Grouping Time-varying Data for Interactive Exploration

Arthur van Goethem
Marc van Kreveld
Bettina Speckmann
Frank Staals
Maarten Löffler

Detecting Patterns in Geometric Data

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Alg(Input)

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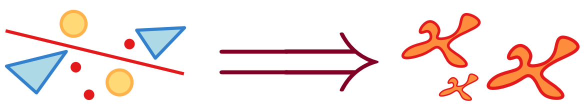

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Detecting maximal groups in trajectory data

✓ (m, ε, δ)-group

m = min size

ε = max dist

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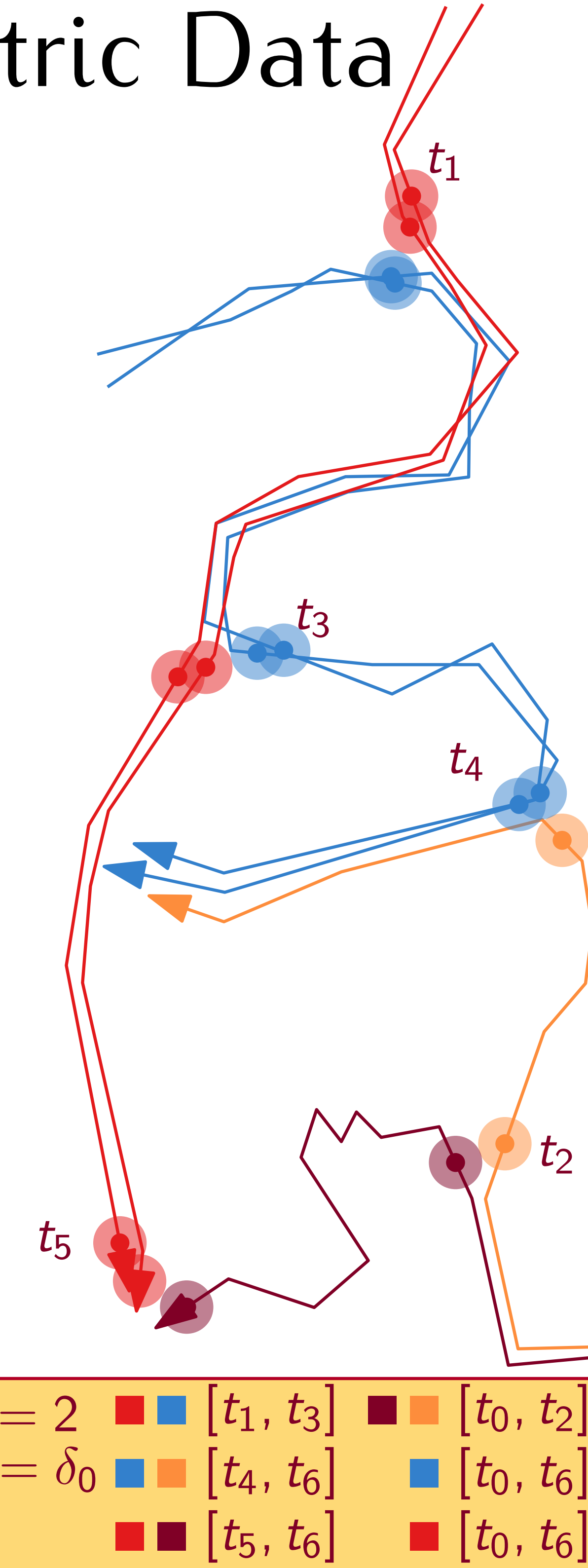

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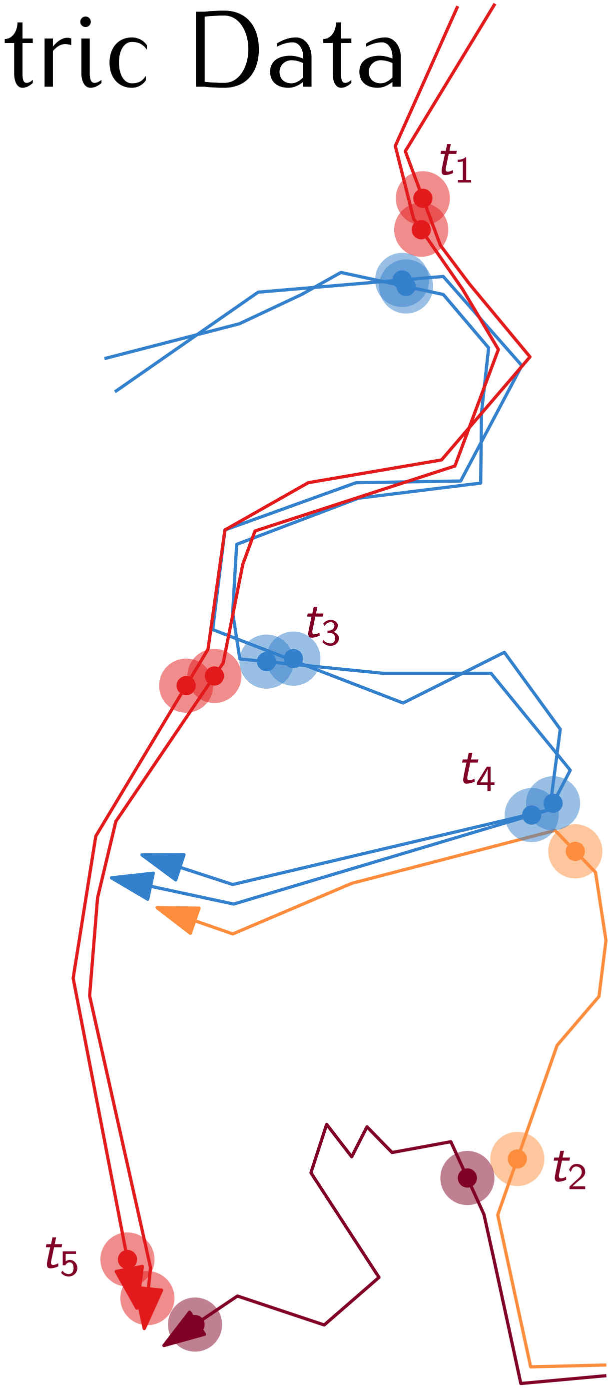

$m = 3$ $[t_1, t_3]$

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$[t_5, t_6]$

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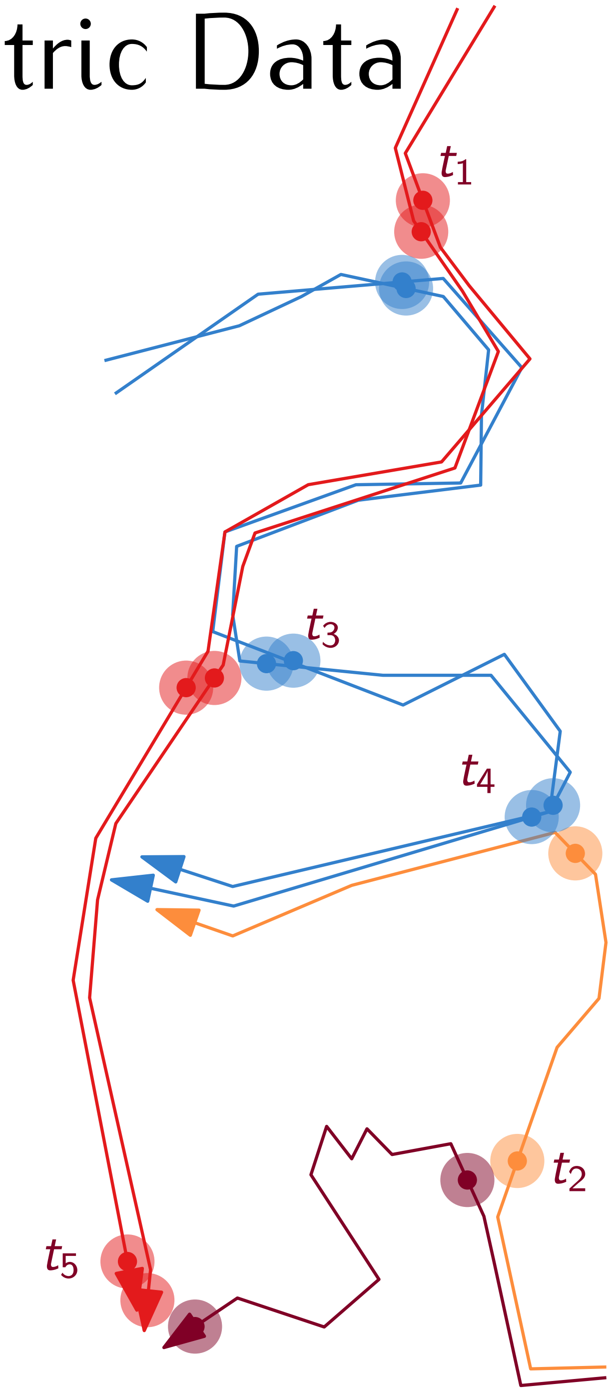

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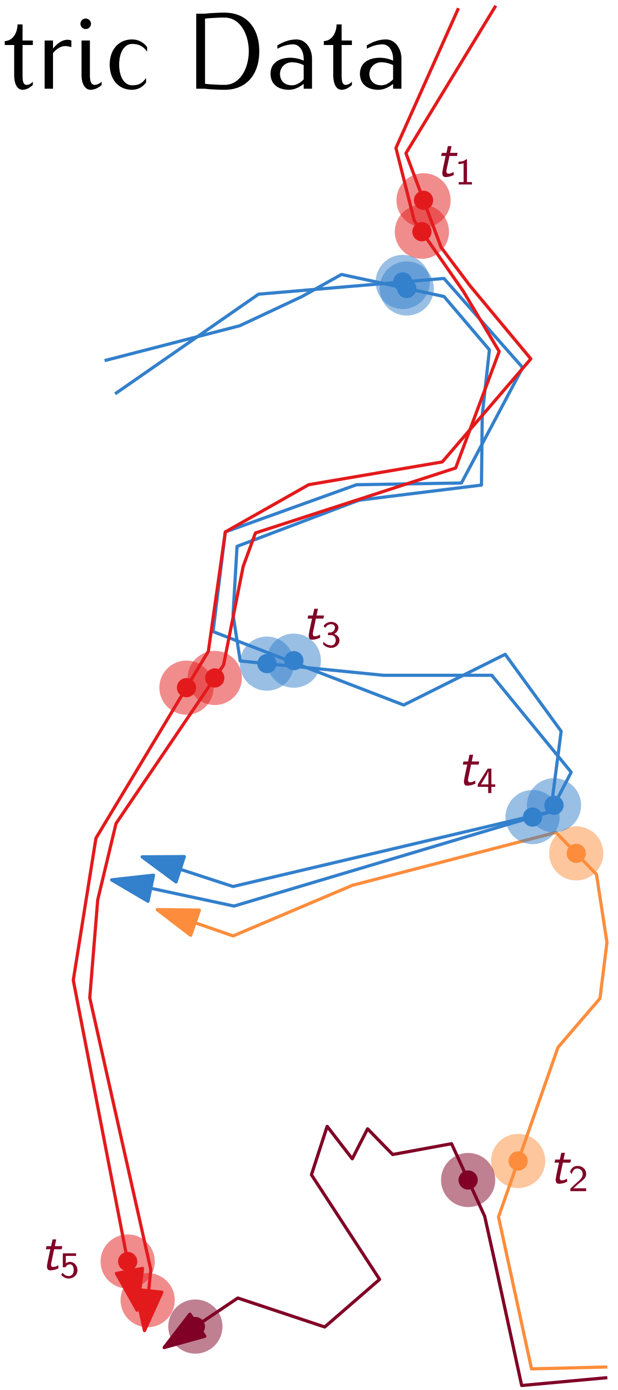
✓ Running time: $O(n^3\tau)$

n = #trajectories

τ = trajectory length

✓ Trajectory Grouping Structure [WADS 2013]



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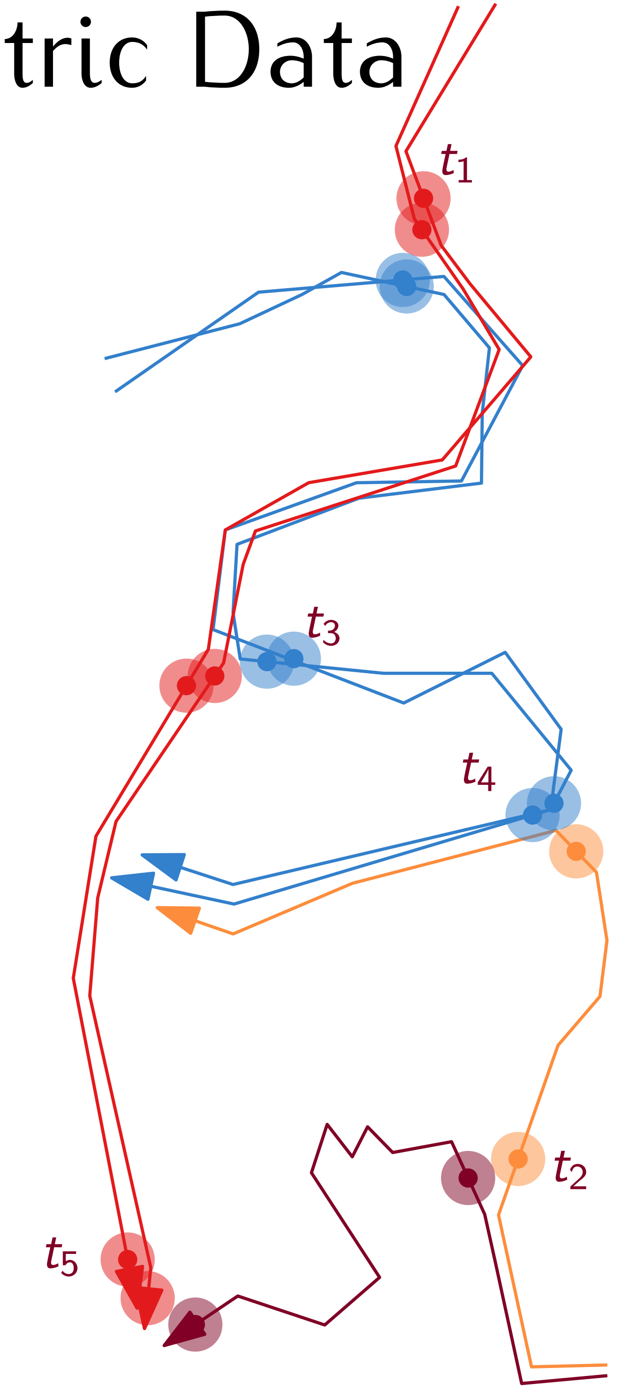

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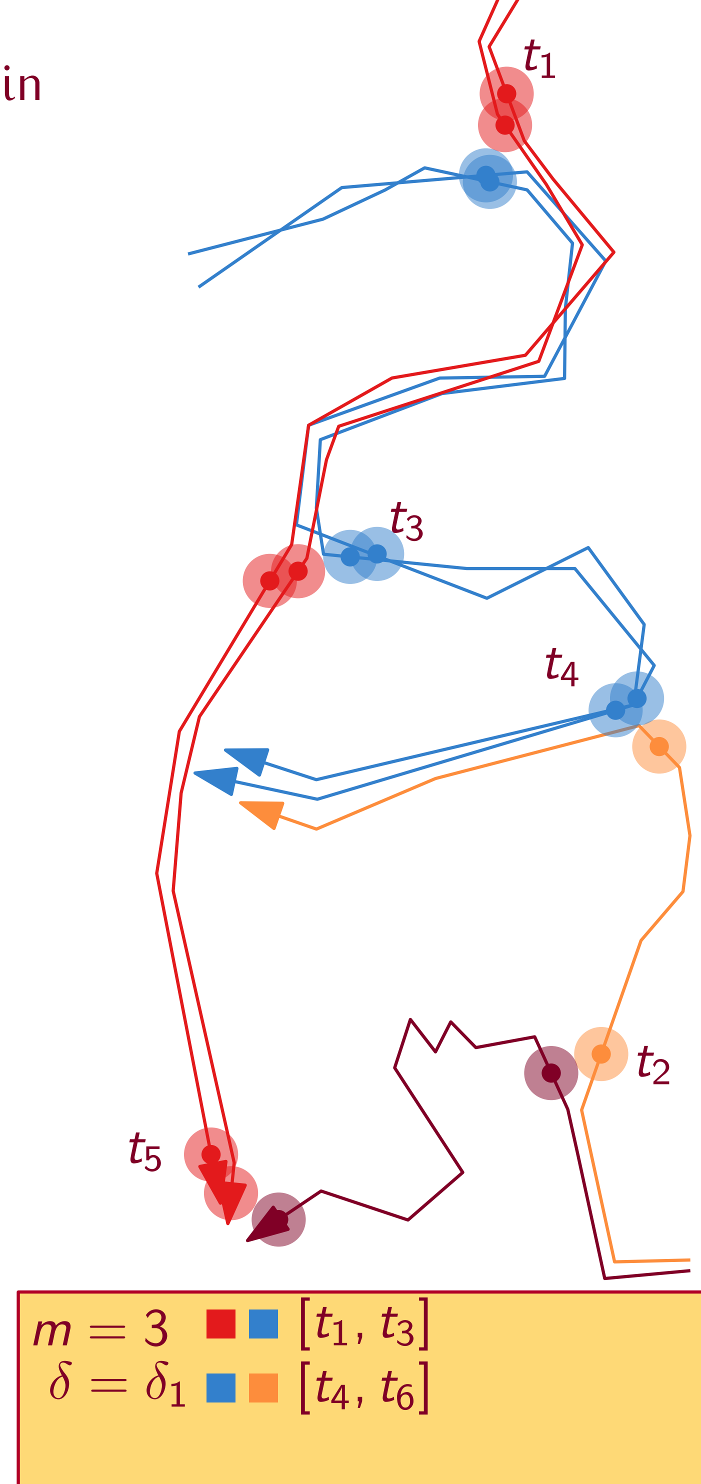

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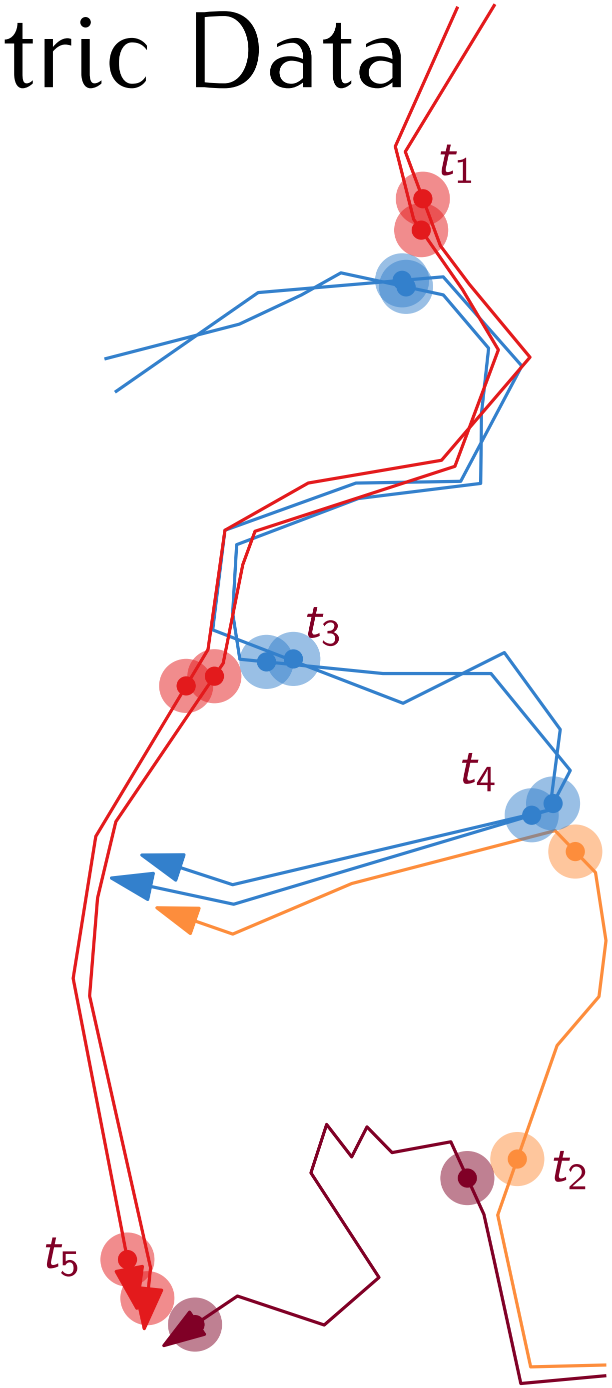
Assumption:
The practicioner knows the right parameter values.



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2) Design an efficient algorithm Alg(Input, $\alpha, .., \eta$)

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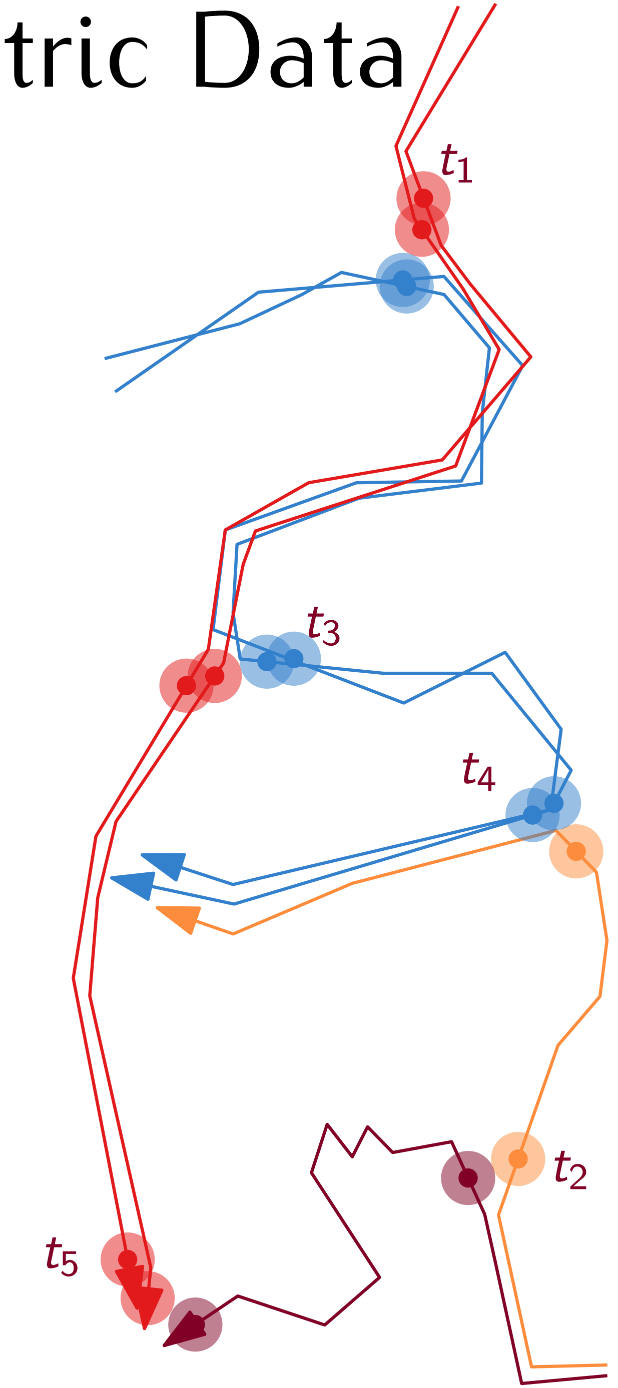

The practicioner does not know the right parameter values

$\Longrightarrow$ We need to be able to change the parameters efficiently

$m = 3$ ■■ $[t_1, t_3]$

$\delta = \delta_1$ ■■ $[t_4, t_6]$

Problems

Goal: Change parameters (m, ε, δ) to $(m', \varepsilon', \delta')$

$\Longrightarrow$ Report only the maximal groups that have changed

- Maximal (m, ε, δ) groups that are not maximal $(m', \varepsilon', \delta')$ groups
- Maximal $(m', \varepsilon', \delta')$ groups that are not maximal (m, ε, δ) groups

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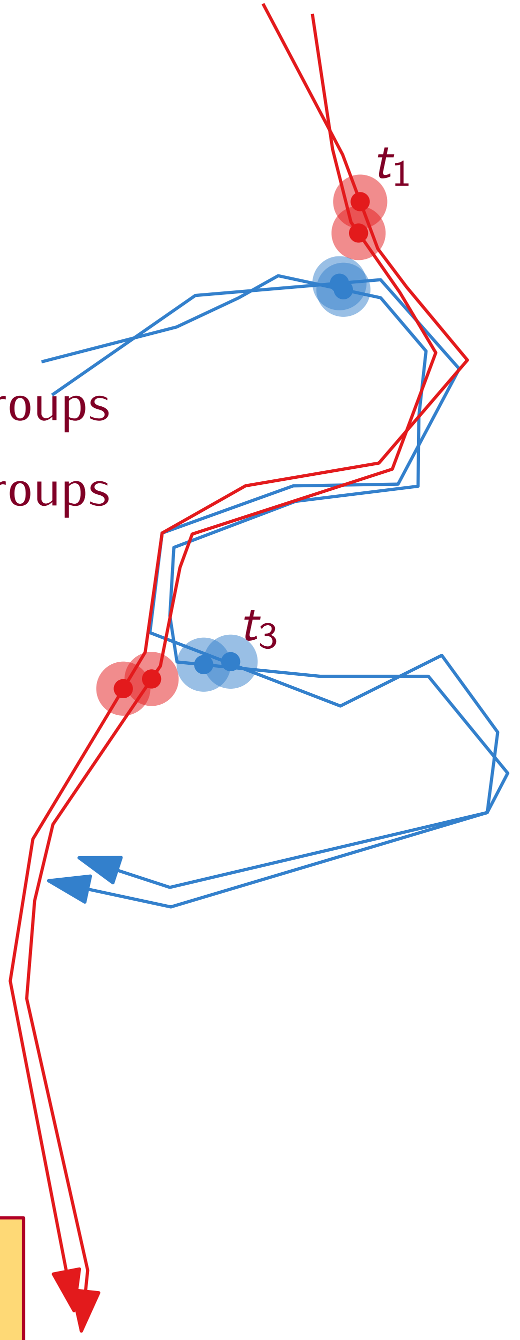

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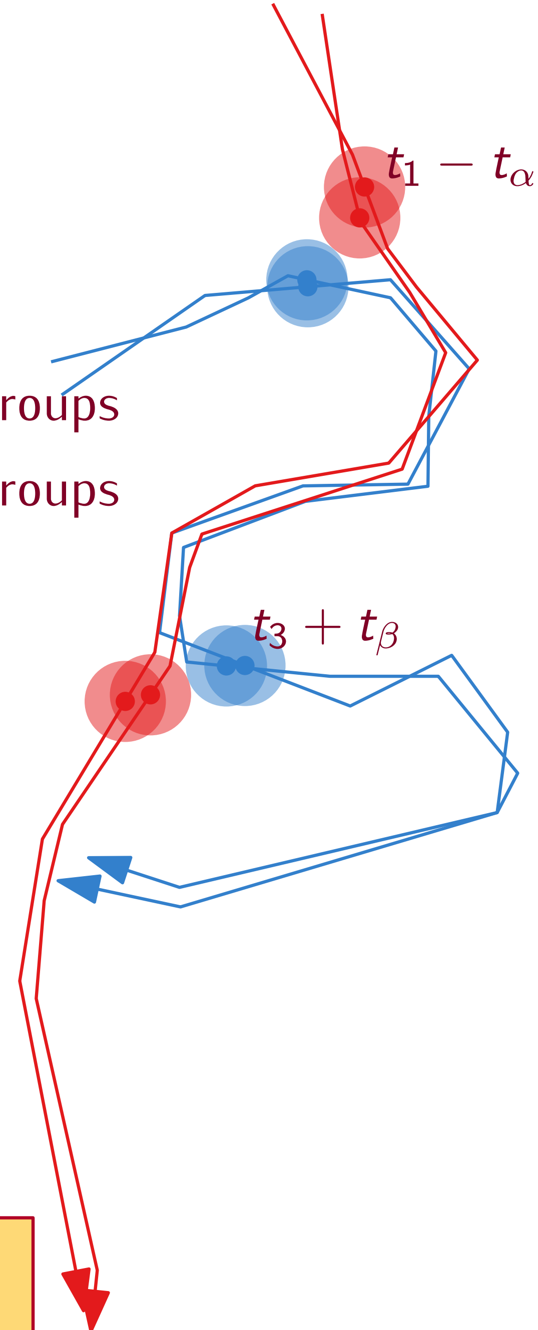
Intuitively, (G, I) and (G, I') are the same group.



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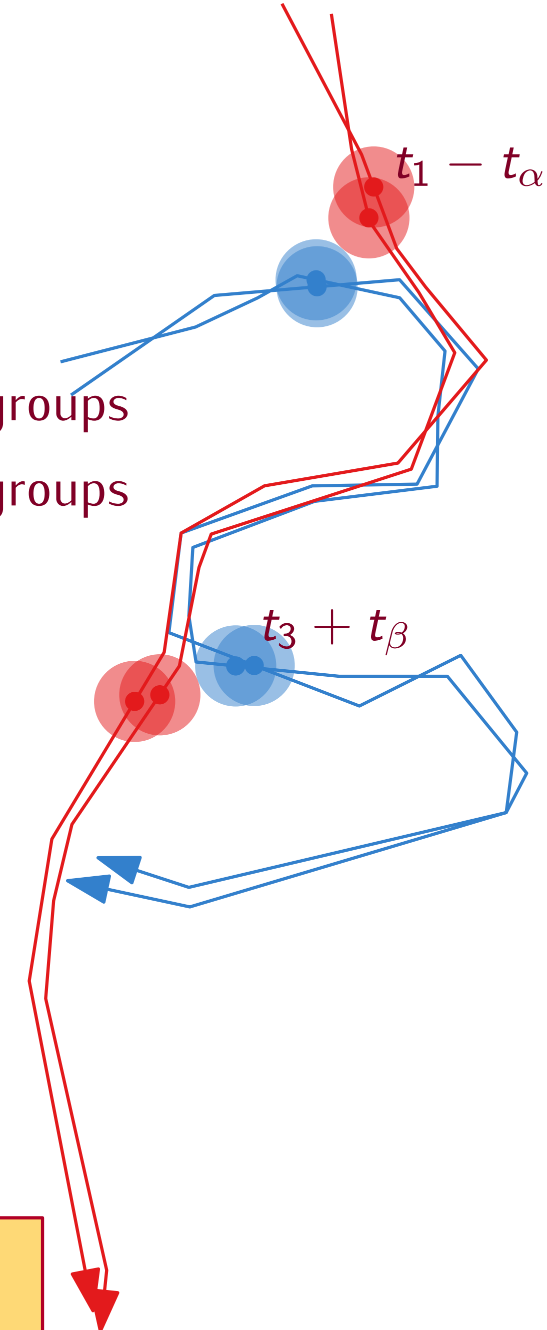

1) How do we represent all maximal groups s.t. we can efficiently change (m, ε, δ)?
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3) How many different maximal groups, over all parameters?

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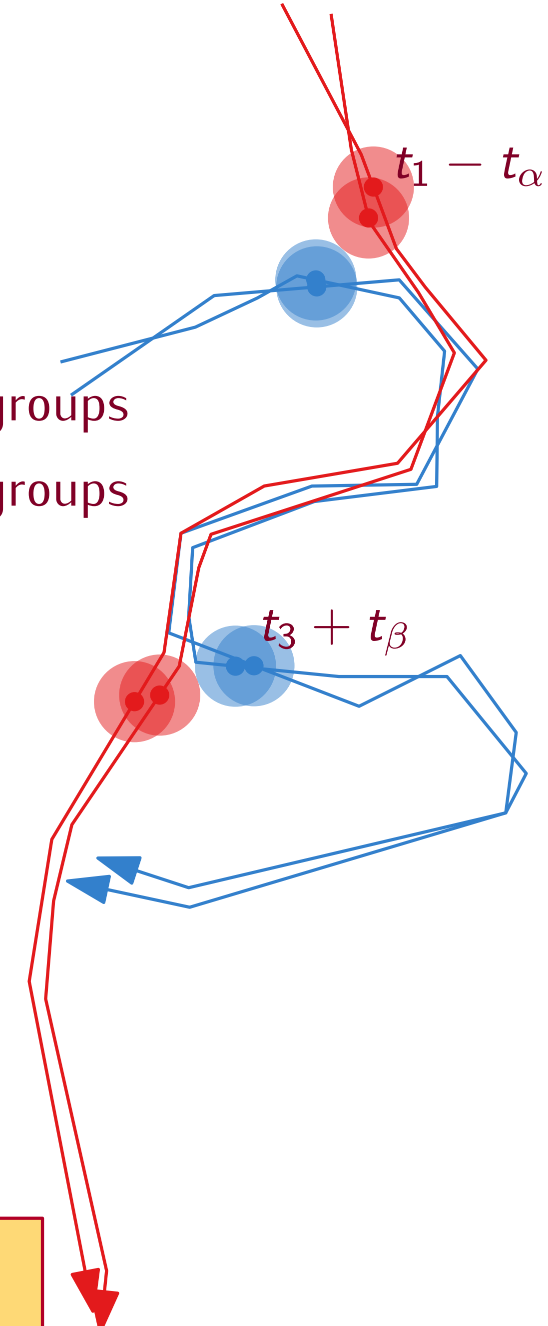

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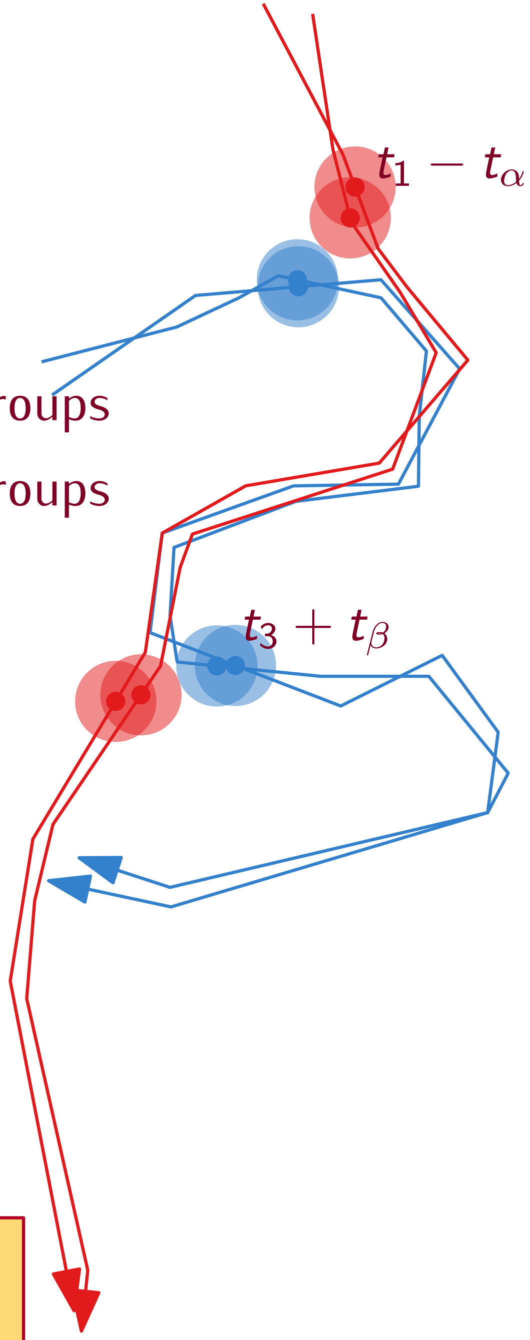

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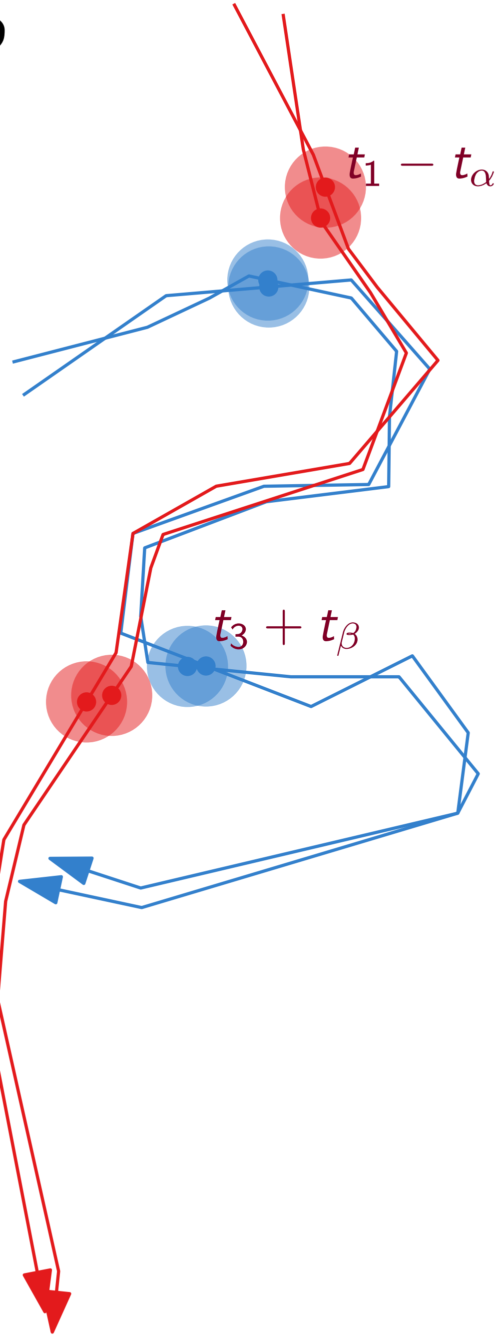

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■■ is a $(m, \varepsilon', \delta)$-group on $I'' = [t, t'']$

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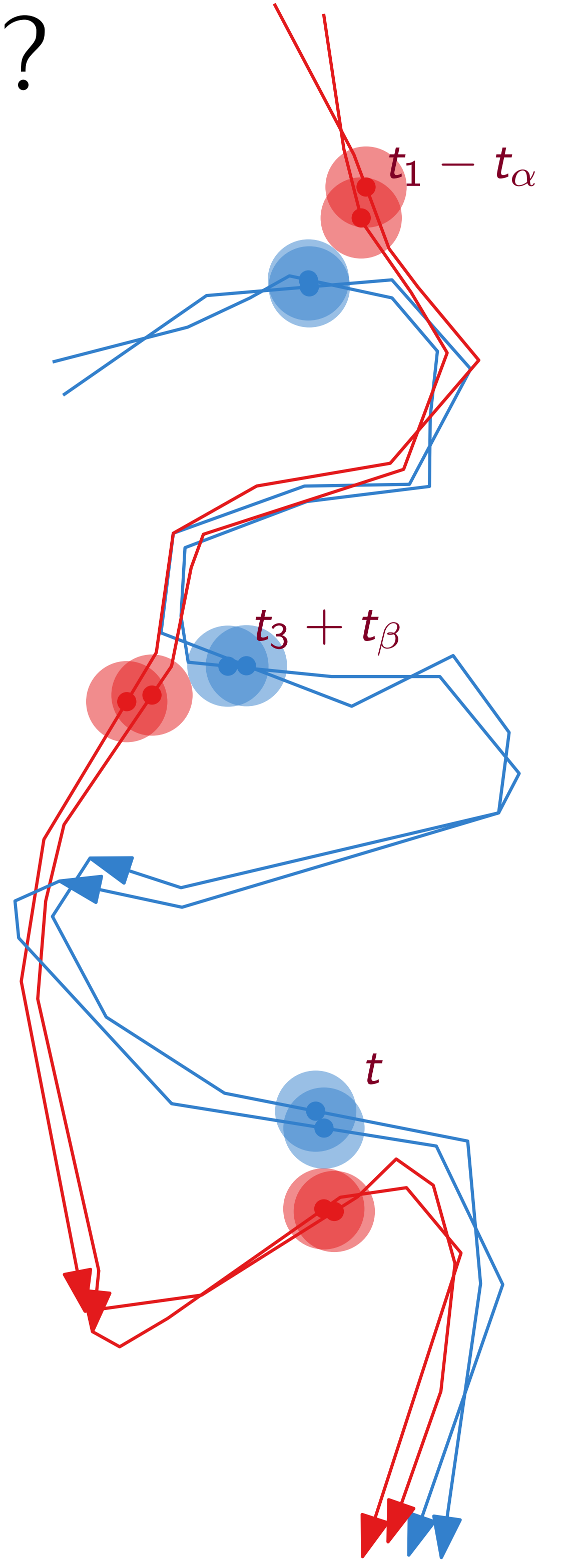

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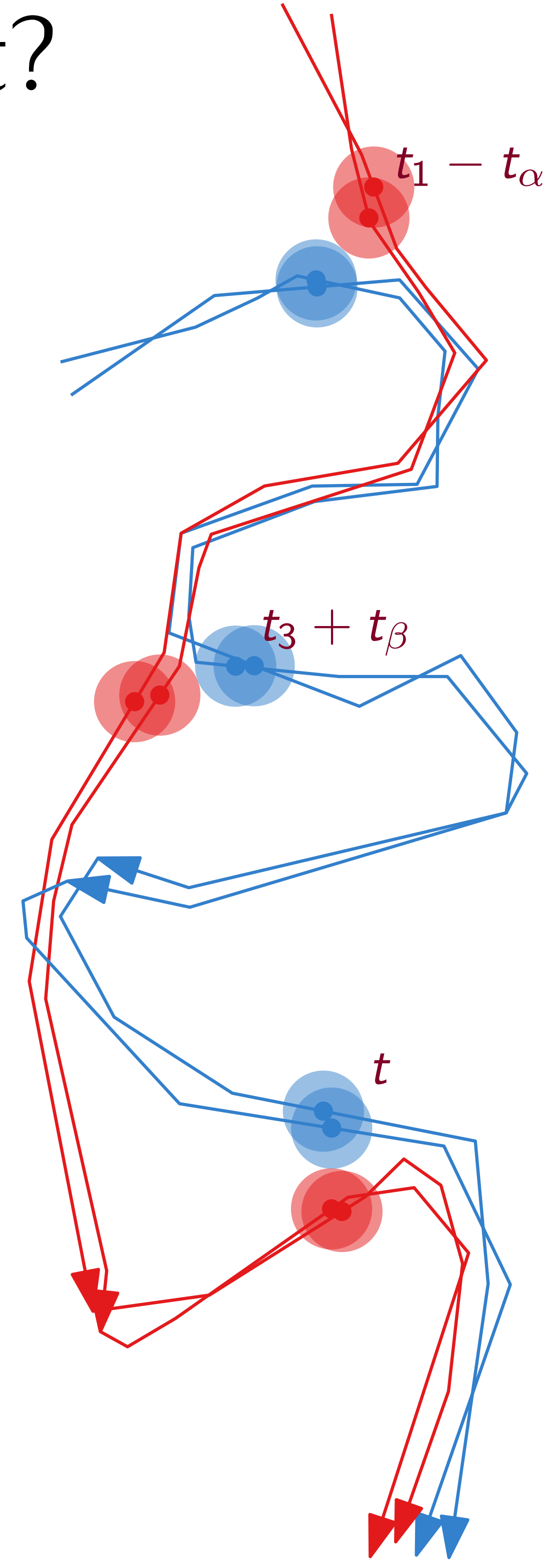
Intuitively, (G, I') and (G, I'') are different groups.

■■ is a $(m, \varepsilon_>, \delta)$-group on $I_> \supset I \cup I'$

Intuitively, $(G, I_>)$ is different from (G, I') and from (G, I'')



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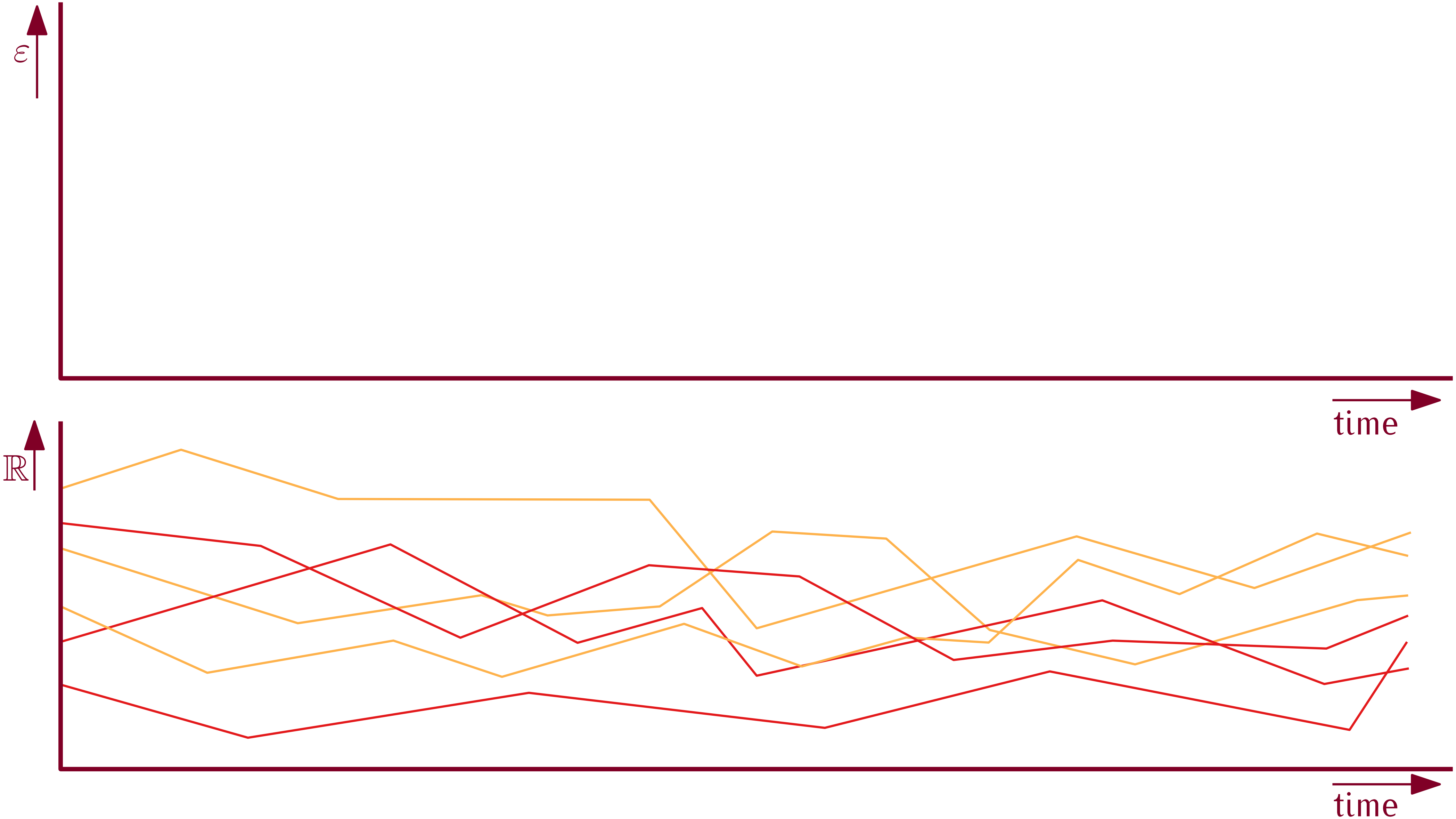

When are two groups different?

$\mathbb{R}^1$
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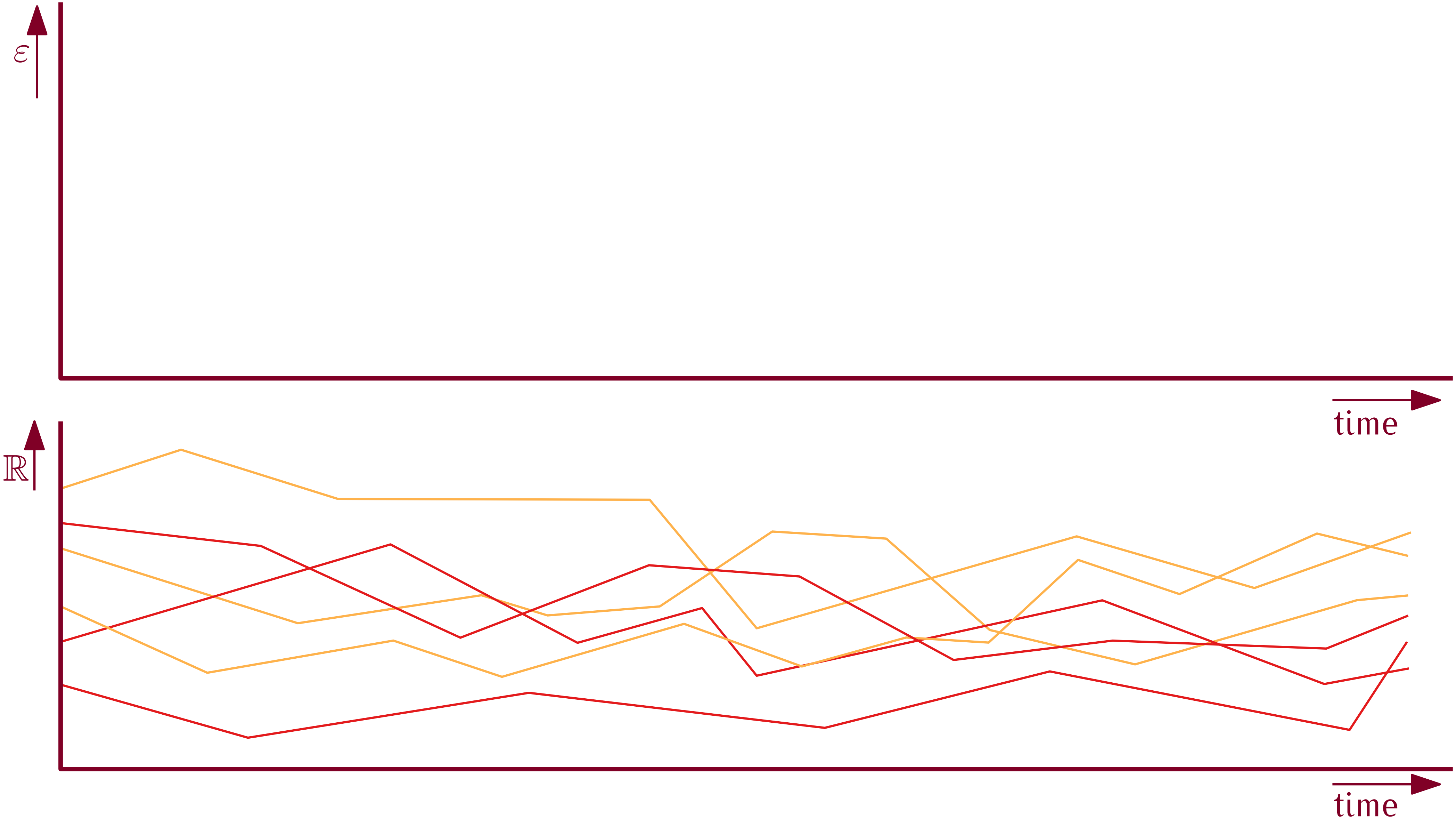
G = some of entites

Consider region A_G s.t.:

$$(\varepsilon, t) \in A_G \iff G \text{ forms an } (m, \varepsilon, \delta)\text{-group at time } t$$



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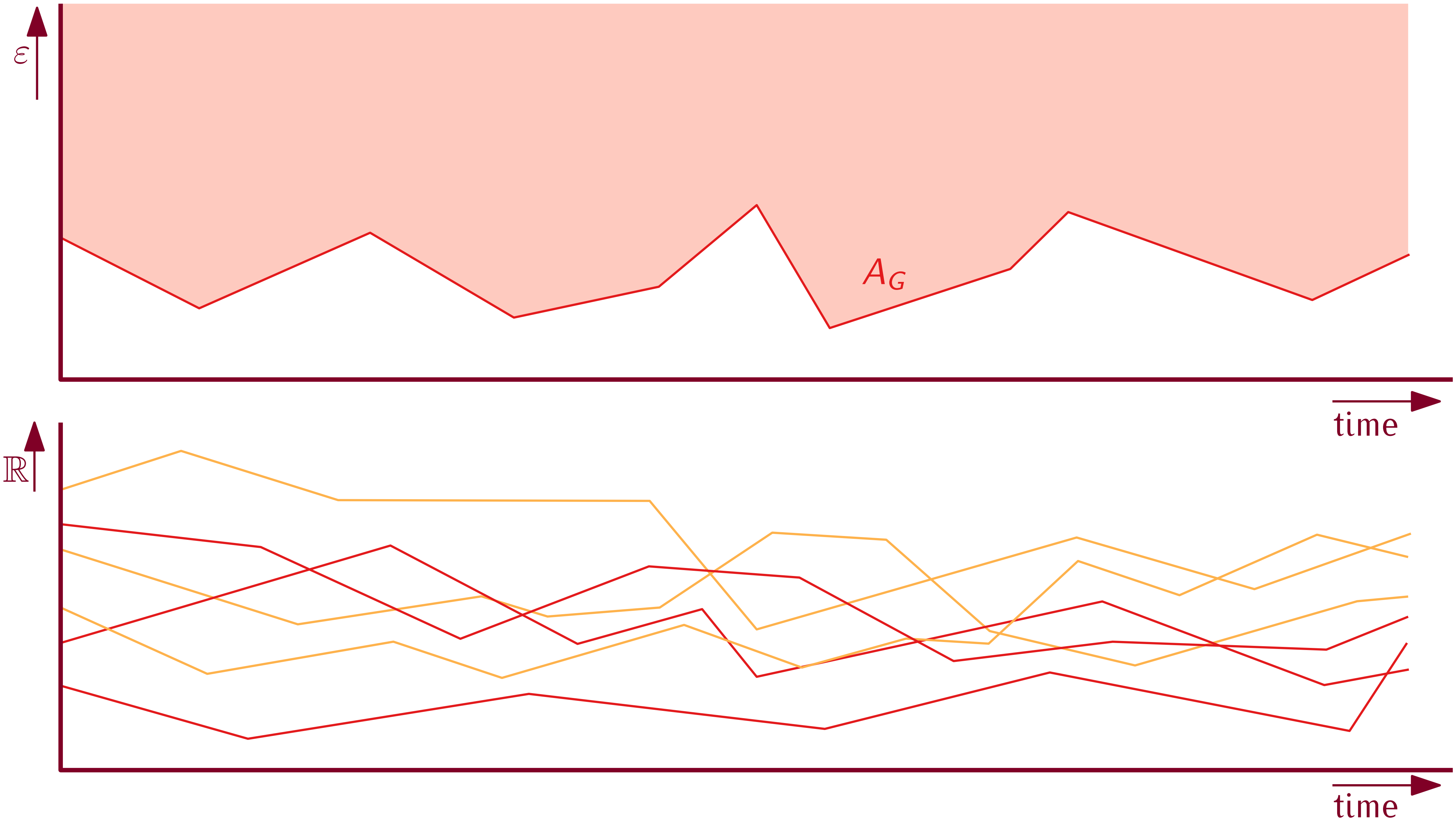

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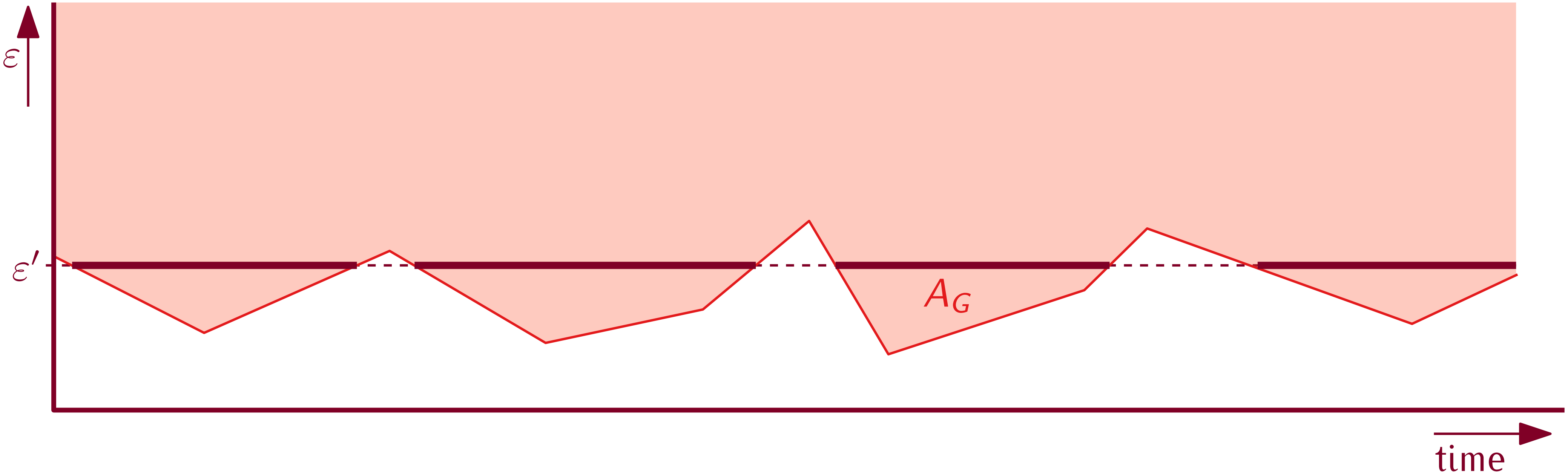

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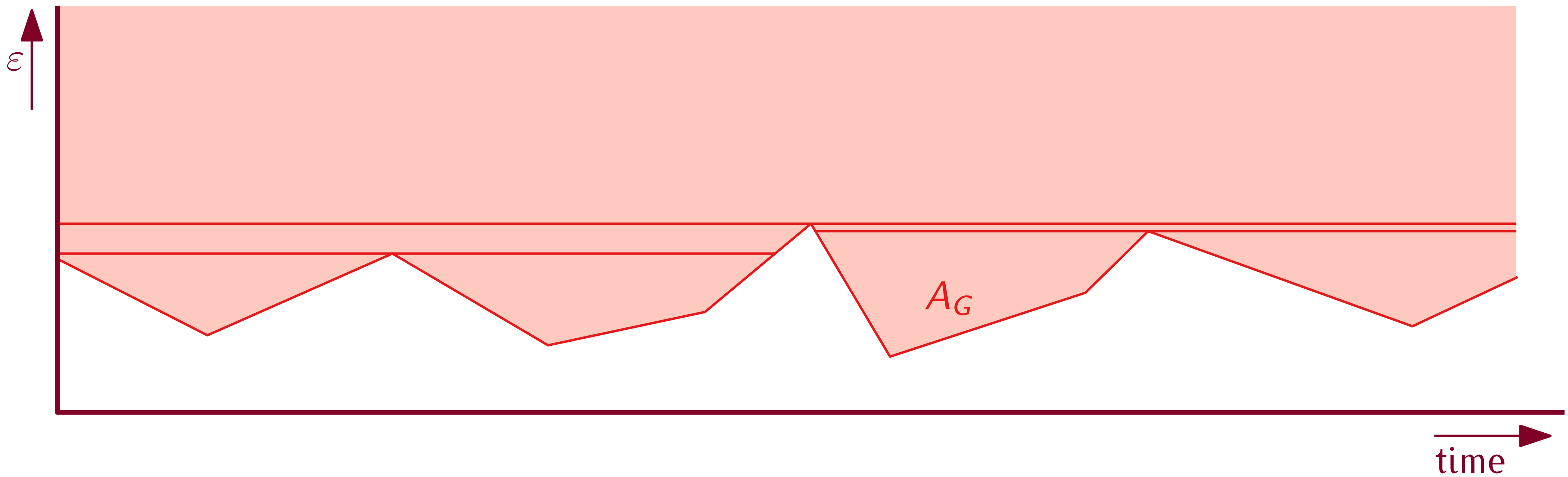

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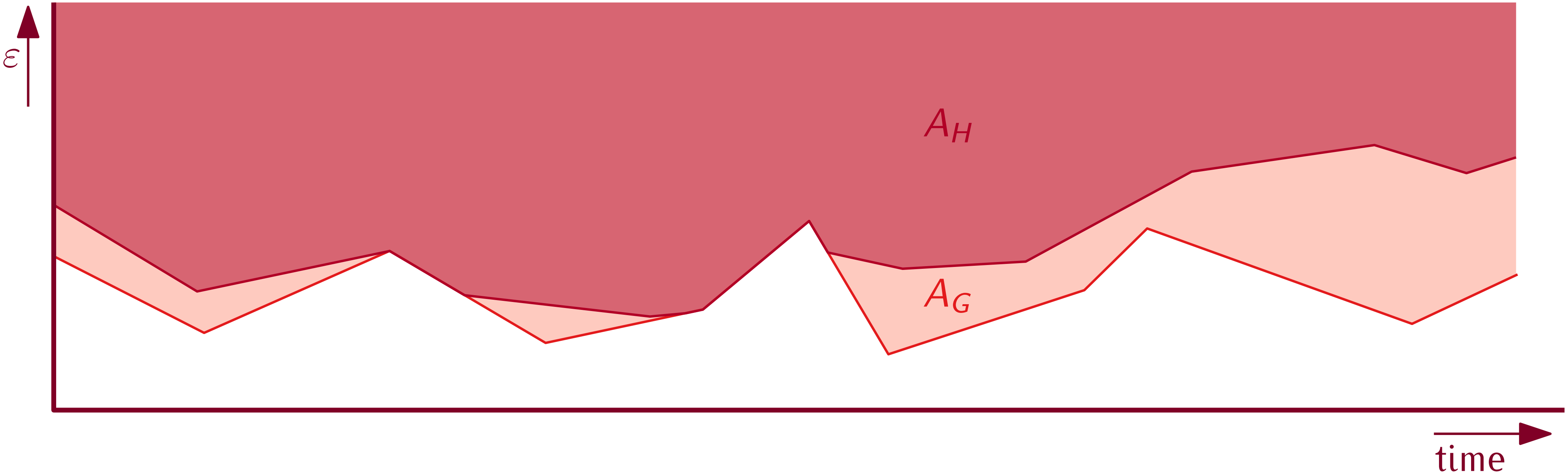

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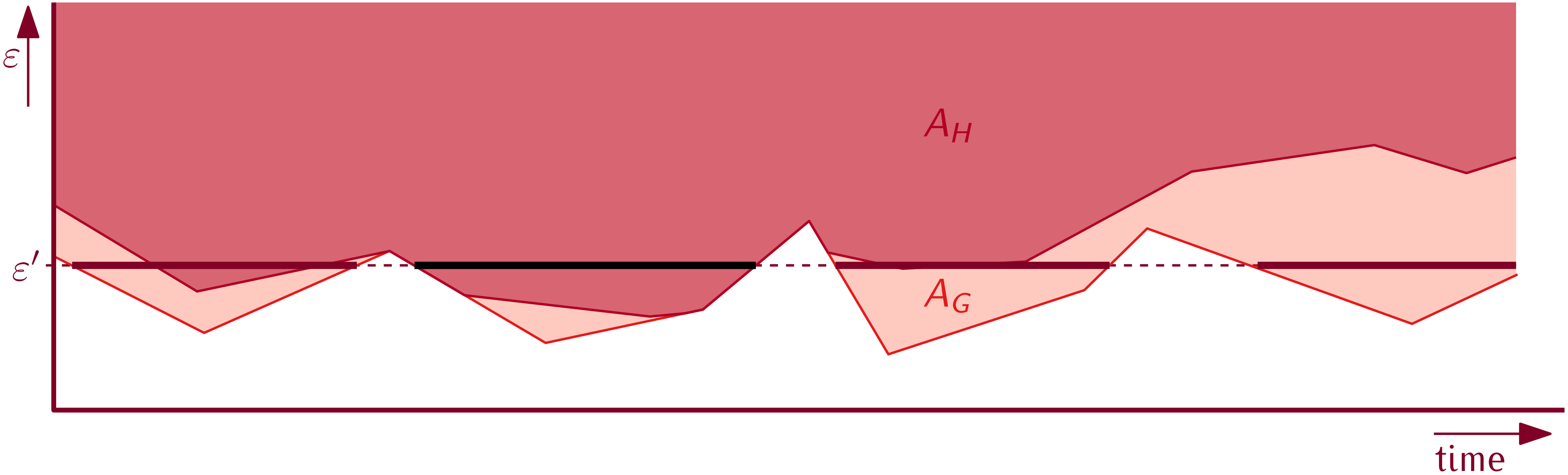

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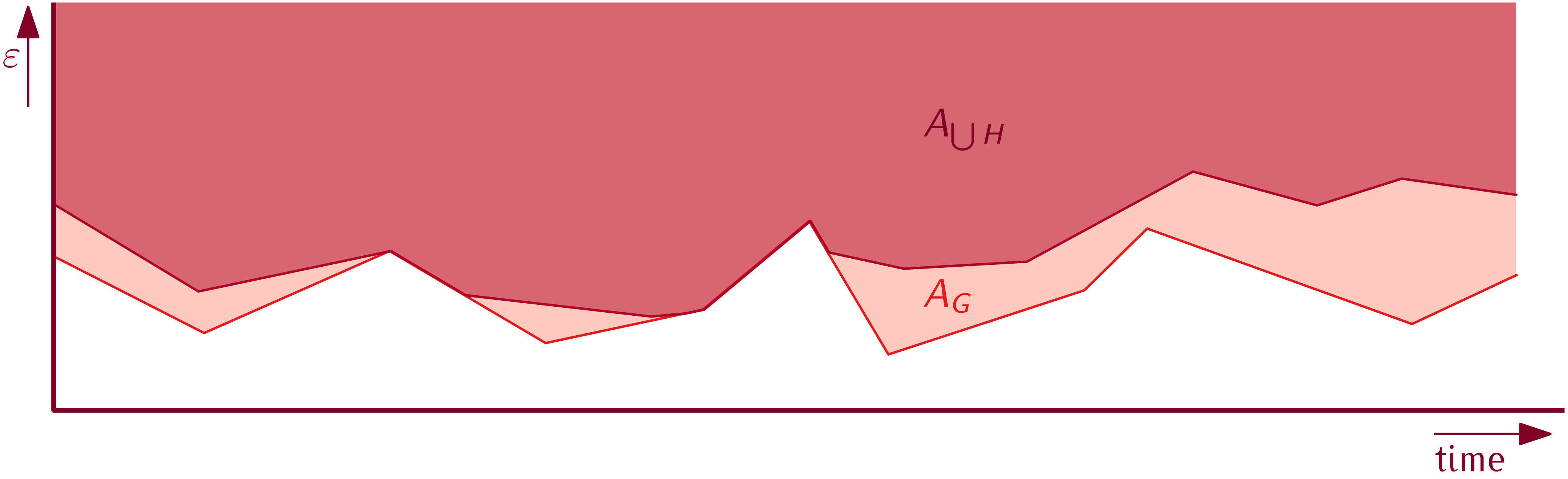

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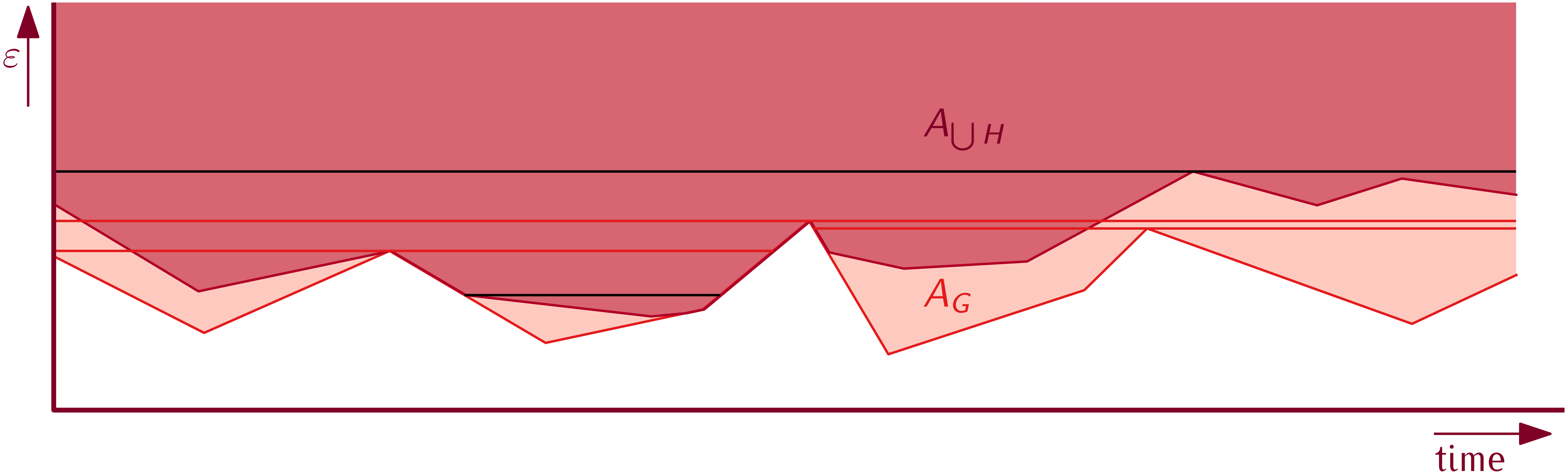

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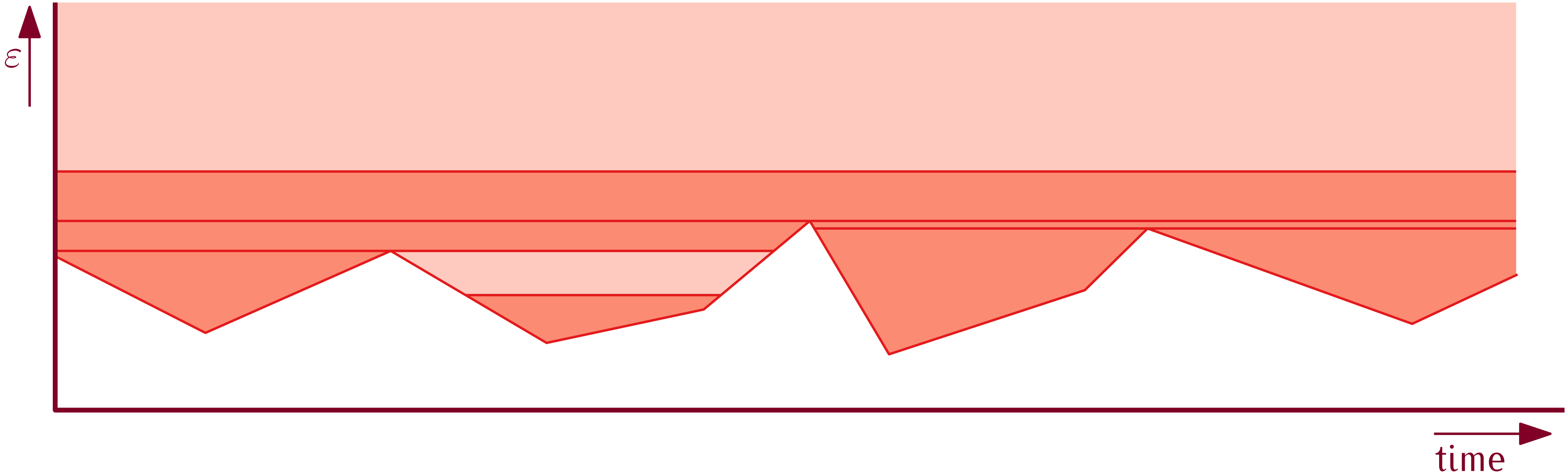

We get a set of regions $\mathcal{P}_G$, each region corresponding to a combinatorially distinct maximal group whose entities are G

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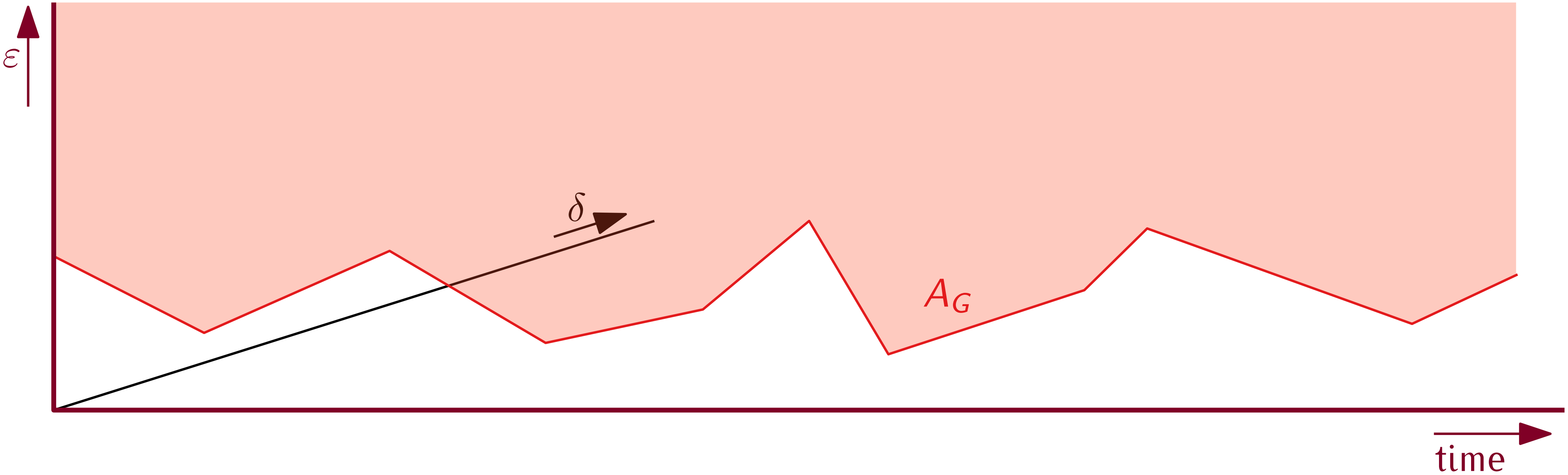

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$\mathbb{R}^1$
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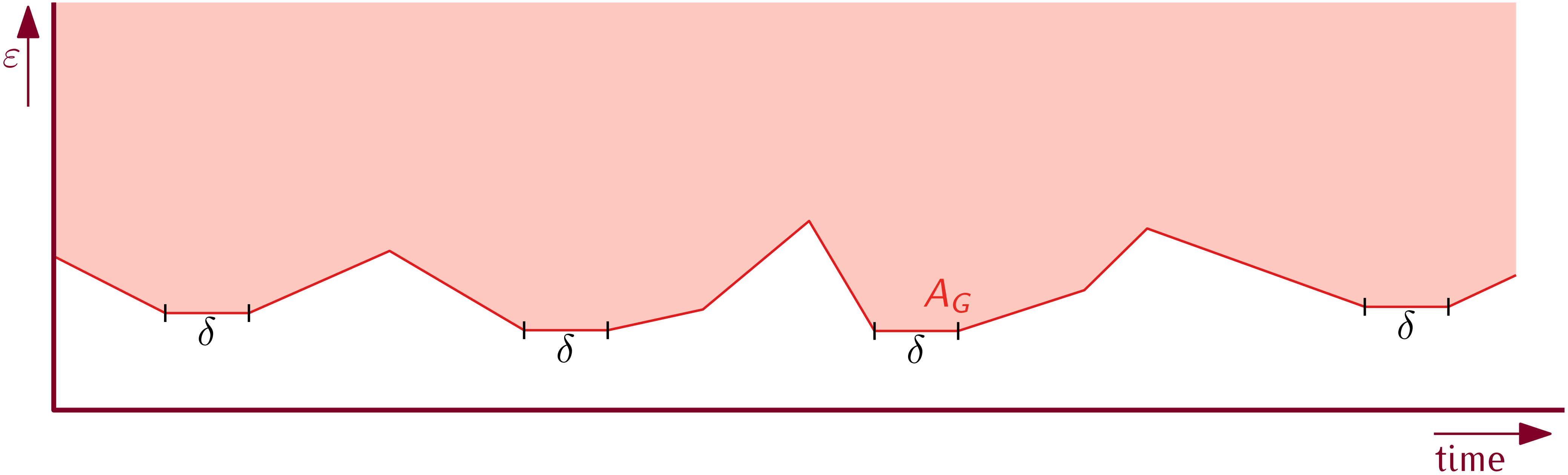

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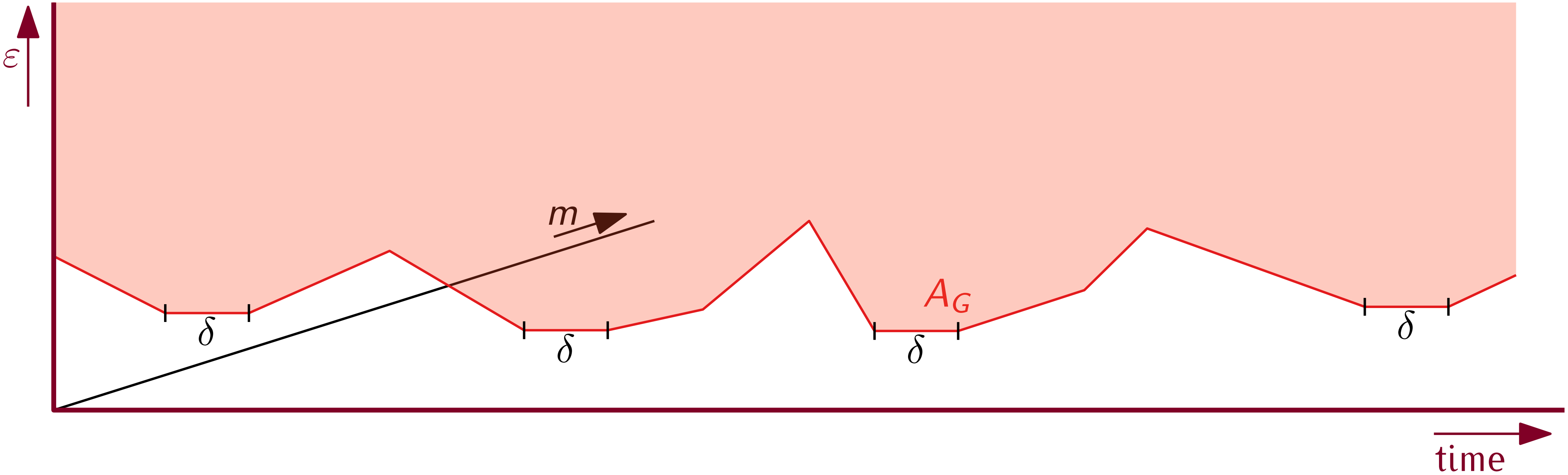

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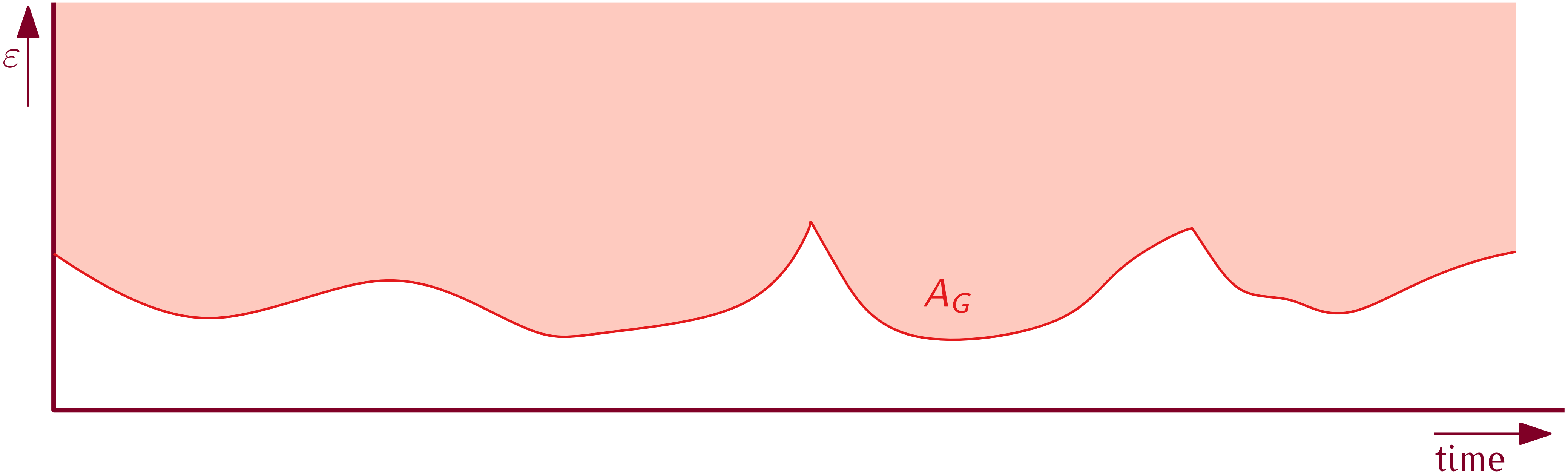

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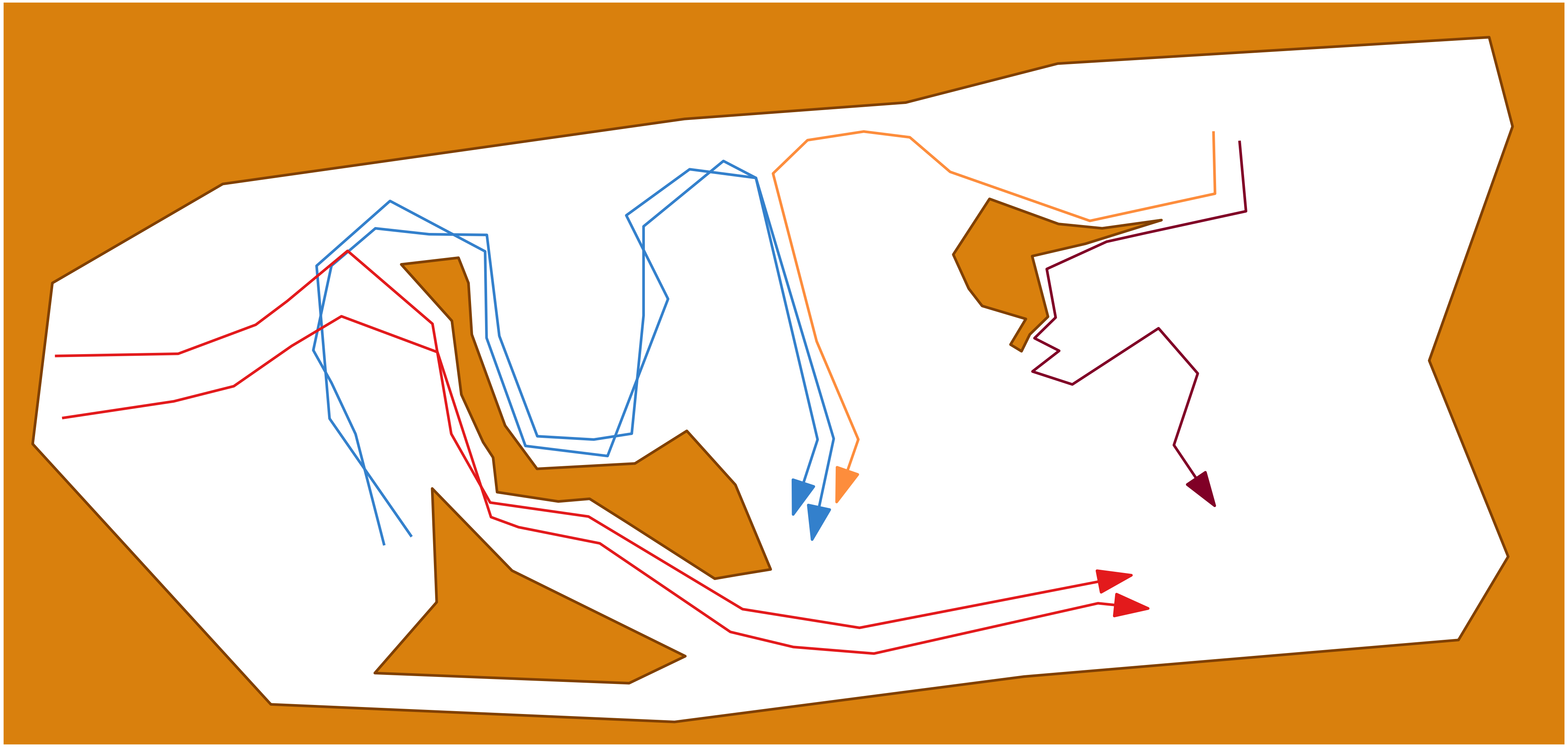

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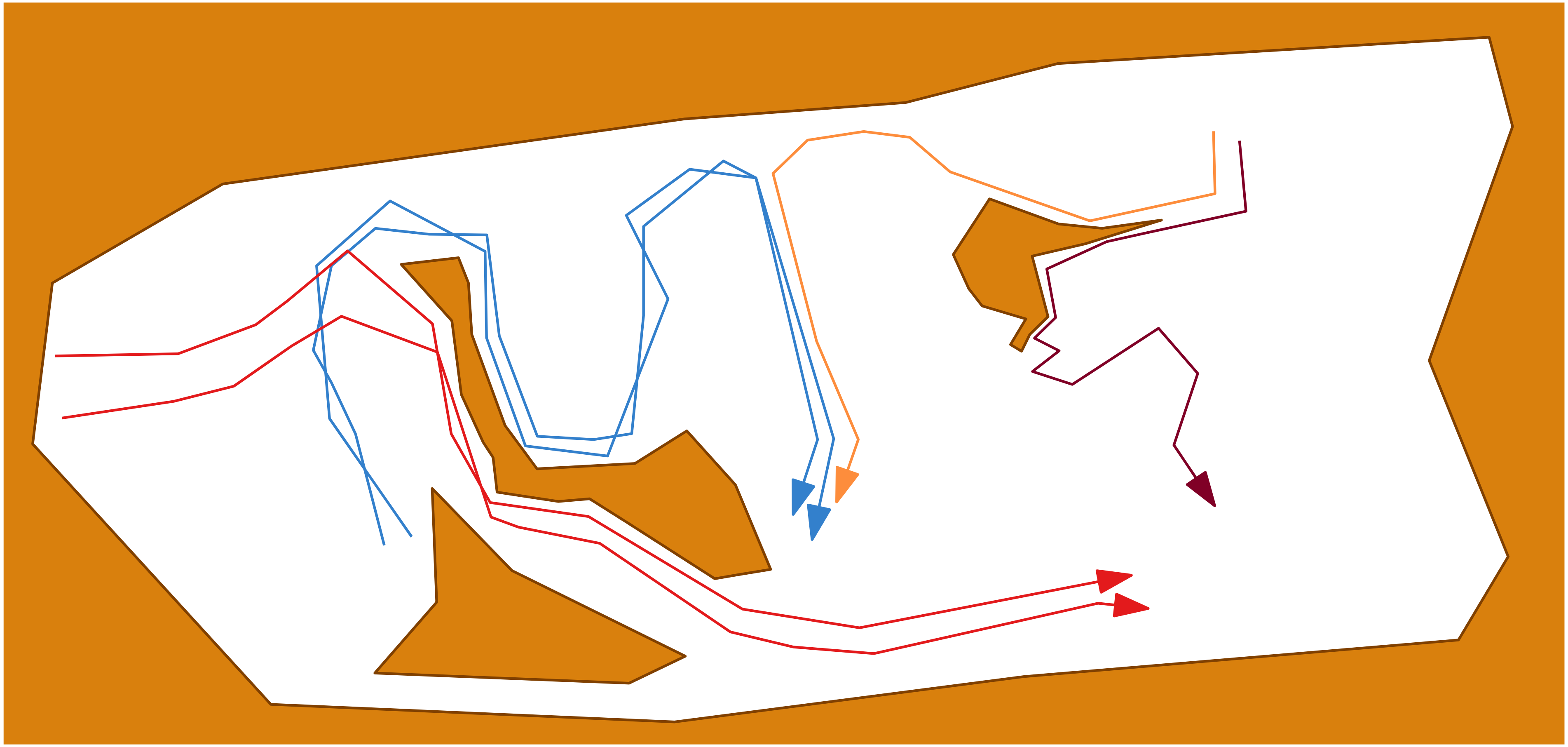

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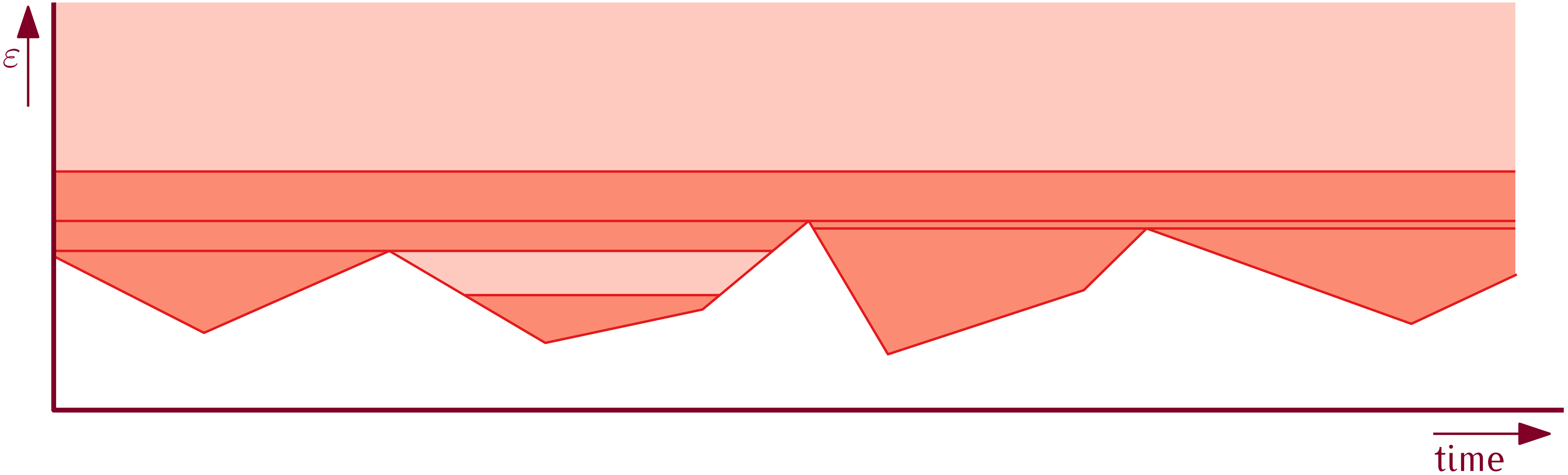

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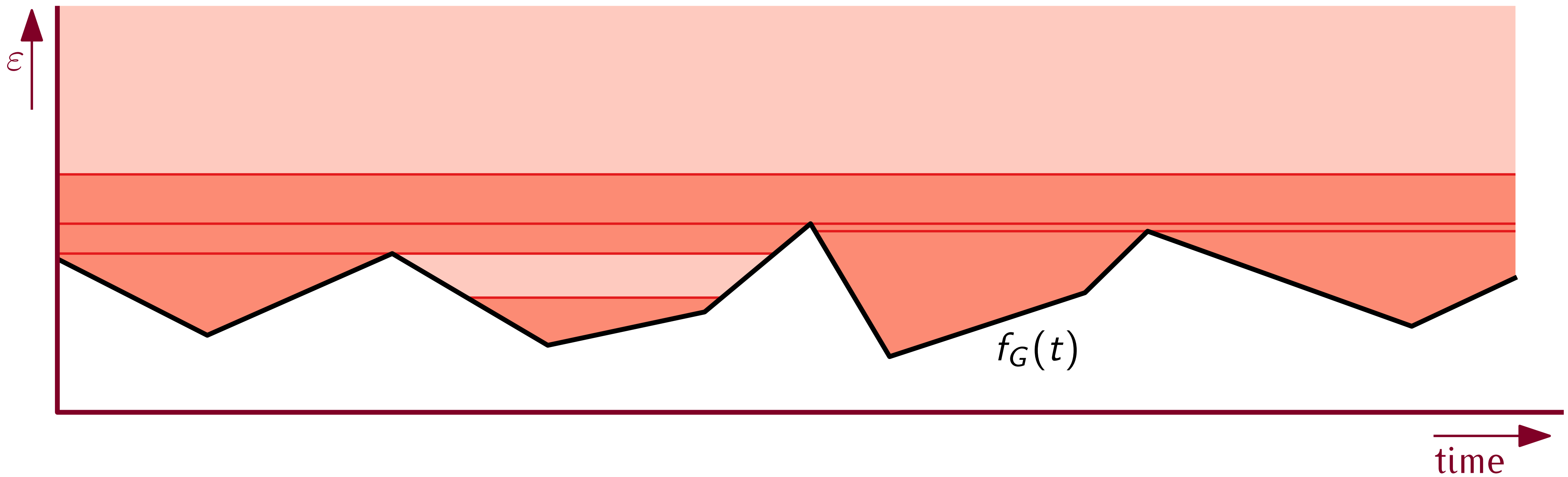

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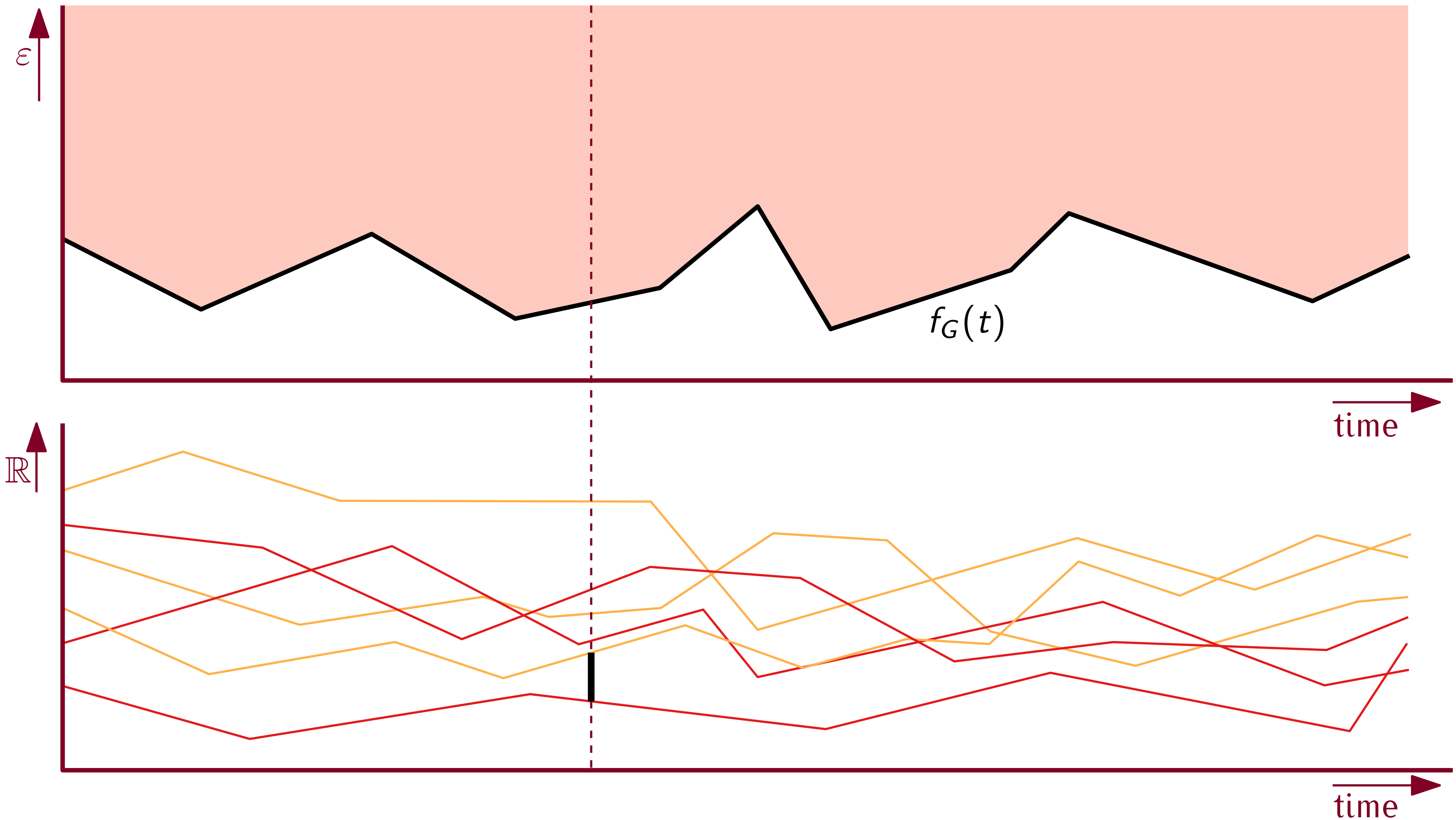

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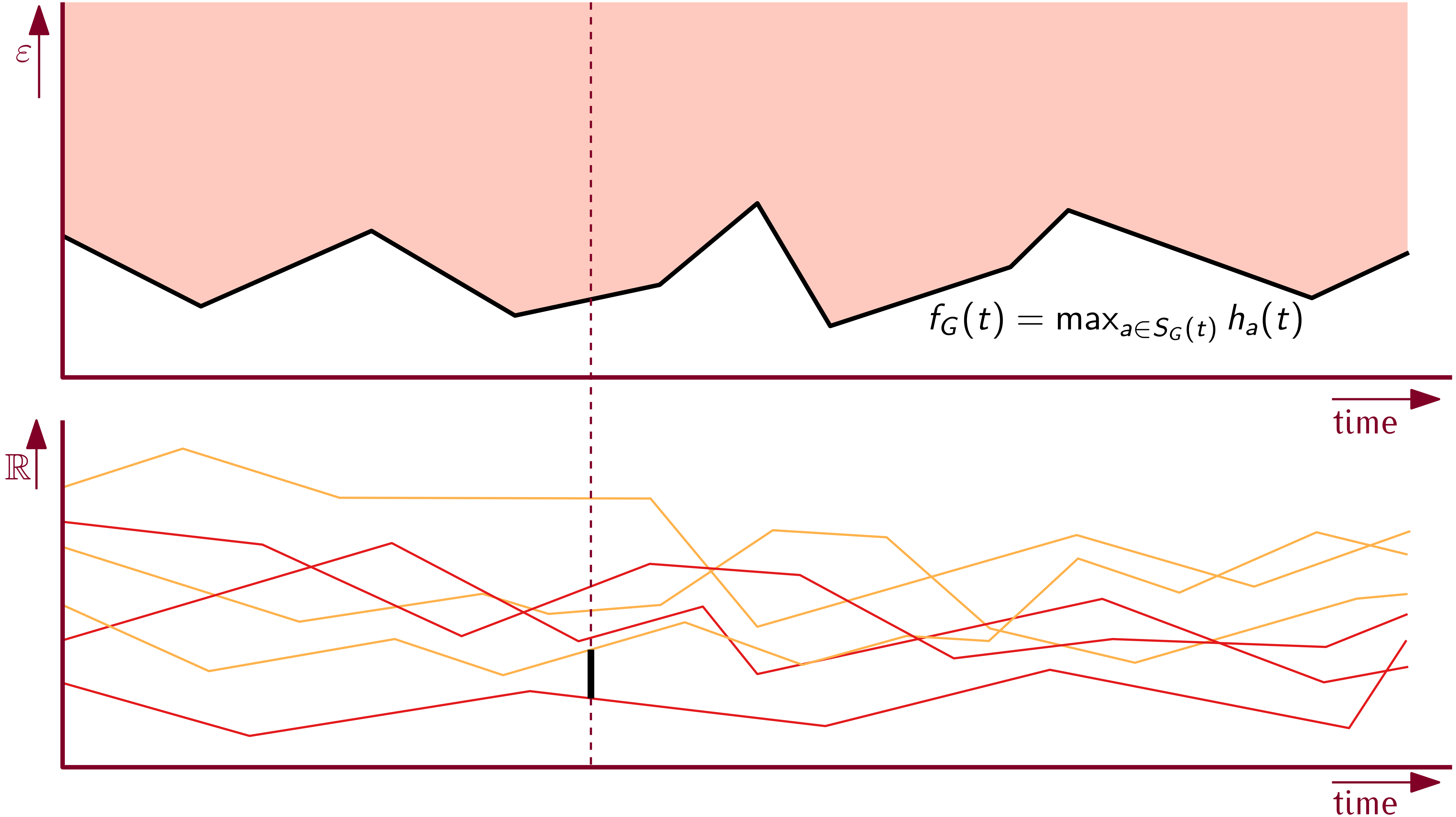

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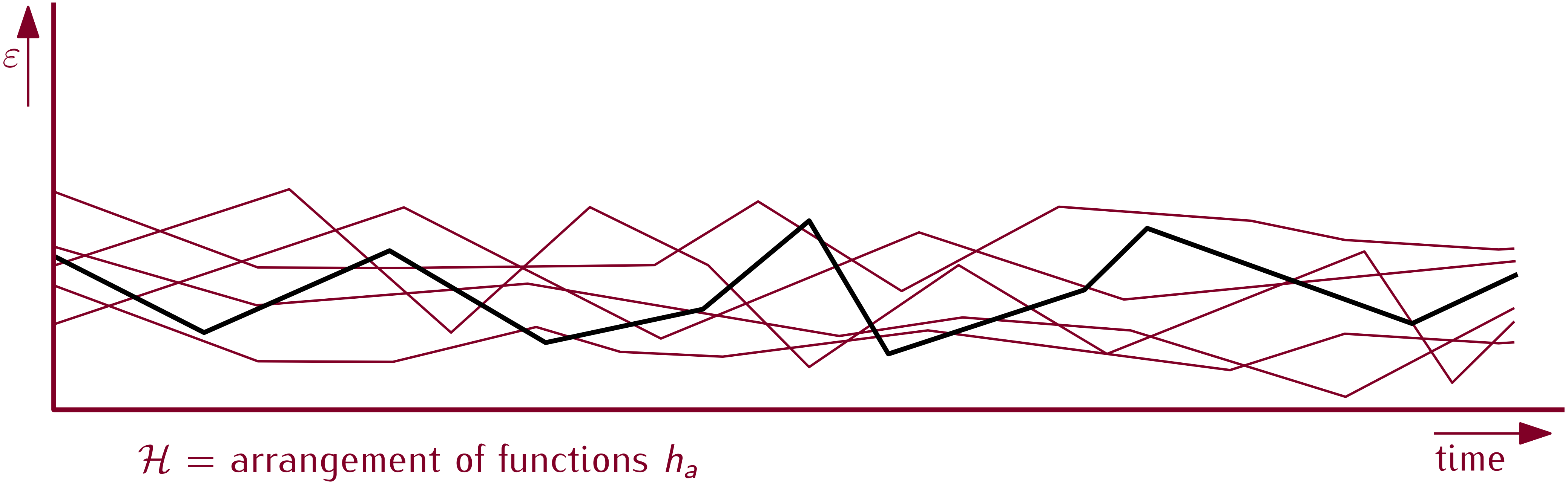

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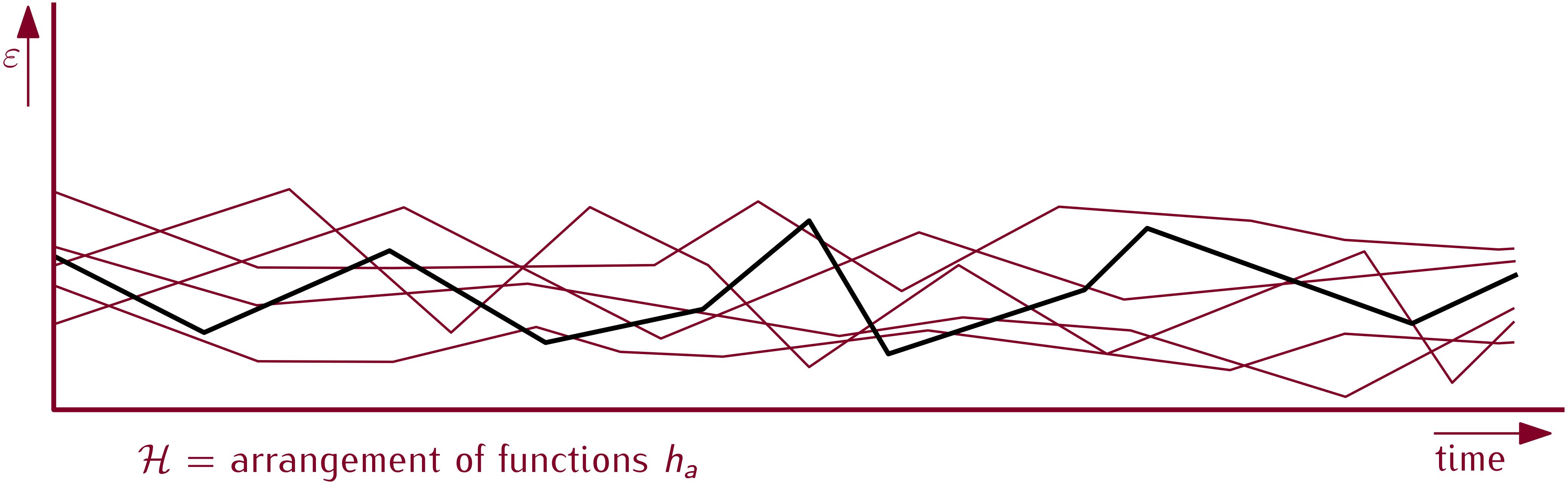

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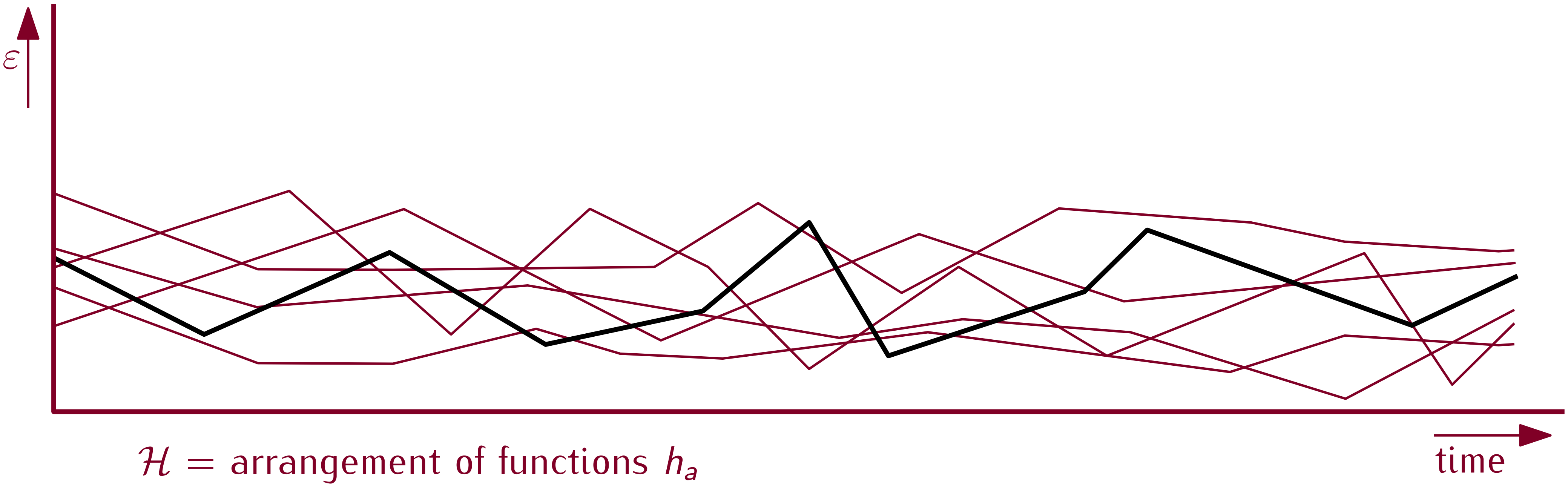

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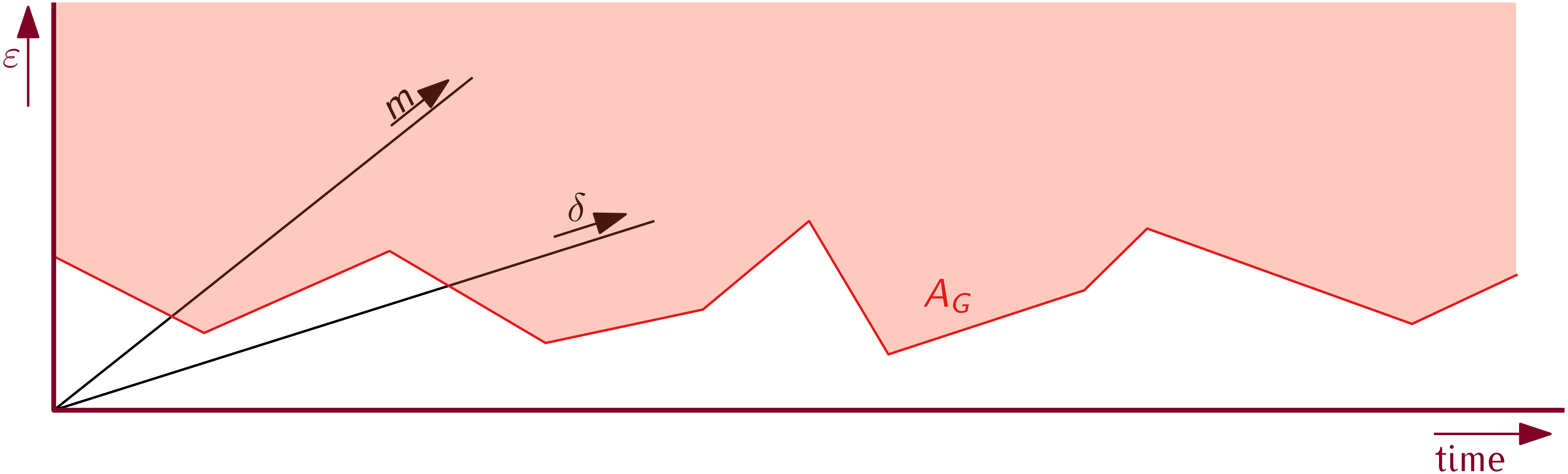

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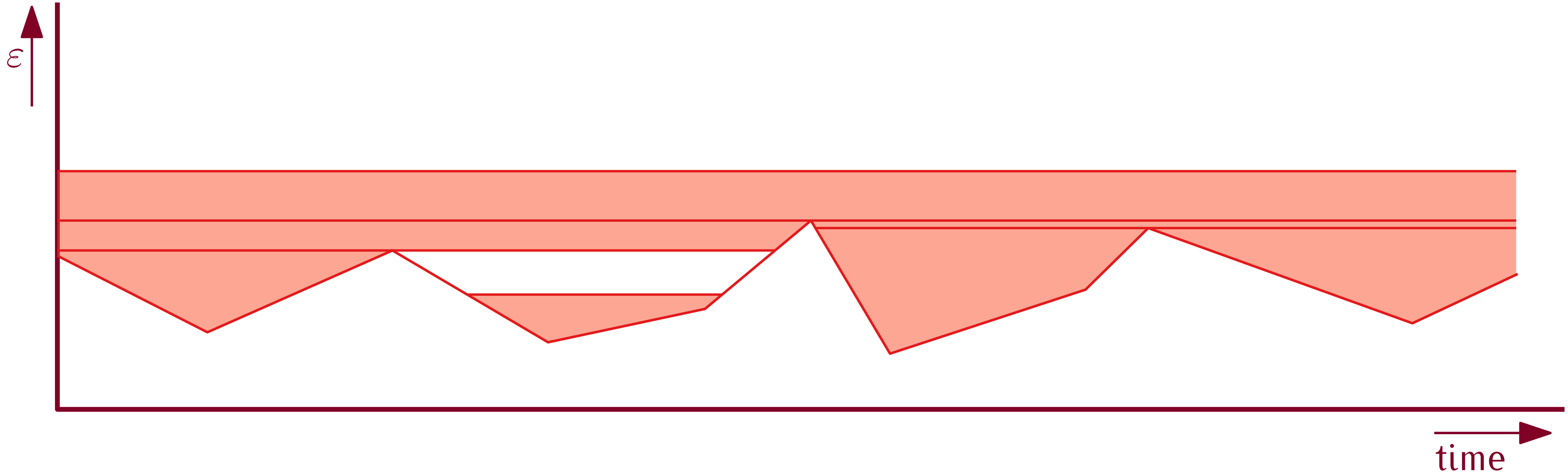

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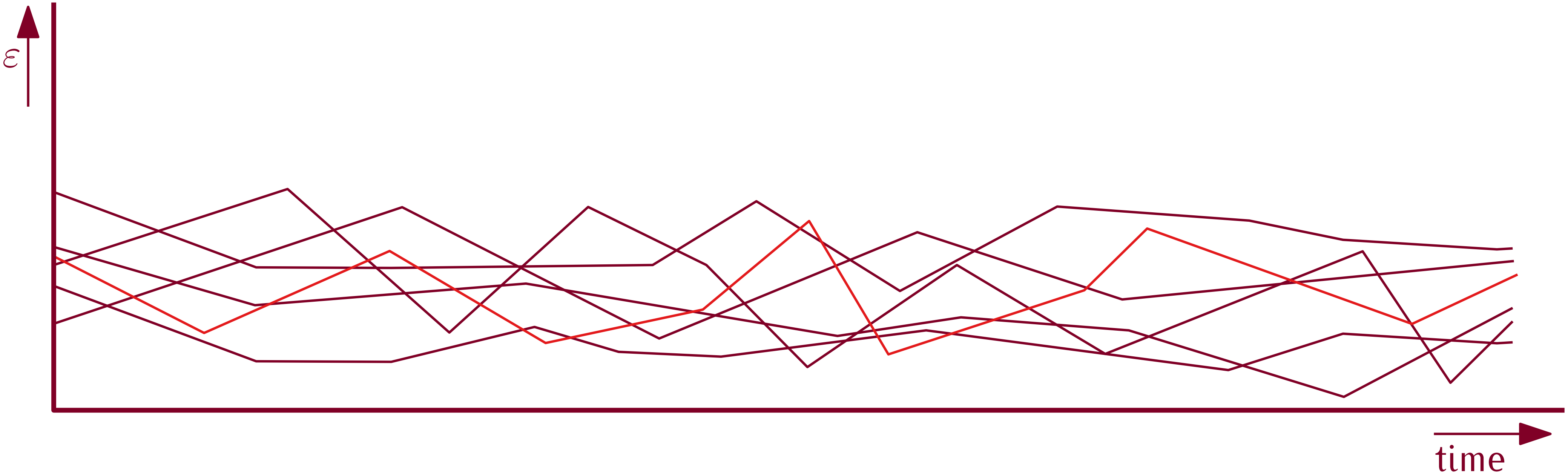

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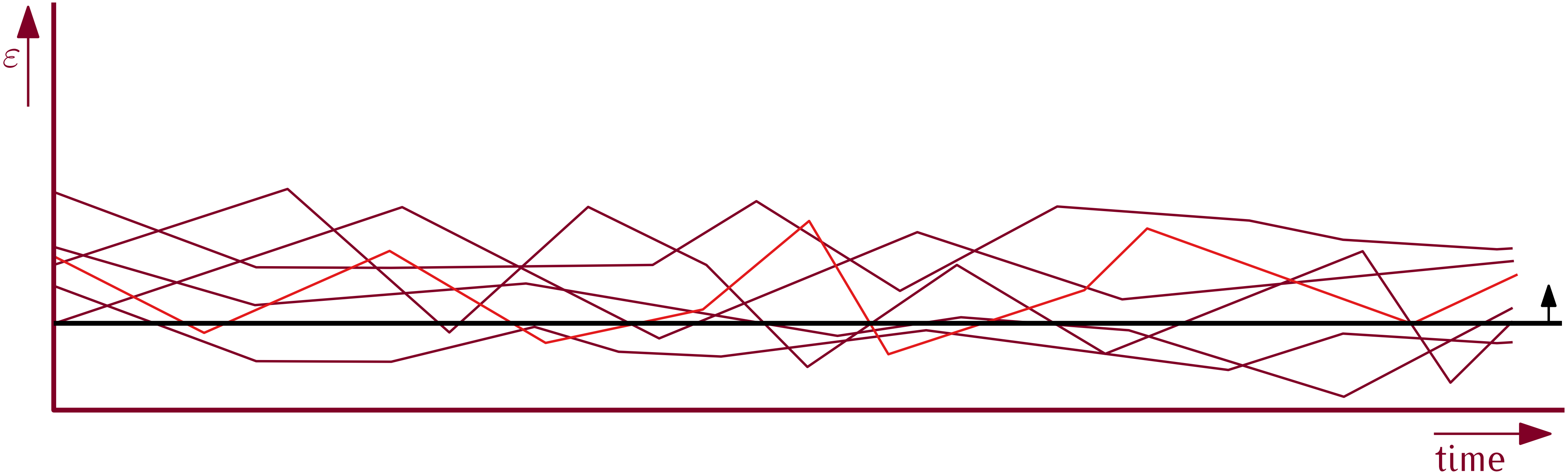

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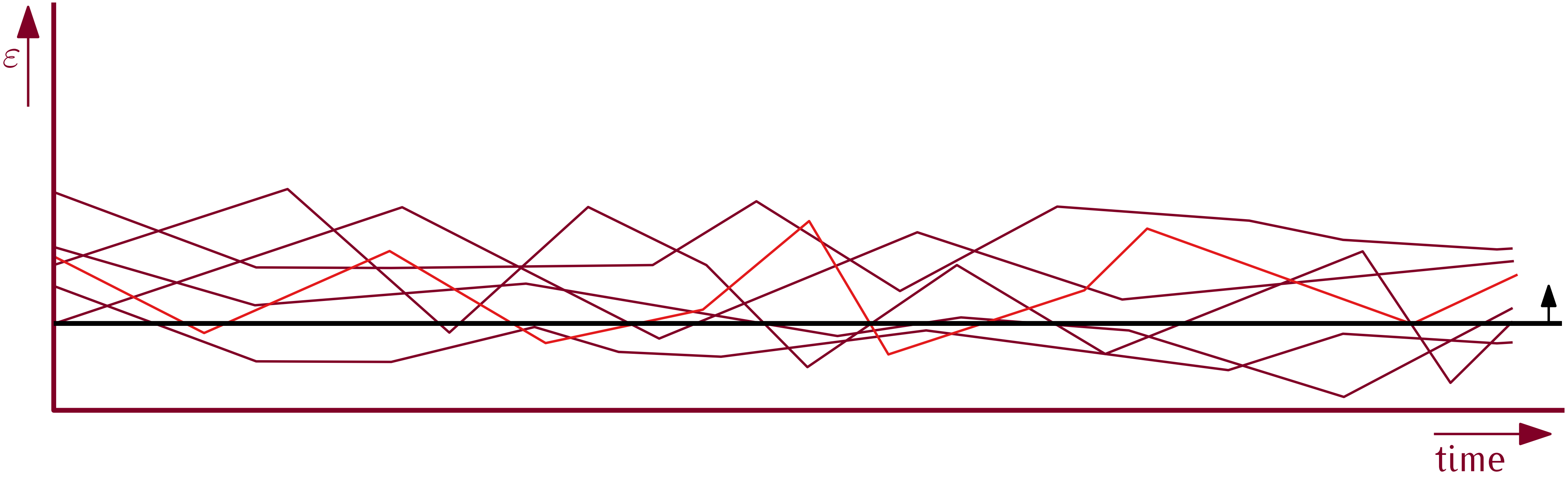

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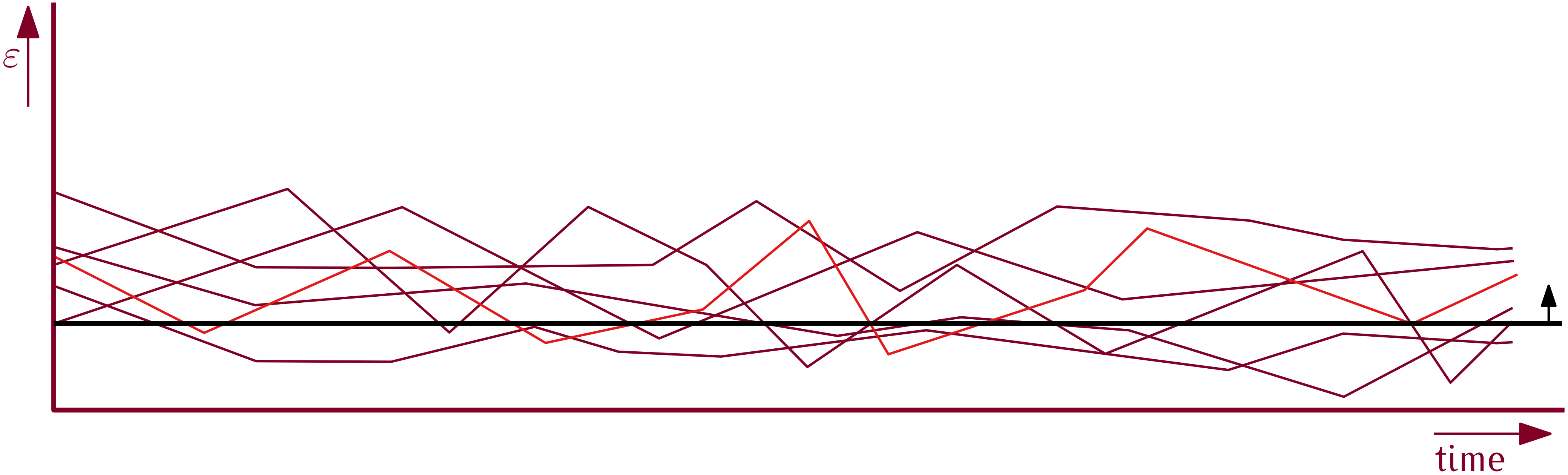

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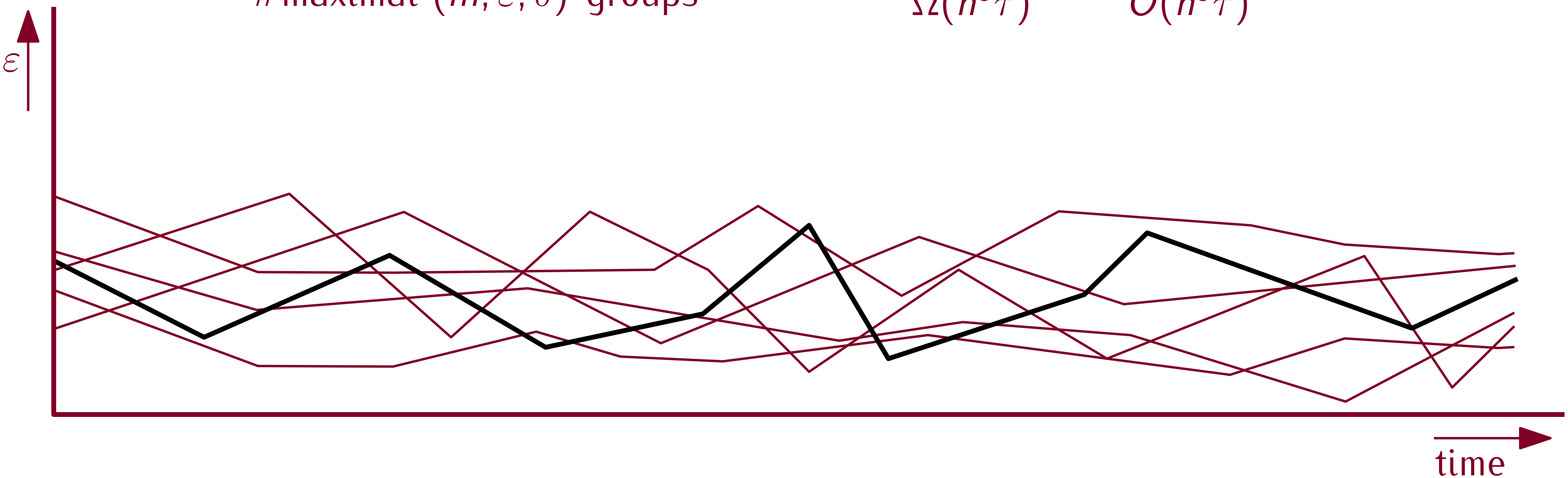

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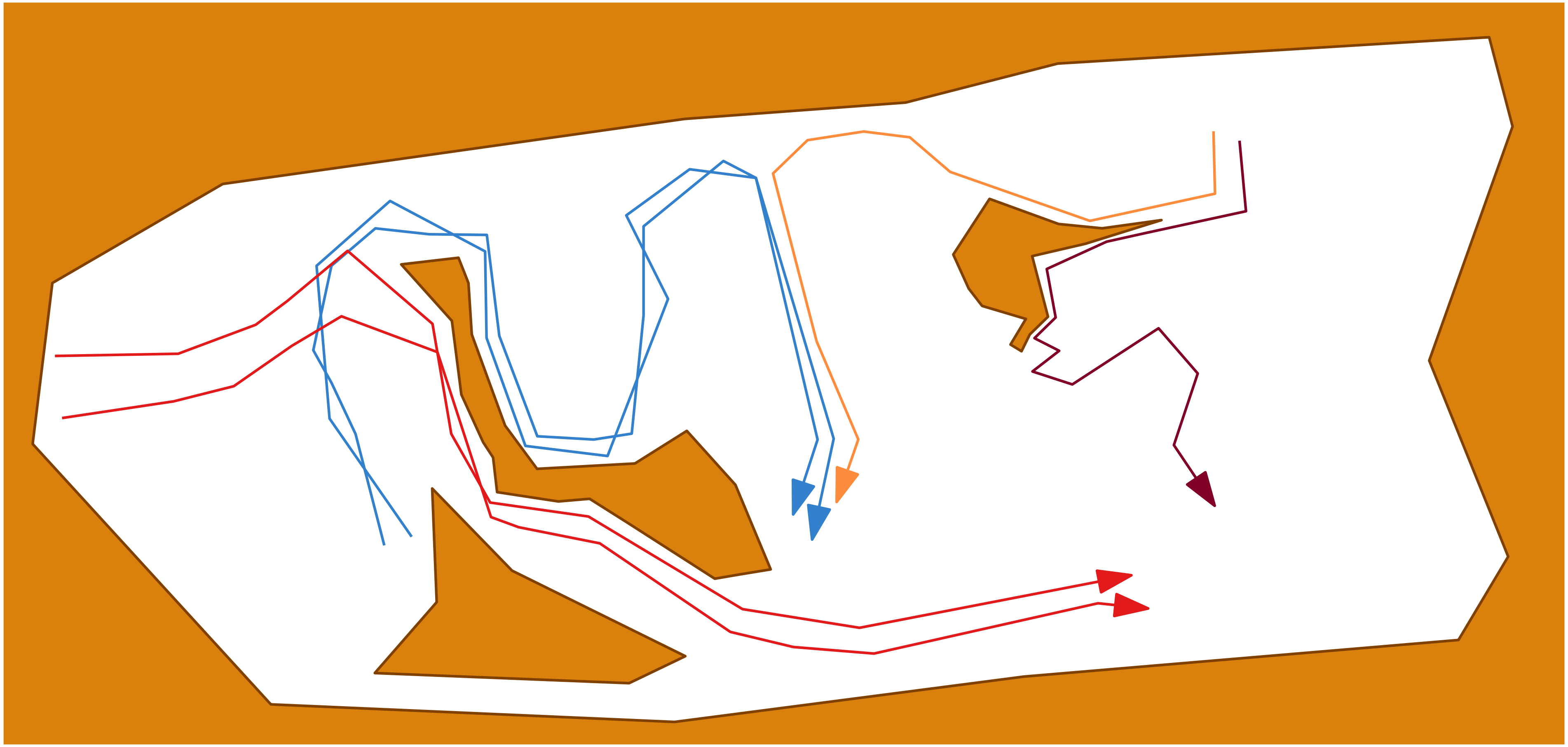

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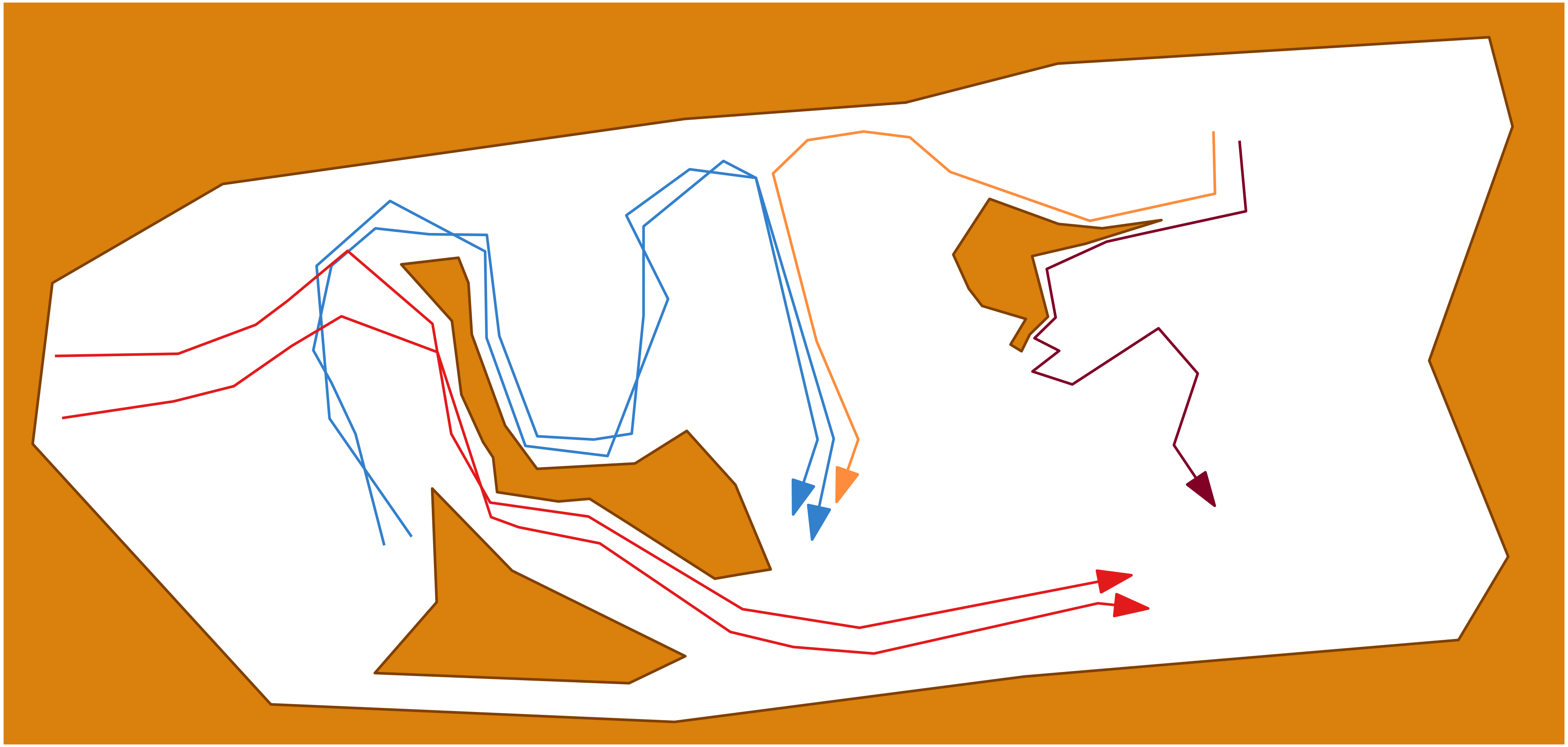

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