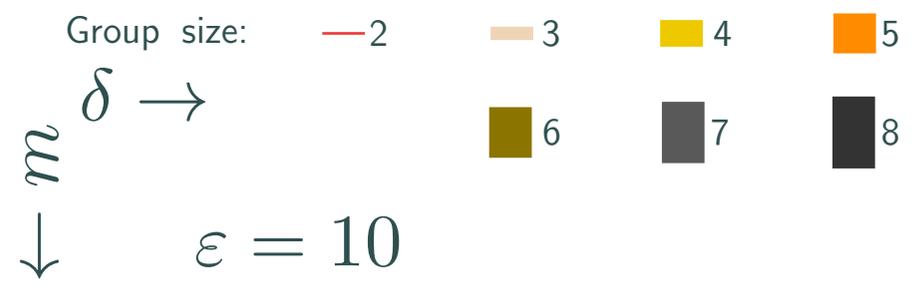
# Different Scales



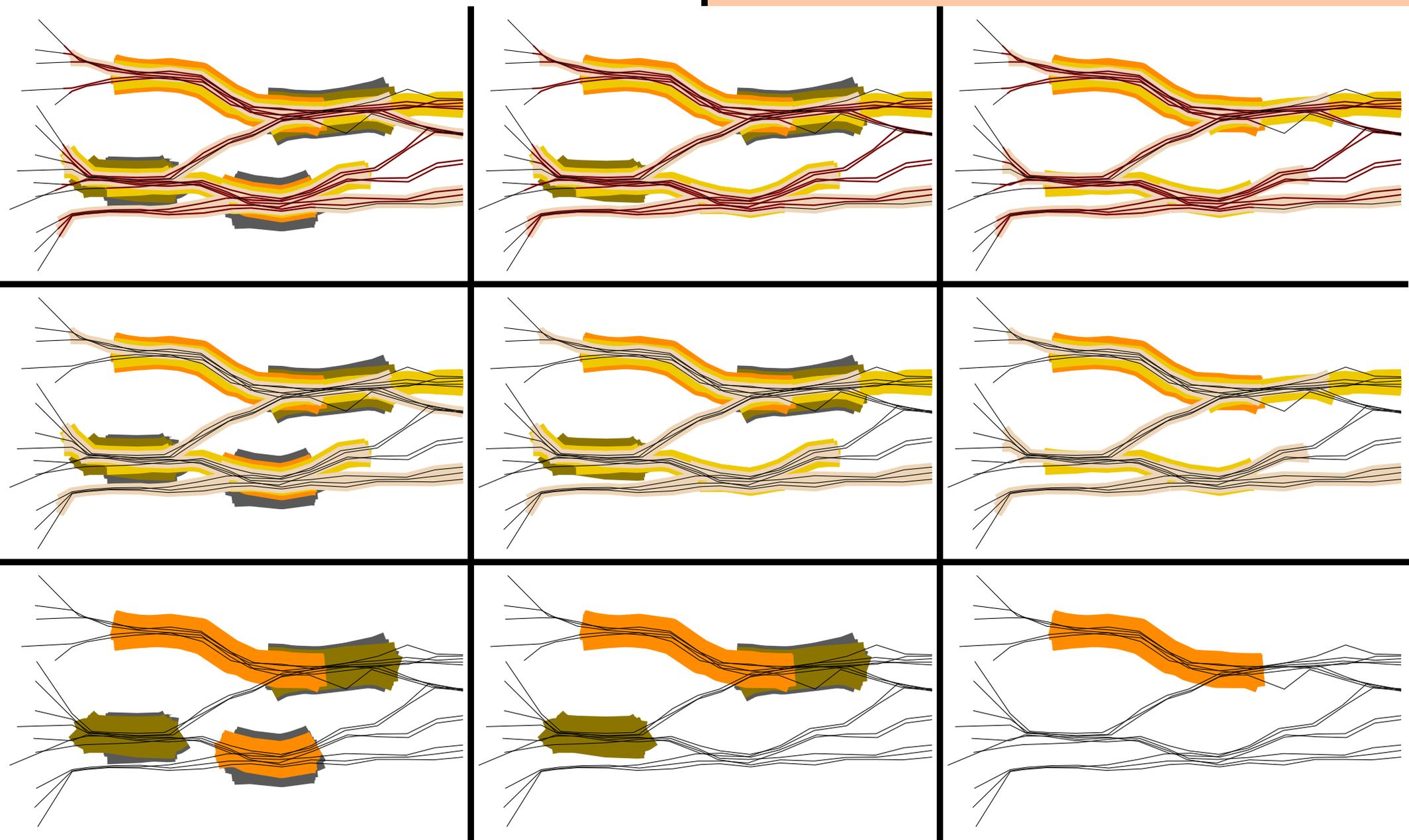
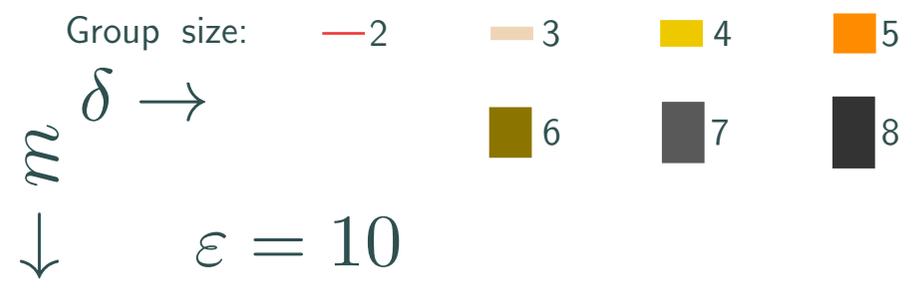
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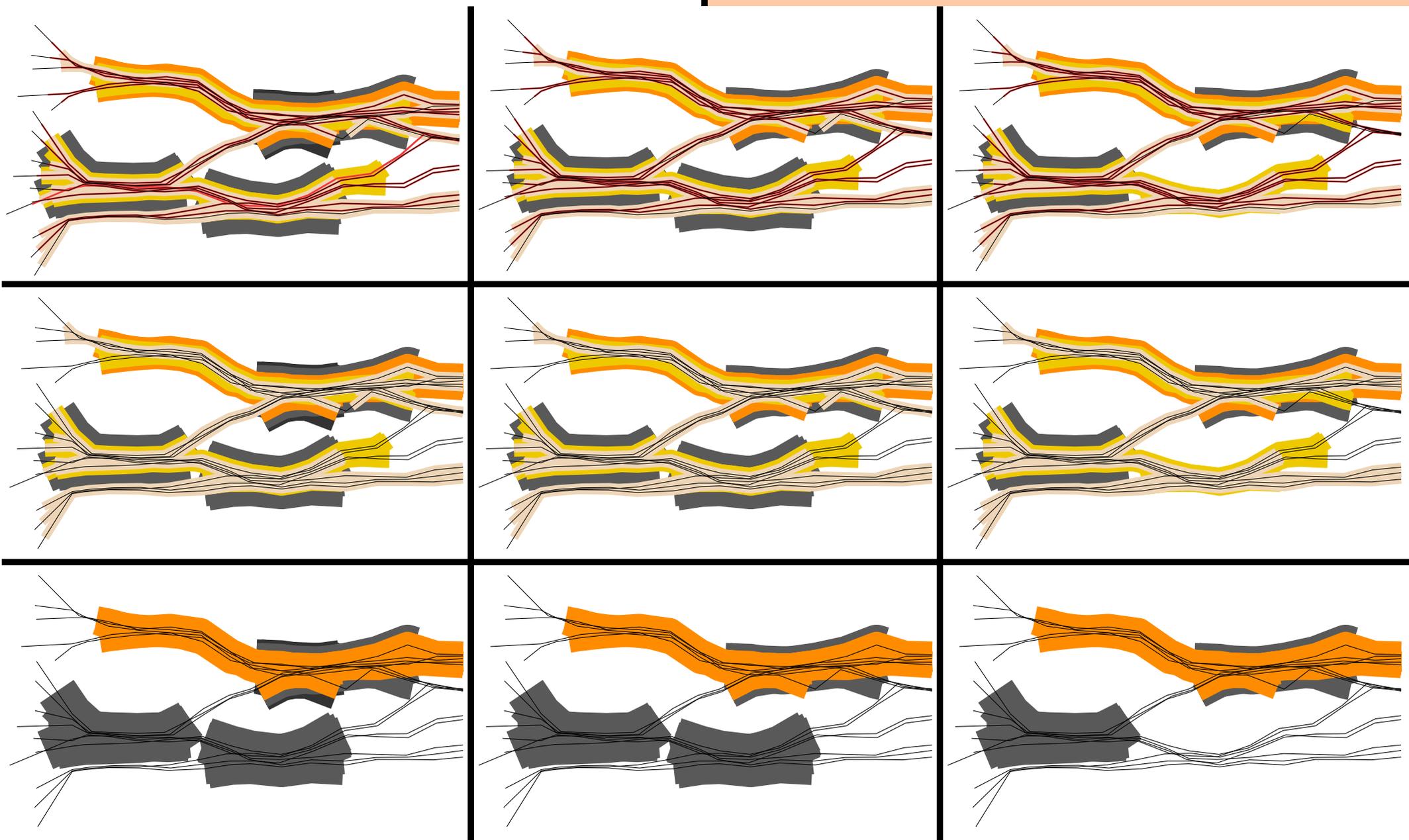
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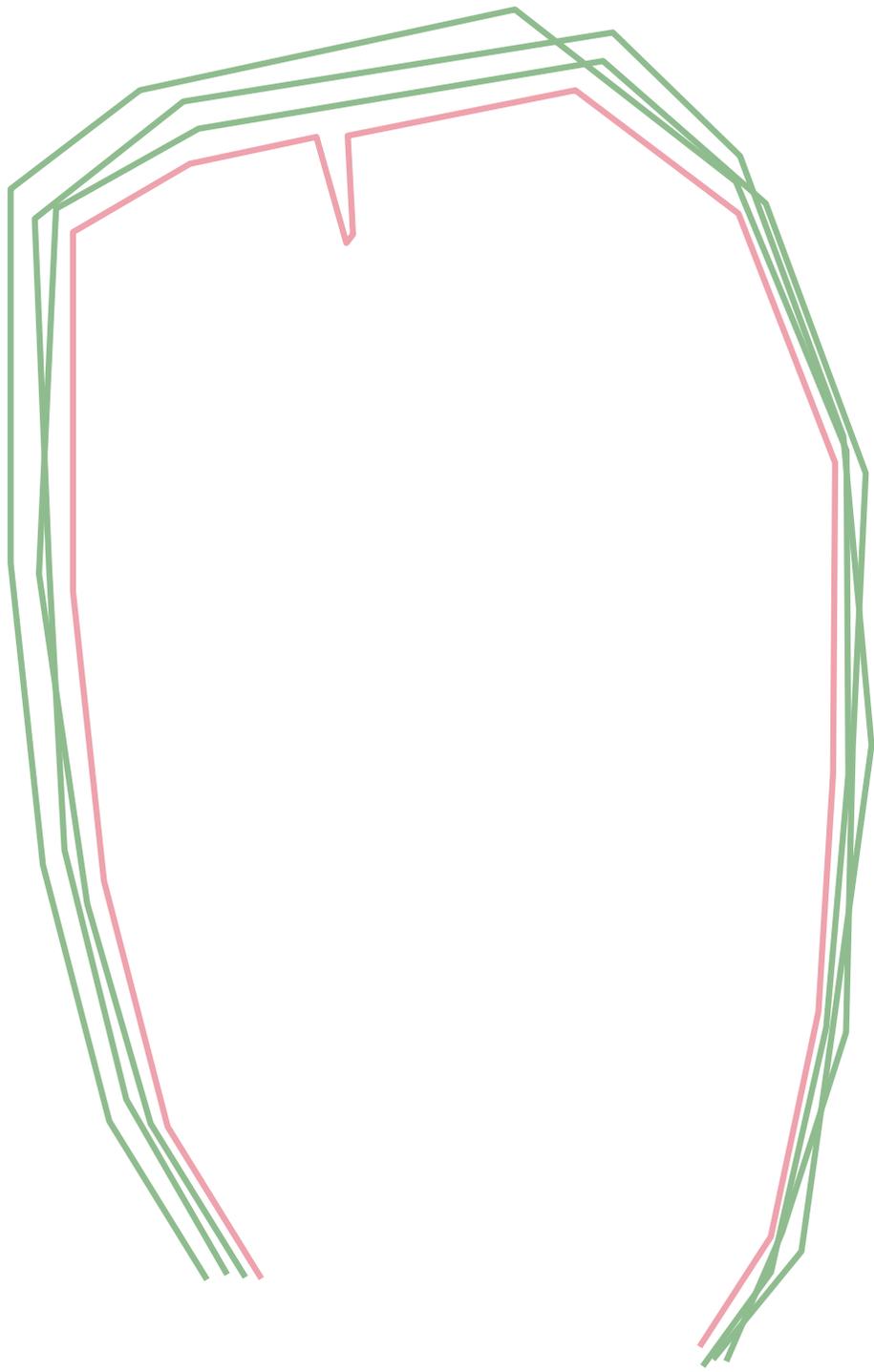
Group size: — 2 — 3 — 4 — 5 — 6 — 7 — 8

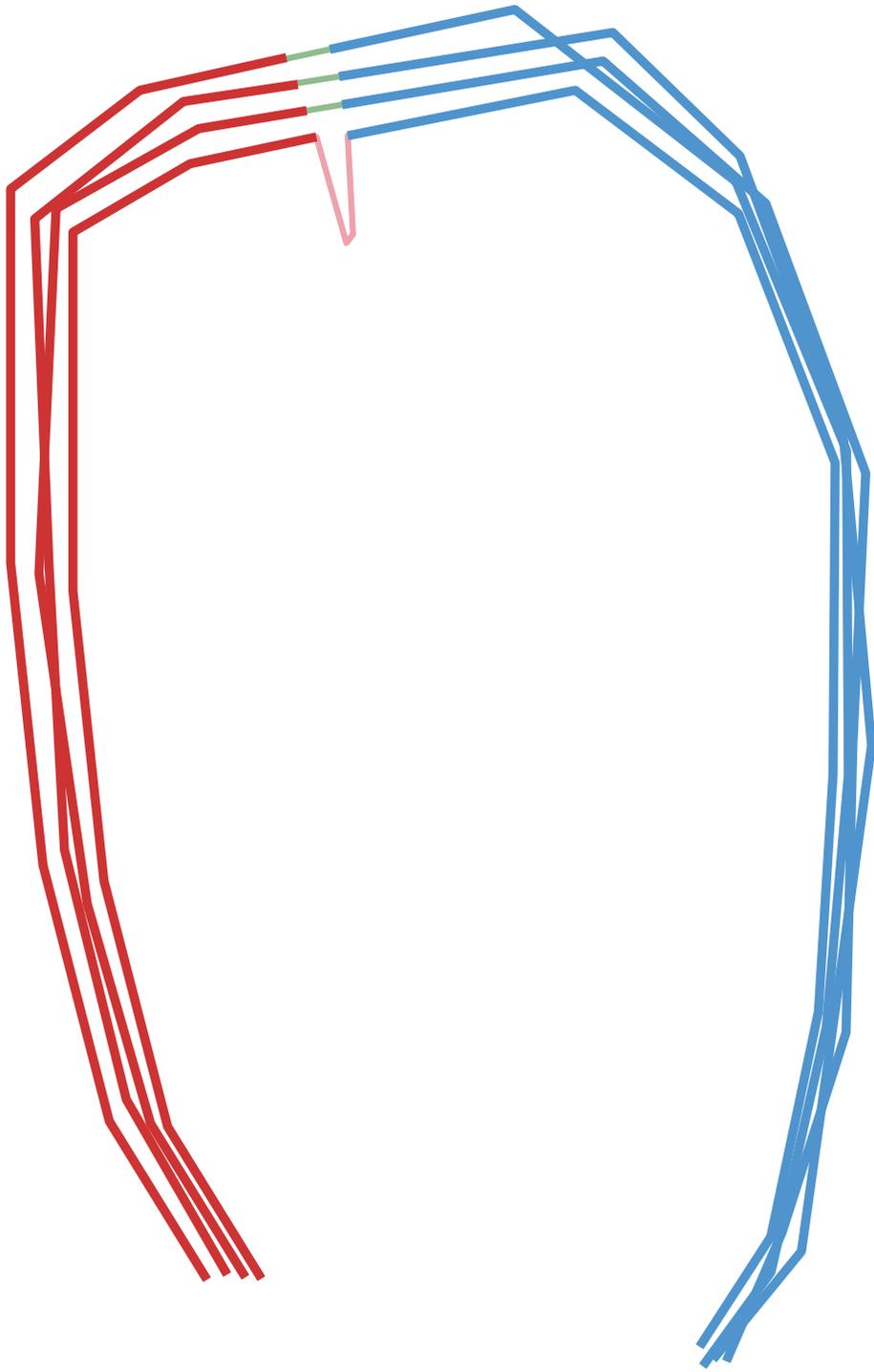
$\delta \rightarrow$

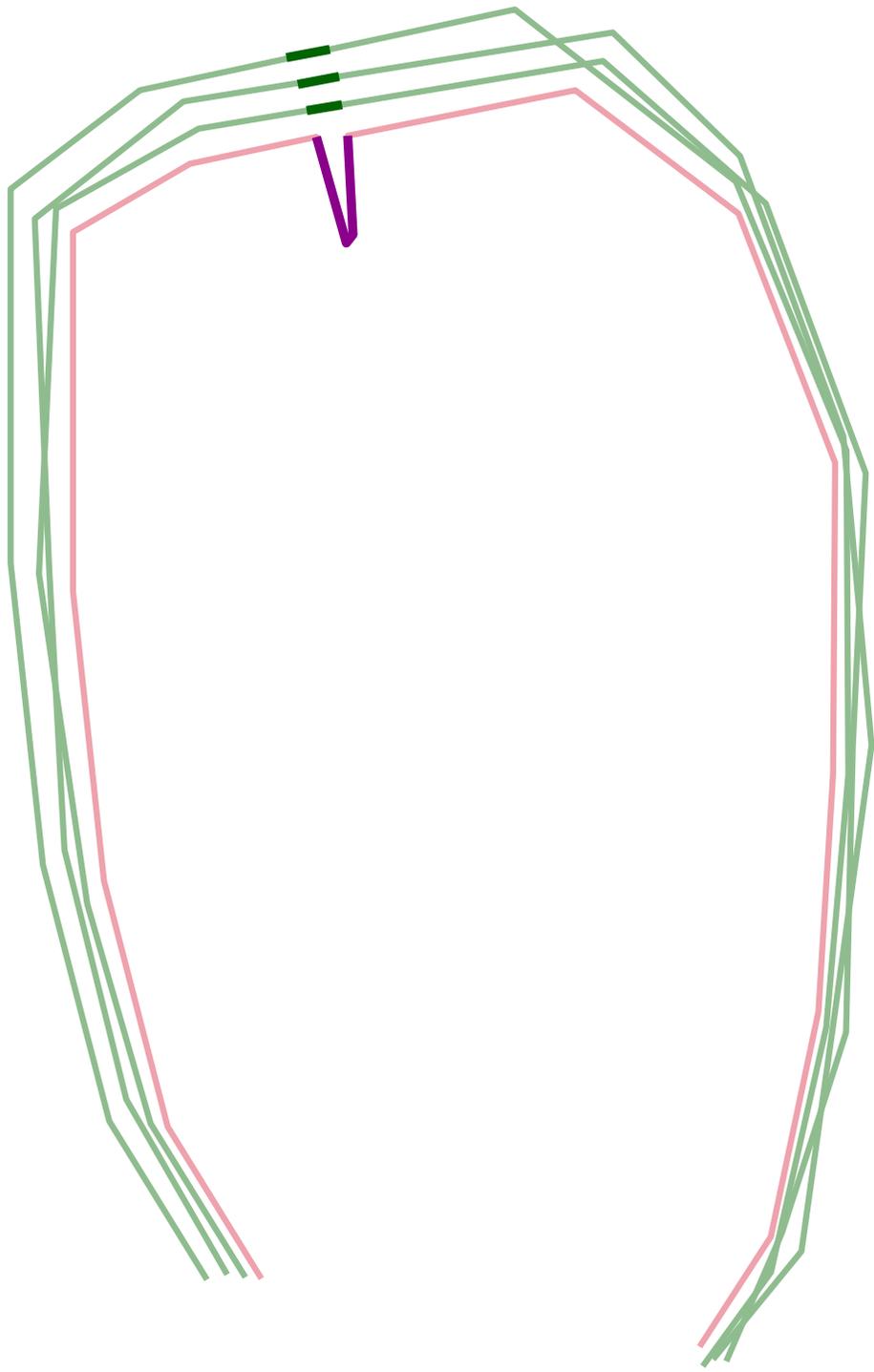
$m \downarrow$

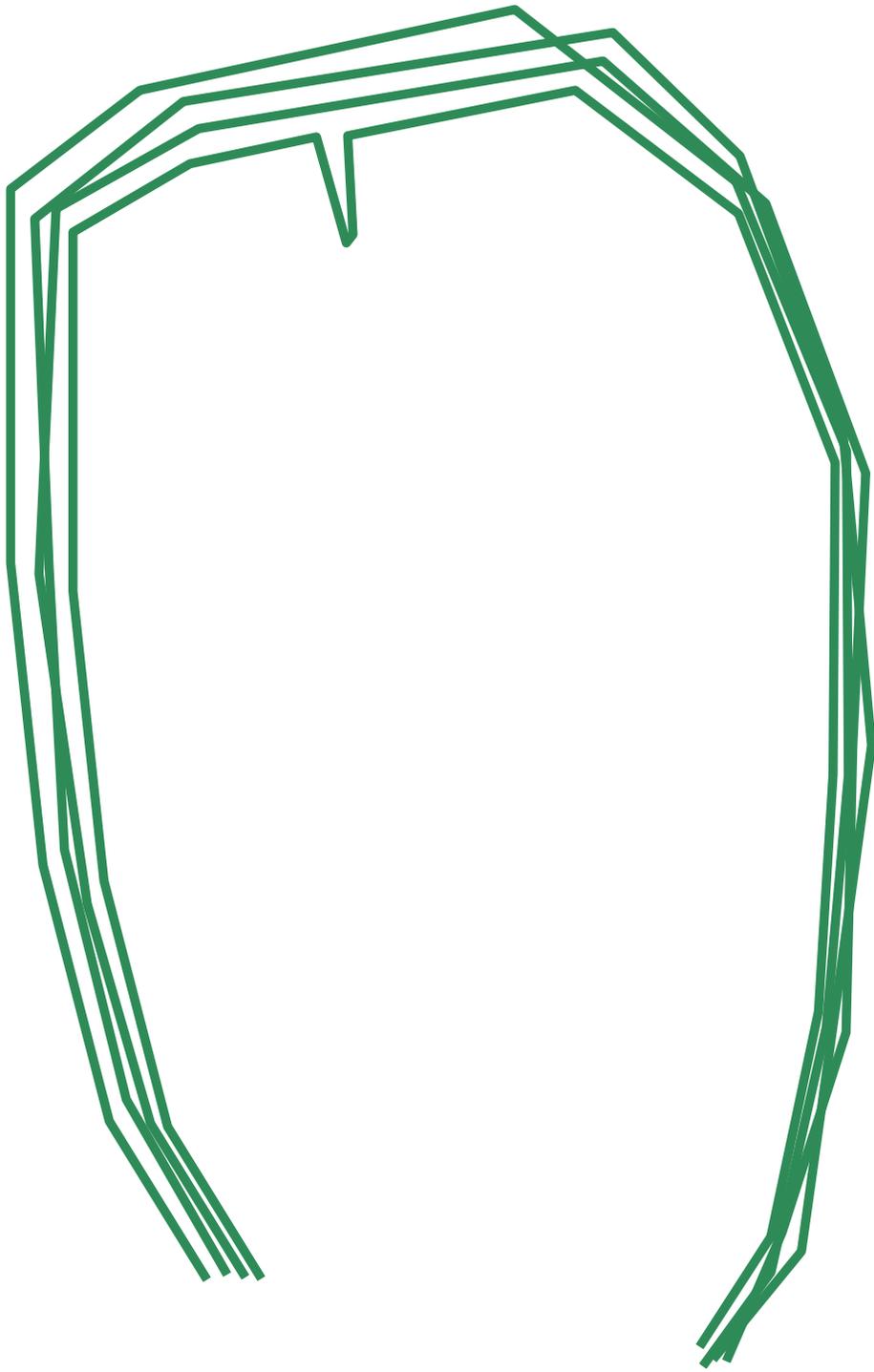
$\varepsilon = 20$



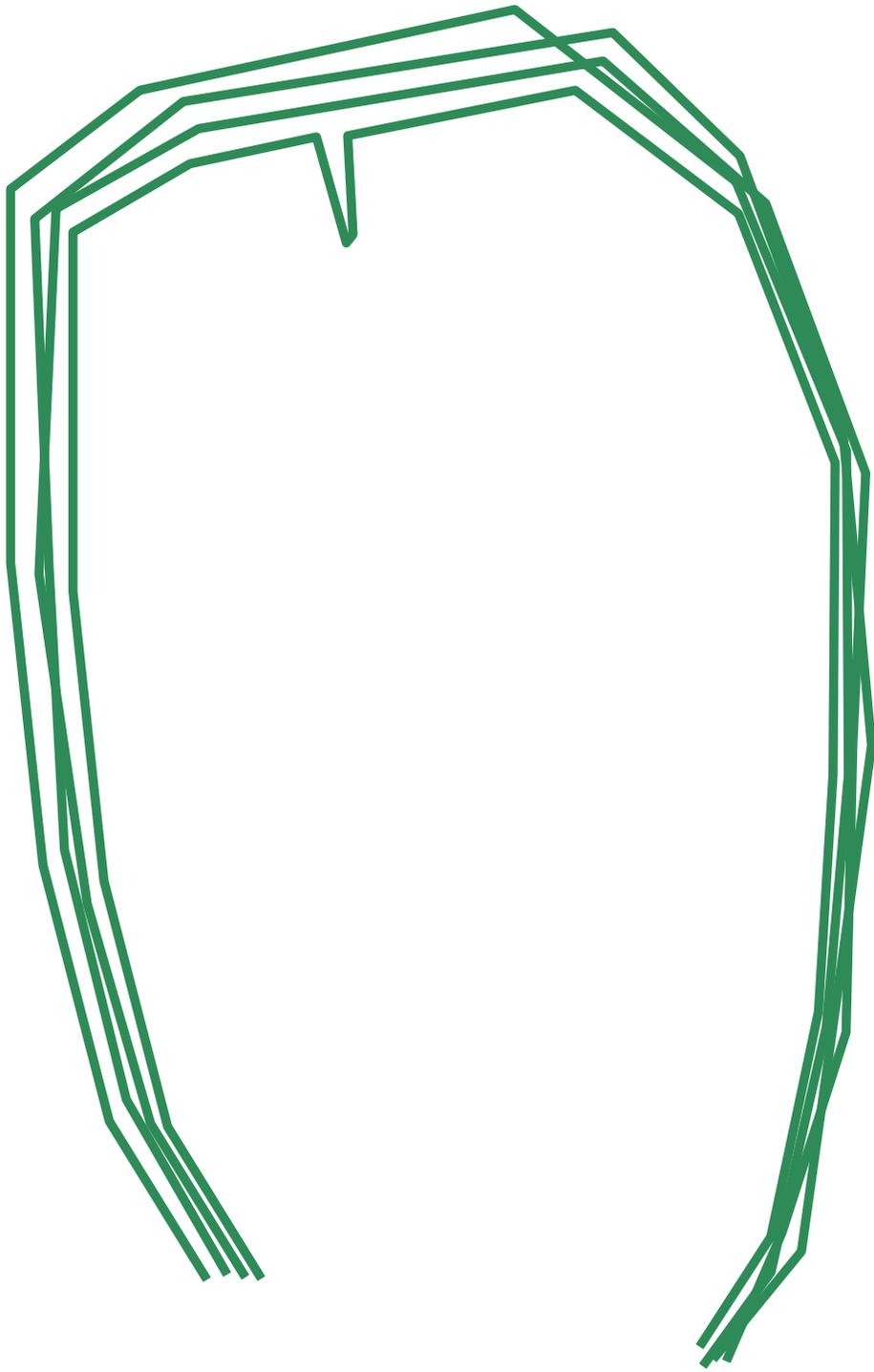








Robustness



## Robustness

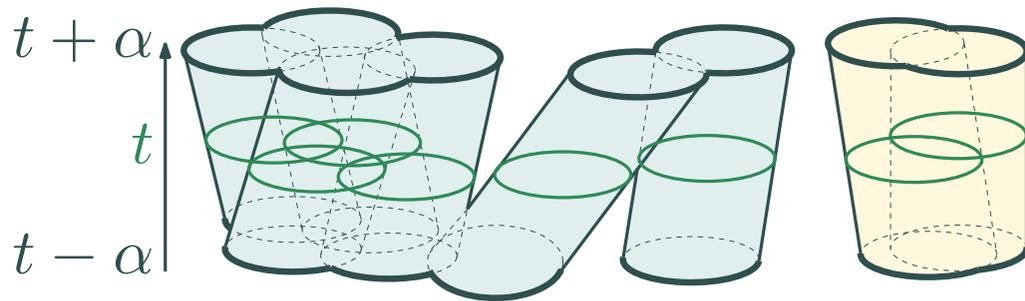
Introduce a new parameter  $\alpha$

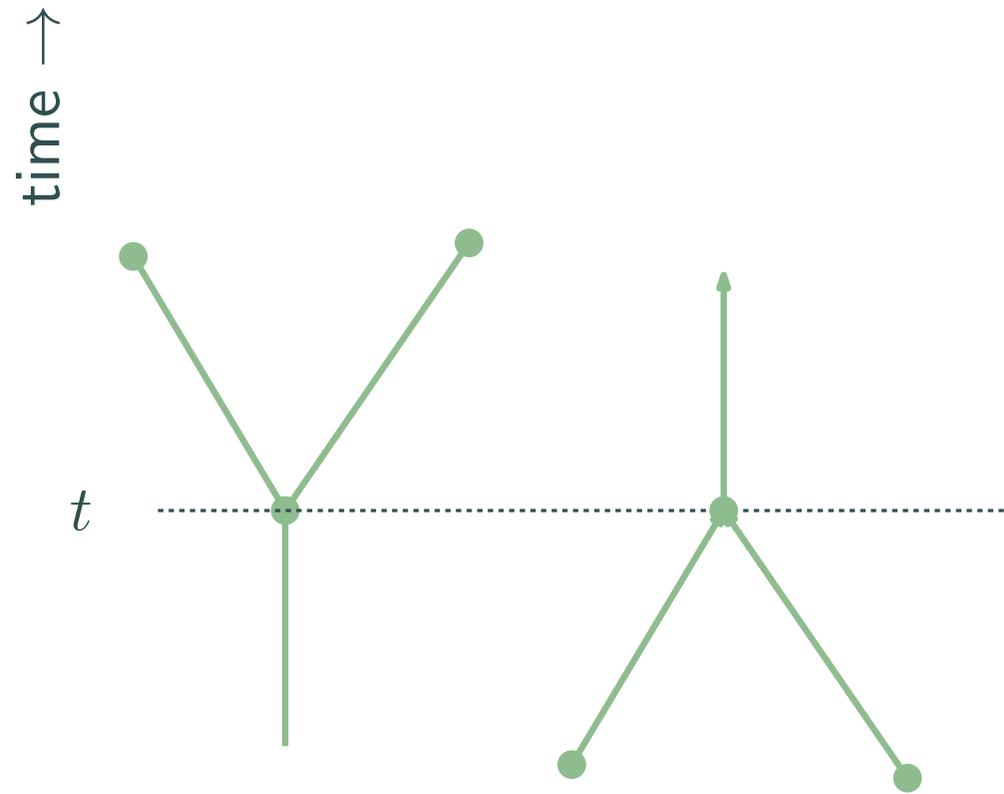
$x$  and  $y$  are in one group at time  $t$  if they are  $\varepsilon$ -connected at a time  $t' \in [t - \alpha, t + \alpha]$

# Robustness

Introduce a new parameter  $\alpha$

$x$  and  $y$  are in one group at time  $t$  if they are  $\varepsilon$ -connected at a time  $t' \in [t - \alpha, t + \alpha]$





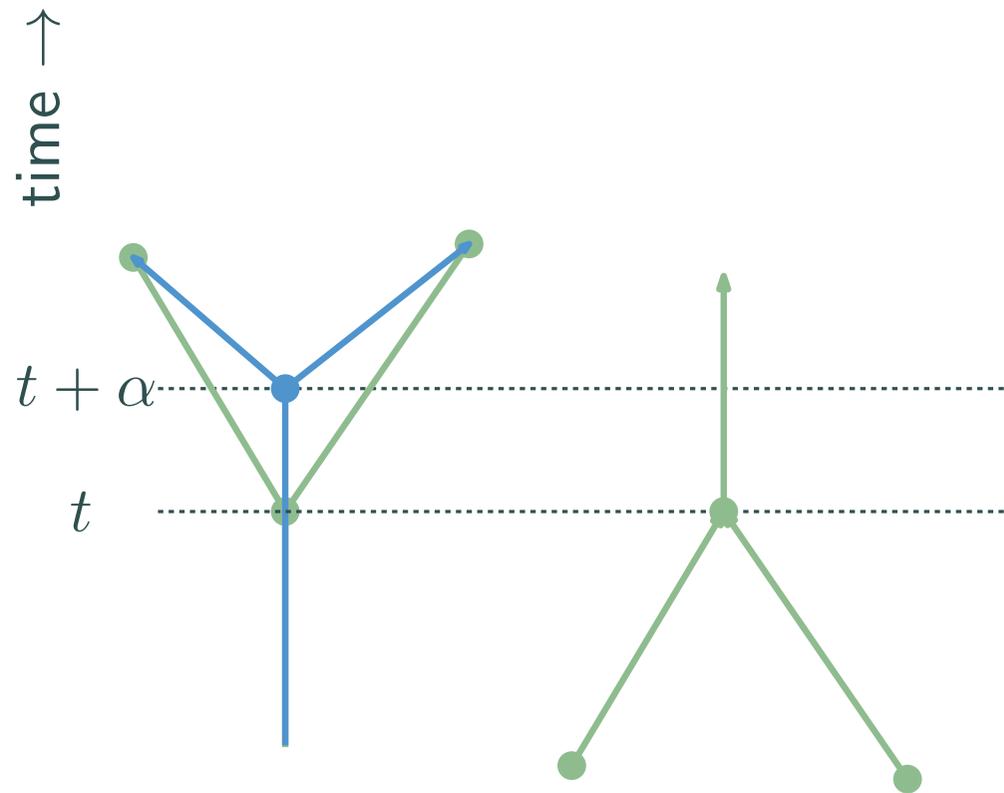
## Robustness

Introduce a new parameter  $\alpha$

$x$  and  $y$  are in one group at time  $t$  if they are  $\varepsilon$ -connected at a time  $t' \in [t - \alpha, t + \alpha]$



The vertices in  $\mathcal{R}$  move in time.



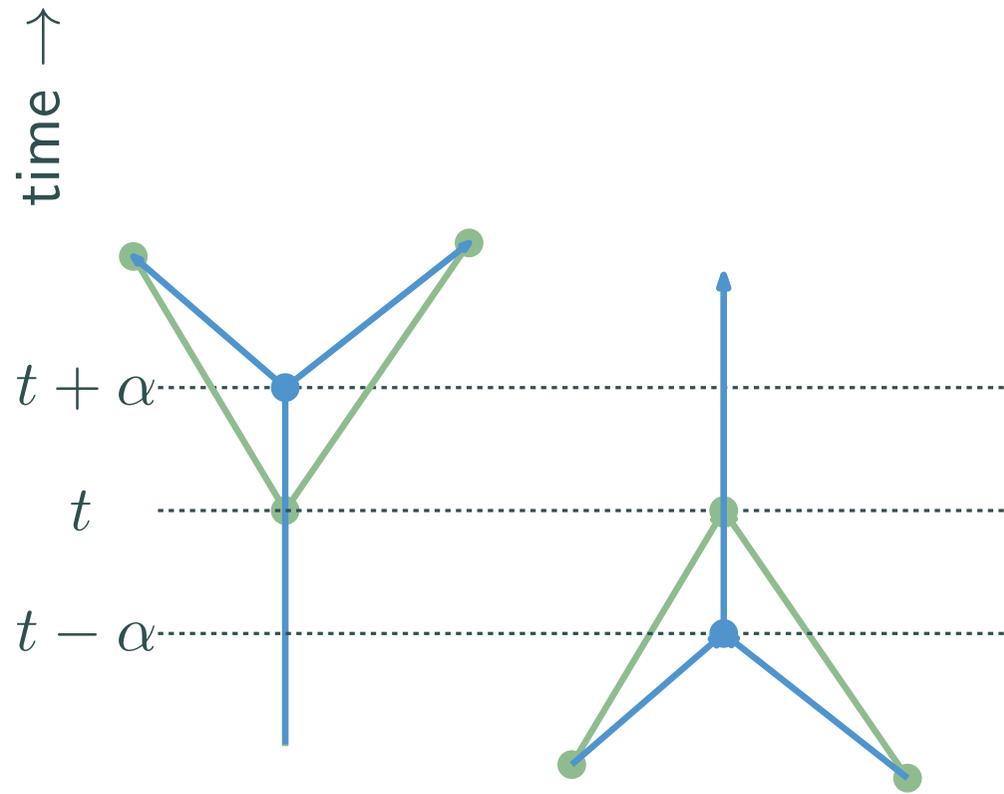
## Robustness

Introduce a new parameter  $\alpha$

$x$  and  $y$  are in one group at time  $t$  if they are  $\varepsilon$ -connected at a time  $t' \in [t - \alpha, t + \alpha]$



The vertices in  $\mathcal{R}$  move in time.



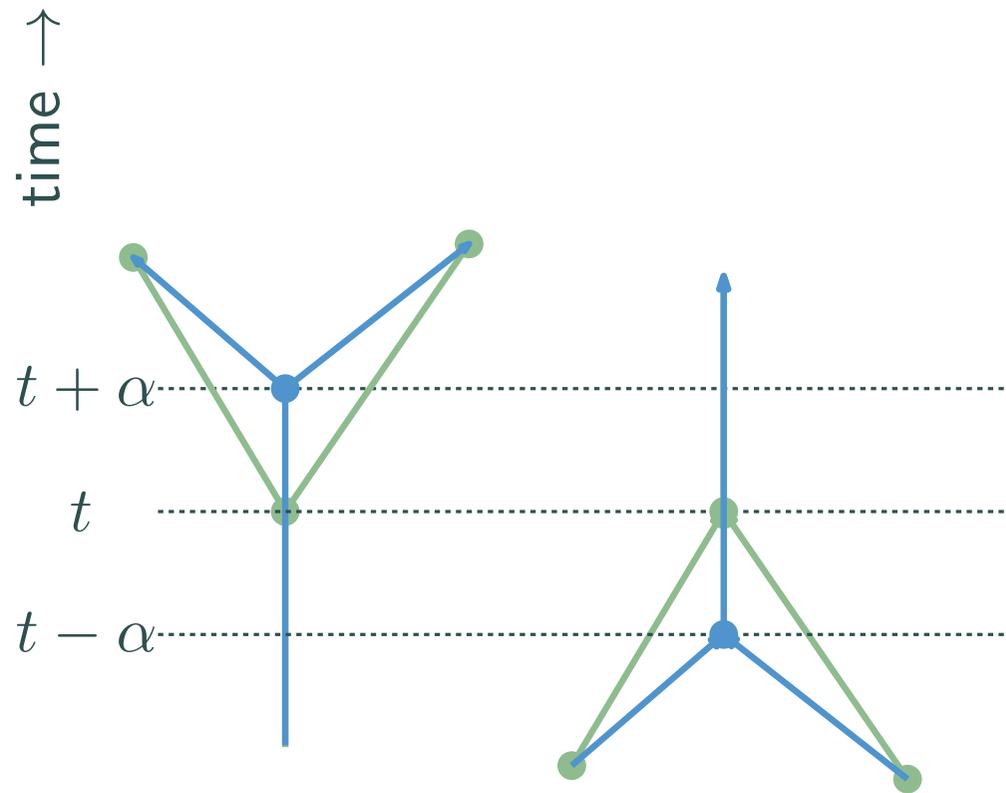
## Robustness

Introduce a new parameter  $\alpha$

$x$  and  $y$  are in one group at time  $t$  if they are  $\varepsilon$ -connected at a time  $t' \in [t - \alpha, t + \alpha]$

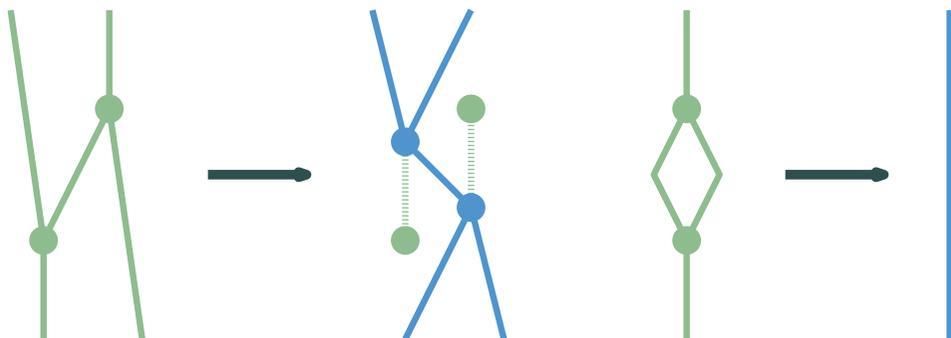


The vertices in  $\mathcal{R}$  move in time.



Passing

Collapse



# Robustness

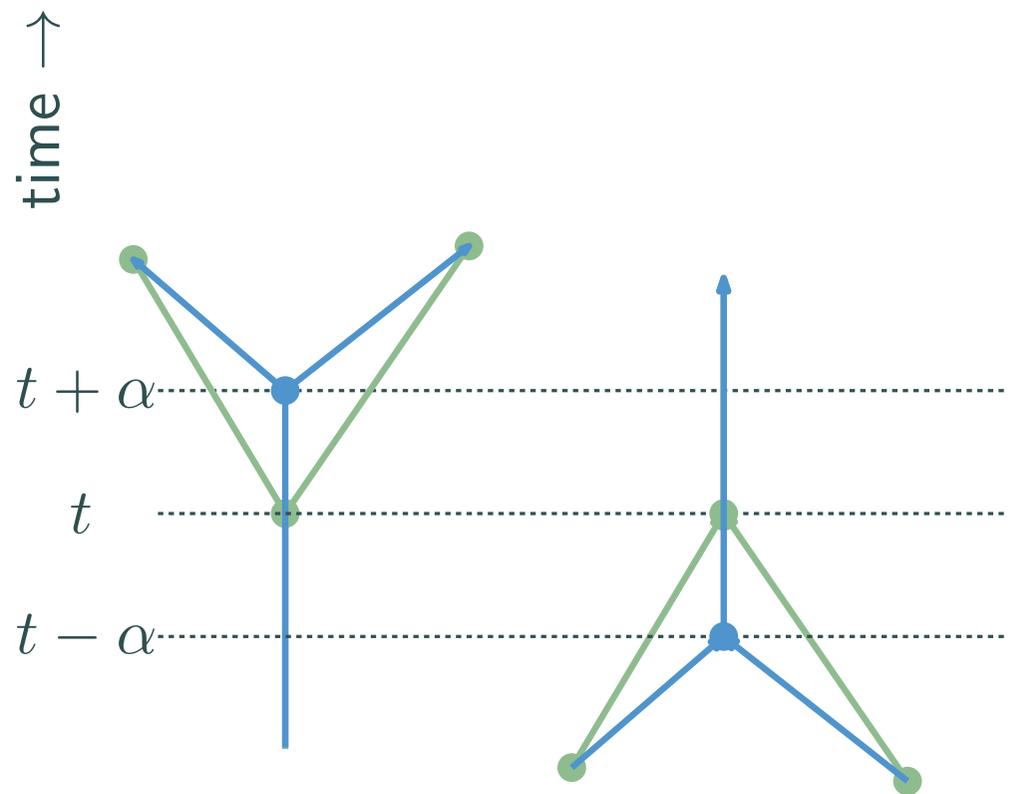
Introduce a new parameter  $\alpha$

$x$  and  $y$  are in one group at time  $t$  if they are  $\varepsilon$ -connected at a time  $t' \in [t - \alpha, t + \alpha]$



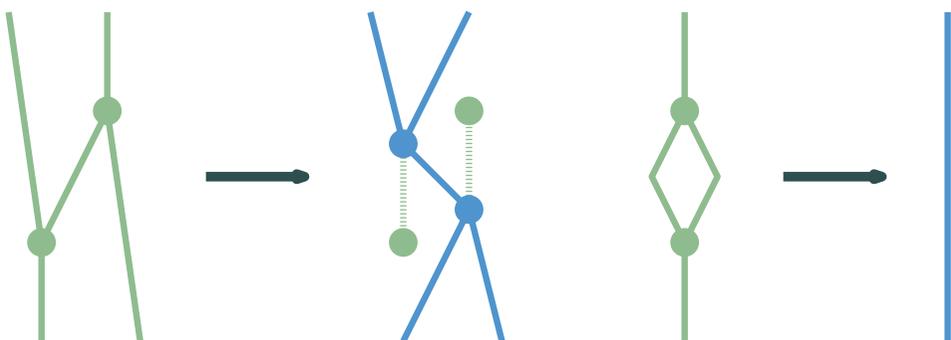
The vertices in  $\mathcal{R}$  move in time.

$\mathcal{R}$  changes at **encounter** events.



Passing

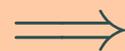
Collapse



## Robustness

Introduce a new parameter  $\alpha$

$x$  and  $y$  are in one group at time  $t$  if they are  $\varepsilon$ -connected at a time  $t' \in [t - \alpha, t + \alpha]$



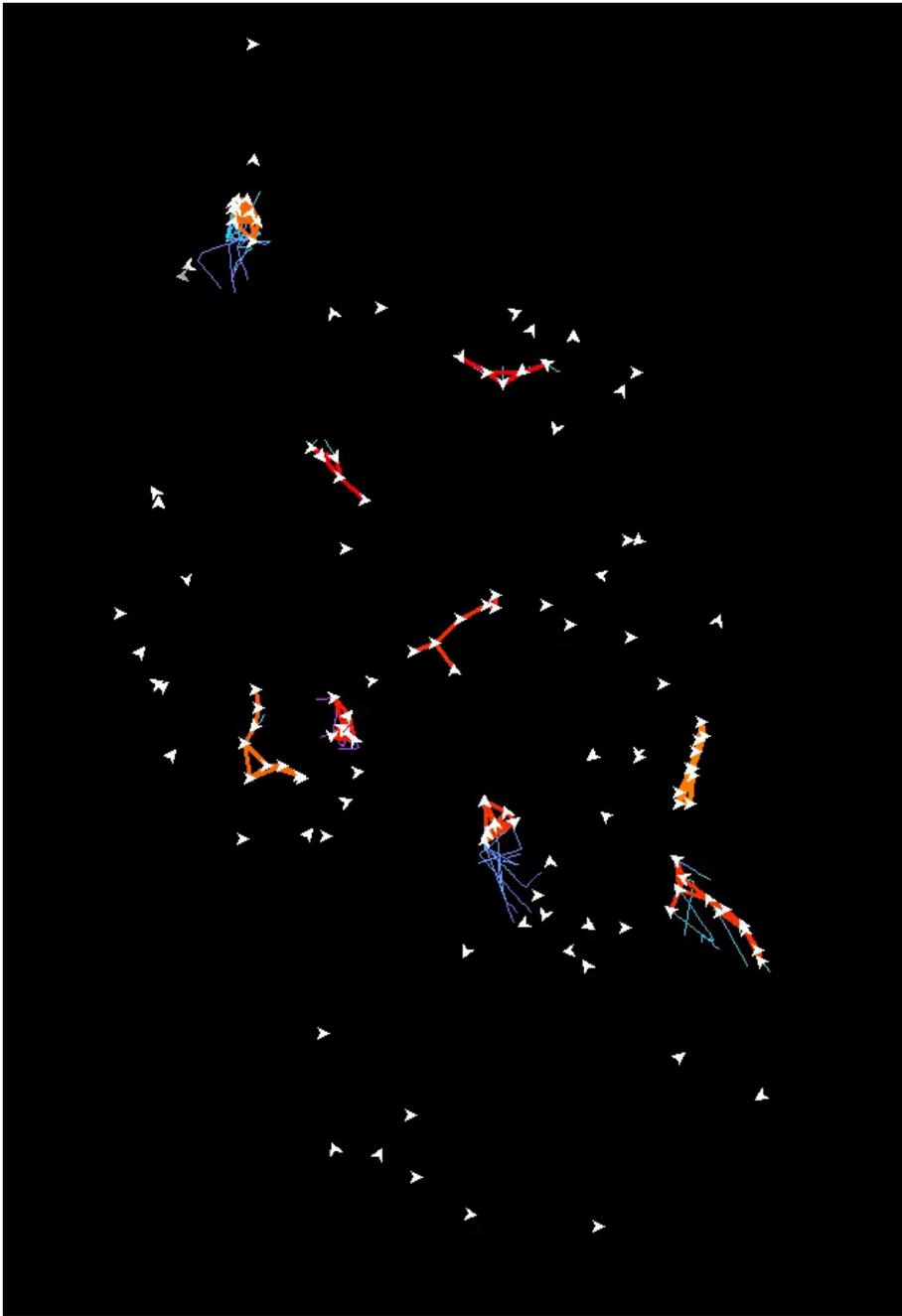
The vertices in  $\mathcal{R}$  move in time.

$\mathcal{R}$  changes at **encounter** events.

#encounter events:  $O(\tau n^3)$

we can compute them in  $O(\tau n^3 \log n)$  time.

# Starkey

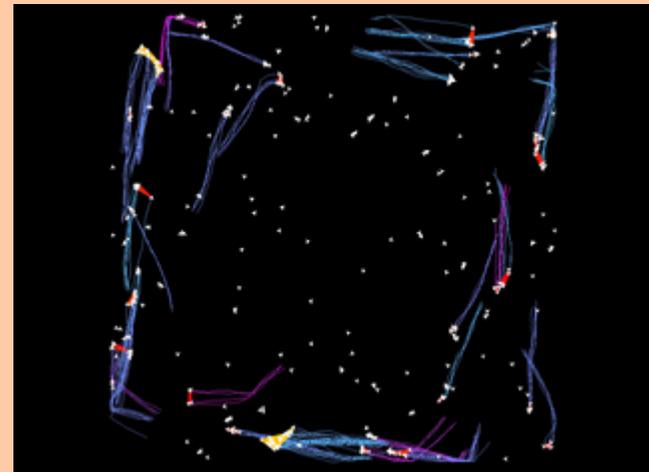


Elk (deer) tracked in Starkey (US).  
 $n = 126, \tau = 1264$

# Evaluation

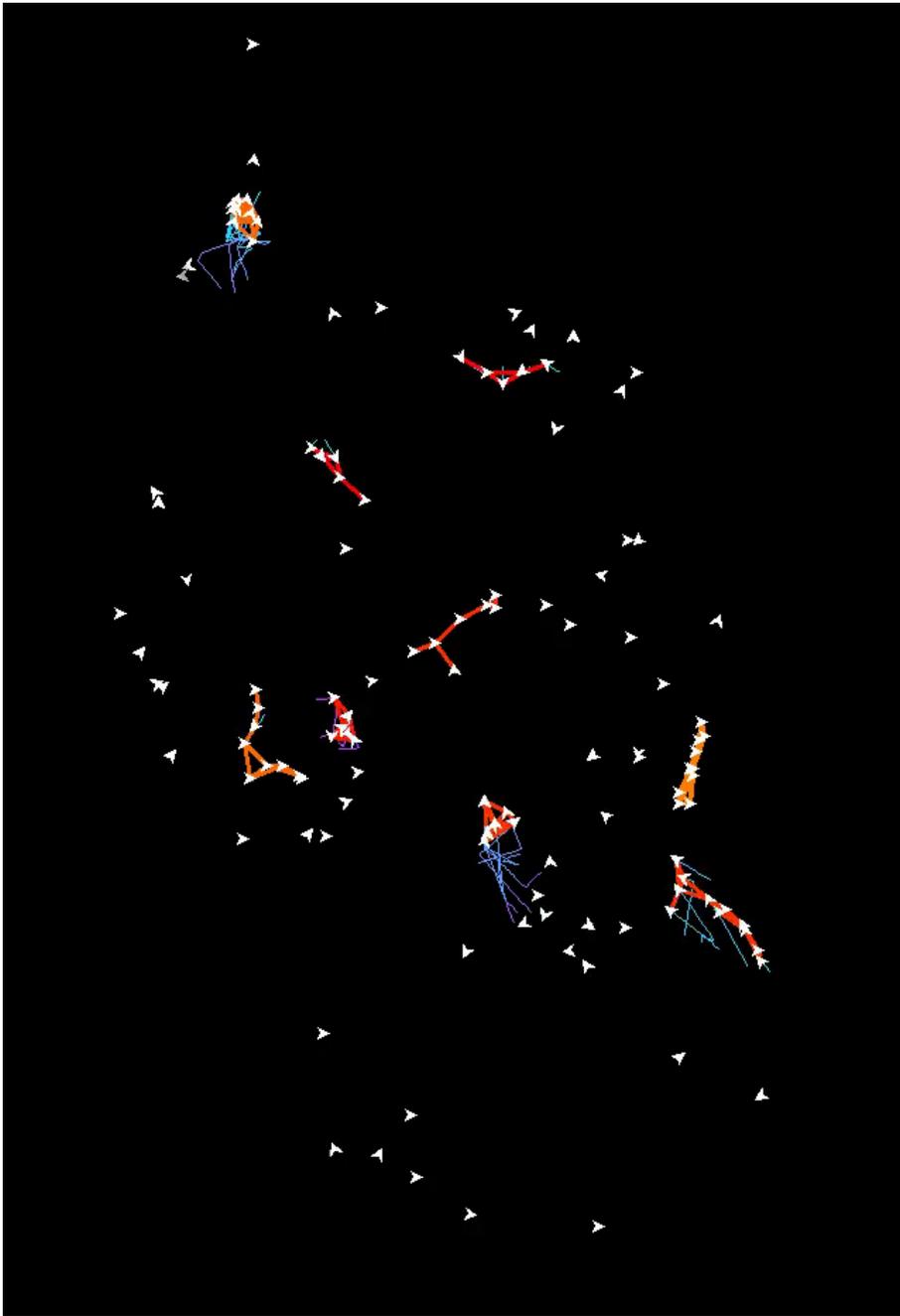
Evaluation on two data sets:  
Starkey and NetLogo.

## NetLogo



Based on the NetLogo flocking model.  
 $n = 400, \tau = 818$

# Starkey



Elk (deer) tracked in Starkey (US).  
 $n = 126, \tau = 1264$

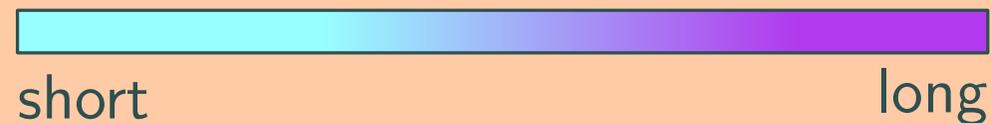
## Evaluation

Evaluation on two data sets:  
Starkey and NetLogo.

Group size:



Group duration:



Videos on:

[www.staff.science.uu.nl/~staa1006/  
grouping](http://www.staff.science.uu.nl/~staa1006/grouping)

# Starkey



Elk (deer) tracked in Starkey (US).  
 $n = 126, \tau = 1264$

## Evaluation

Evaluation on two data sets:  
Starkey and NetLogo.

Thank you!

small

large

Group duration:



short

long

Videos on:

[www.staff.science.uu.nl/~staa1006/  
grouping](http://www.staff.science.uu.nl/~staa1006/grouping)