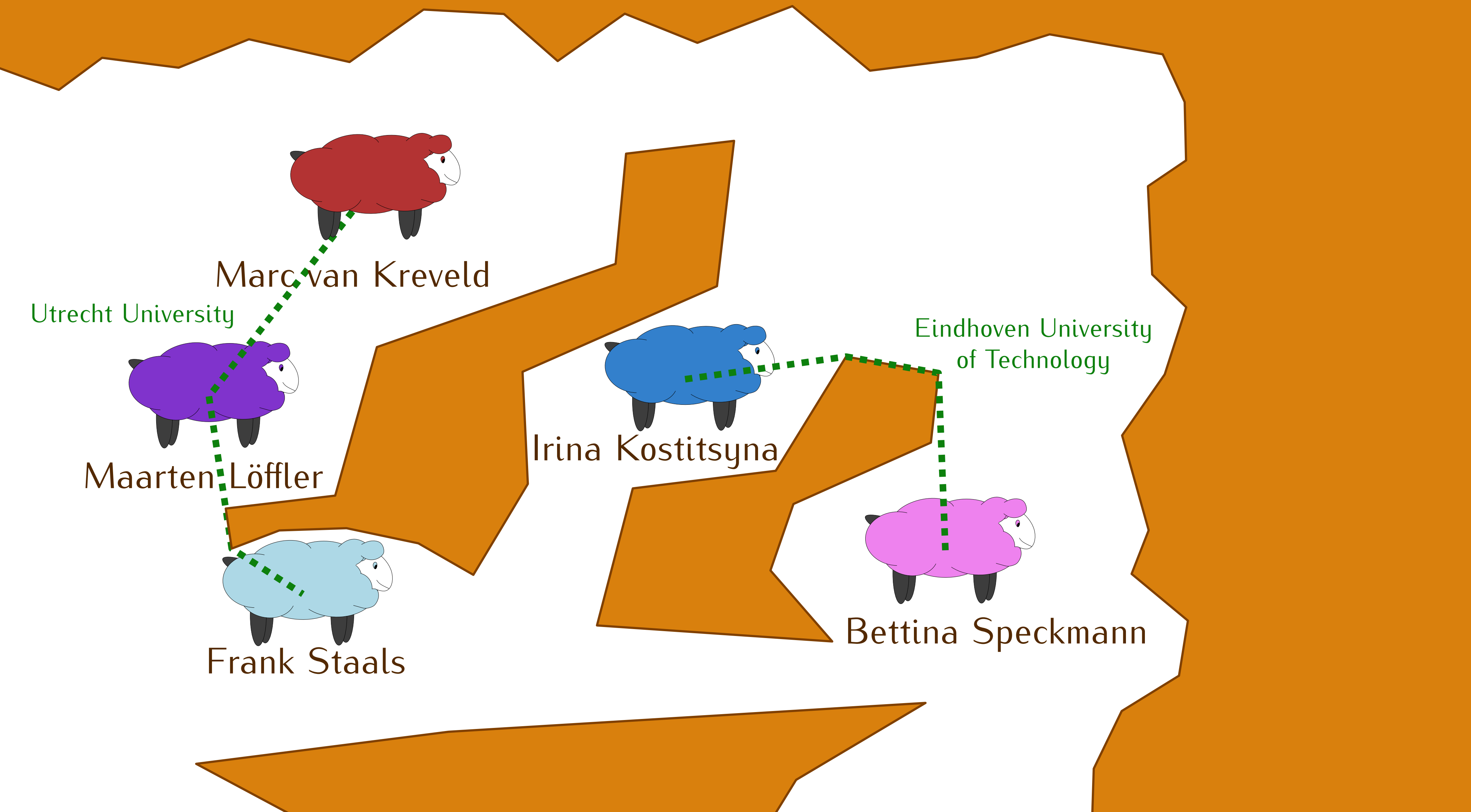
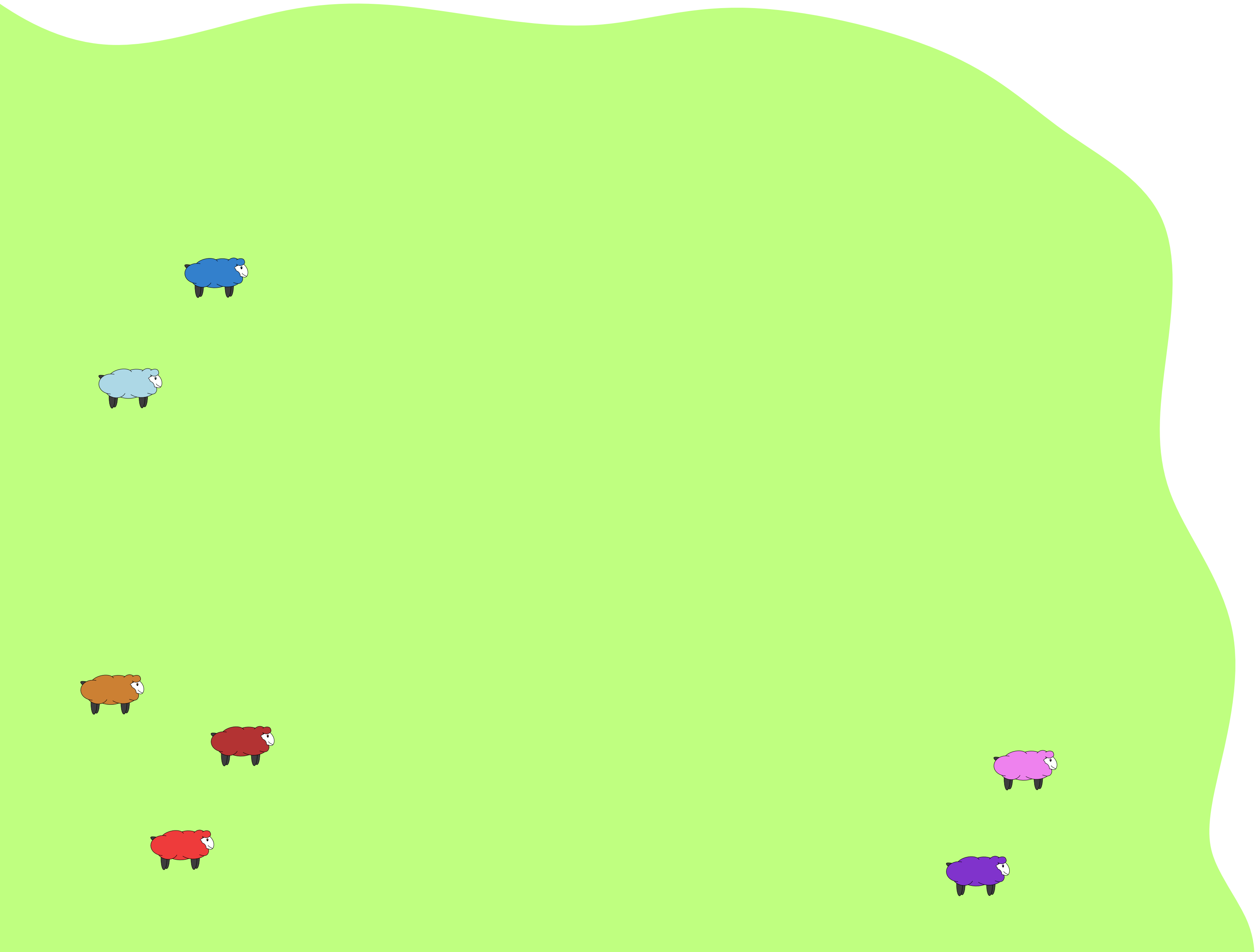
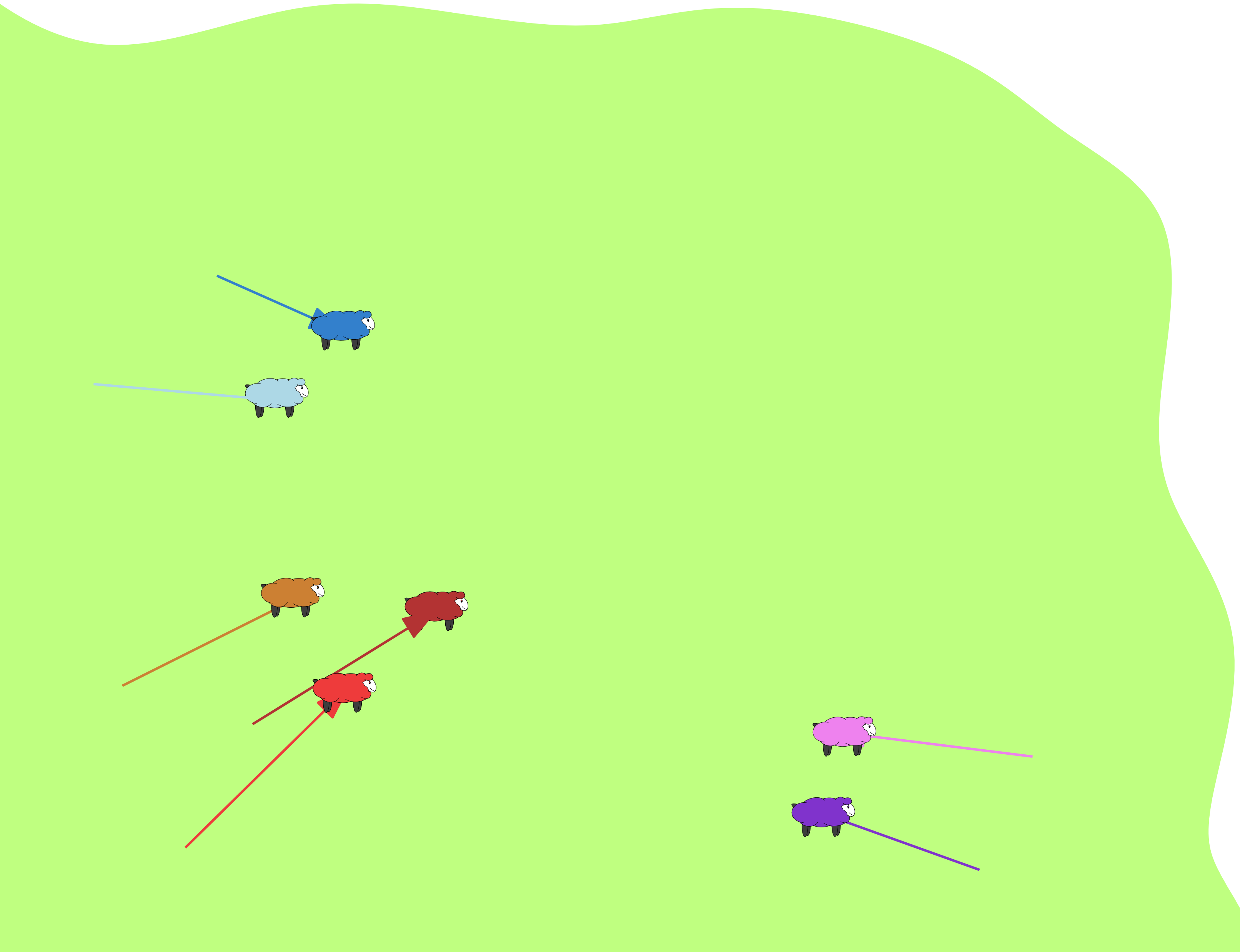
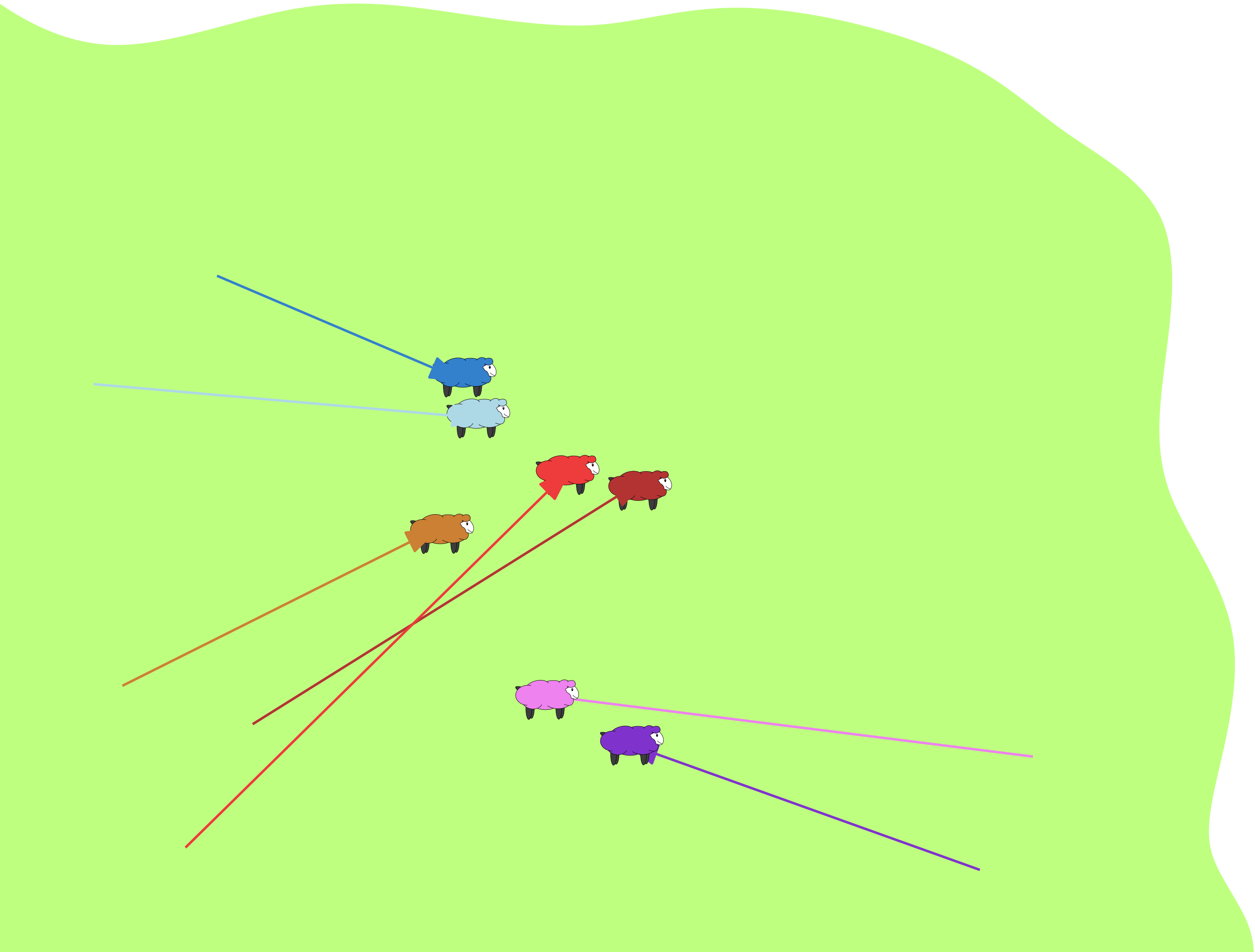
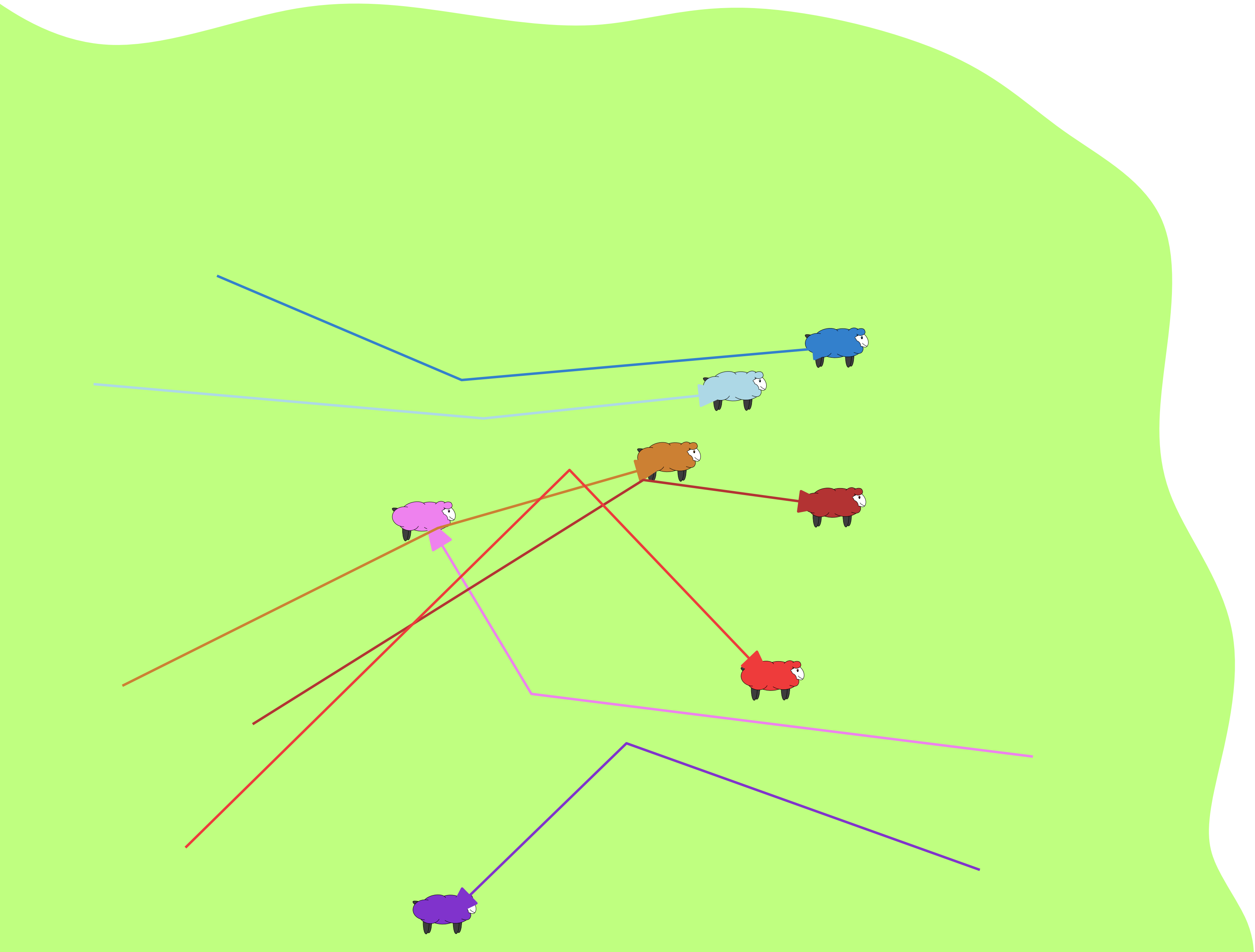
Trajectory Grouping Structure under Geodesic Distance

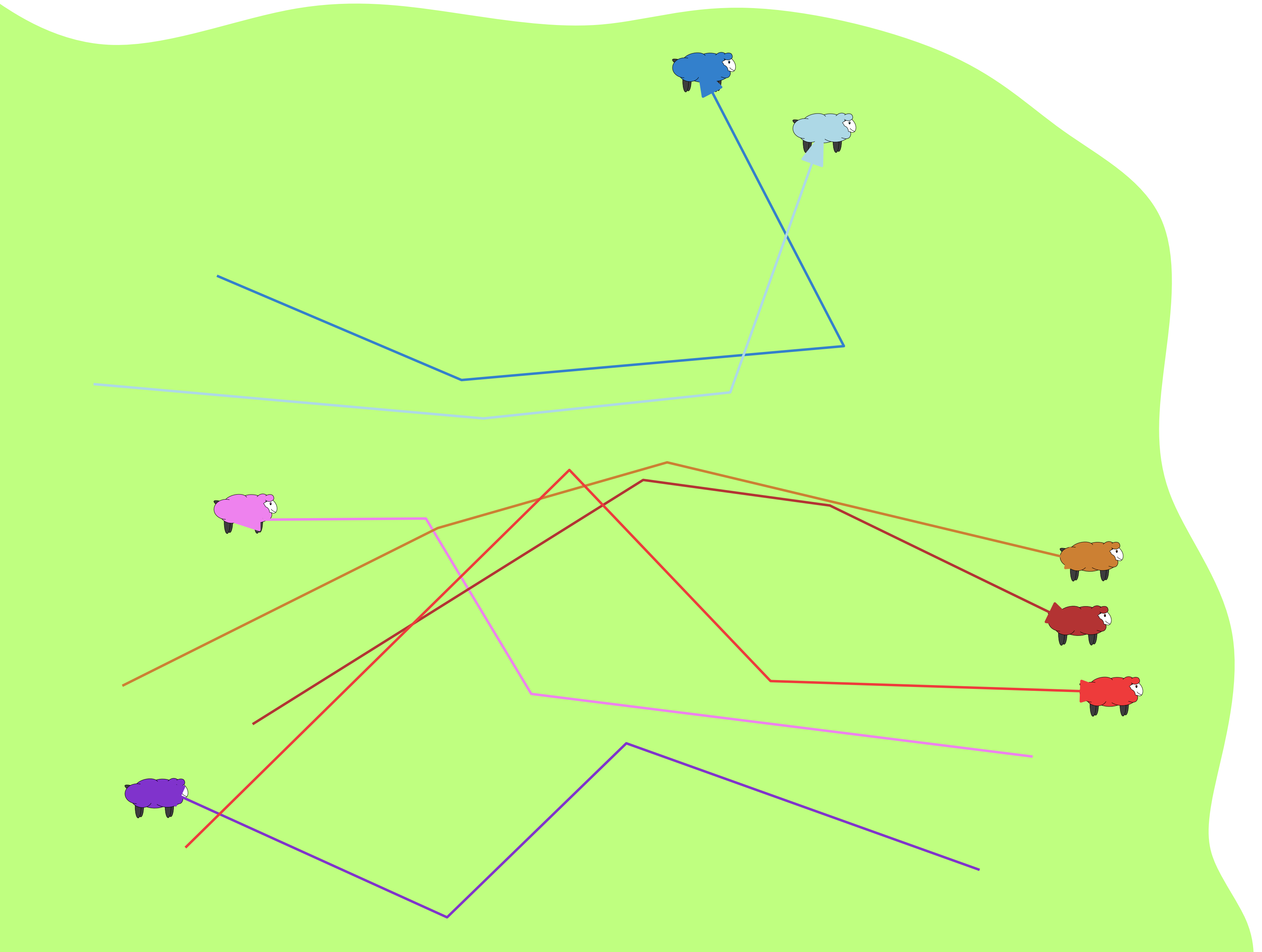


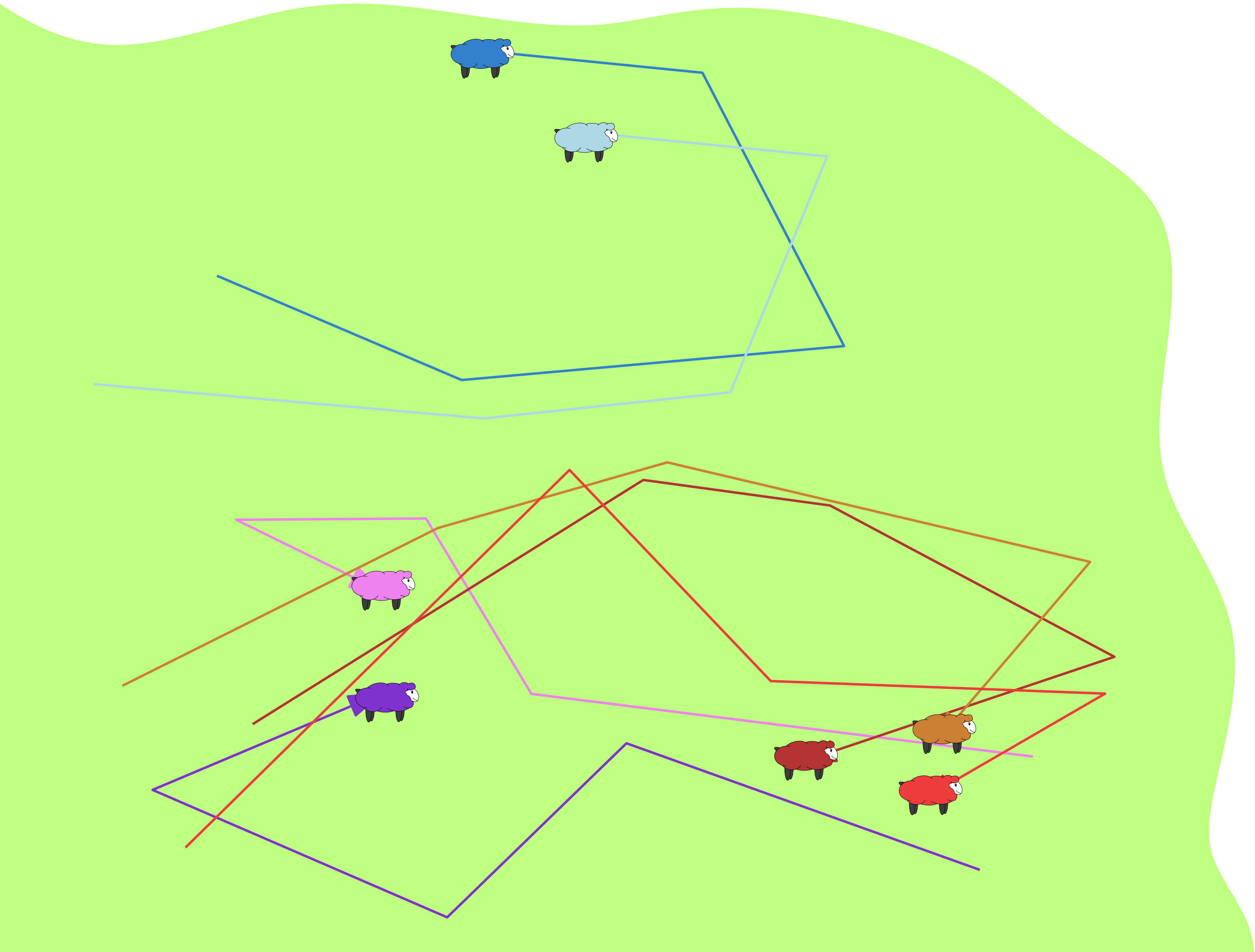




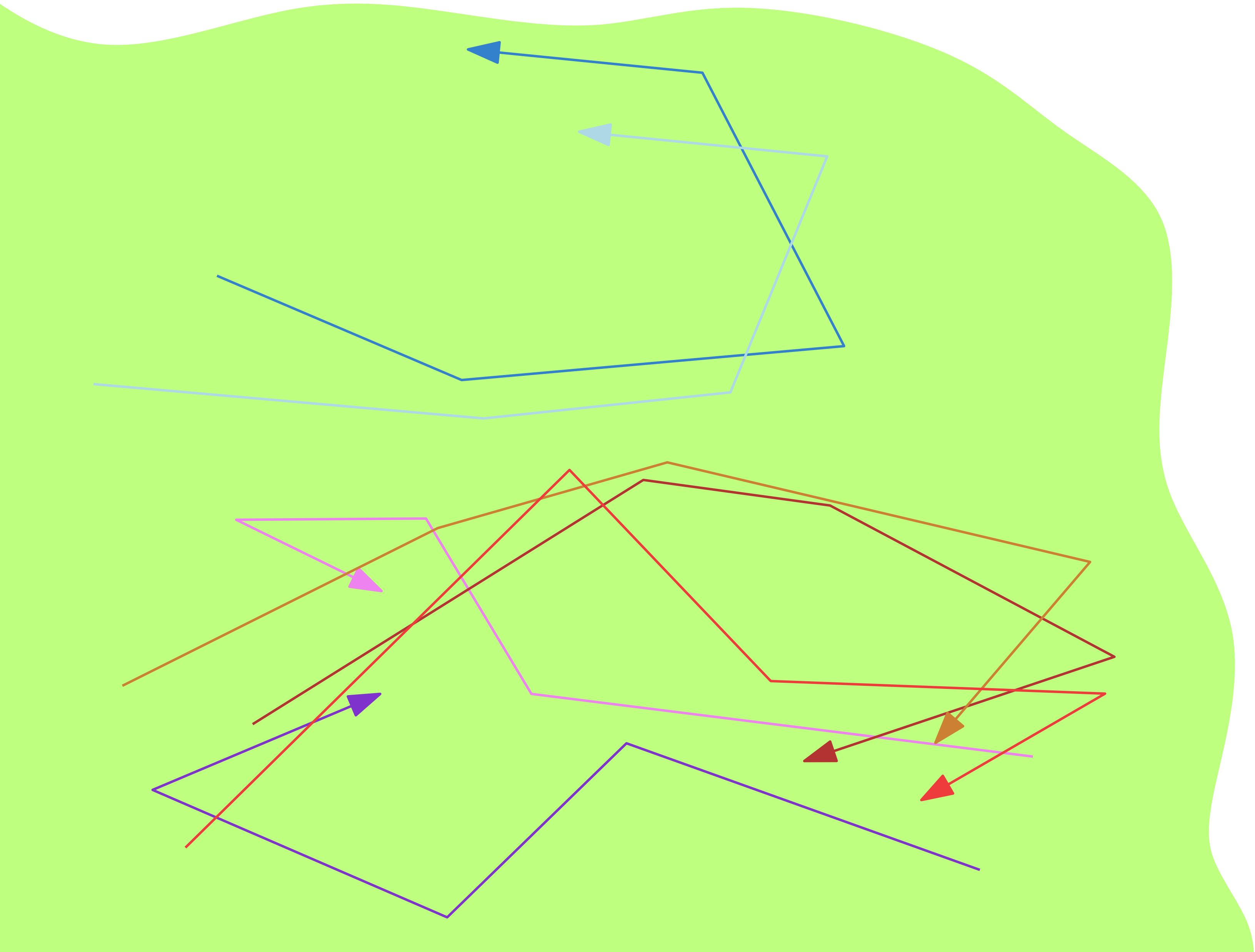






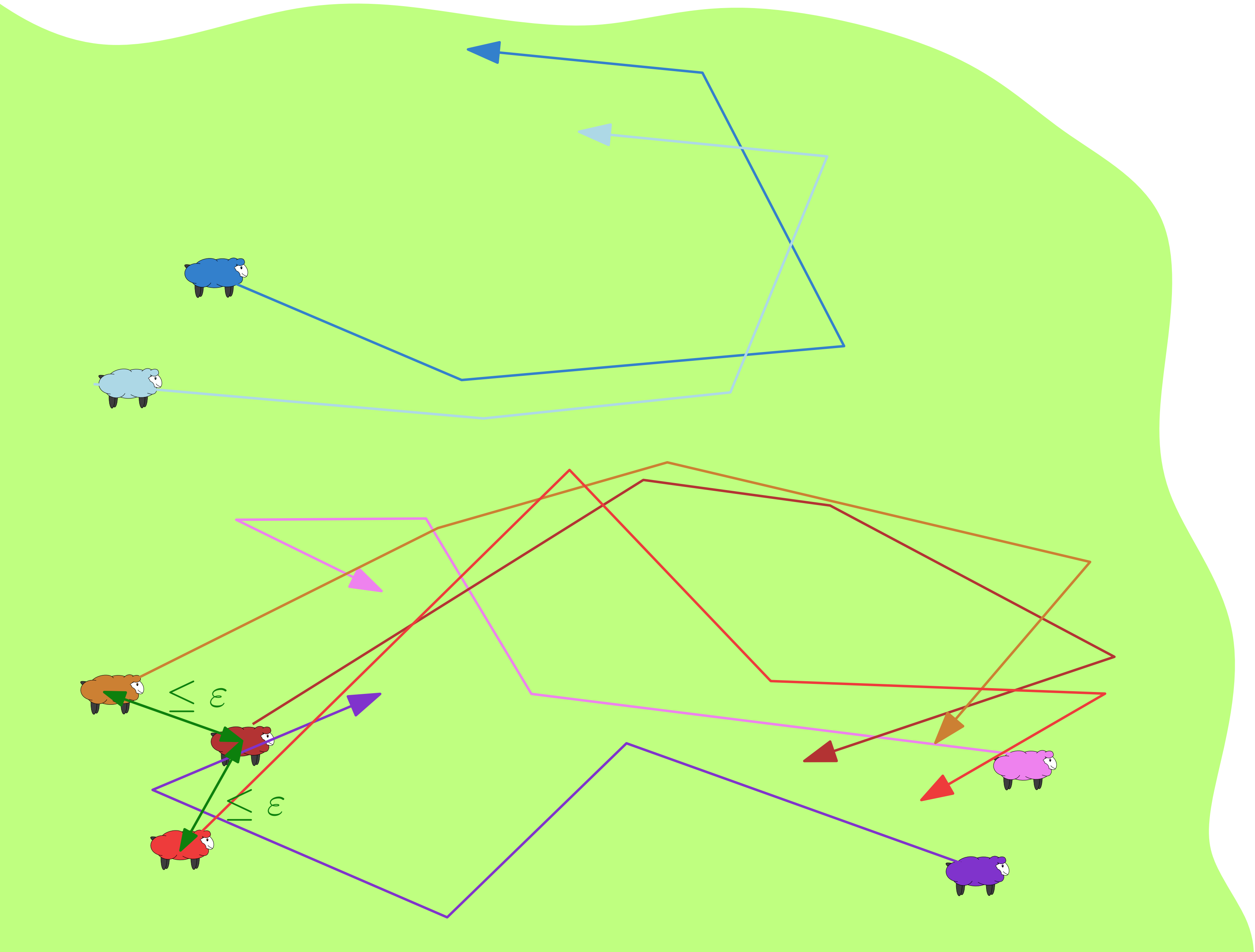


Find all maximal groups: sets of entities that travel together during a time interval of length at least δ



Find all maximal groups: sets of entities that are ε-connected during a time interval of length at least δ

a and b ε-connected iff
$\exists$ path $a = p_1, .., p_k = b$ s.t. $\|p_i p_{i+1}\| \leq \varepsilon$



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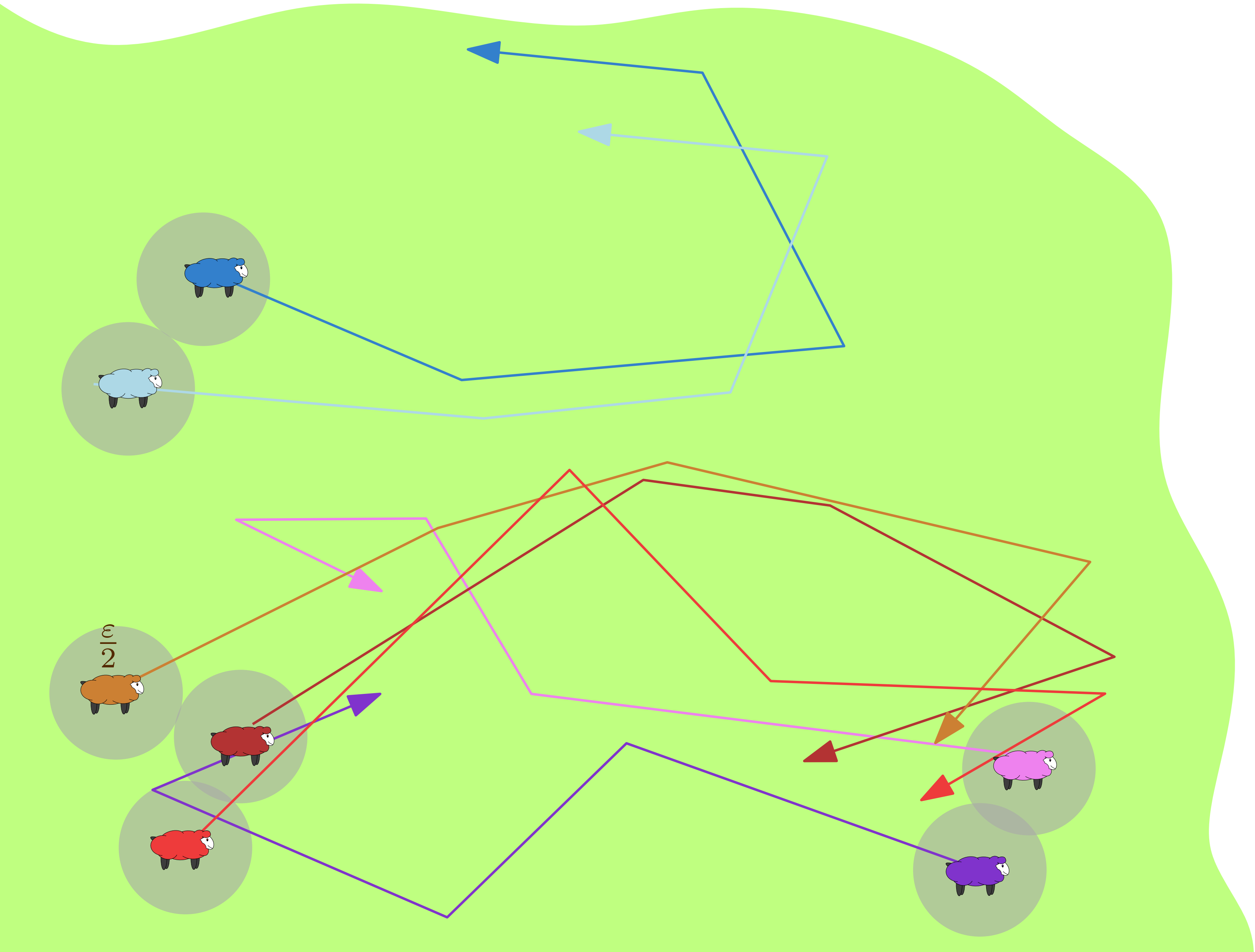

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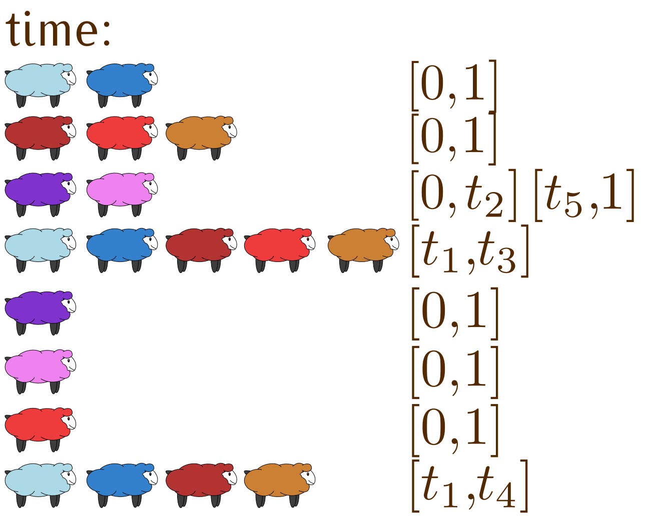

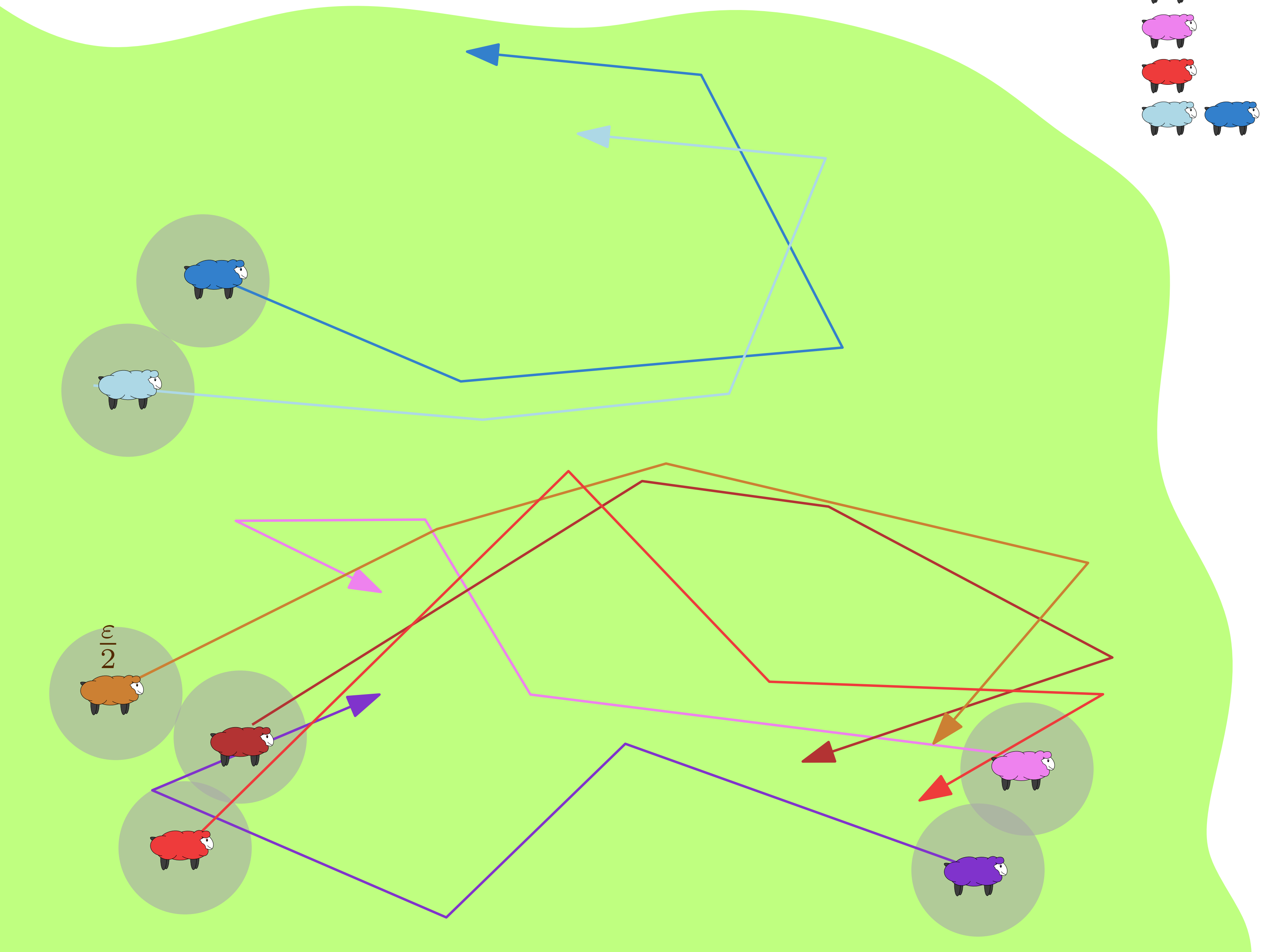

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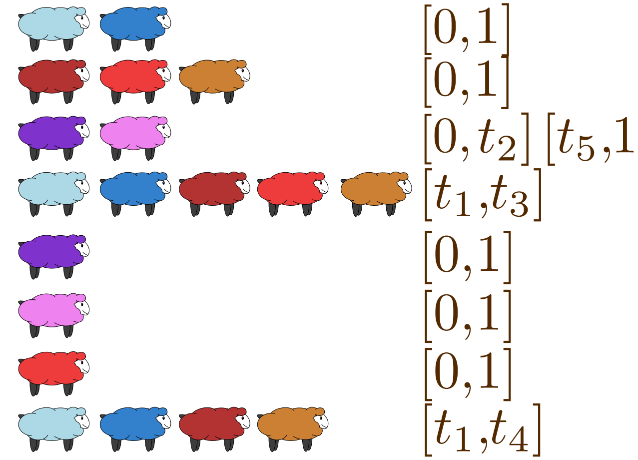
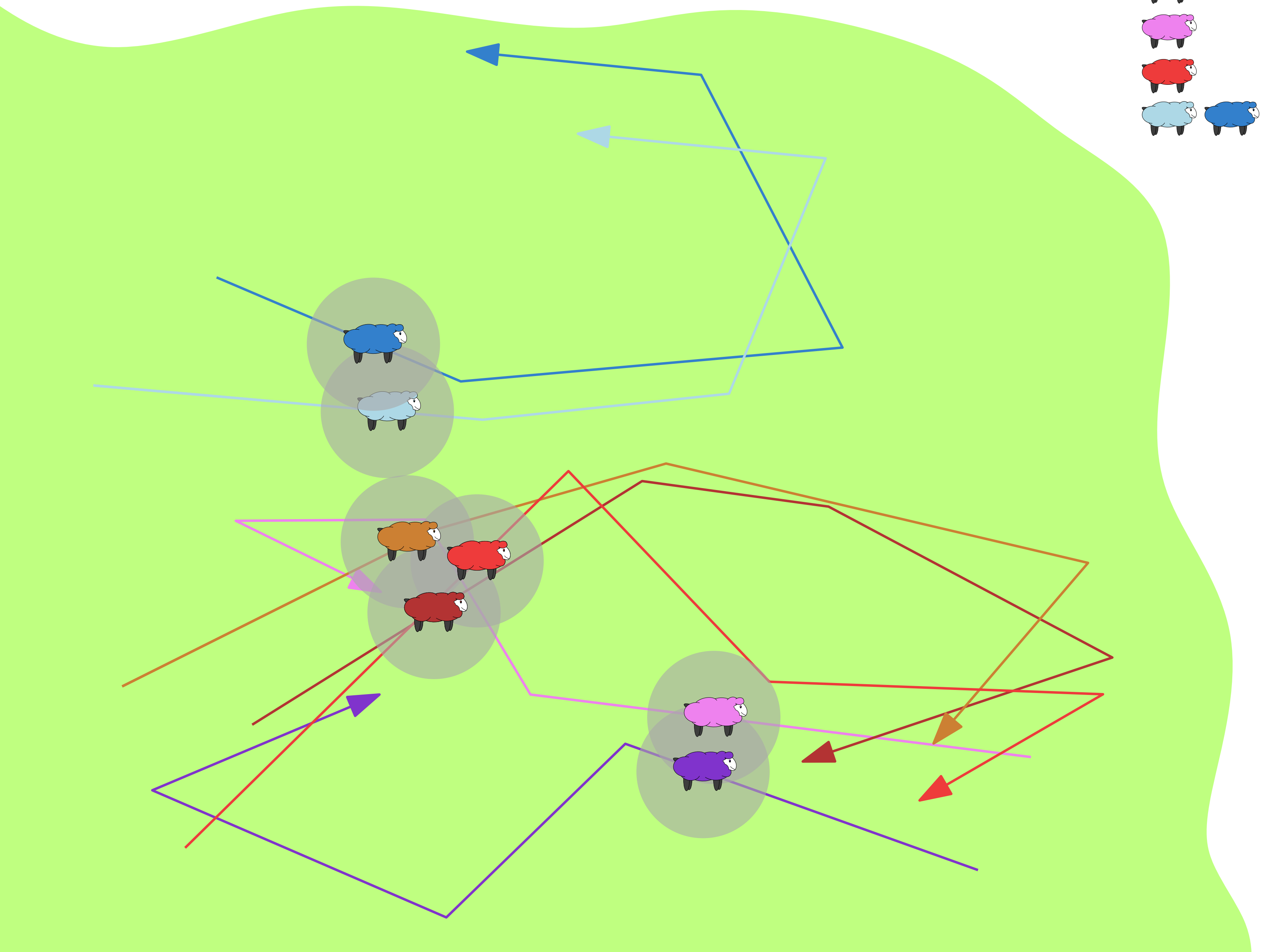
time: t_1





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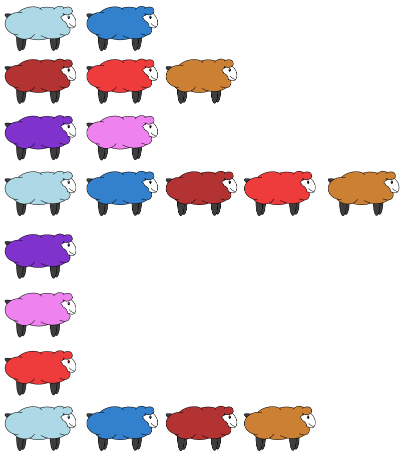
time:



$[0,1]$

$[0,1]$

$[0,t_2]$ $[t_5,1]$

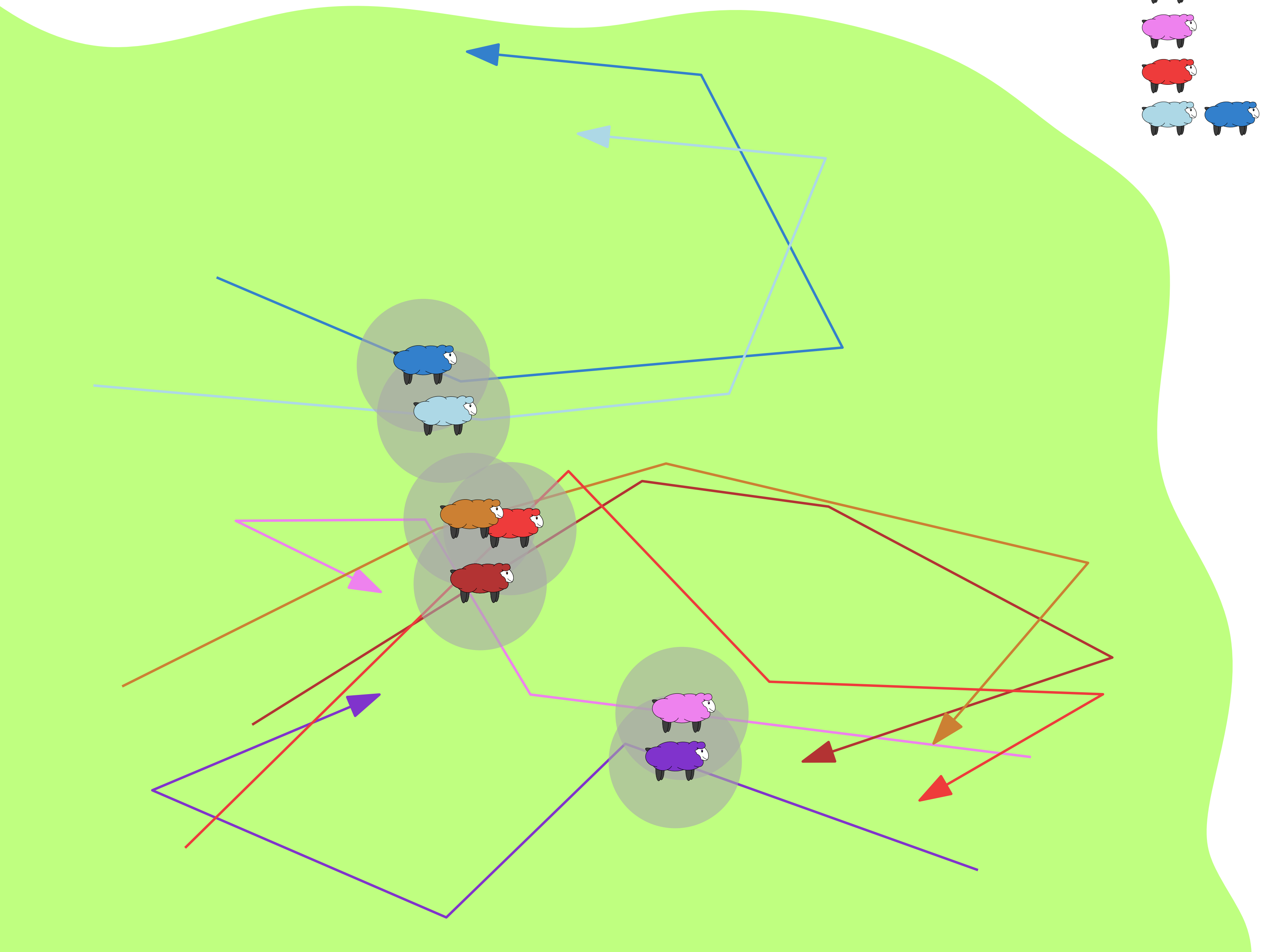
$[t_1,t_3]$

$[0,1]$

$[0,1]$

$[0,1]$

$[t_1,t_4]$



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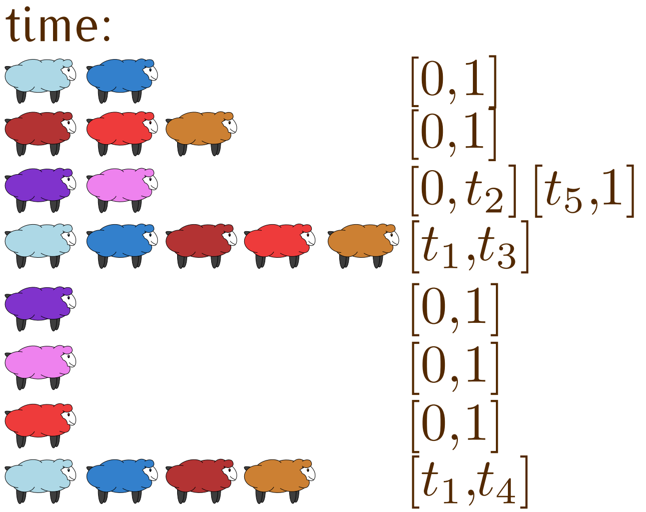

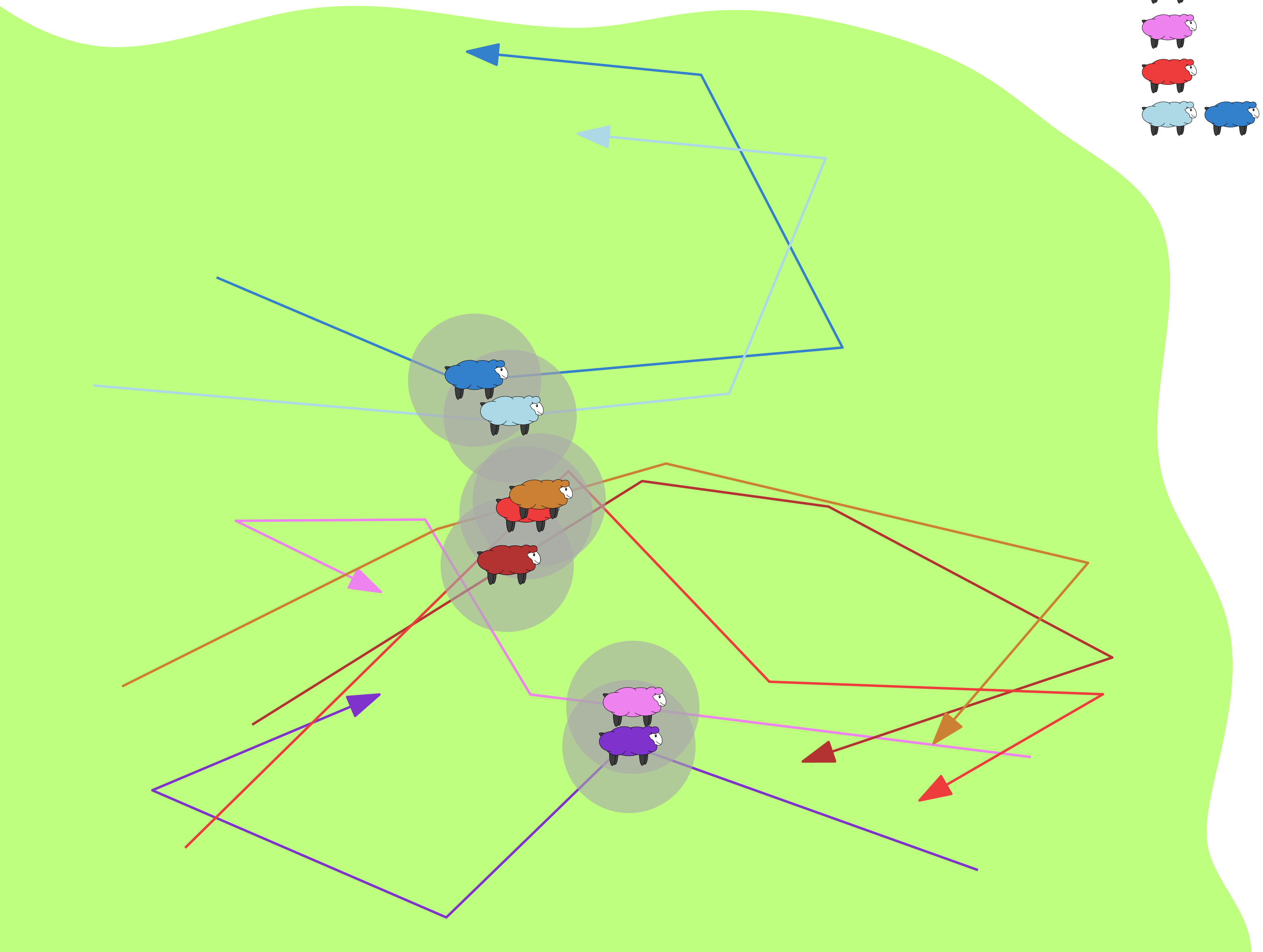

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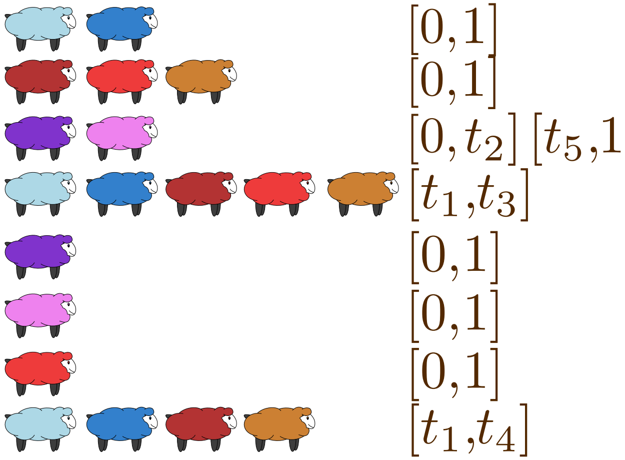
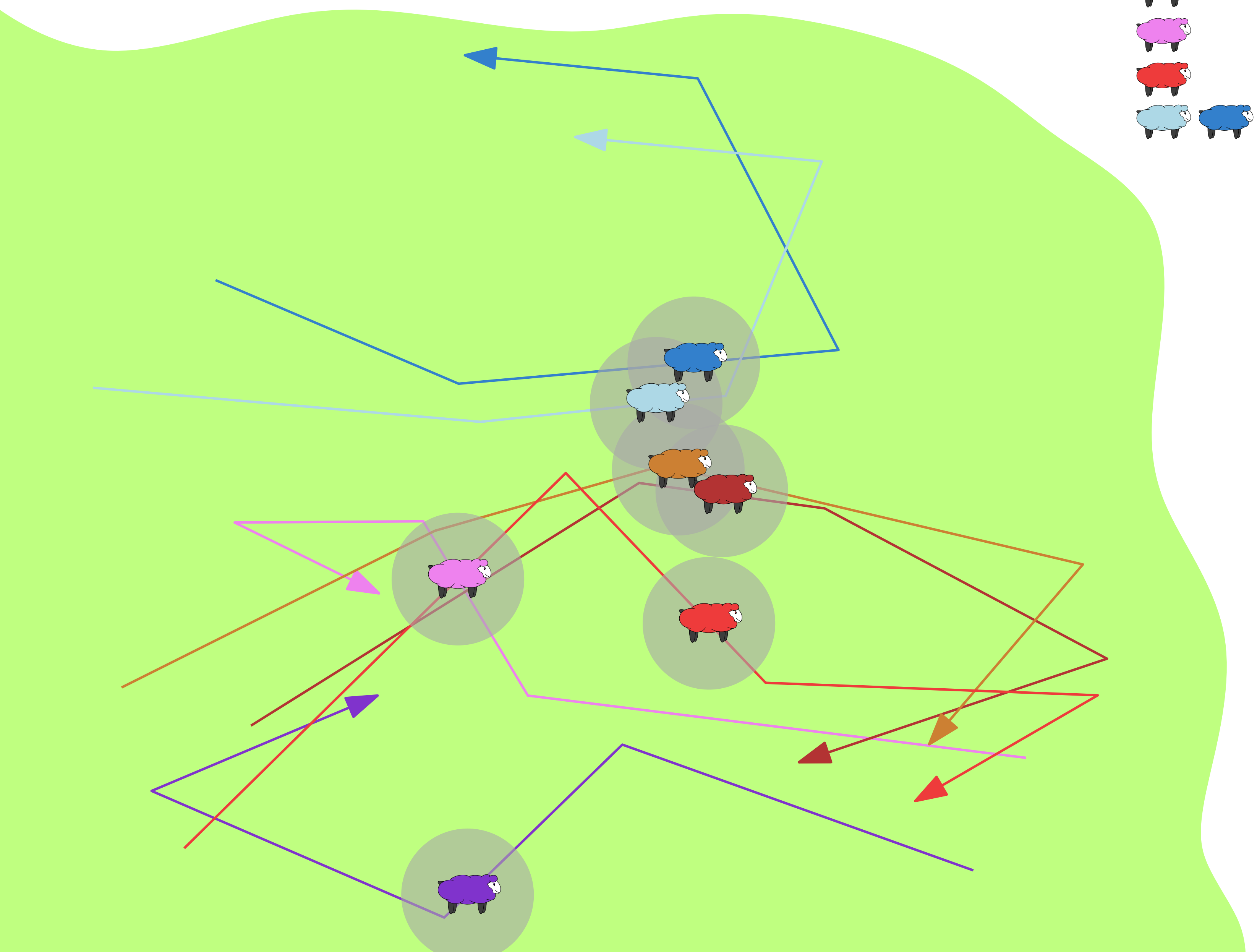
time: t_3





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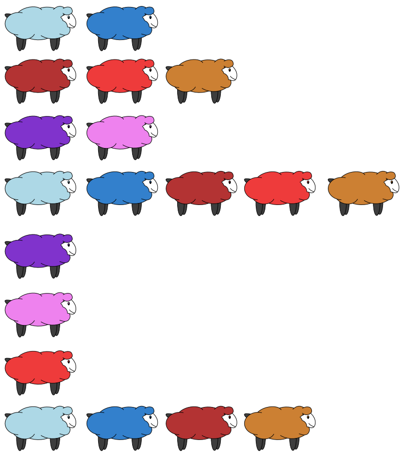
time: t_4



$[0,1]$
$[0,1]$
$[0,t_2]$ $[t_5,1]$
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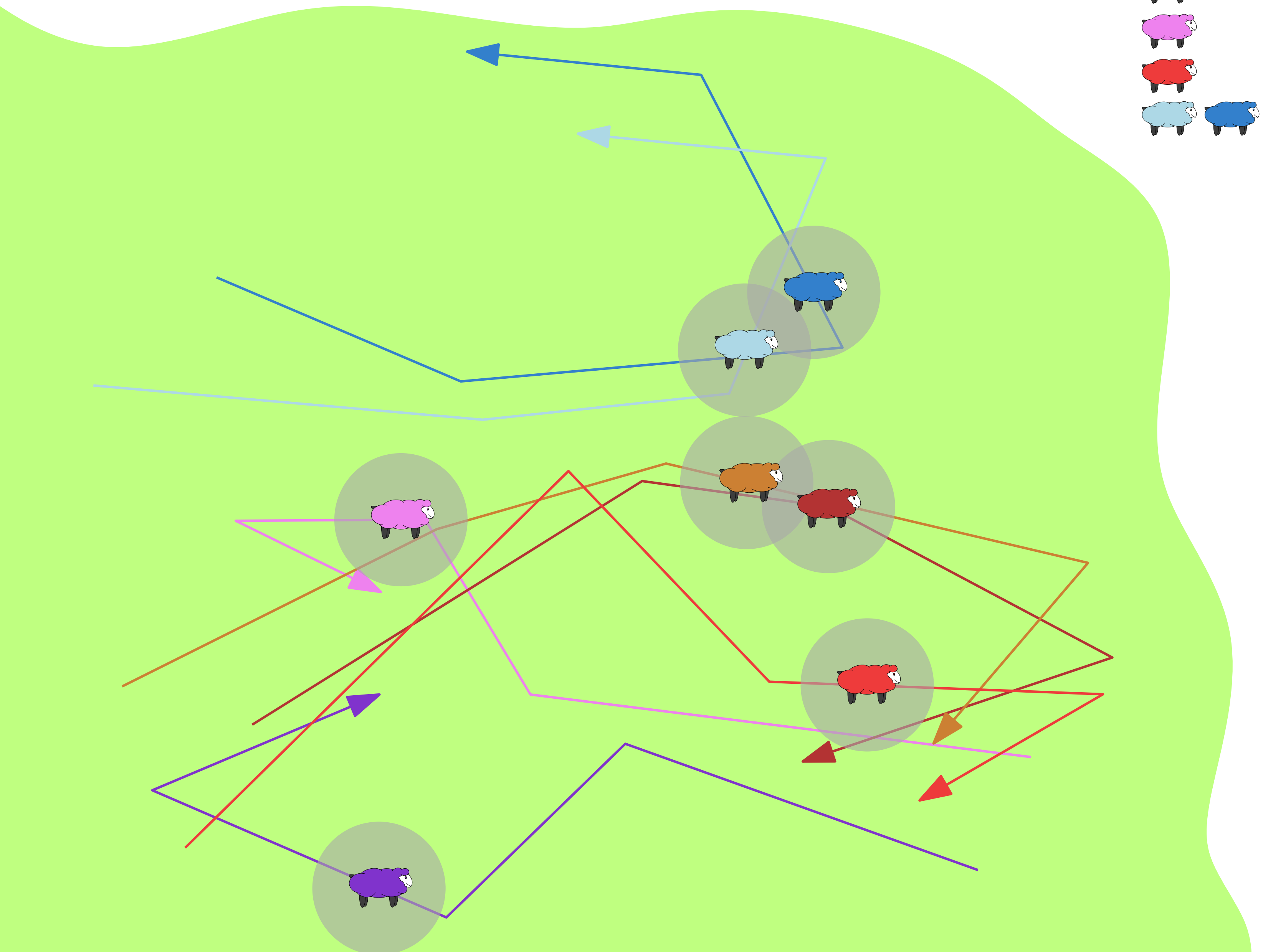
$[t_1,t_4]$



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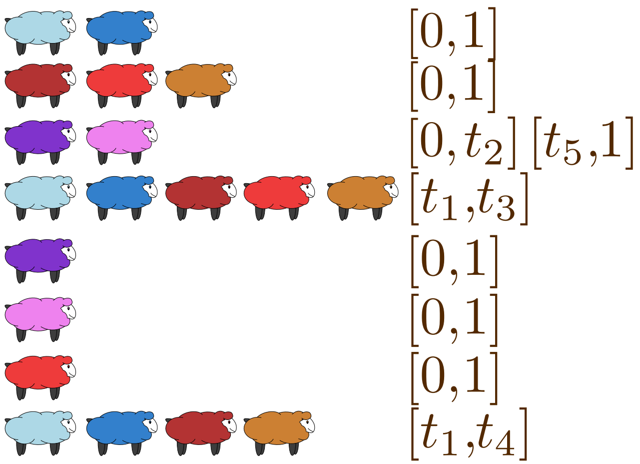
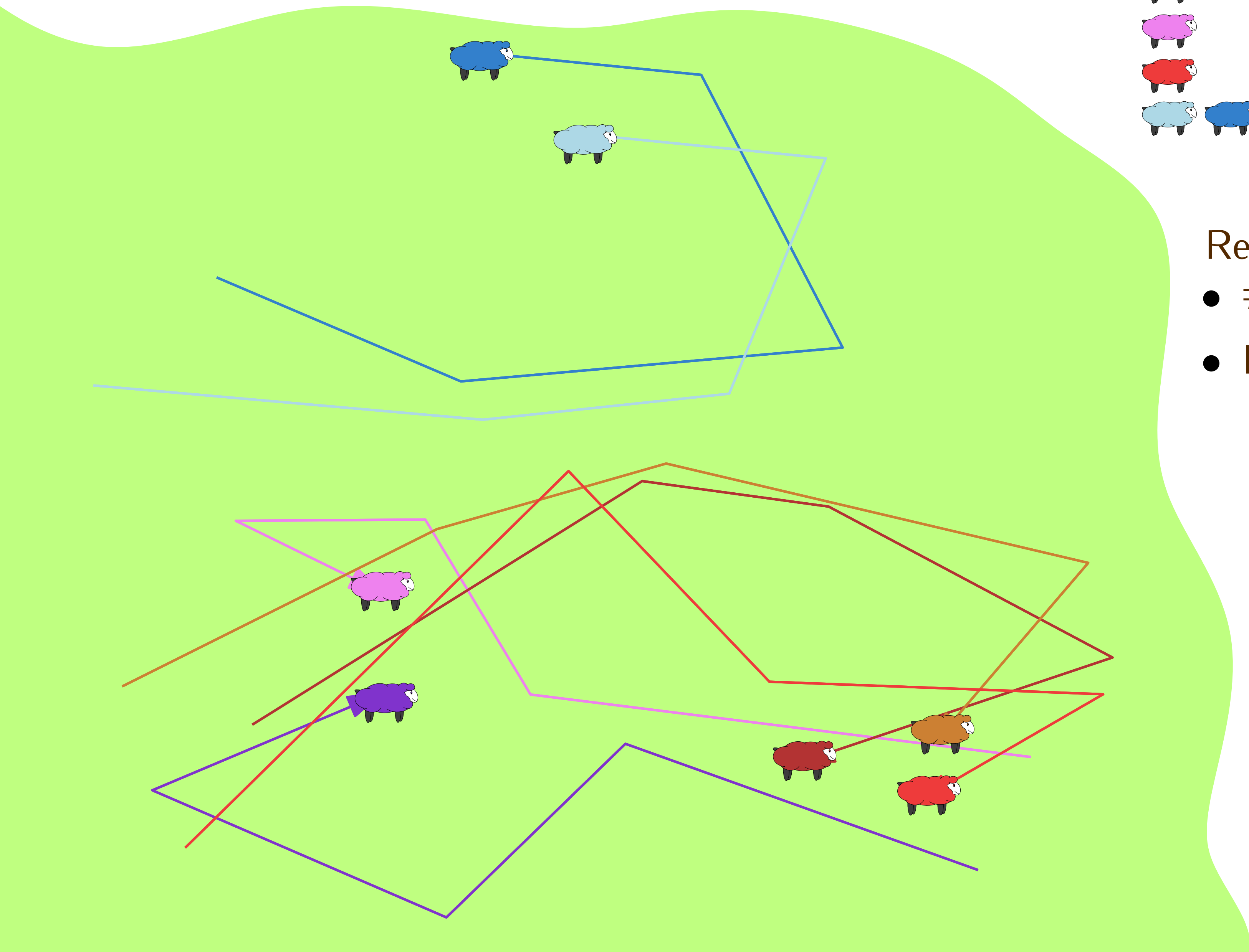
time:





Results on:

- # maximal groups
- How to compute them

Find all maximal groups: sets of entities that are ε-connected during a time interval of length at least δ

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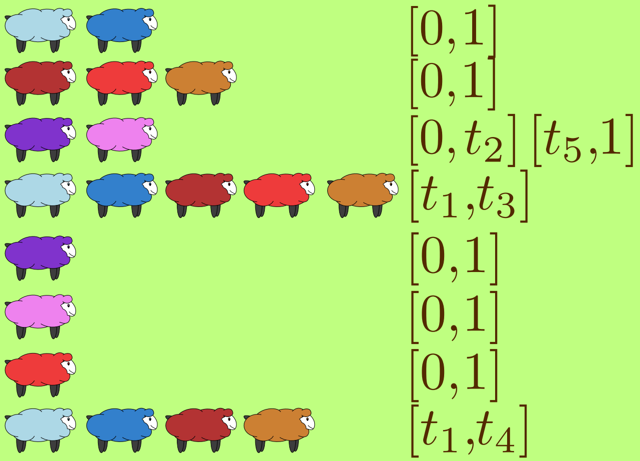
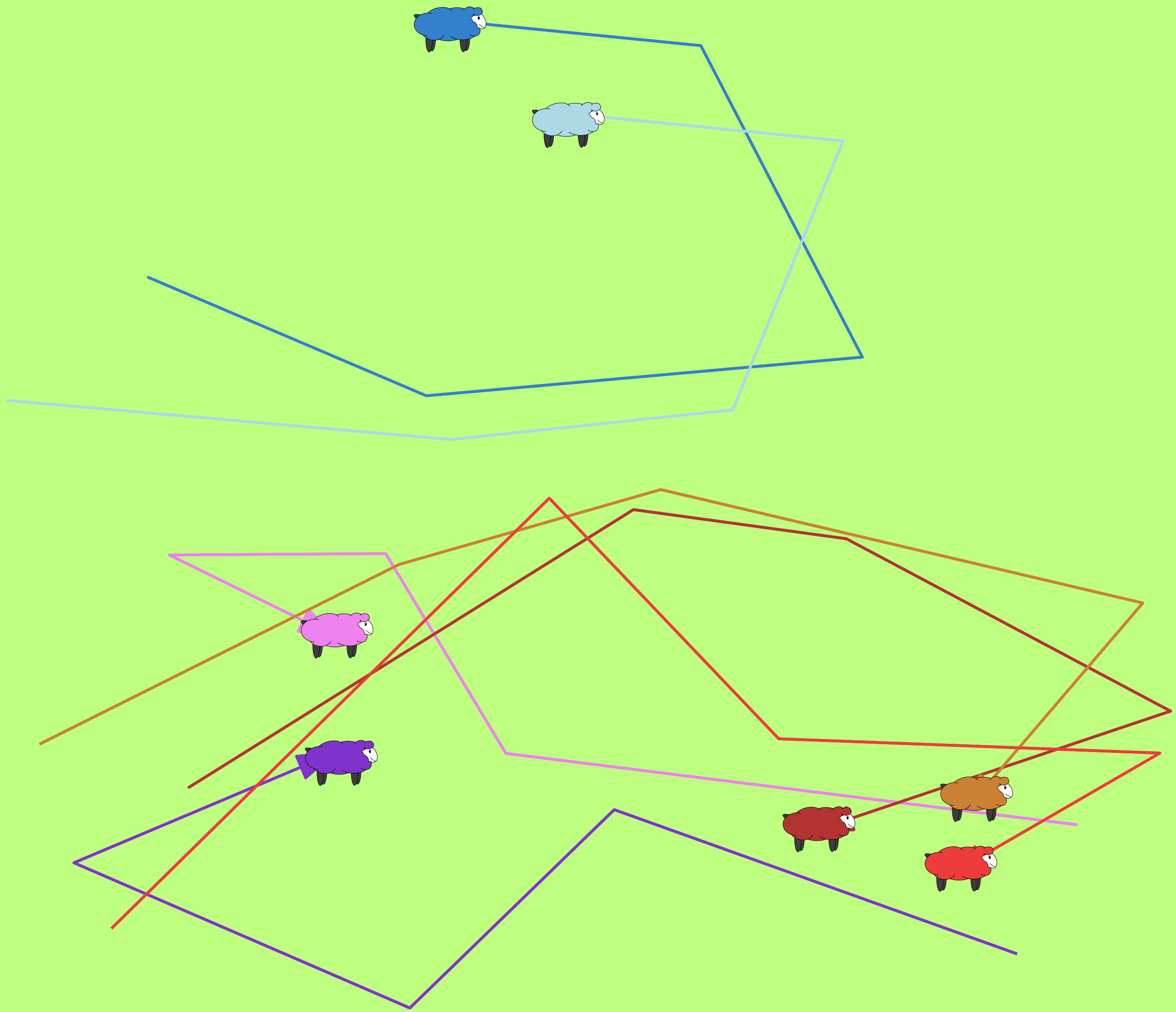
time:





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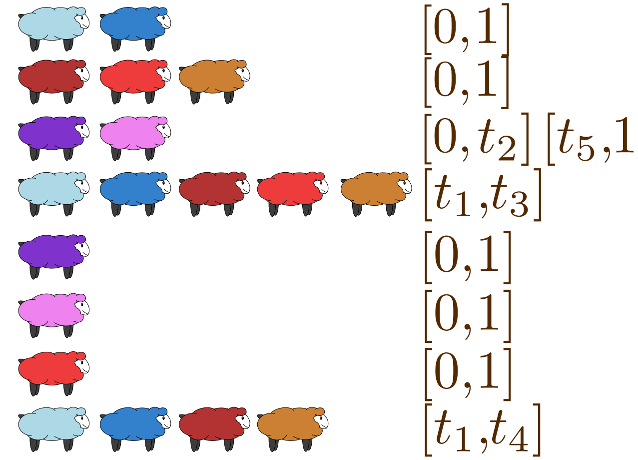

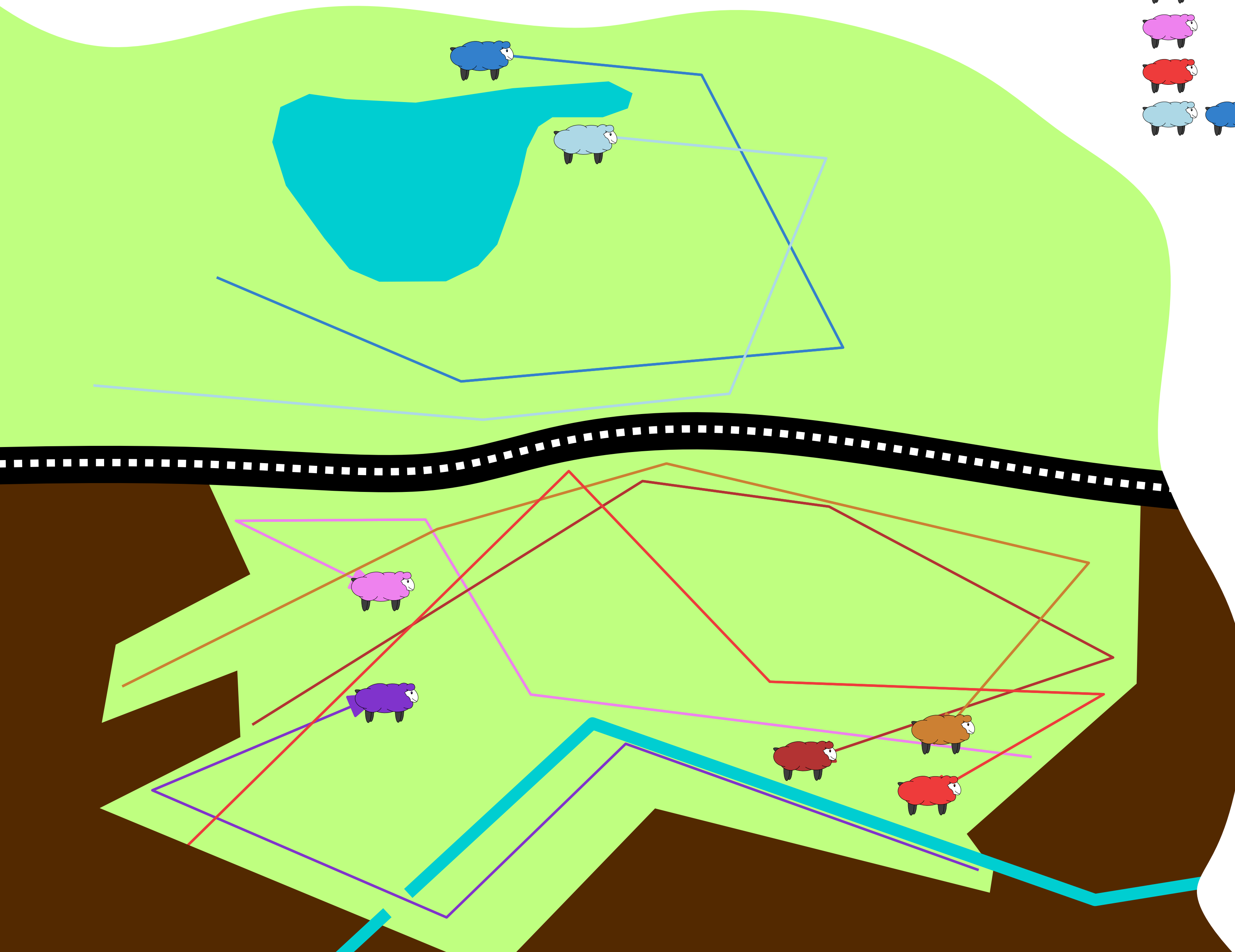

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time:

$[0, \cancel{1}]$

$[\cancel{0}, 1]$

$\cancel{[0, t_2]}\ [t_5, 1]$

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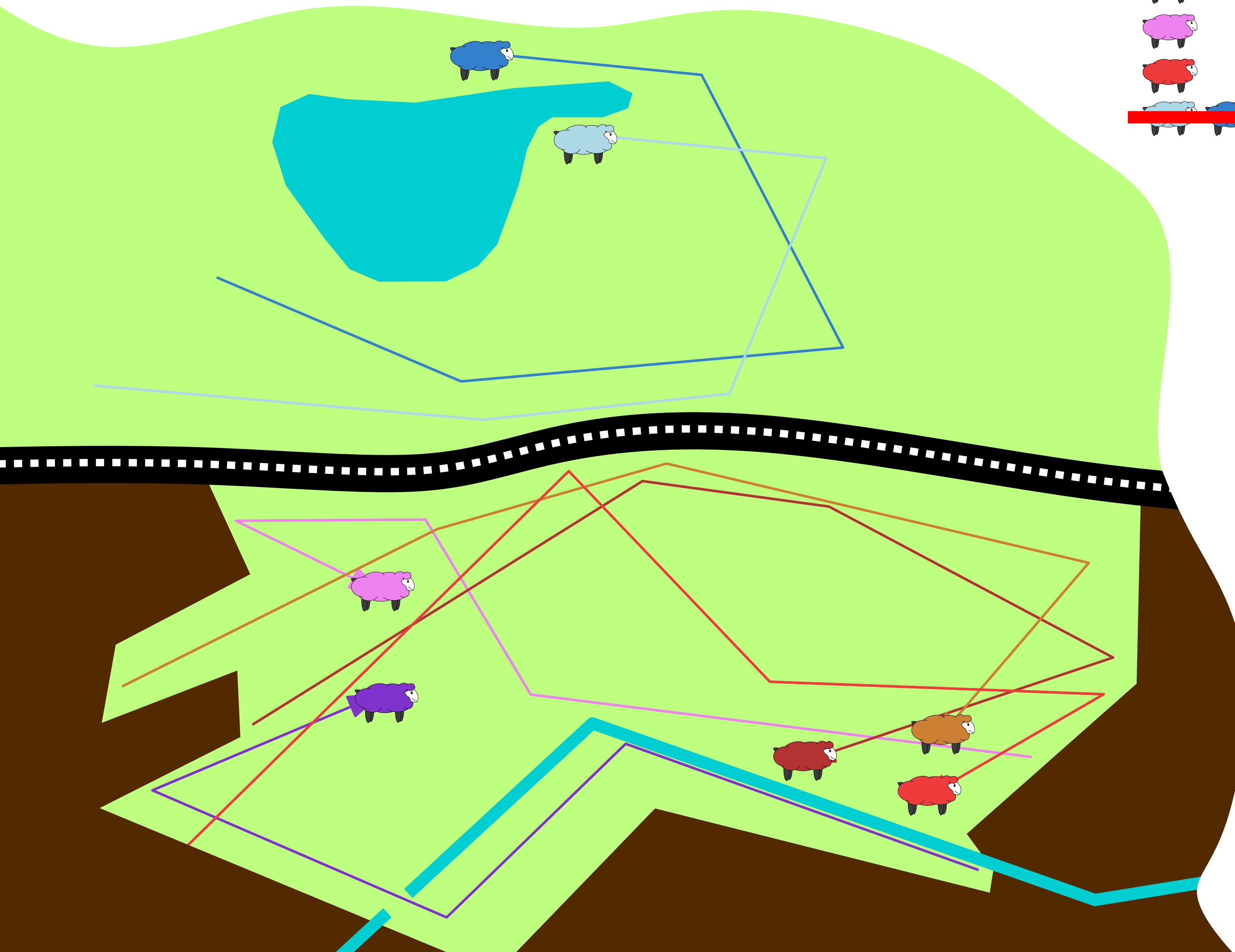

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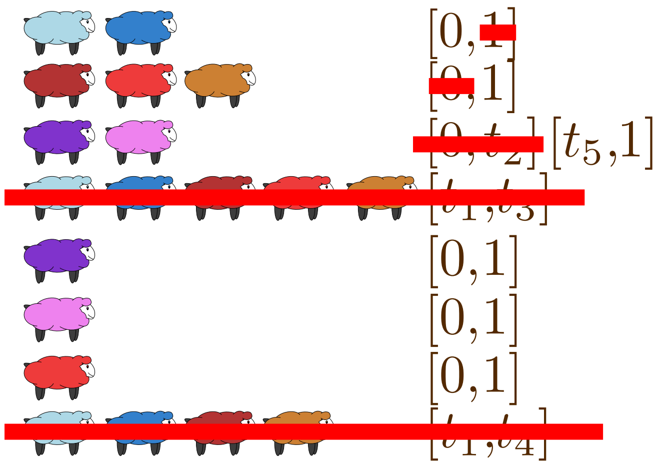
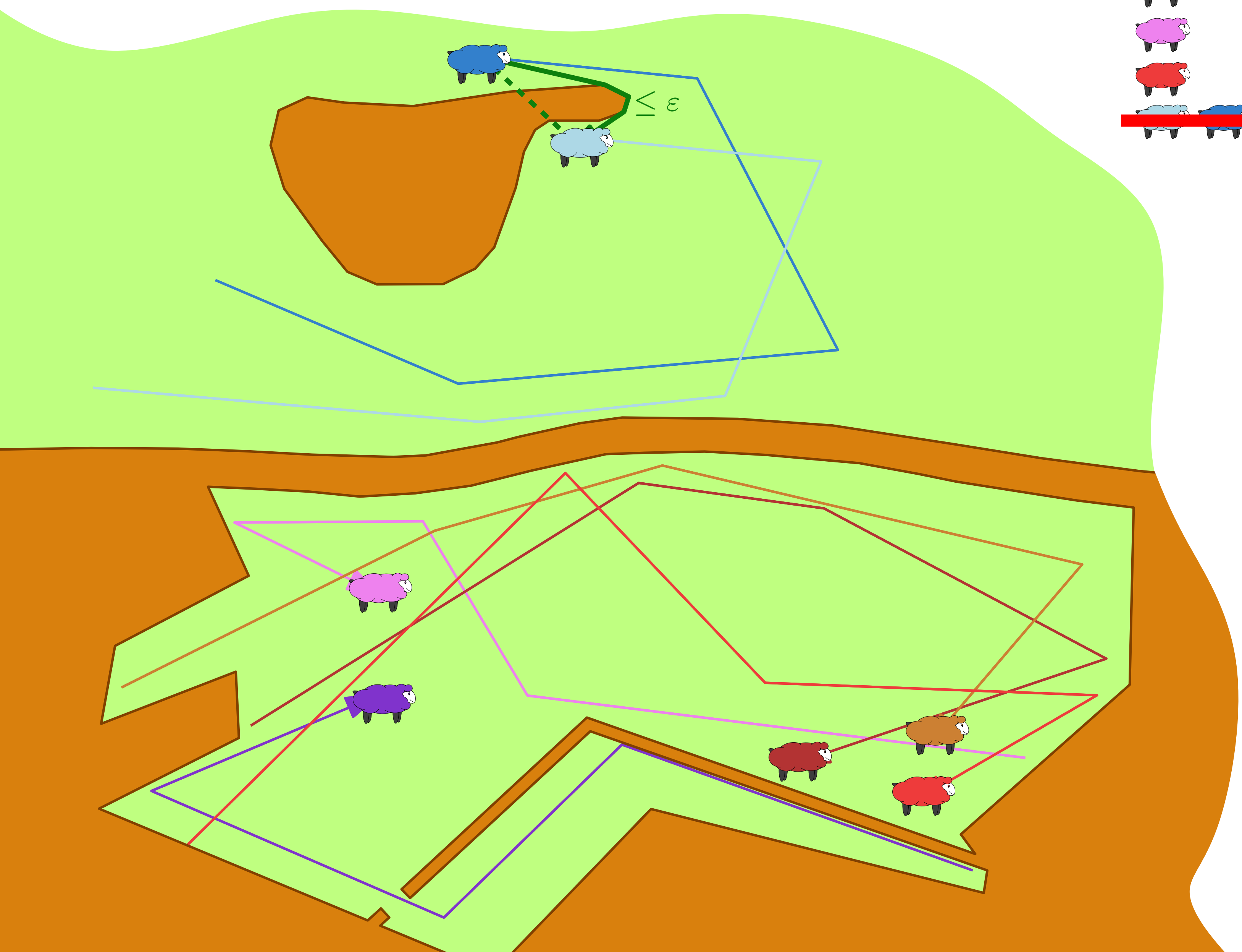
time:





Trajectory Grouping Structure

1) Compute when and how the partition into ε-connected sets changes

2) Compute the groups: which sets stay together long enough

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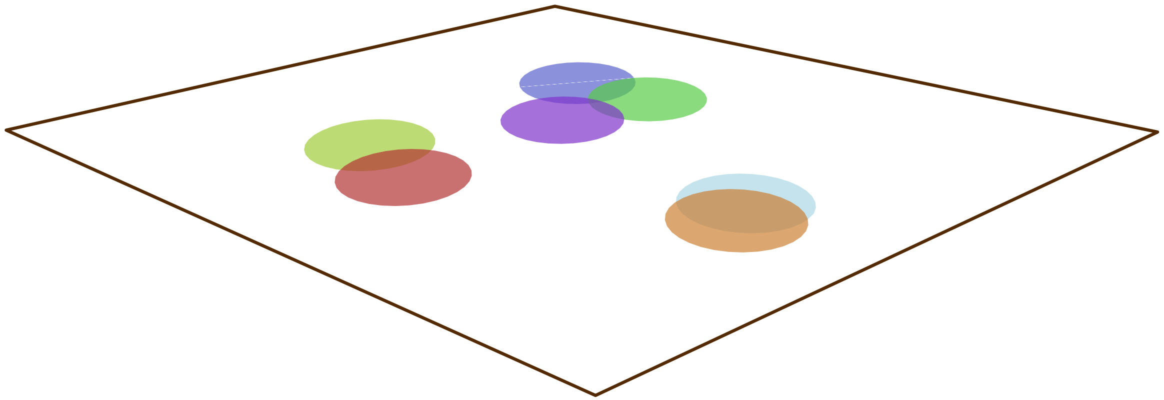

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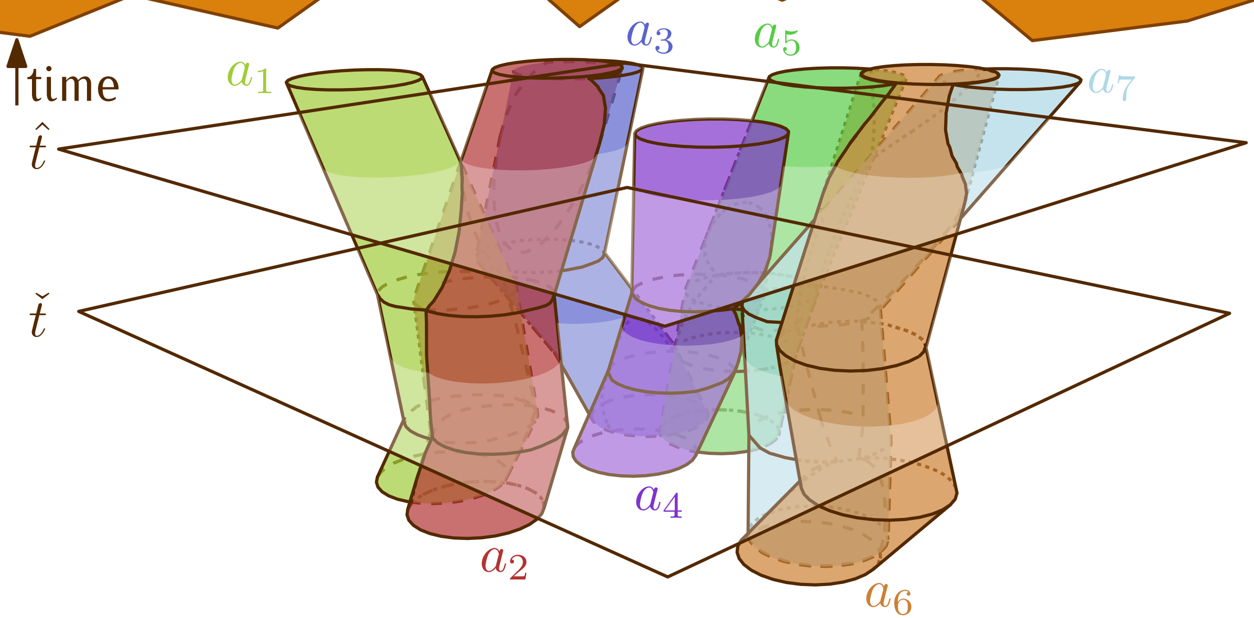
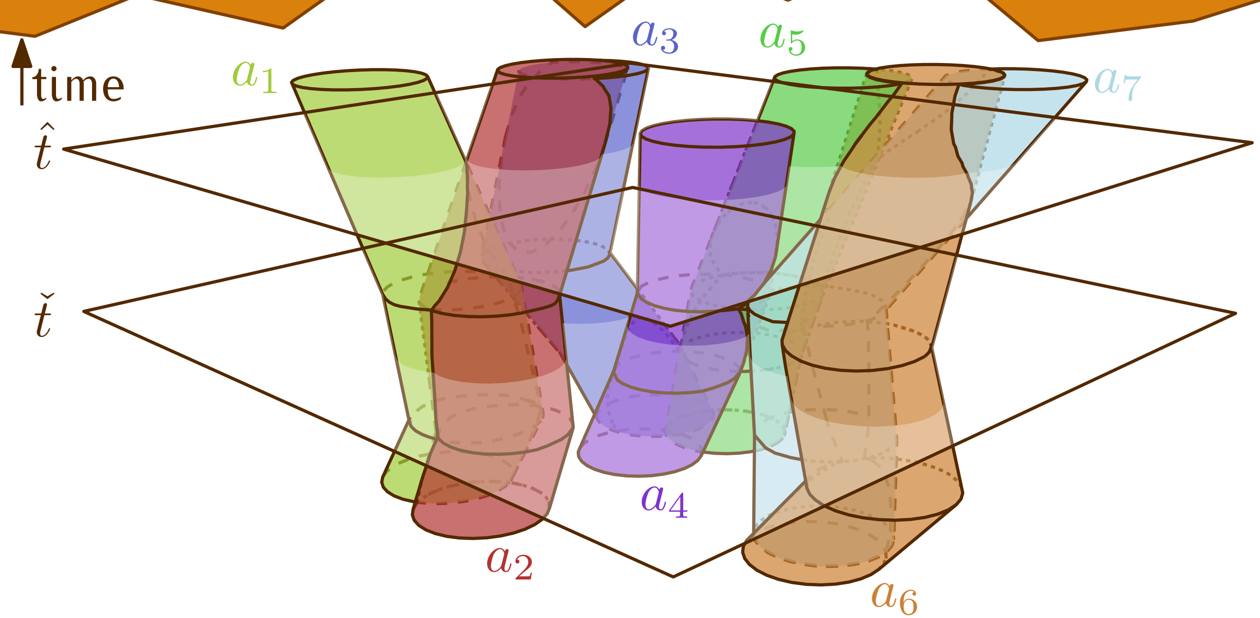
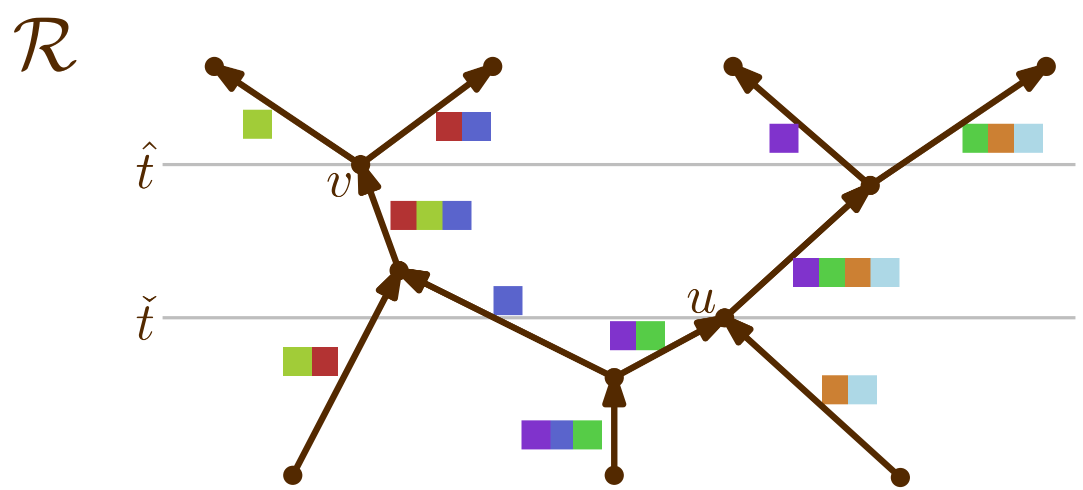


Trajectory Grouping Structure

1) Compute when and how the partition into ε-connected sets changes
$\implies$ Construct the Reeb graph $\mathcal{R}$

2) Compute the groups: which sets stay together long enough





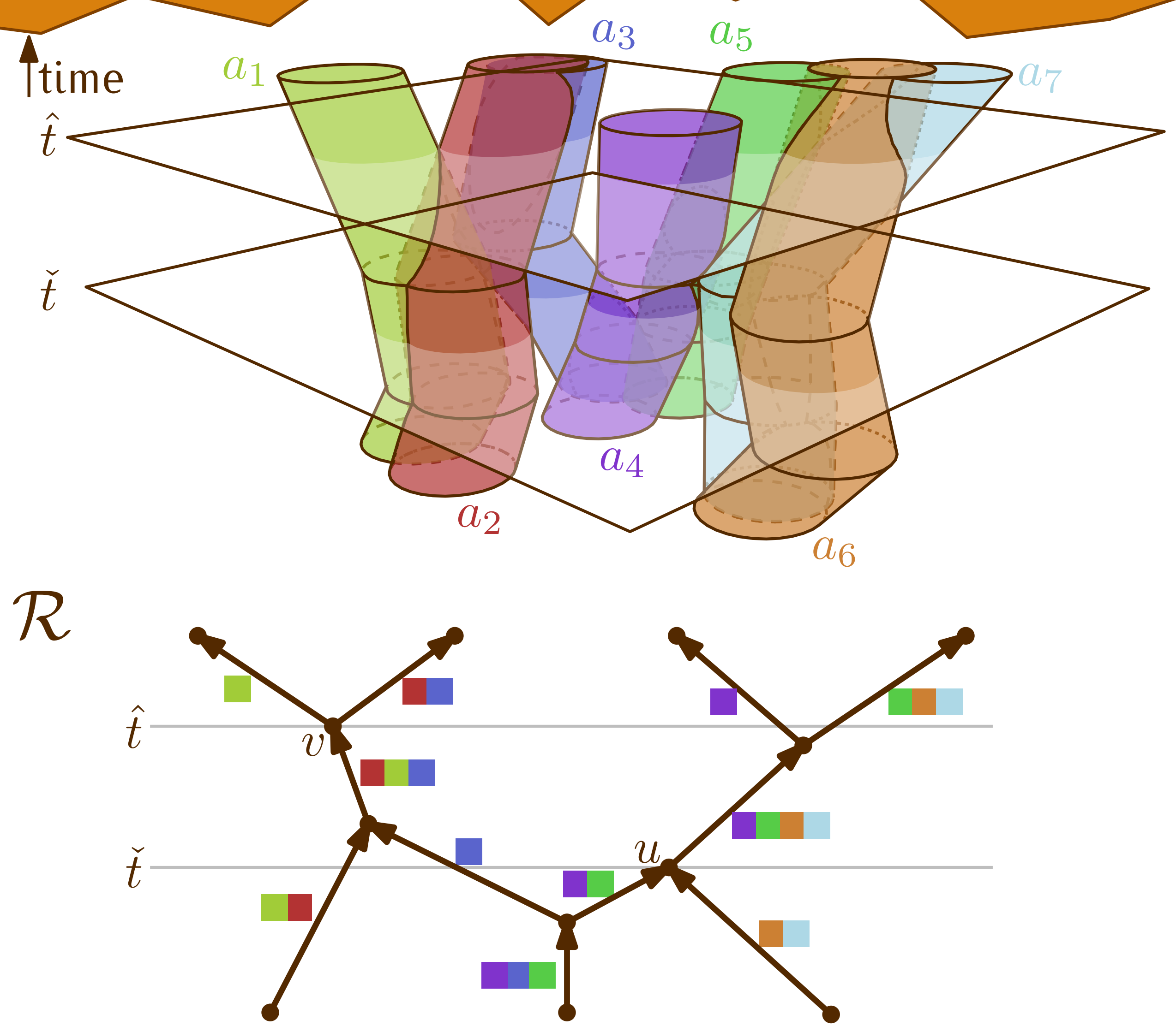
Trajectory Grouping Structure

1) Compute when and how the partition into ε-connected sets changes

$\Longrightarrow$ Construct the Reeb graph $\mathcal{R}$

2) Compute the groups: which sets stay together long enough

$\Longrightarrow$ Compute groups from $\mathcal{R}$

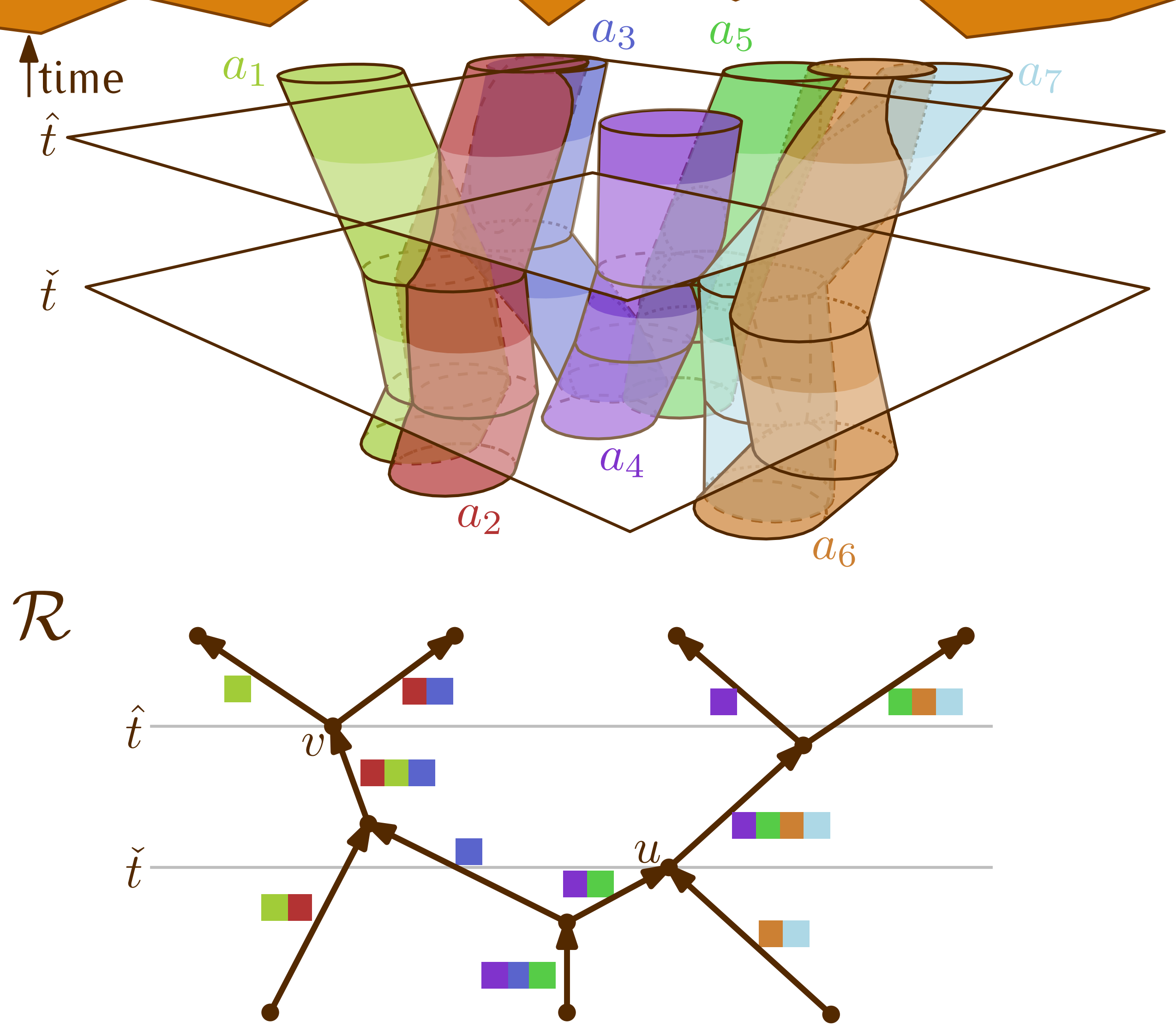


Trajectory Grouping Structure

1) Compute when and how the partition into ε-connected sets changes

$\Longrightarrow$ Construct the Reeb graph $\mathcal{R}$

a) When are two ε-connected sets at (geodesic) distance exactly ε?



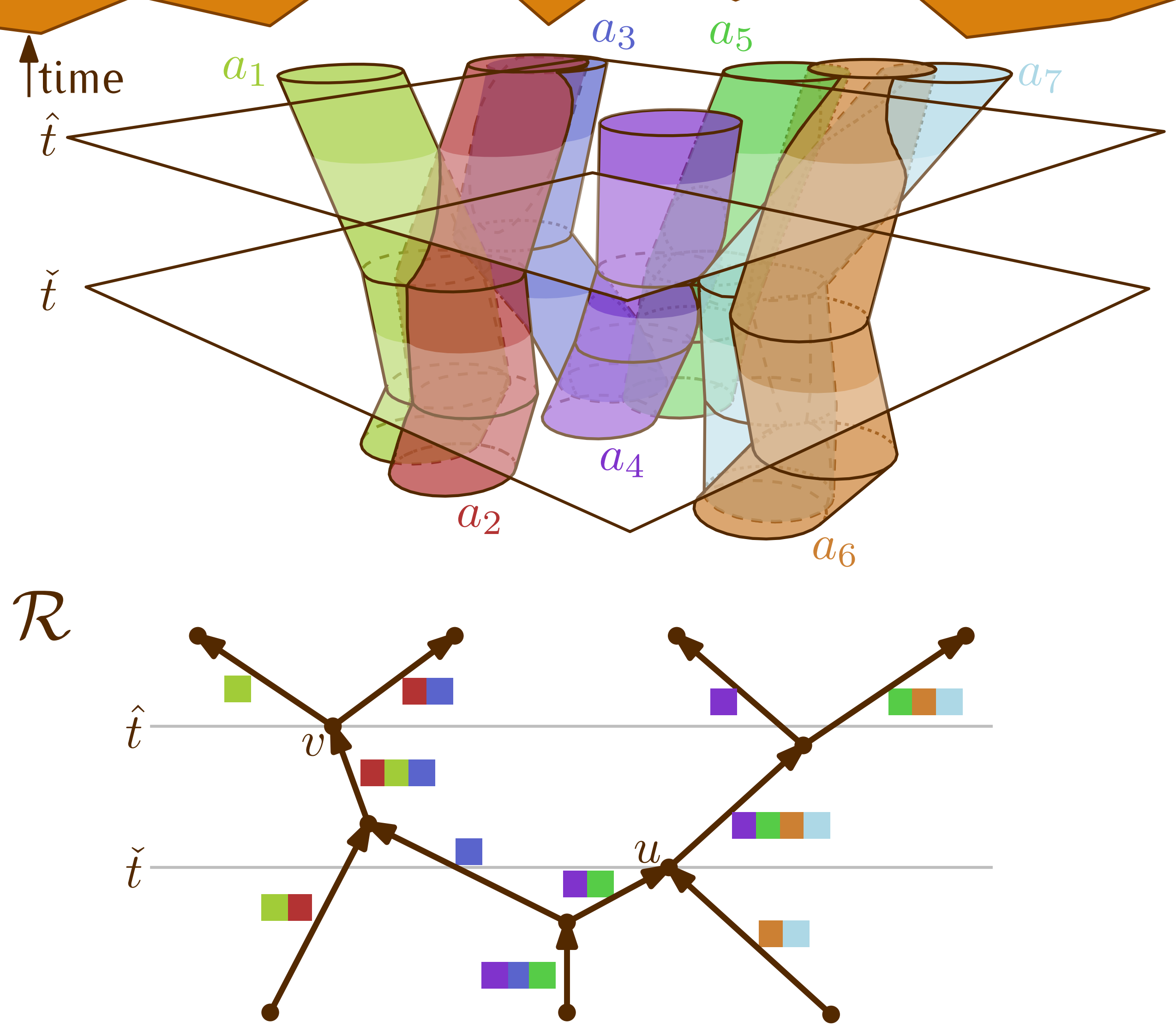
Trajectory Grouping Structure

1) Compute when and how the partition into ε-connected sets changes

$\Longrightarrow$ Construct the Reeb graph $\mathcal{R}$

a) When are two ε-connected sets at (geodesic) distance exactly ε?

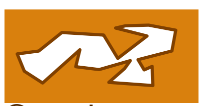
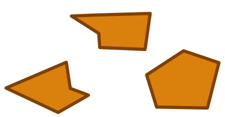
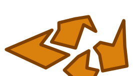
b) How many such critical events are there?



Results

Bounds on the number of critical events:

times at which two maximal sets of ε-connected entities are at geodesic distance ε

	Lower bound	Upper bound	Algorithm
No obstacles	$\Omega(\tau n^2)$	$O(\tau n^2)$	$O(\tau n^2 \log n)$
 Simple polygon	$\Omega(\tau n^2)$	$O(\tau n^2)$	$O(\tau n^2 (\log^2 m + \log n) + m)$
 Well-spaced obstacles	$\Omega(\tau(n^2 + nm))$	$O(\tau(n^2 + m\lambda_4(n)))$	$O(\tau n^2 m \log n)$
 General obstacles	$\Omega(\tau(n^2 + nm \min\{n, m\}))$	$O(\tau \min\{n^2 + m^3\lambda_4(n), n^2 m^2\})$	$O(\tau n^2 m^2 \log n + m^2 \log m)$

$n = \#$ entities

$\tau = \#$ vertices in each trajectory

$m = \#$ obstacle vertices

$\lambda_4(n)$ = max length of a Davenport-Schinzel sequence of order 4 on n symbols.

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Bounds on the number of critical events:

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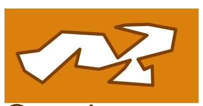
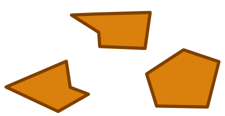
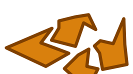
 Simple polygon	$\Omega(\tau n^2)$	$O(\tau n^2)$	$O(\tau n^2(\log^2 m + \log n) + m)$
 Well-spaced obstacles	$\Omega(\tau(n^2 + nm))$	$\boxed{O(\tau(n^2 + m\lambda_4(n)))}$	$\boxed{O(\tau n^2 m \log n)}$
 General obstacles	$\Omega(\tau(n^2 + nm \min\{n, m\}))$	$O(\tau \min\{n^2 + m^3\lambda_4(n), n^2m^2\})$	$O(\tau n^2 m^2 \log n + m^2 \log m)$

$n = \#$ entities

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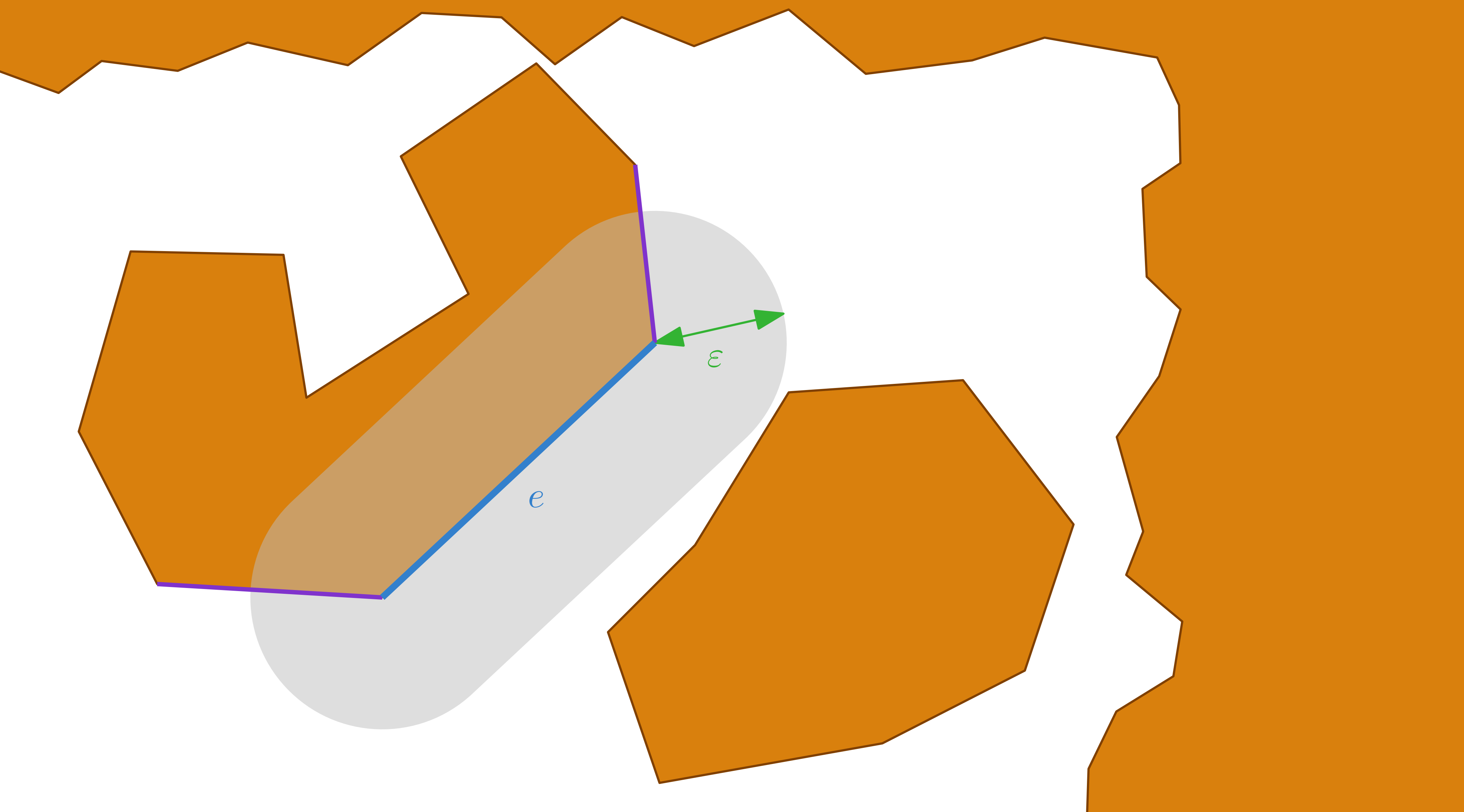
$\lambda_4(n)$ = max length of a Davenport-Schinzel sequence of order 4 on n symbols.

Upperbound for Well Spaced Obstacles

The obstacles are well-spaced

$\iff$

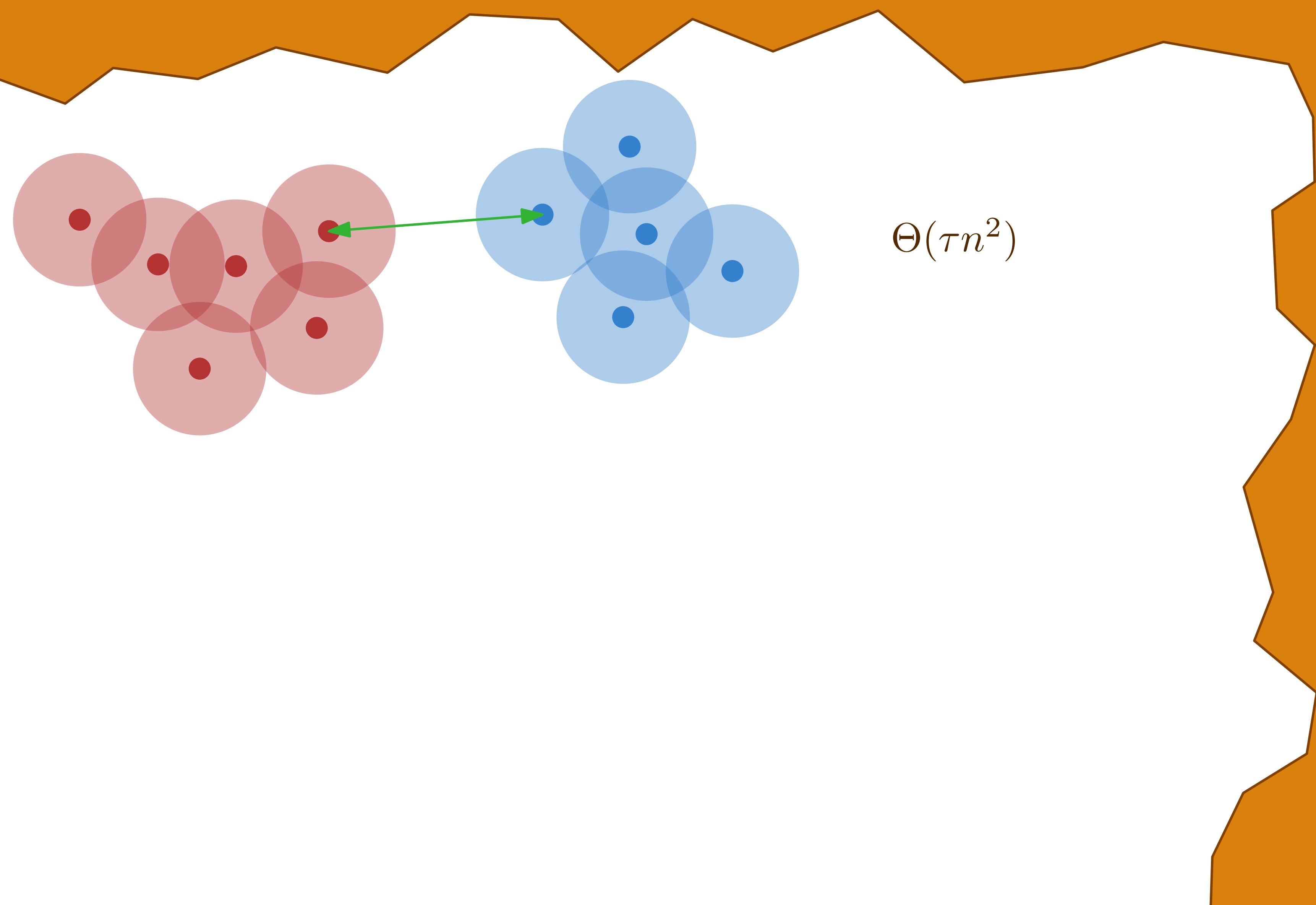
The distance between any pair of non-adjacent edges is $\geq \varepsilon$



Upperbound for Well Spaced Obstacles

Two types of critical events:

1) no vertices on geodesic
2) geodesic via obstacle vertex



$\Theta(\tau n^2)$

Upperbound for Well Spaced Obstacles

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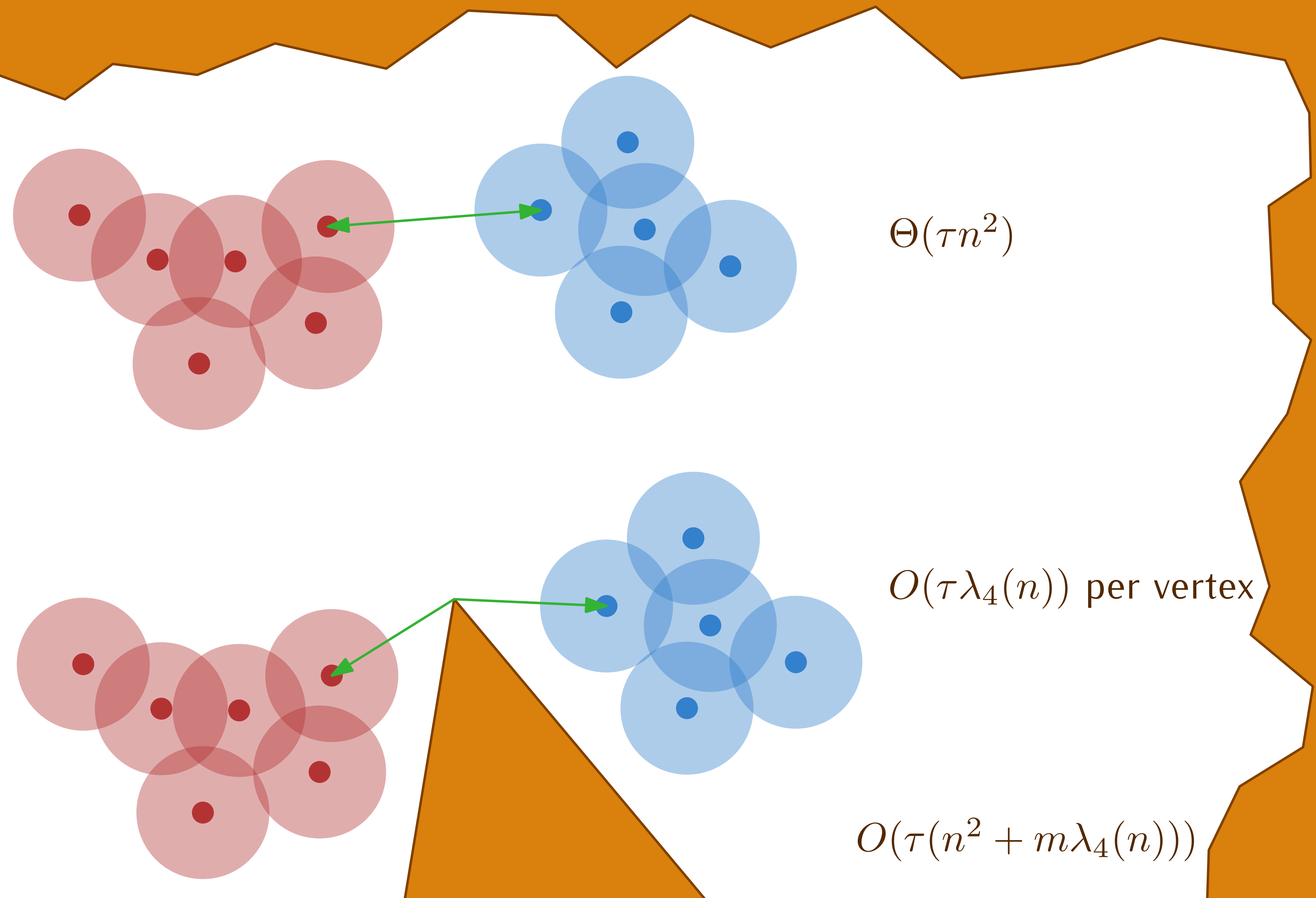
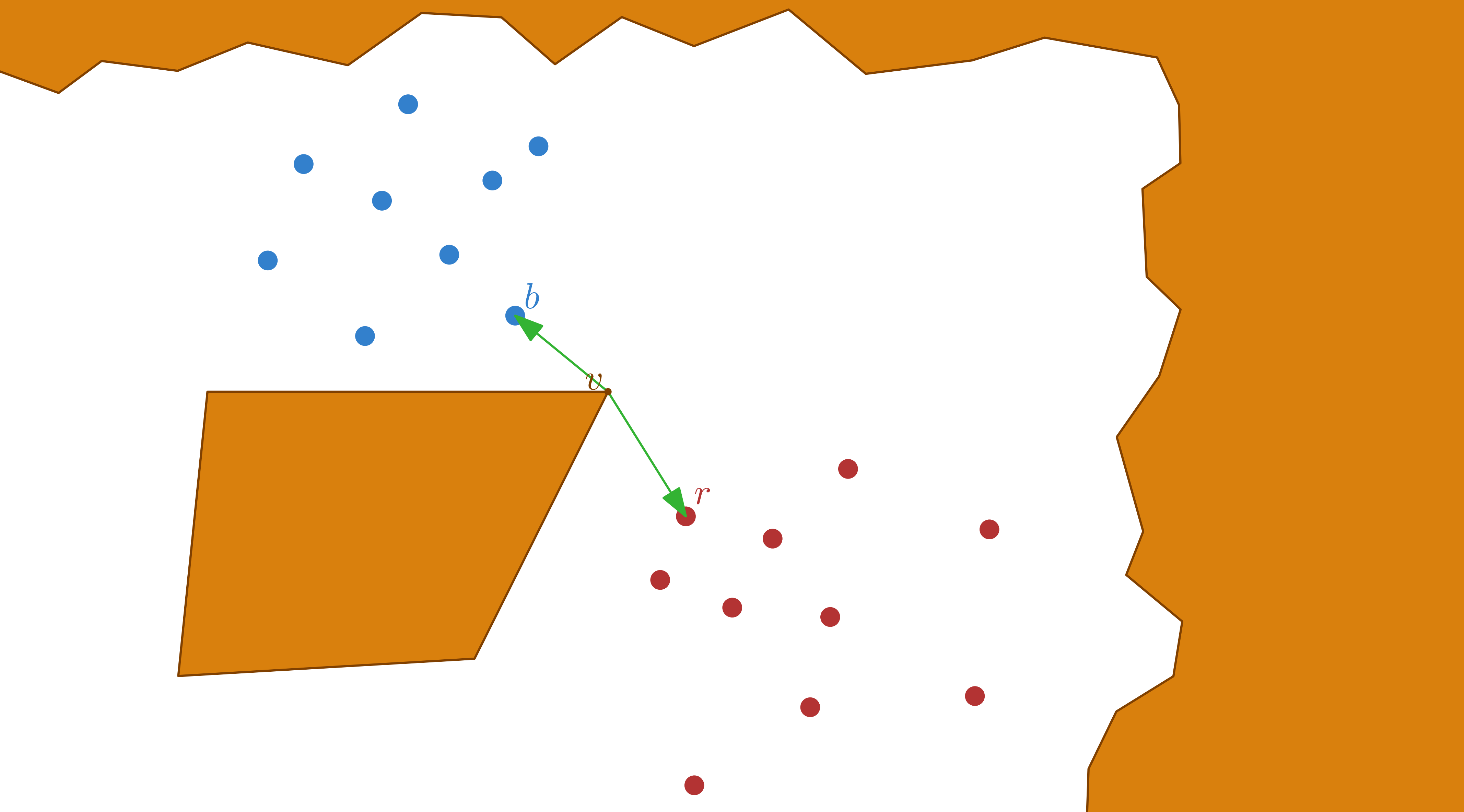


Upperbound for Well Spaced Obstacles

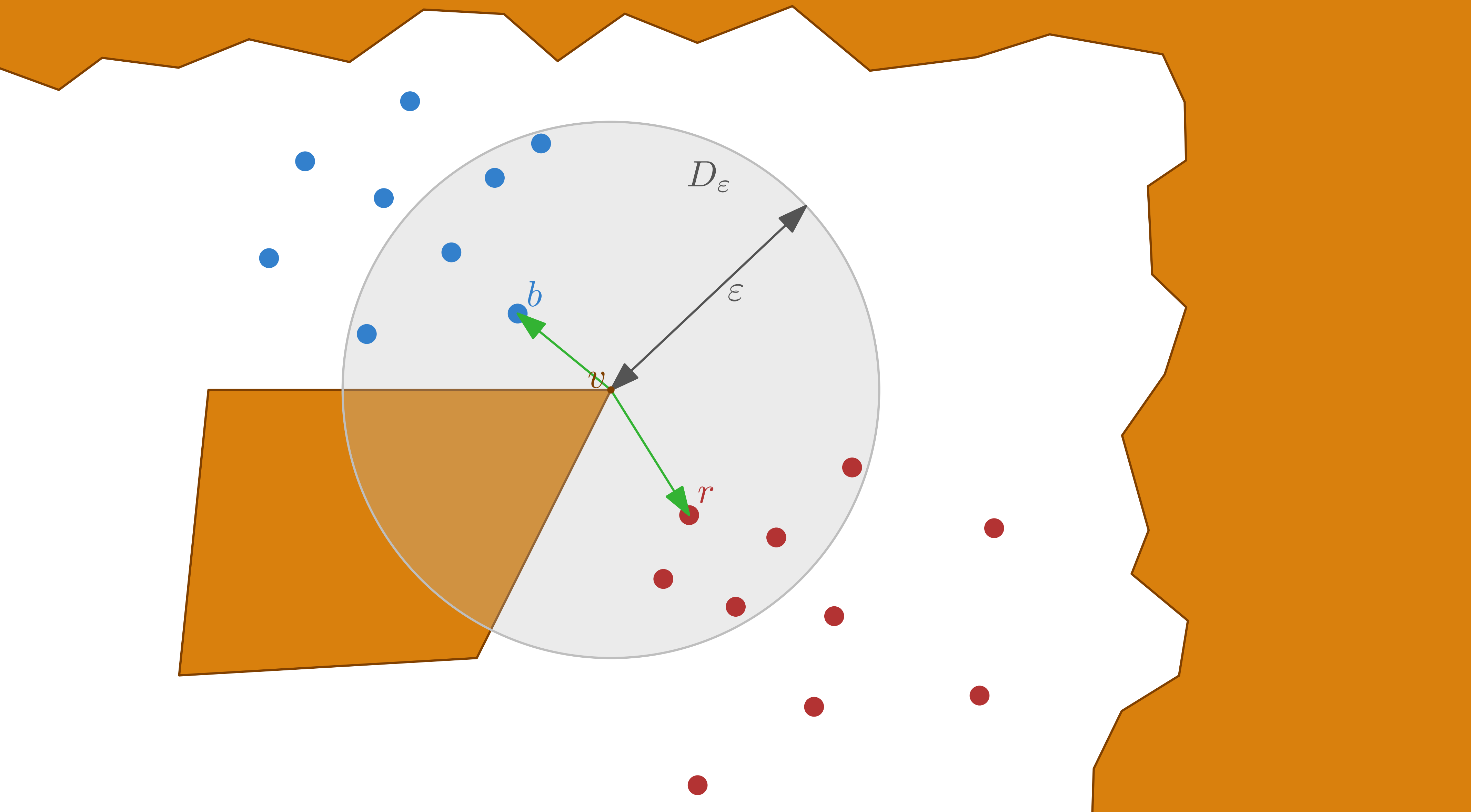
Consider a critical event between sets R and B s.t. v on geodesic



Upperbound for Well Spaced Obstacles

Consider a critical event between sets R and B s.t. v on geodesic

$\implies$ closest pair $r \in R$ and $b \in B$ in $\mathcal{D}_\varepsilon$

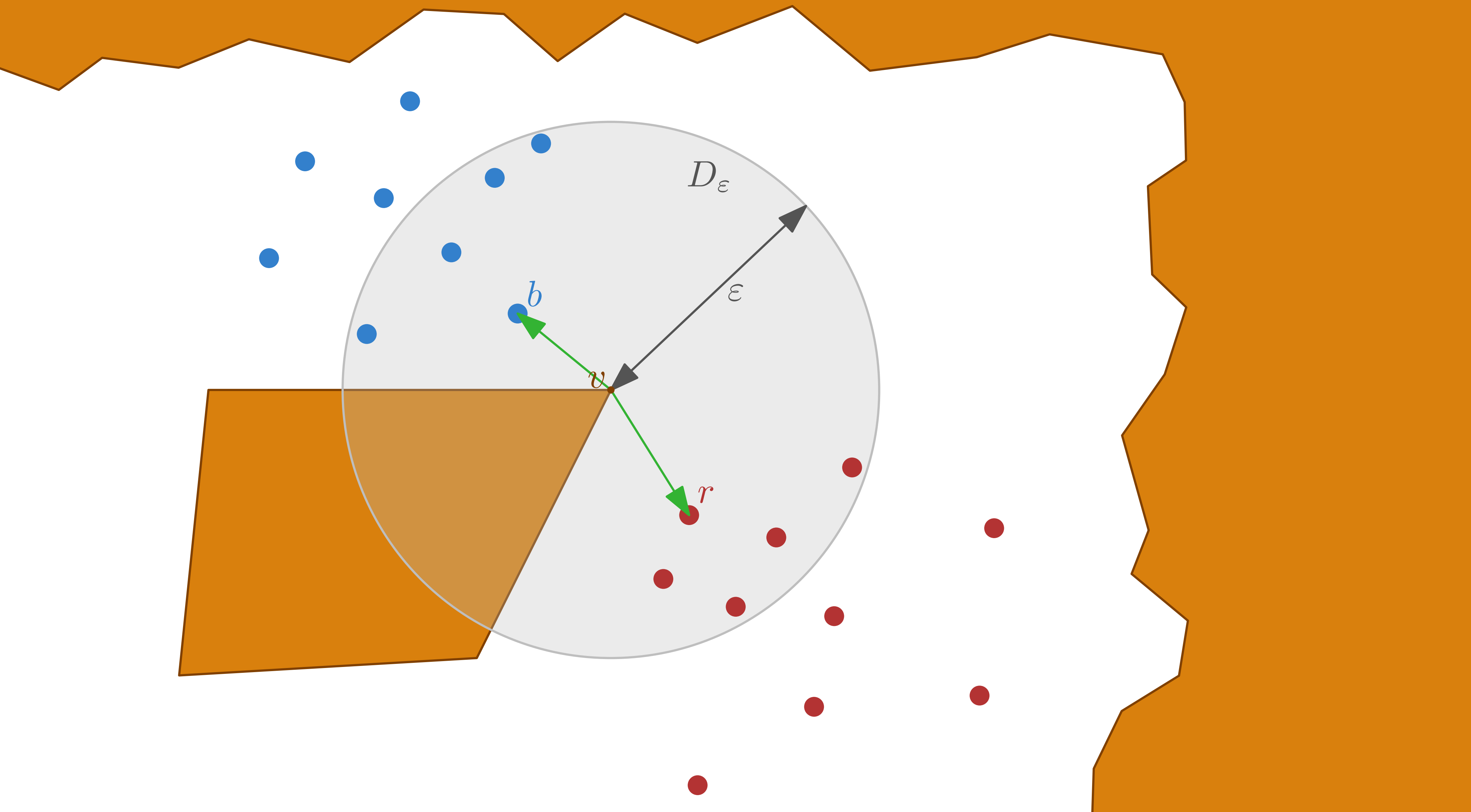


Upperbound for Well Spaced Obstacles

Consider a critical event between sets R and B s.t. v on geodesic

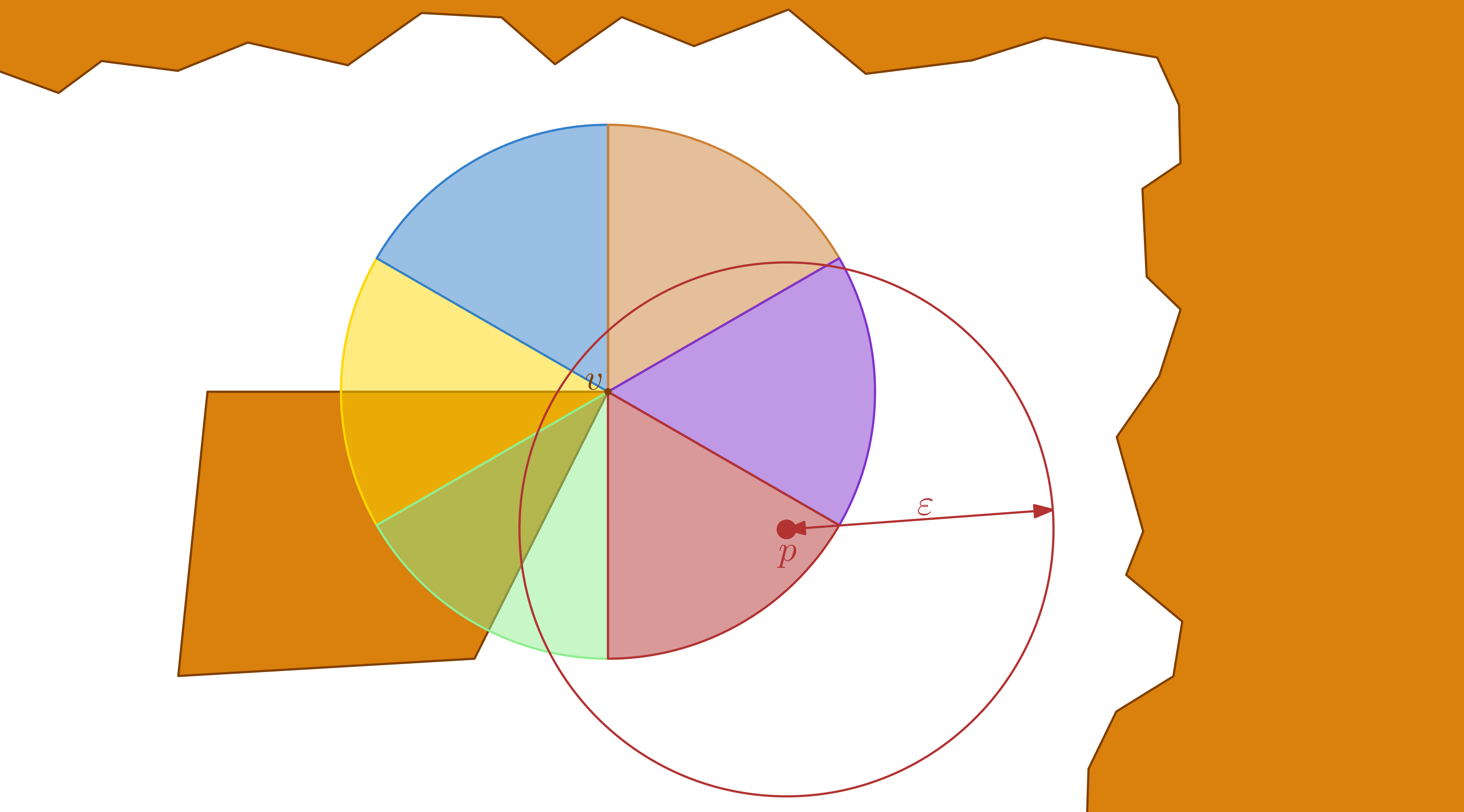
$\implies$ closest pair $r \in R$ and $b \in B$ in $\mathcal{D}_\varepsilon$

$\implies$ $r \in R$ ($b \in B$) is the point from R (B) closest to v.



Upperbound for Well Spaced Obstacles

Partition $\mathcal{D}_\varepsilon$ into 6 regions $W_1, .., W_6$ s.t. $\forall p, q \in W_i$, $\|pq\| \leq \varepsilon$

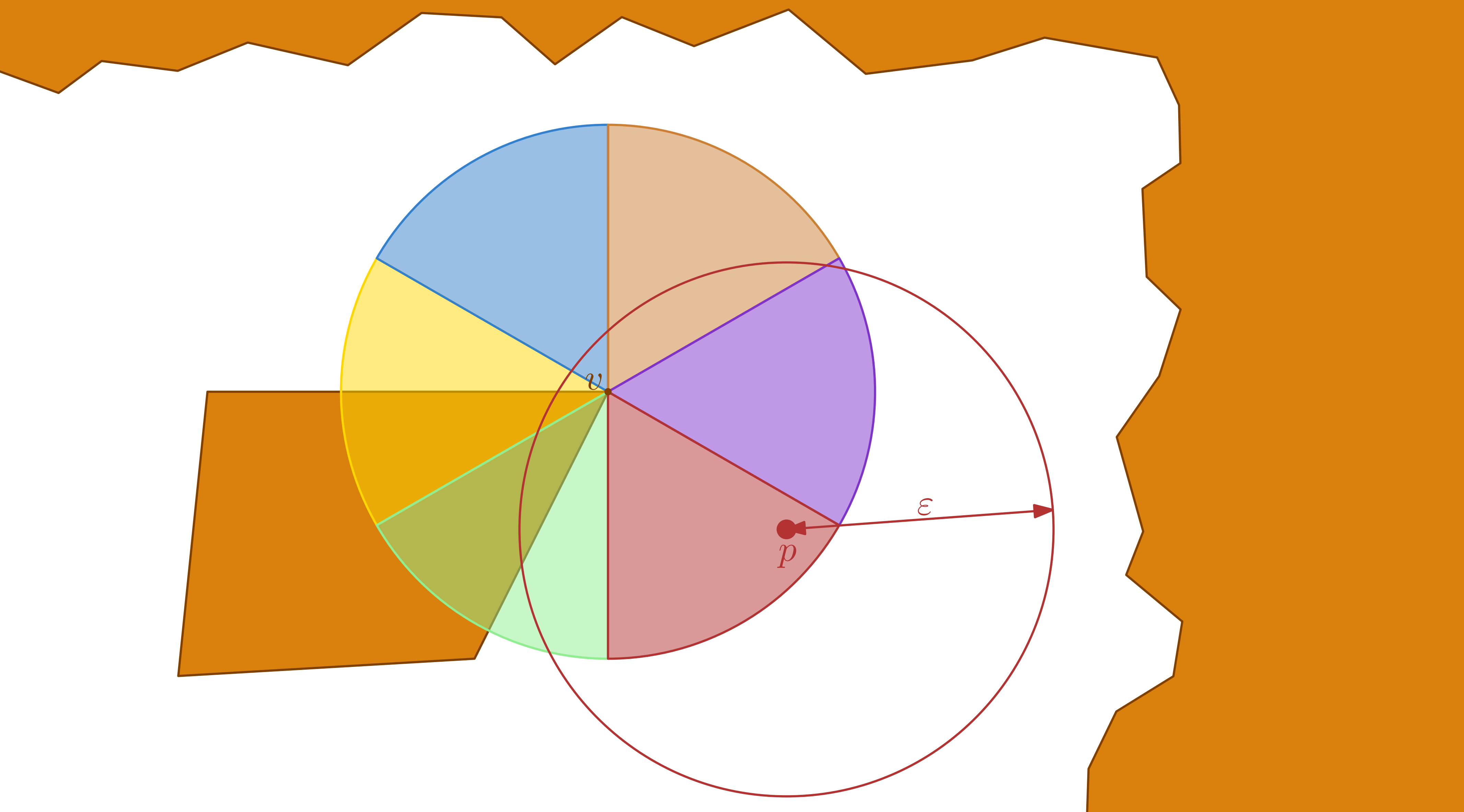


Upperbound for Well Spaced Obstacles

Partition $\mathcal{D}_\varepsilon$ into 6 regions $W_1, .., W_6$ s.t. $\forall p, q \in W_i$, $\|pq\| \leq \varepsilon$

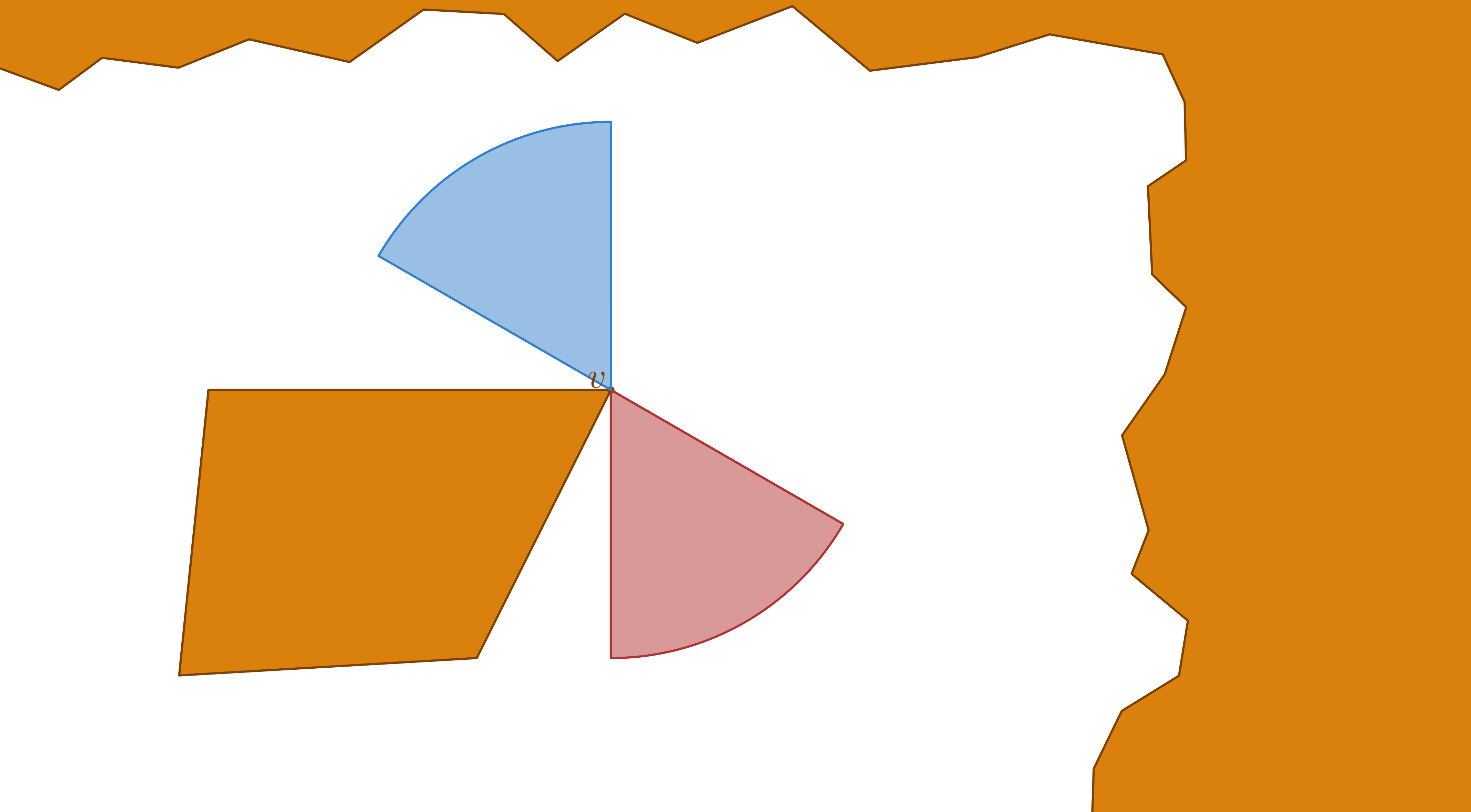
At any time there is at most one ε-connected set interseting a region W_i

$\implies$ At any time, W_i corresponds to at most one ε-connected set



Upperbound for Well Spaced Obstacles

Fix two regions W_R and W_B, count #critical events via v between the ε-connected sets of W_R and W_B.

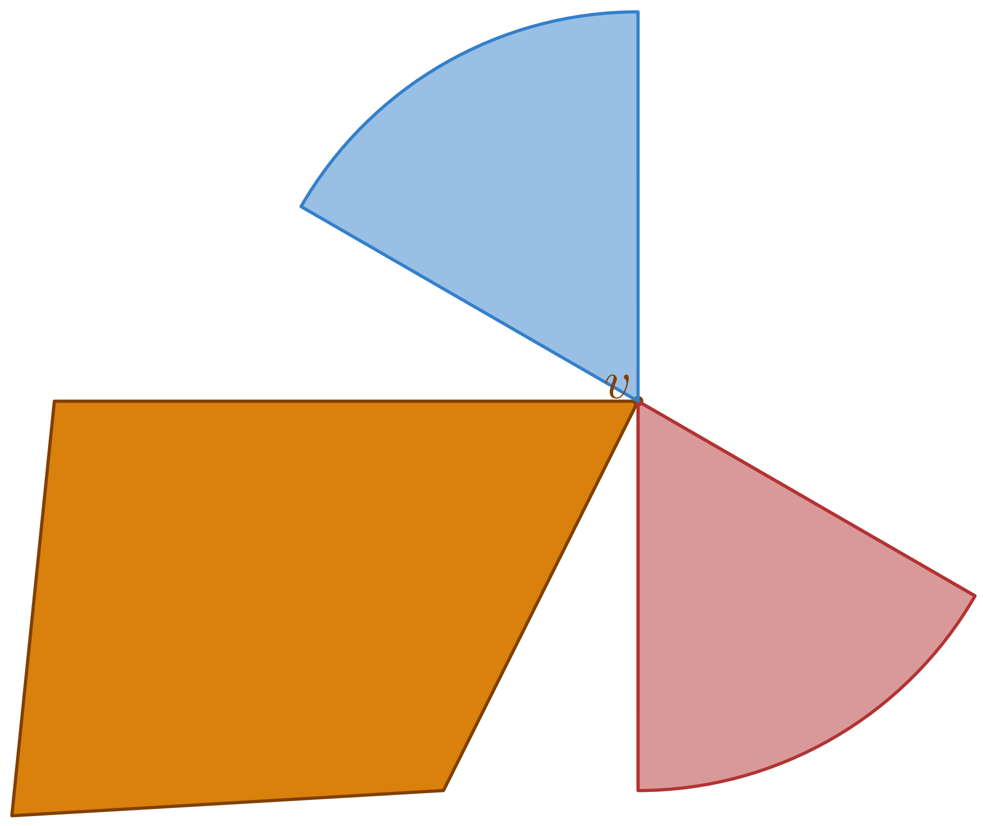


Upperbound for Well Spaced Obstacles

Fix two regions W_R and W_B, count #critical events via v between the ε-connected sets of W_R and W_B.

Critical event $\implies \|bv\| + \|rv\| = \varepsilon$

where $b \in B$ ($r \in R$) is the point from B (R) closest to v.



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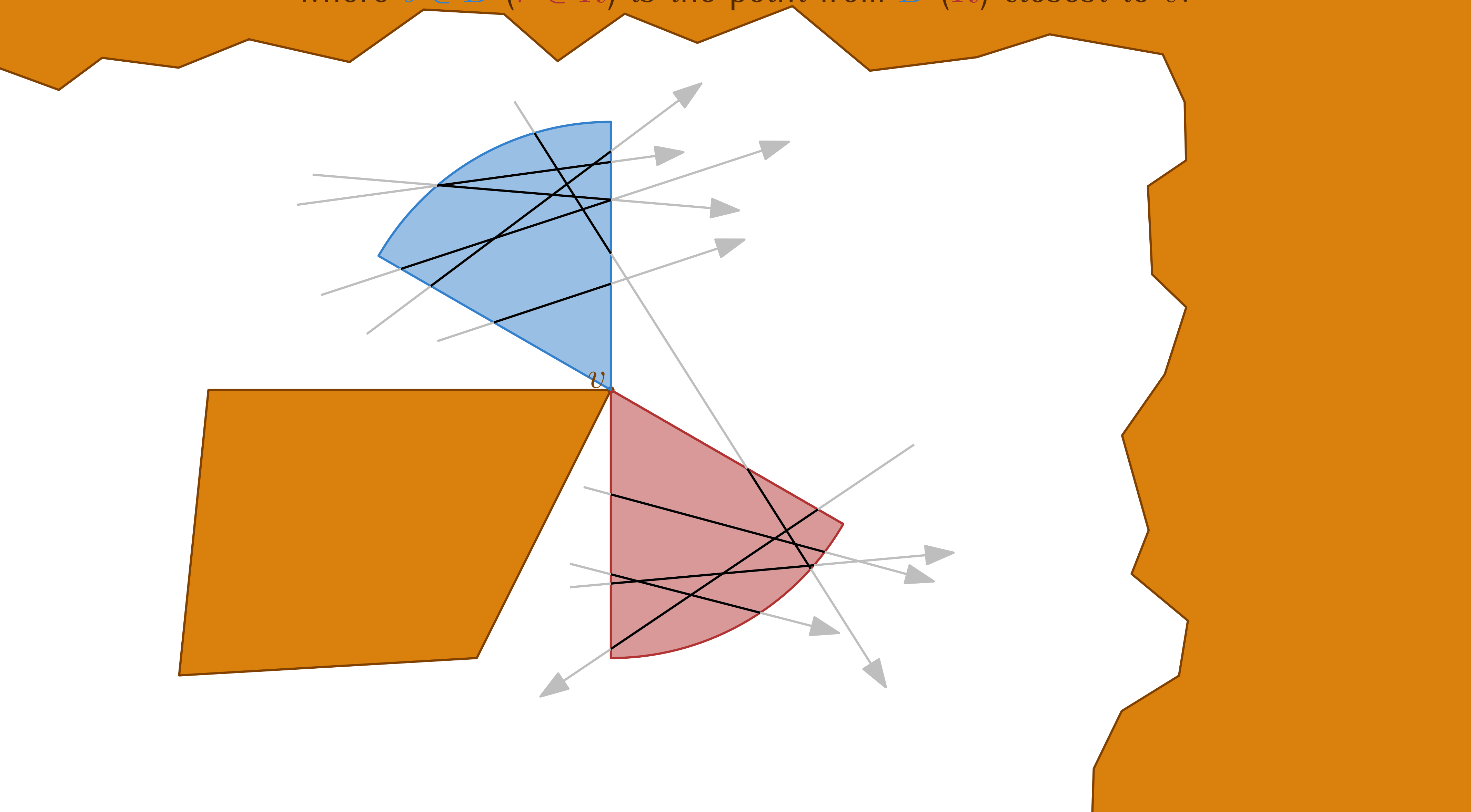

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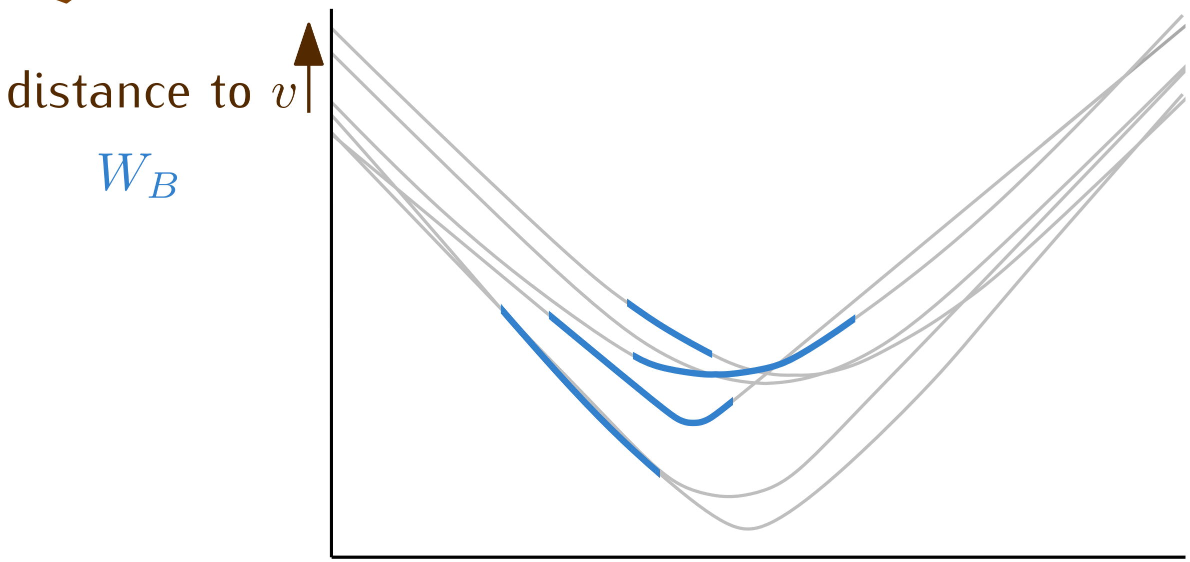

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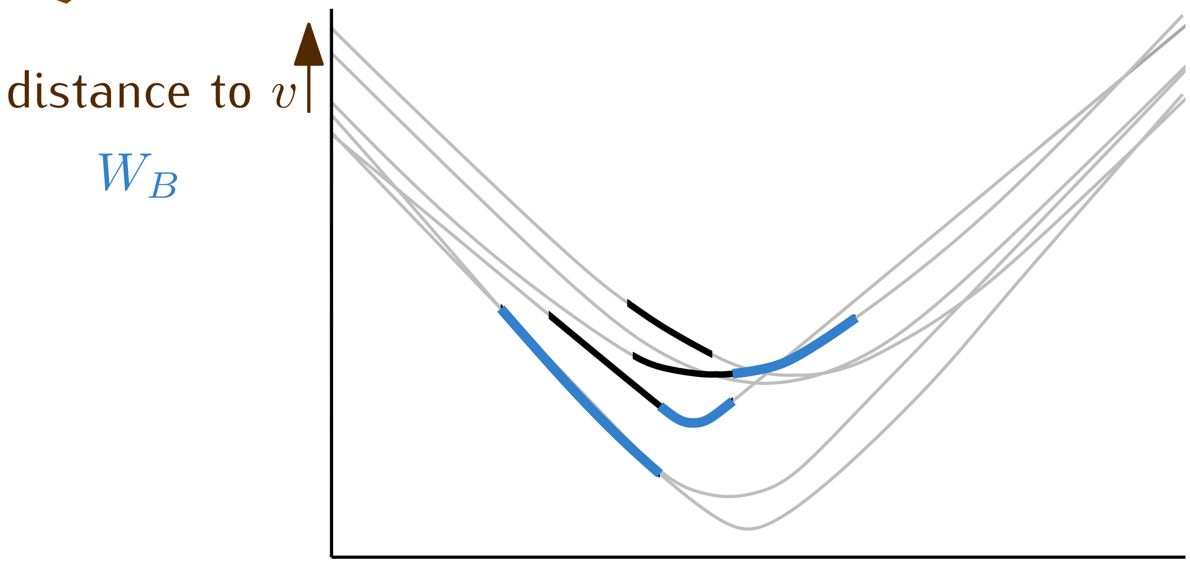

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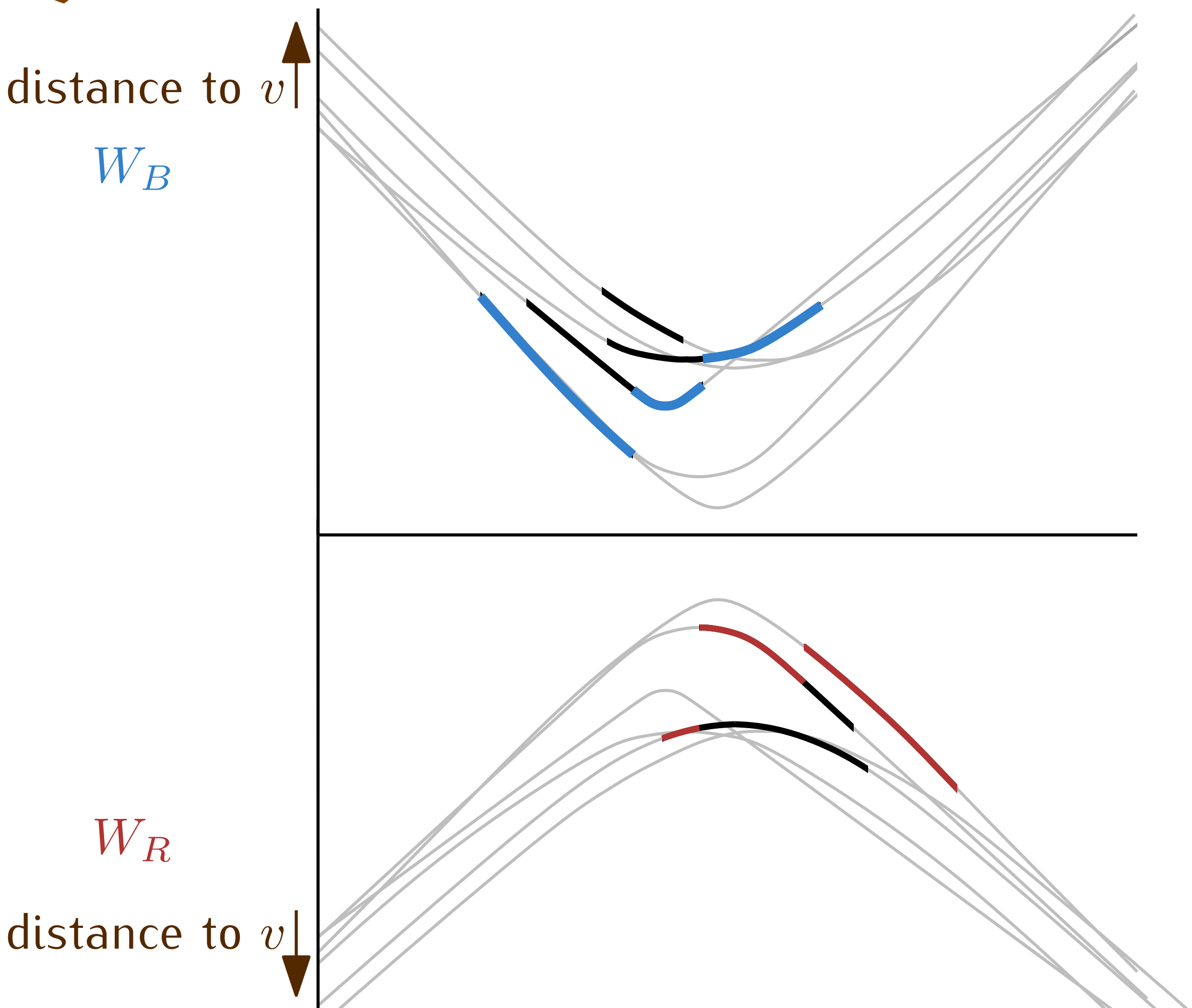

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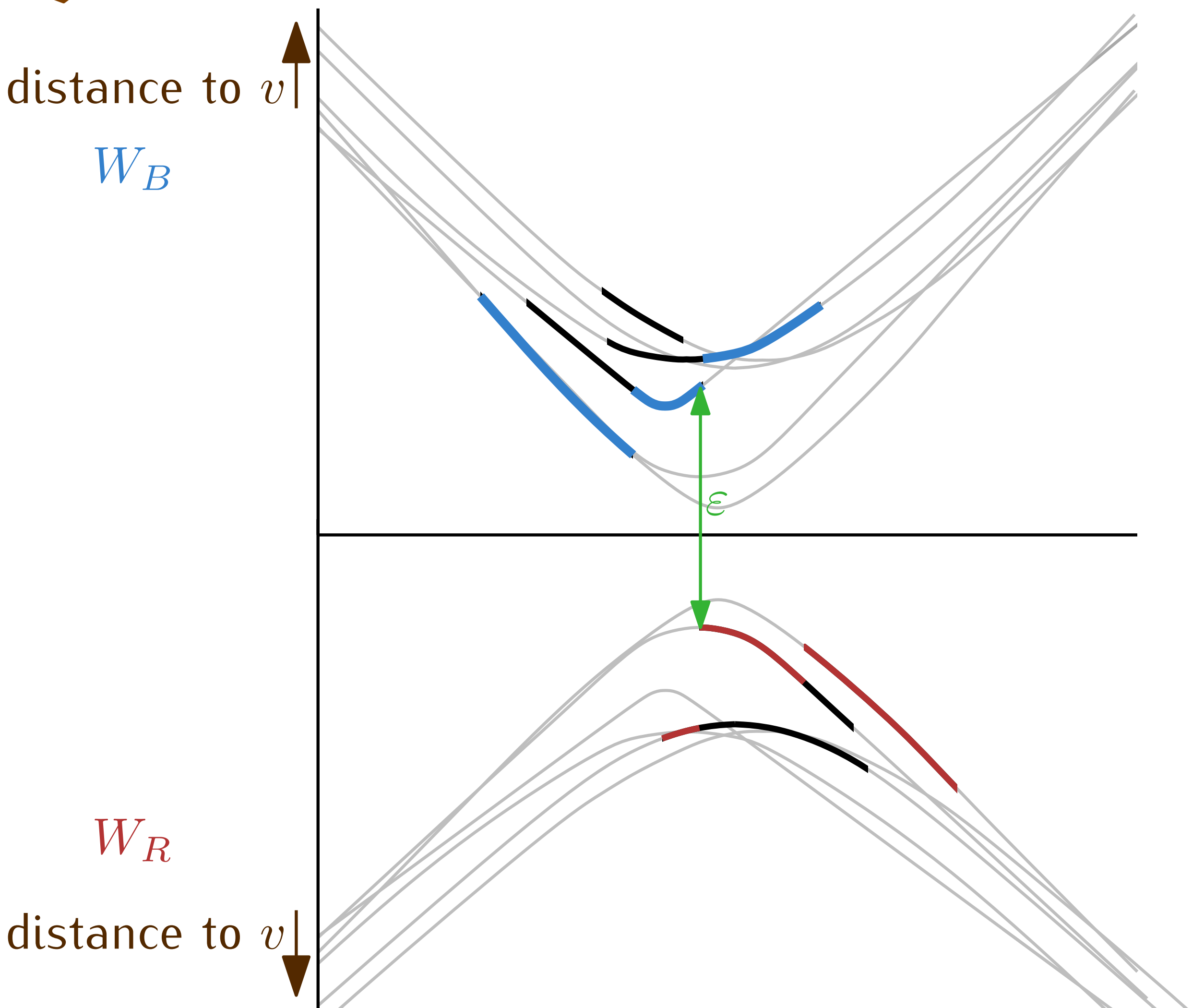
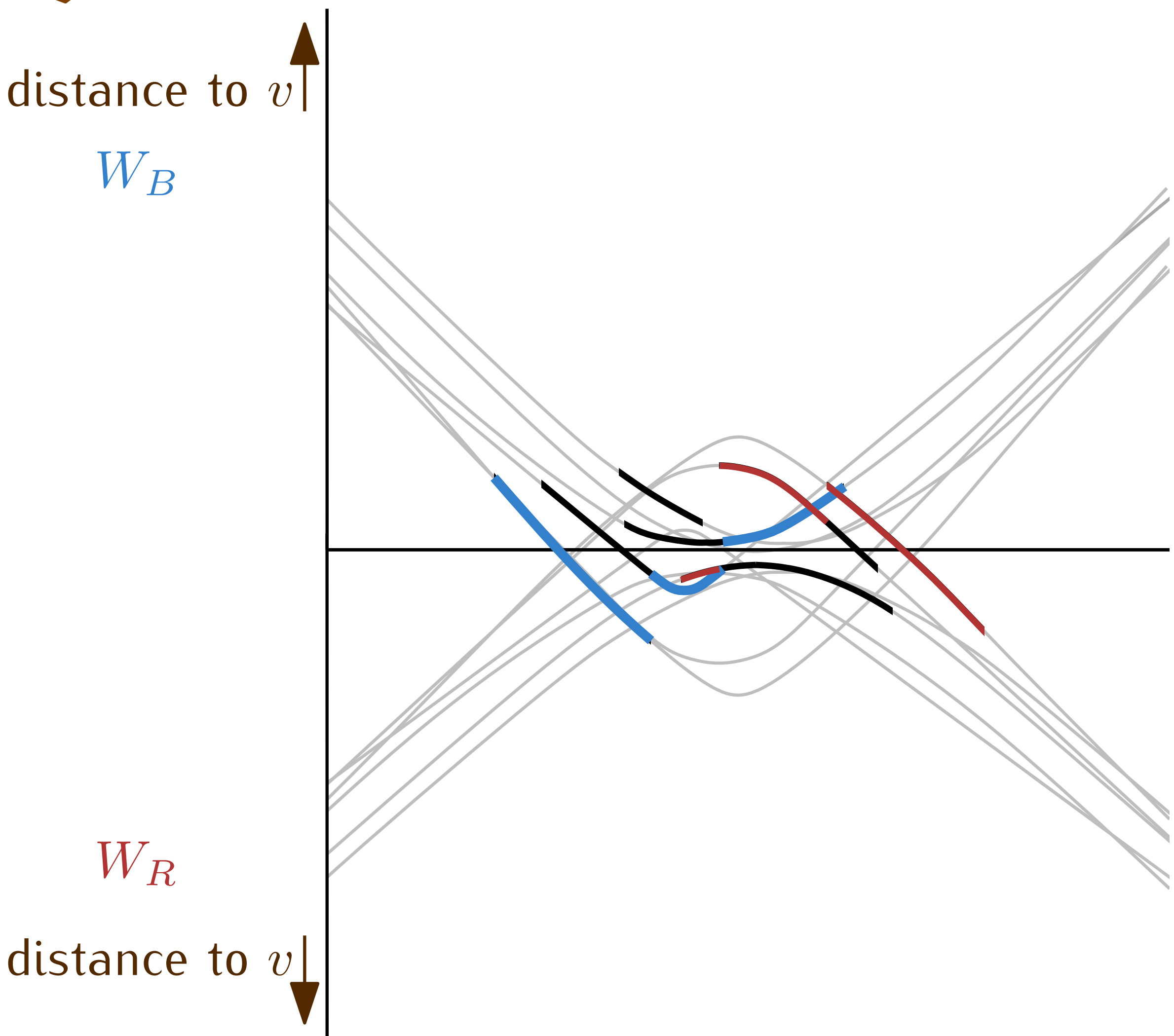


Upperbound for Well Spaced Obstacles

Fix two regions W_R and W_B, count #critical events via v between the ε-connected sets of W_R and W_B.

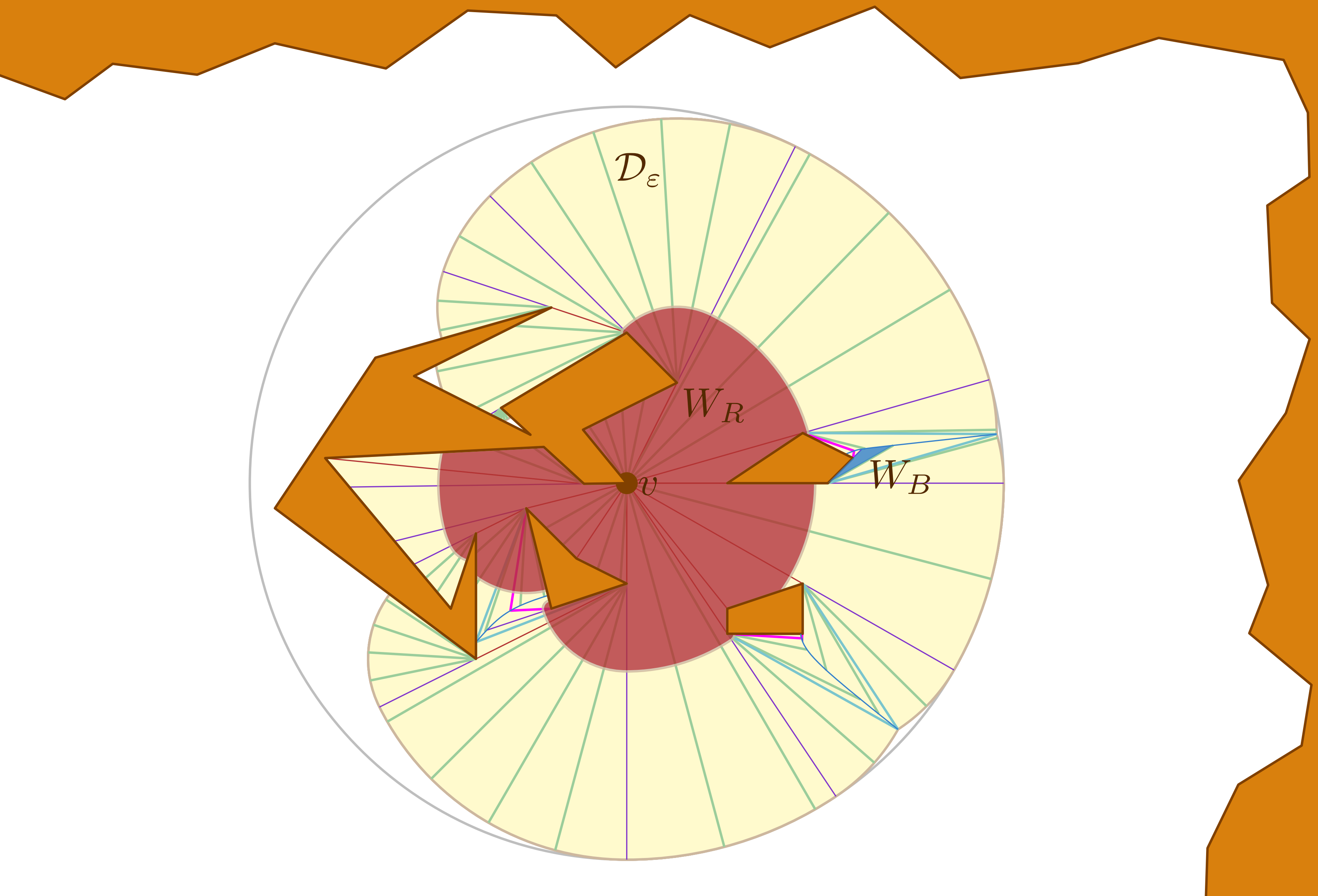
Critical event $\Longrightarrow$ intersection between upper and lower envelope

There are at most $O(\tau\lambda_4(n))$ such intersections.



Upperbound for General Obstacles

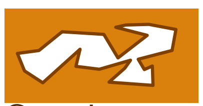
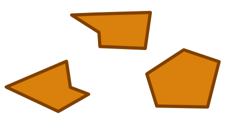
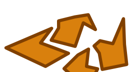
Fix two regions W_R and W_B, count #critical events via v between the ε-connected sets of W_R and W_B.



Algorithm Well-Spaced Obstacles

1) Compute when and how the partition into ε-connected sets changes

$\Longrightarrow$ Construct the Reeb graph $\mathcal{R}$

	Lower bound	Upper bound	Algorithm
No obstacles	$\Omega(\tau n^2)$	$O(\tau n^2)$	$O(\tau n^2 \log n)$
 Simple polygon	$\Omega(\tau n^2)$	$O(\tau n^2)$	$O(\tau n^2(\log^2 m + \log n) + m)$
 Well-spaced obstacles	$\Omega(\tau(n^2 + nm))$	$\boxed{O(\tau(n^2 + m\lambda_4(n)))}$	$\boxed{O(\tau n^2 m \log n)}$
 General obstacles	$\Omega(\tau(n^2 + nm \min\{n, m\}))$	$O(\tau \min\{n^2 + m^3\lambda_4(n), n^2m^2\})$	$O(\tau n^2 m^2 \log n + m^2 \log m)$

$n = \#$ entities

$\tau = \#$ vertices in each trajectory

$m = \#$ obstacle vertices

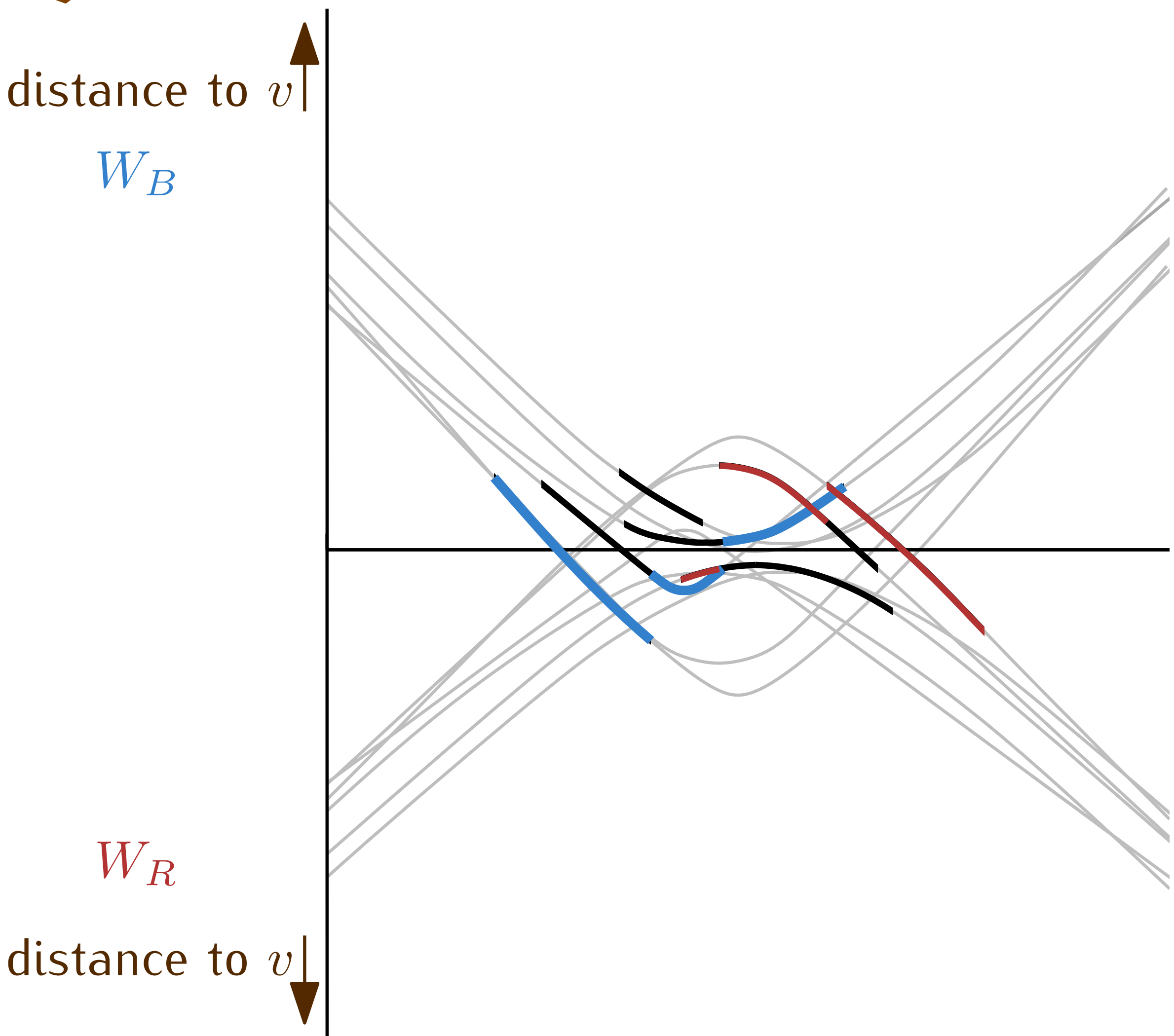
$\lambda_4(n)$ = max length of a Davenport-Schinzel sequence of order 4 on n symbols.

Algorithm Well-Spaced Obstacles

1) Compute when and how the partition into ε-connected sets changes

$\Longrightarrow$ Construct the Reeb graph $\mathcal{R}$

Compute all critical events t

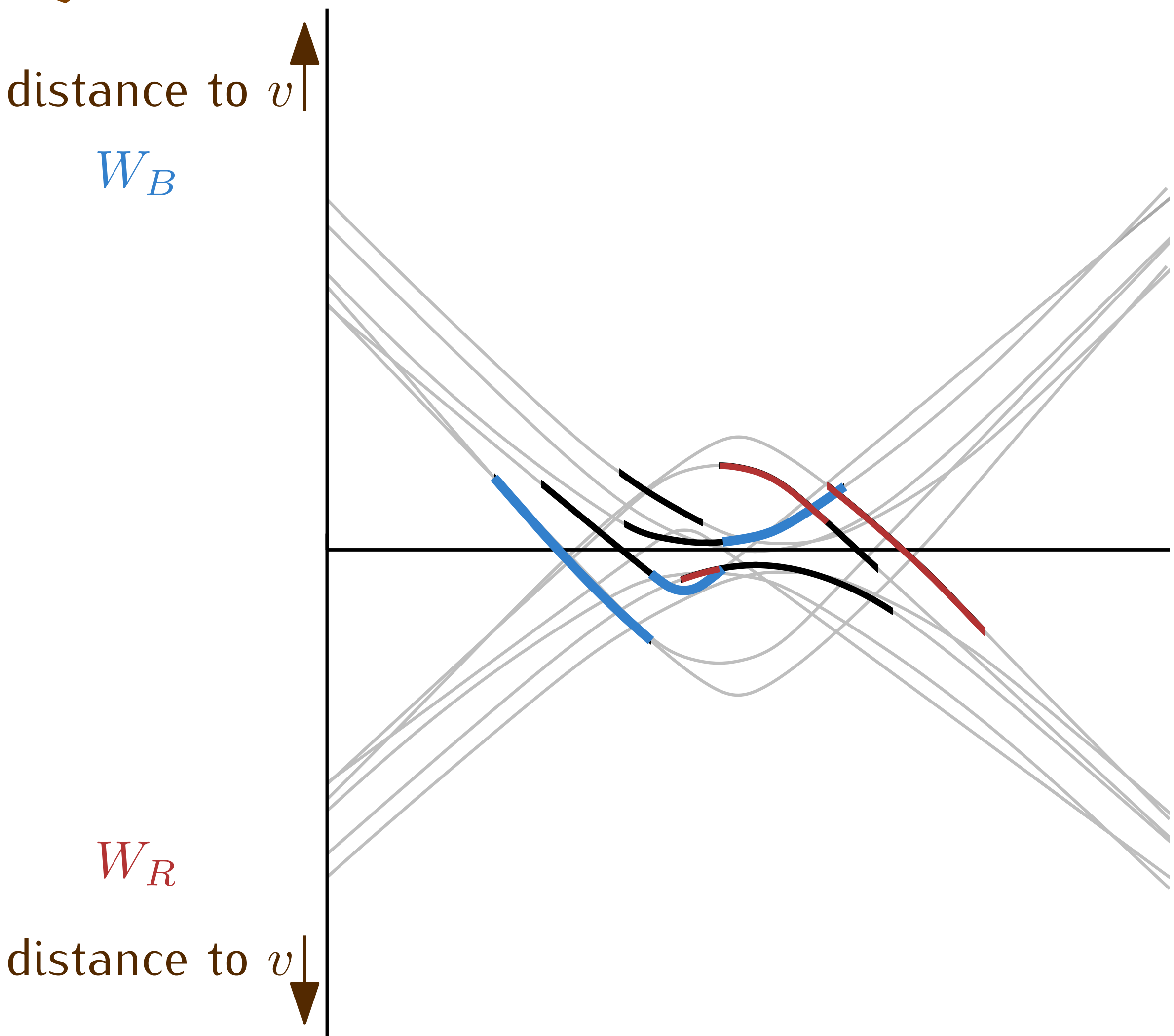


Algorithm Well-Spaced Obstacles

1) Compute when and how the partition into ε-connected sets changes

$\implies$ Construct the Reeb graph $\mathcal{R}$

Compute all critical events (t, R, B)



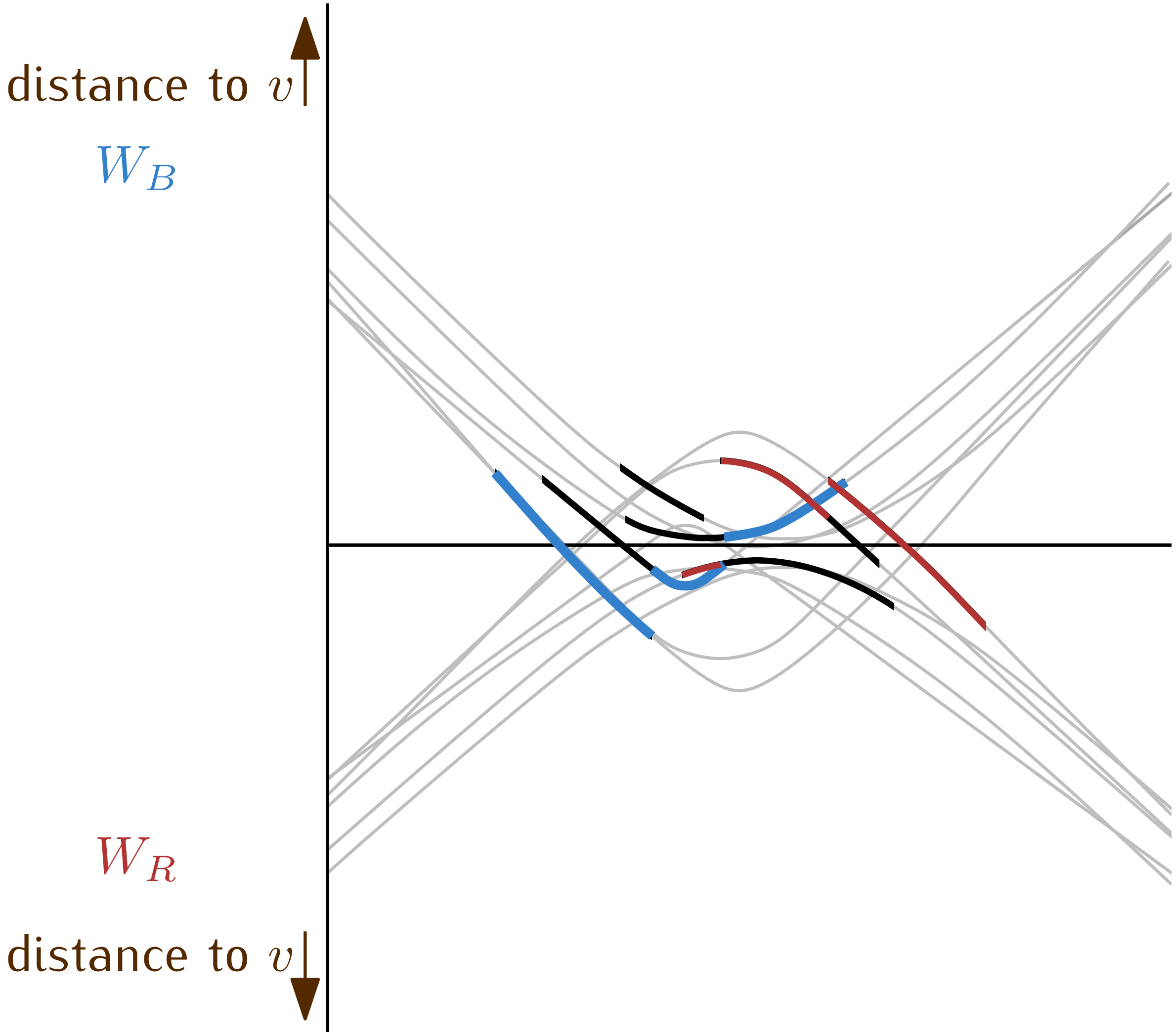
Algorithm Well-Spaced Obstacles

1) Compute when and how the partition into ε-connected sets changes

$\implies$ Construct the Reeb graph $\mathcal{R}$

a) Compute all ε-events: times s.t. $\varsigma(a,b) = \varepsilon$

b) Maintain dynamic connectivity graph $G = (\mathcal{X}, \{(a,b) \mid \varsigma(a,b) \leq \varepsilon\})$



Algorithm Well-Spaced Obstacles

1) Compute when and how the partition into ε-connected sets changes

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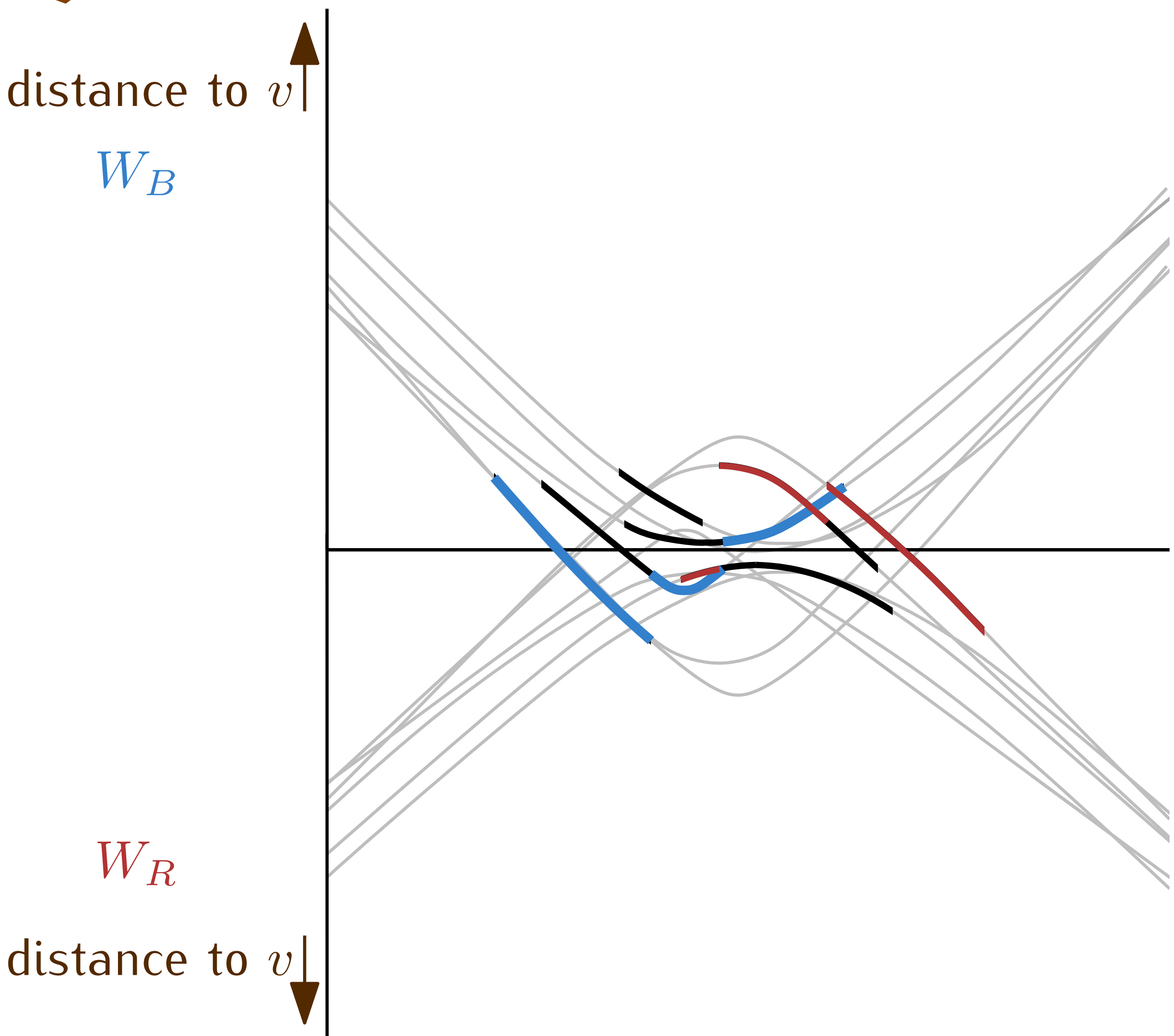
$\#\varepsilon$-events is $\Theta(\tau n^2 m)$



Future work

1) Compute $\mathcal{R}$ output sensitively

2) Improve upper bound general obstacles

we believe $O(\tau(n^2 + m^2\lambda_4(n))$ should be possible

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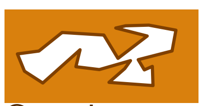
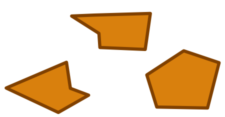
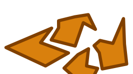
 Simple polygon	$\Omega(\tau n^2)$	$O(\tau n^2)$	$O(\tau n^2(\log^2 m + \log n) + m)$
 Well-spaced obstacles	$\Omega(\tau(n^2 + nm))$	$O(\tau(n^2 + m\lambda_4(n)))$	$O(\boxed{\tau n^2 m}\log n)$
 General obstacles	$\Omega(\tau(n^2 + nm\min\{n, m\}))$	$O(\tau\min\{n^2 + \boxed{m^3\lambda_4(n)}, n^2m^2\})$	$O(\boxed{\tau n^2 m^2}\log n + m^2\log m)$

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