Practical Approaches to Partially Guarding a Polyhedral Terrain

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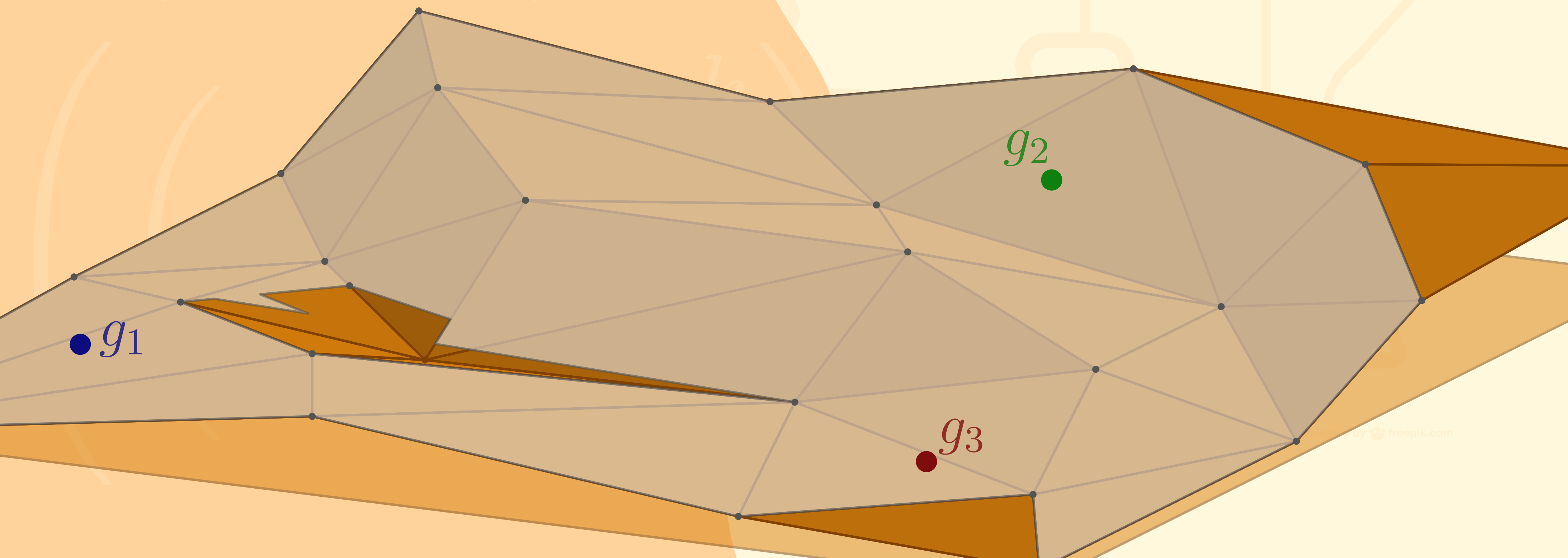

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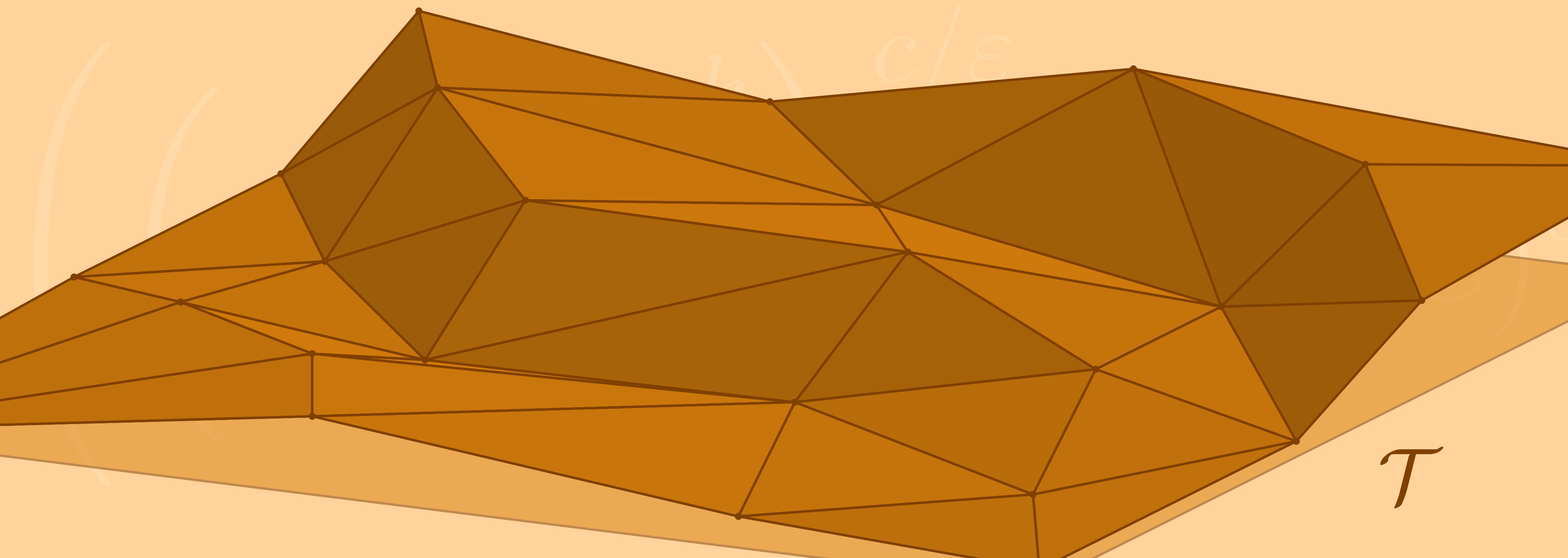

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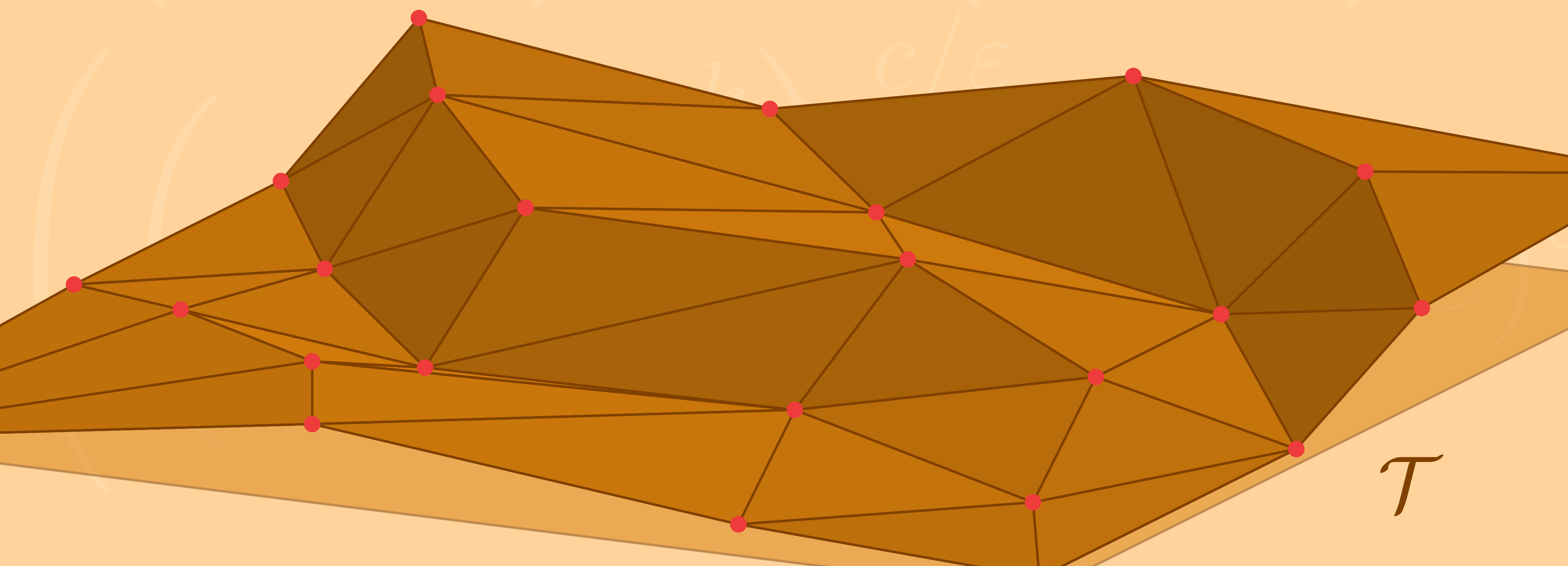

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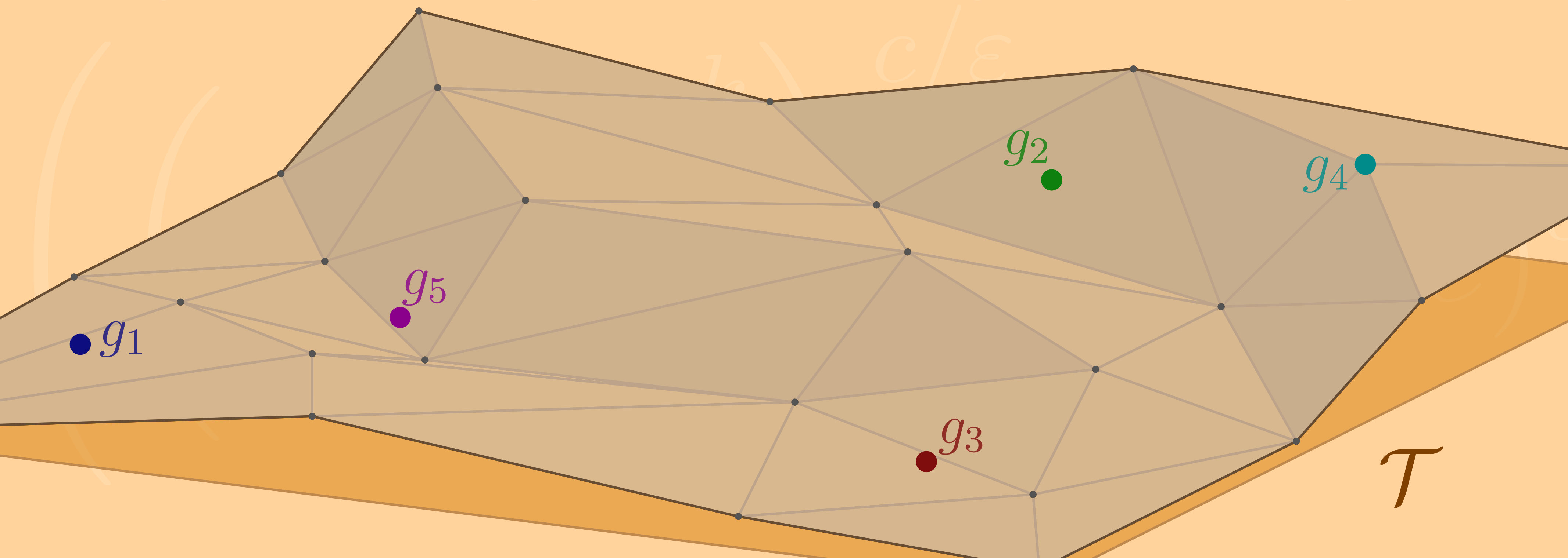
Find a set of guards $\mathcal{G}$ that can together completely see $\mathcal{T}$



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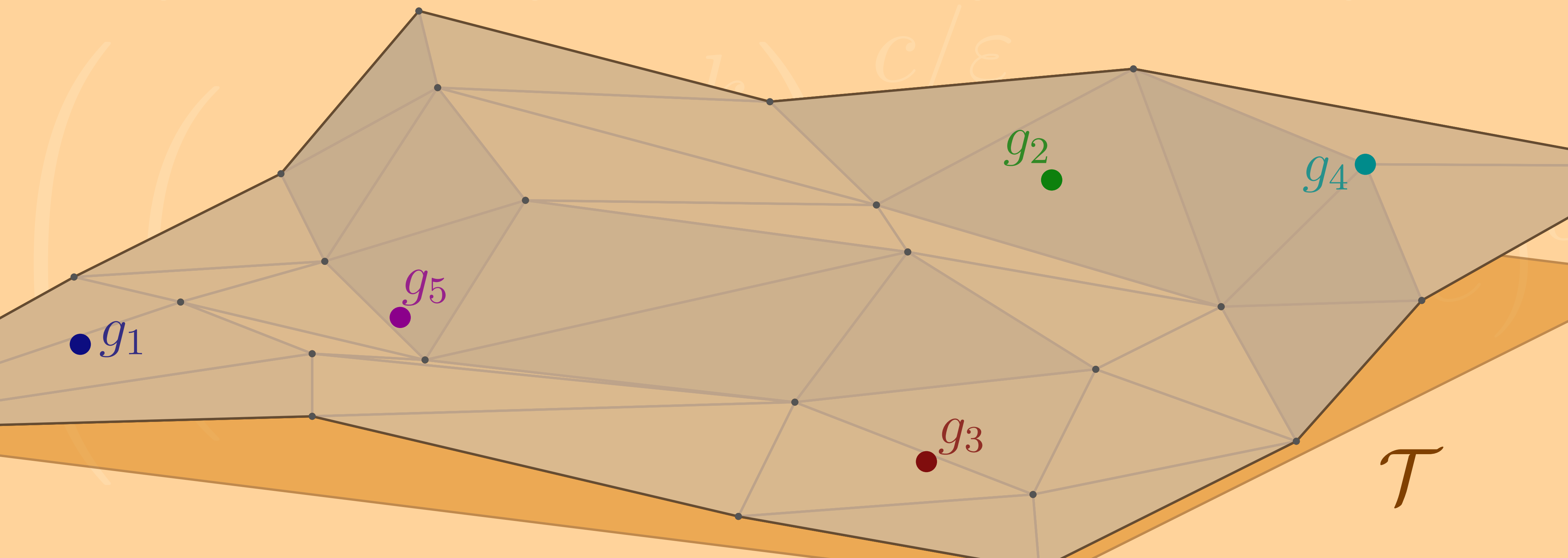

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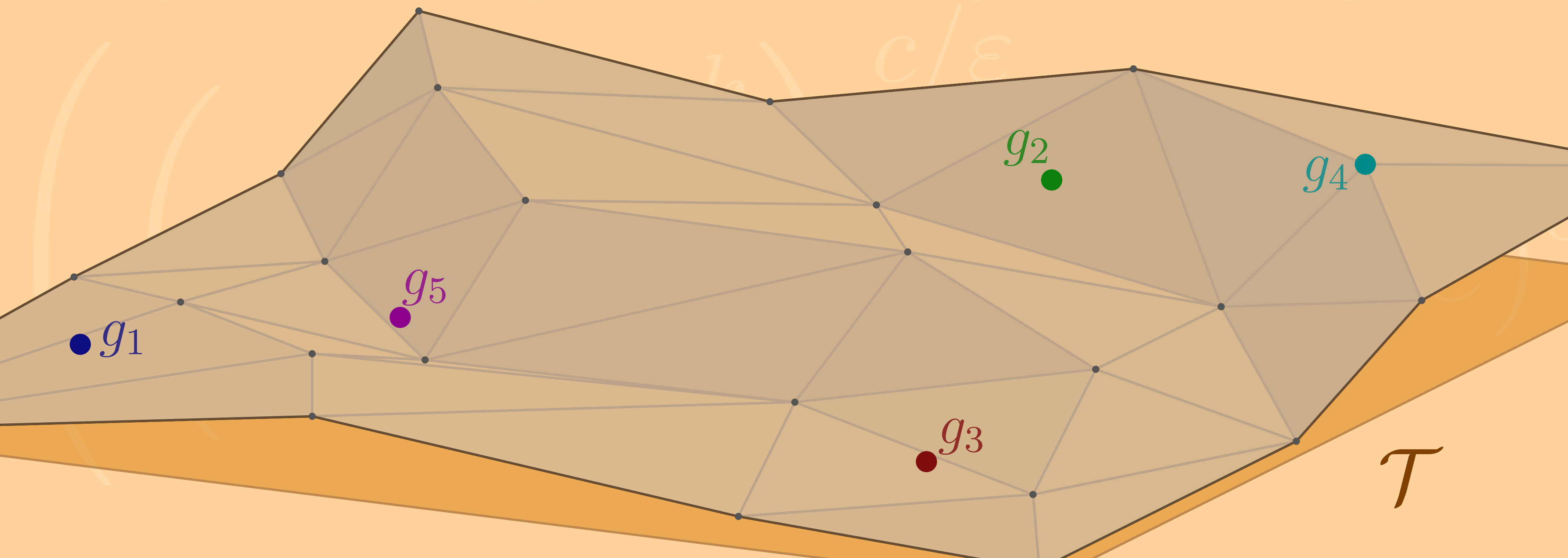
Select $\mathcal{G}$ from a set of potential guards $\mathcal{P}$.
Guards are placed at height h above the terrain.



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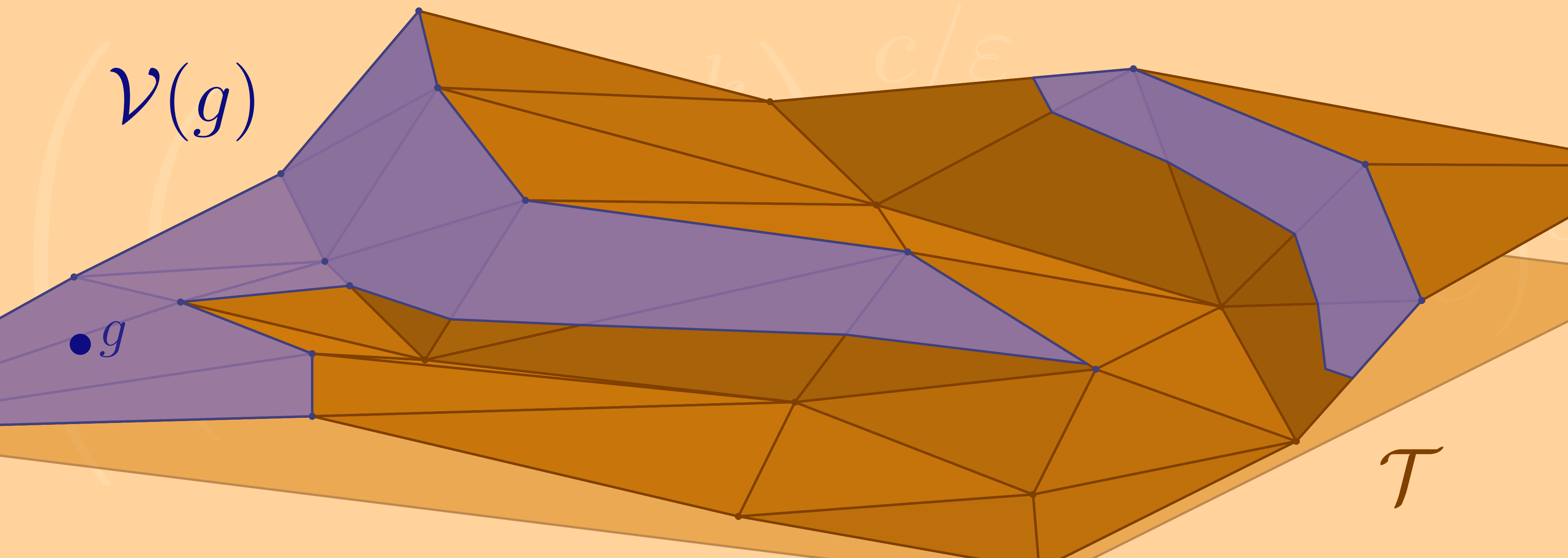
The part of $\mathcal{T}$ visible from a point (guard) g = The viewshed $\mathcal{V}(g)$ of g



Practical Approaches to Partially Guarding a Polyhedral Terrain

Find a smallest set of guards $\mathcal{G}$ that can together completely see $\mathcal{T}$

i.e. such that: $\mathcal{T} = \mathcal{V}(\mathcal{G})$ $\qquad \mathcal{V}(\mathcal{G}) = \bigcup_{g \in \mathcal{G}} \mathcal{V}(g)$

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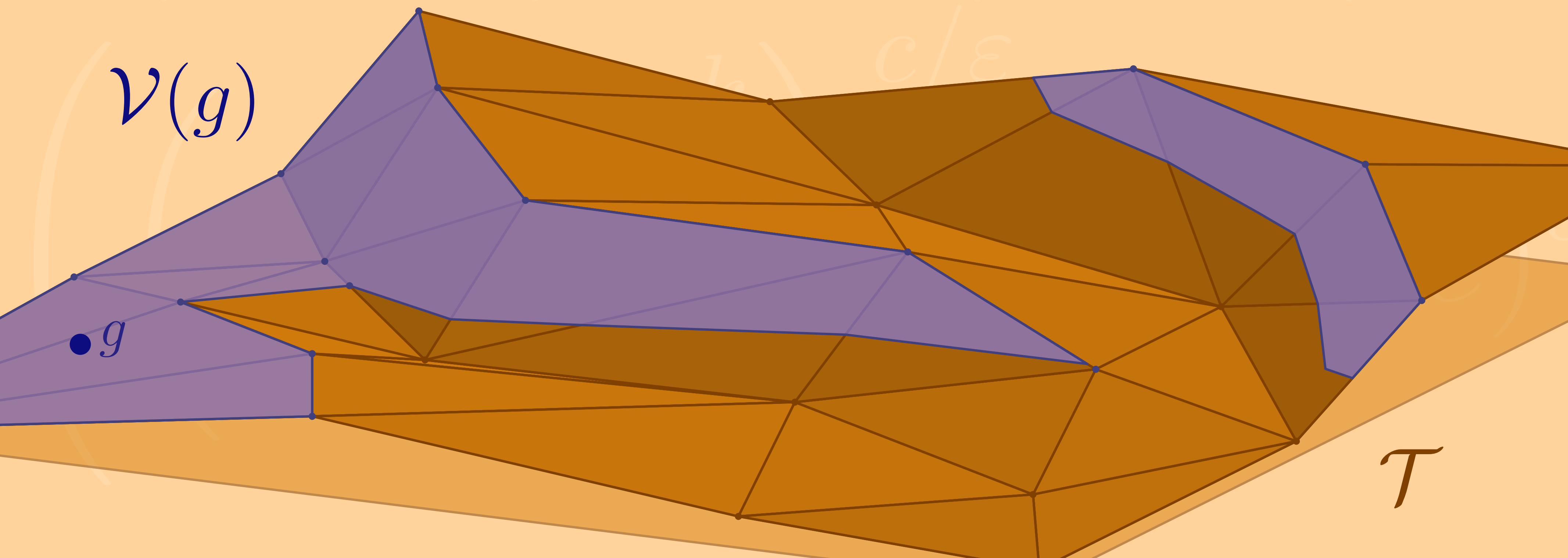
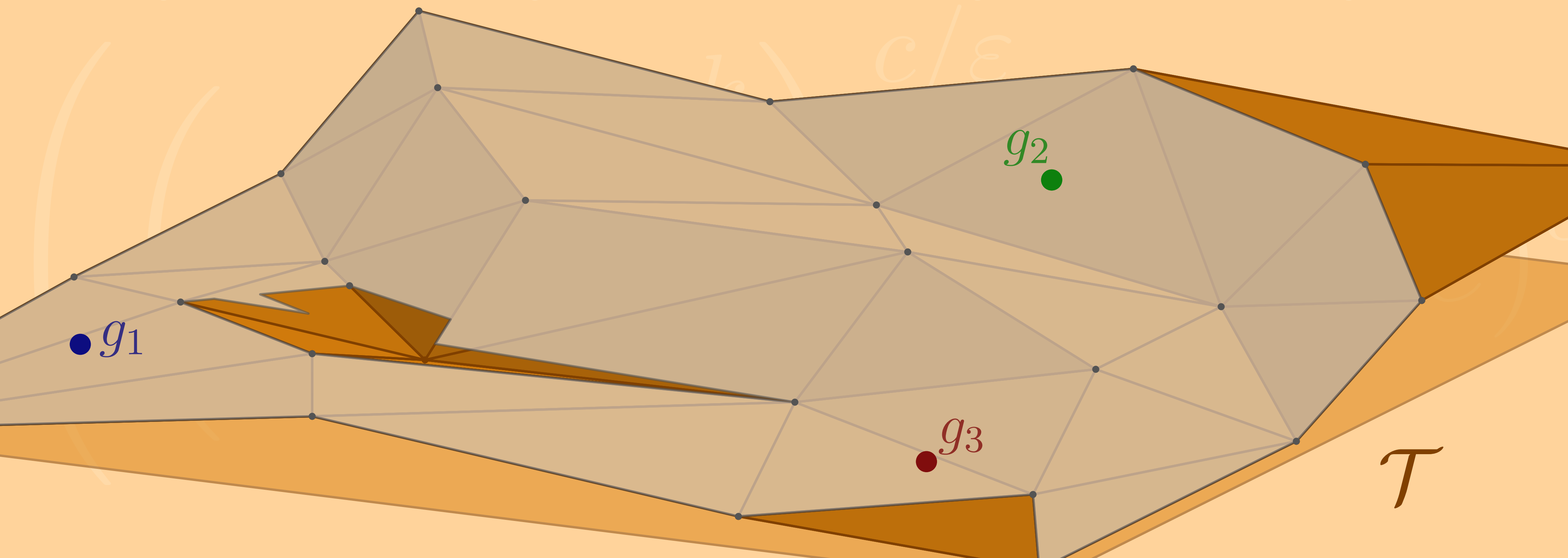


Practical Approaches to Partially Guarding a Polyhedral Terrain

- $\mathcal{T}$ is often imprecise.
- Vegetation, weather, etc influence visibility.

So, it may be sufficient to see a large part of $\mathcal{T}$.



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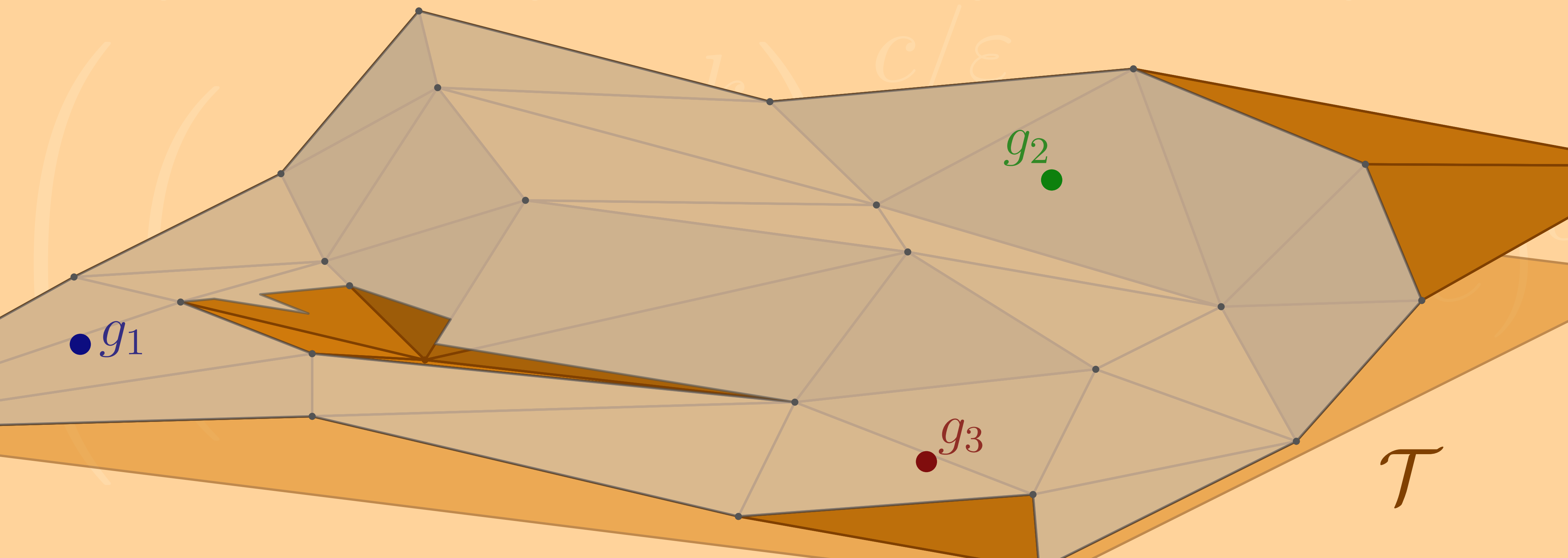
So, it may be sufficient to see a large part of $\mathcal{T}$.

Find a smallest set of guards $\mathcal{G}$ such that

$$[\![\mathcal{V}(\mathcal{G})]\!] \geq (1-\varepsilon)\,[\![\mathcal{T}]\!], \qquad \text{for a given } \varepsilon$$

$[\![\mathcal{T}']\!] =$ the size of $\mathcal{T}'$



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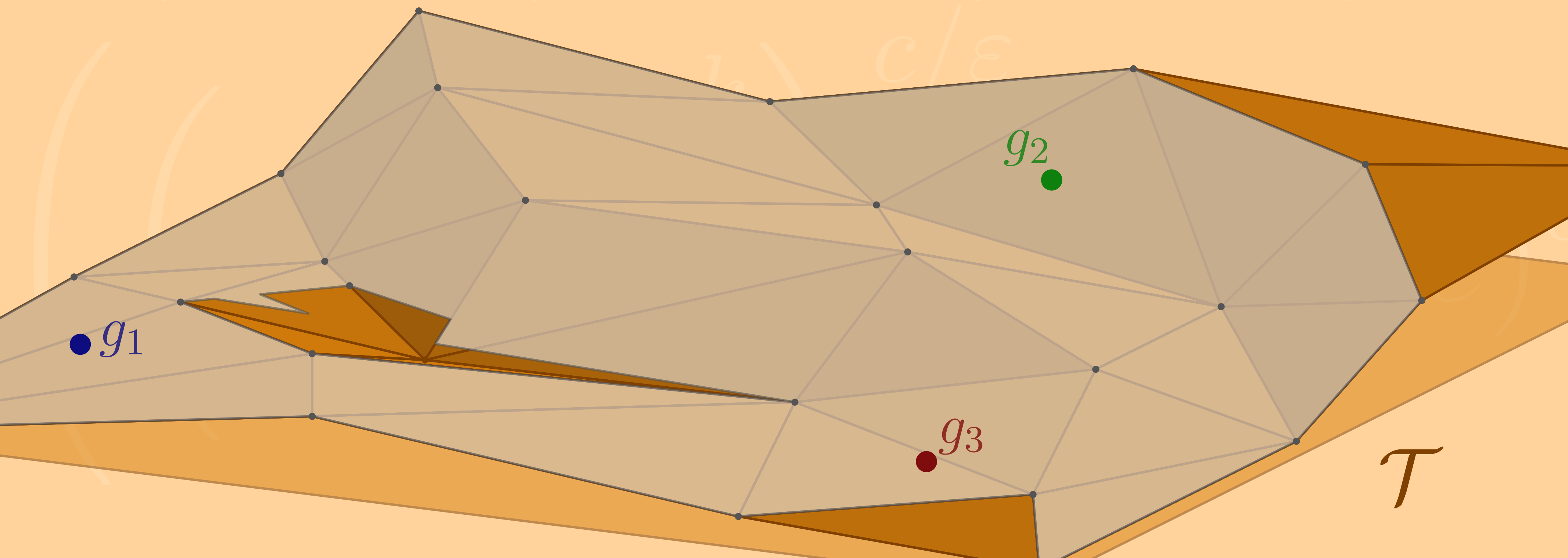
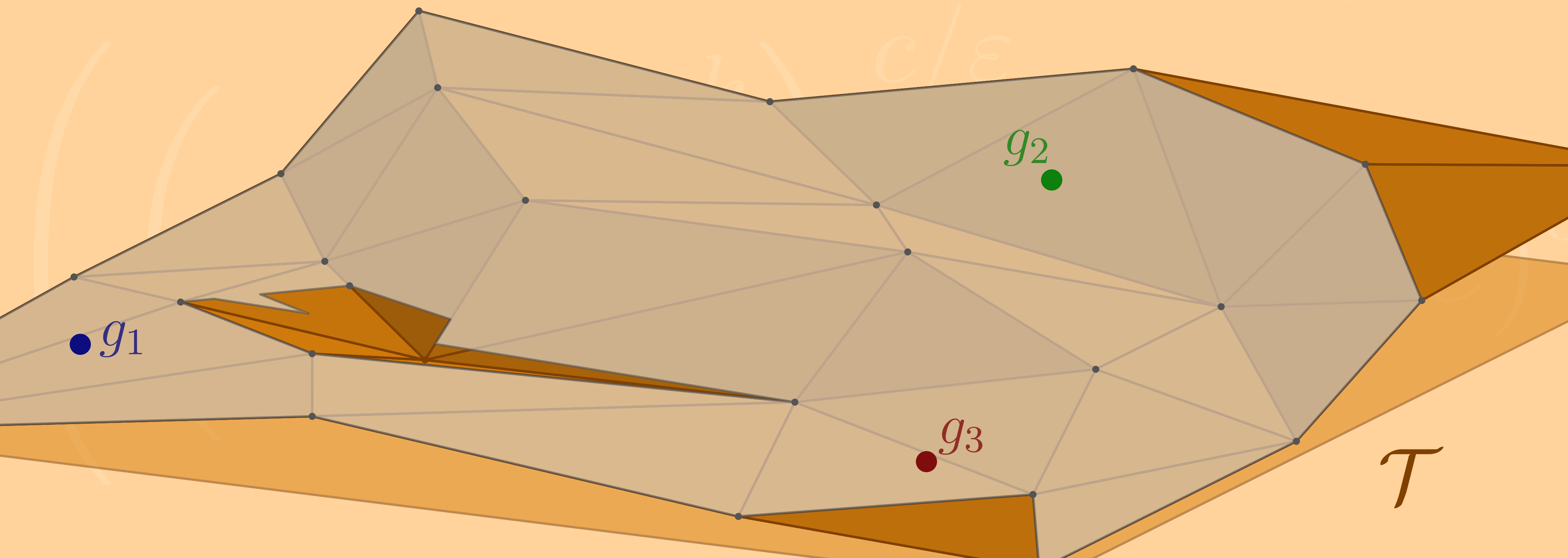
- Terrain Guarding is NP-hard [Cole & Sharir, J. Sym. Comp '89]



Practical Approaches to Partially Guarding a Polyhedral Terrain

- $\mathcal{T}$ is often imprecise.
- Vegetation, weather, etc influence visibility.

- Terrain Guarding is NP-hard [Cole & Sharir, J. Sym. Comp '89]
- NP-Hard to approximate #guards within a factor $O(\log n)$ [Eidenbenz *et al.*, Algoritmica '00]



Results

NP-Hard to approximate the amount of terrain covered within a factor $O(\log n)$

Quality guarantees for a simple greedy algorithm

Observations to reduce the number of potential guards in $\mathcal{P}$

Experiments on real terrains showing:

the #guards used for an $(1-\varepsilon)$-cover

the reduction of the #potential guards

A simple Greedy Algorithm

Algorithm $\textsc{GreedyGuard}(\mathcal{T}, \varepsilon, \mathcal{P})$

```
Compute the viewsheds for all guards in 𝒫.
Let 𝒢 = ∅ and ℛ = 𝒫.
while ⟦𝒱(𝒢)⟧ < (1 − ε) ⟦𝒱(𝒫)⟧ and ℛ ≠ ∅ do
    Select a guard g ∈ ℛ that maximizes the size ⟦𝒱(g) \ 𝒱(𝒢)⟧,
    i.e., the size of the region it can cover but is not covered by 𝒢
    yet.
    Remove g from ℛ and add it to 𝒢.
return 𝒢
```

A simple Greedy Algorithm

Algorithm $\textsc{GreedyGuard}(\mathcal{T}, \varepsilon, \mathcal{P})$

1. Compute the viewsheds for all guards in $\mathcal{P}$.
2. Let $\mathcal{G} = \emptyset$ and $\mathcal{R} = \mathcal{P}$.
3. **while** $[\![\mathcal{V}(\mathcal{G})]\!] < (1-\varepsilon)\,[\![\mathcal{V}(\mathcal{P})]\!]$ **and** $\mathcal{R} \neq \emptyset$ **do**
4. Select a guard $g \in \mathcal{R}$ that maximizes the size $[\![\mathcal{V}(g) \setminus \mathcal{V}(\mathcal{G})]\!]$, i.e., the size of the region it can cover but is not covered by $\mathcal{G}$ yet.
5. Remove g from $\mathcal{R}$ and add it to $\mathcal{G}$.
6. **return** $\mathcal{G}$

Lemma 1. $\textsc{GreedyGuard}$ *computes an ε-cover of $\mathcal{T}' = \mathcal{V}(\mathcal{P})$ of at most $O(k/\varepsilon)$ guards, where k is the size of an optimal 0-cover of $\mathcal{T}'$.*

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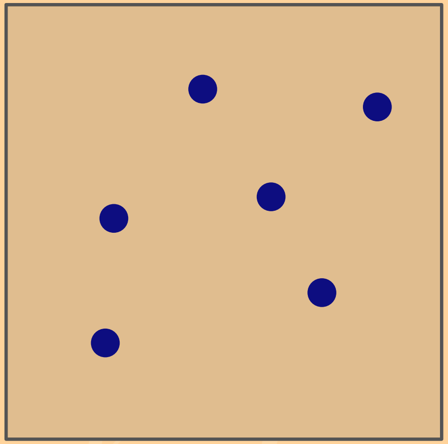
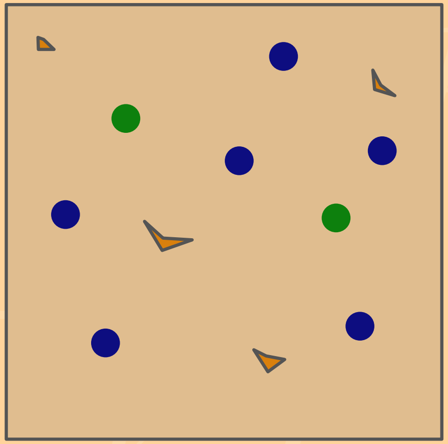
"If OPT can cover $\mathcal{T}'$ with k guards, we can cover a $(1-\varepsilon)$ fraction of $\mathcal{T}'$ with ck/ε guards."

A simple Greedy Algorithm





	OPT	GreedyGuard
fraction of $\mathcal{T}'$ covered	1	$1-\varepsilon$
$\lvert\mathcal{G}\rvert$	k	ck/ε

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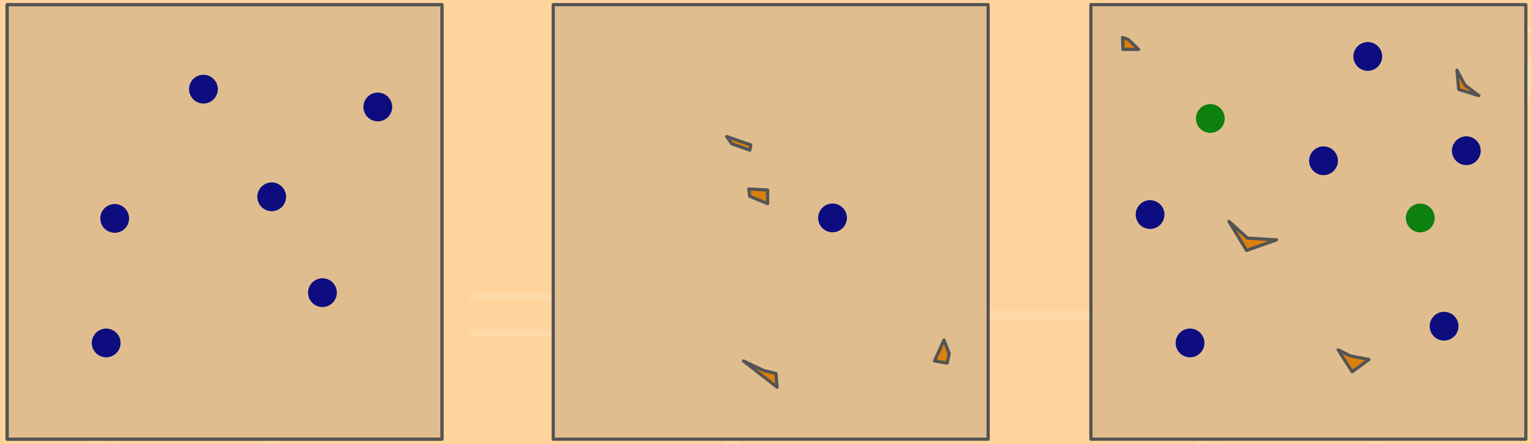

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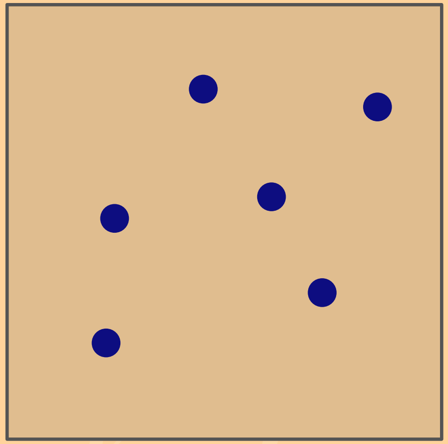

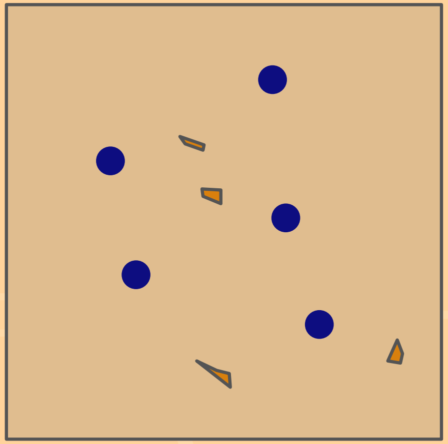

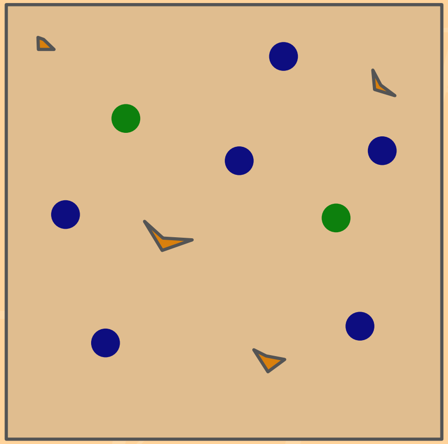

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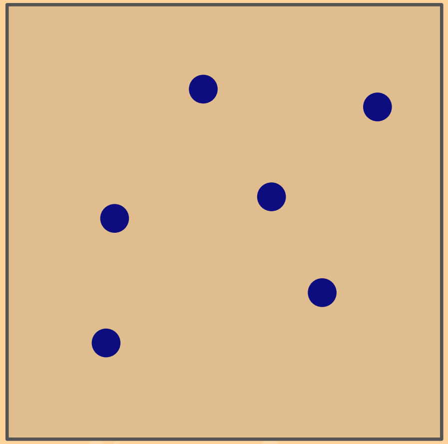 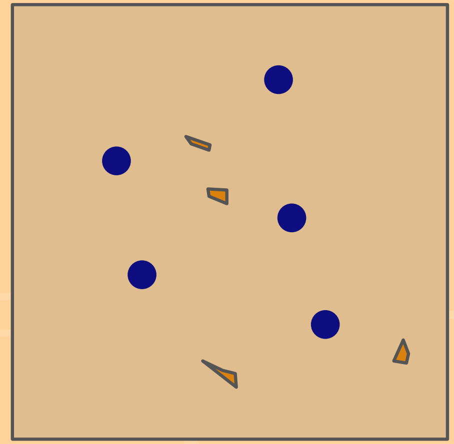 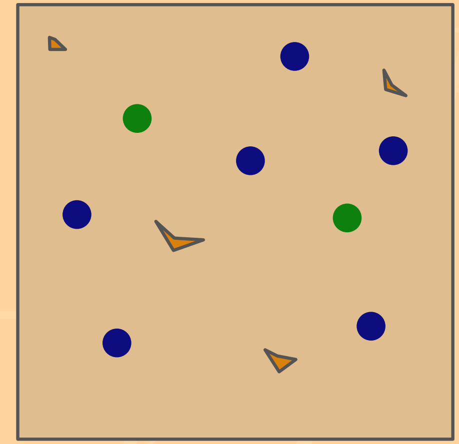

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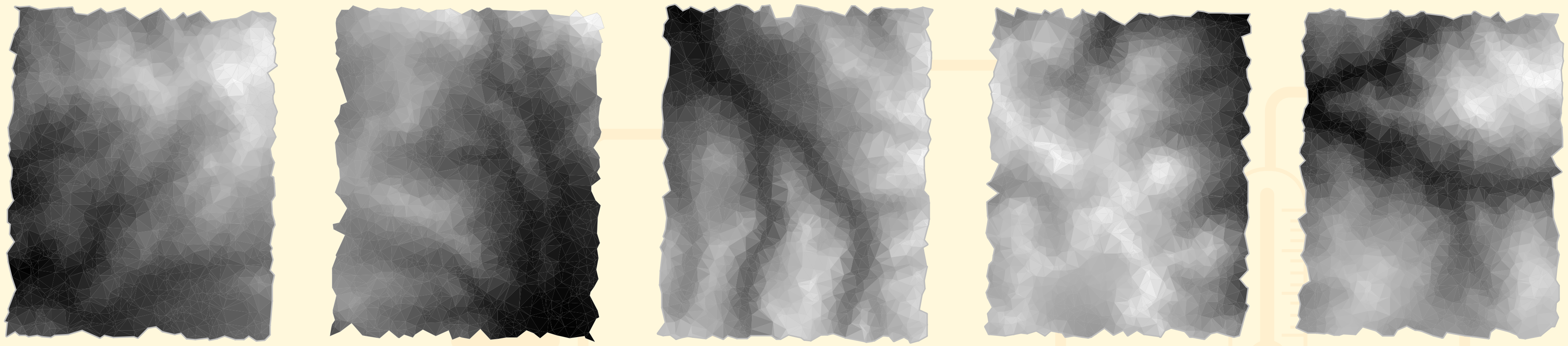
A simple Greedy Algorithm

Hot Springs Quinn Pk Sphinx Lakes Split Mountain Wren Peak



#vertices in $\mathcal{T}$

coarse $\approx$ 1700 fine $\approx$ 16 000

covered area $\approx 11.5\text{km} \times 14\text{km}$

$\mathcal{P}$ = the set of vertices of $\mathcal{T}$

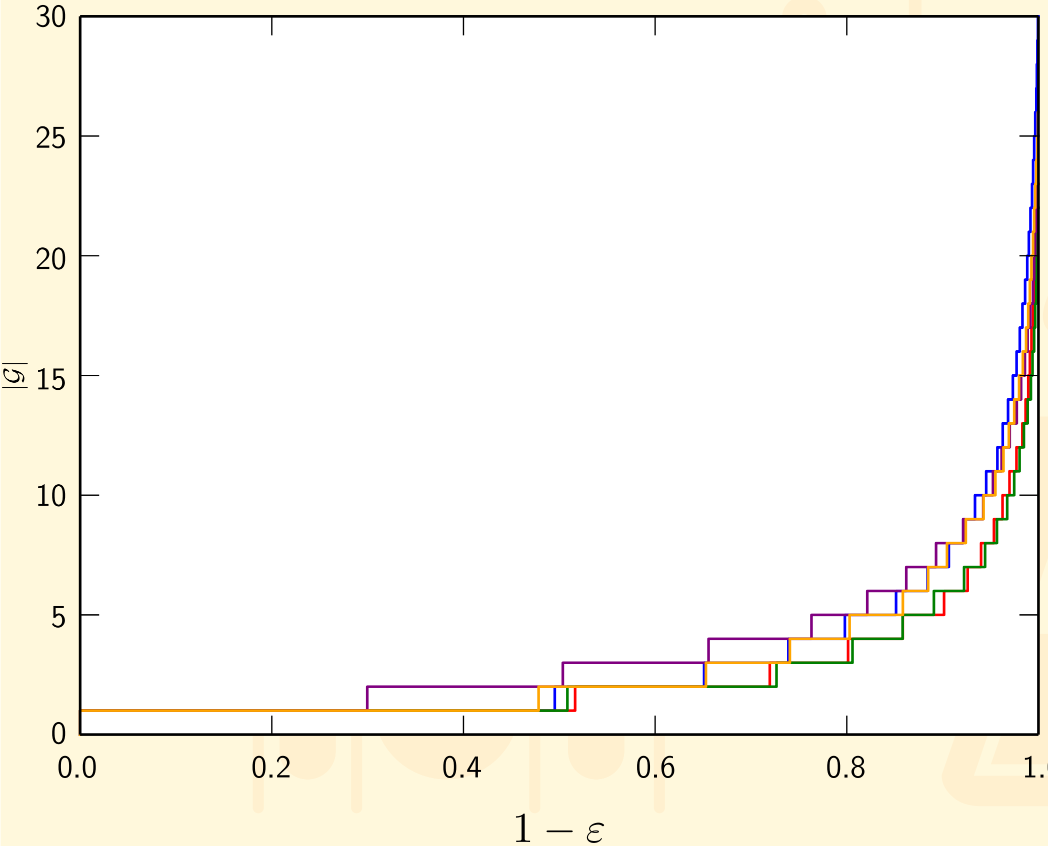
$[\![\mathcal{V}(g)]\!]$ = #terrain vertices in $\mathcal{V}(g)$

h = 15 meter

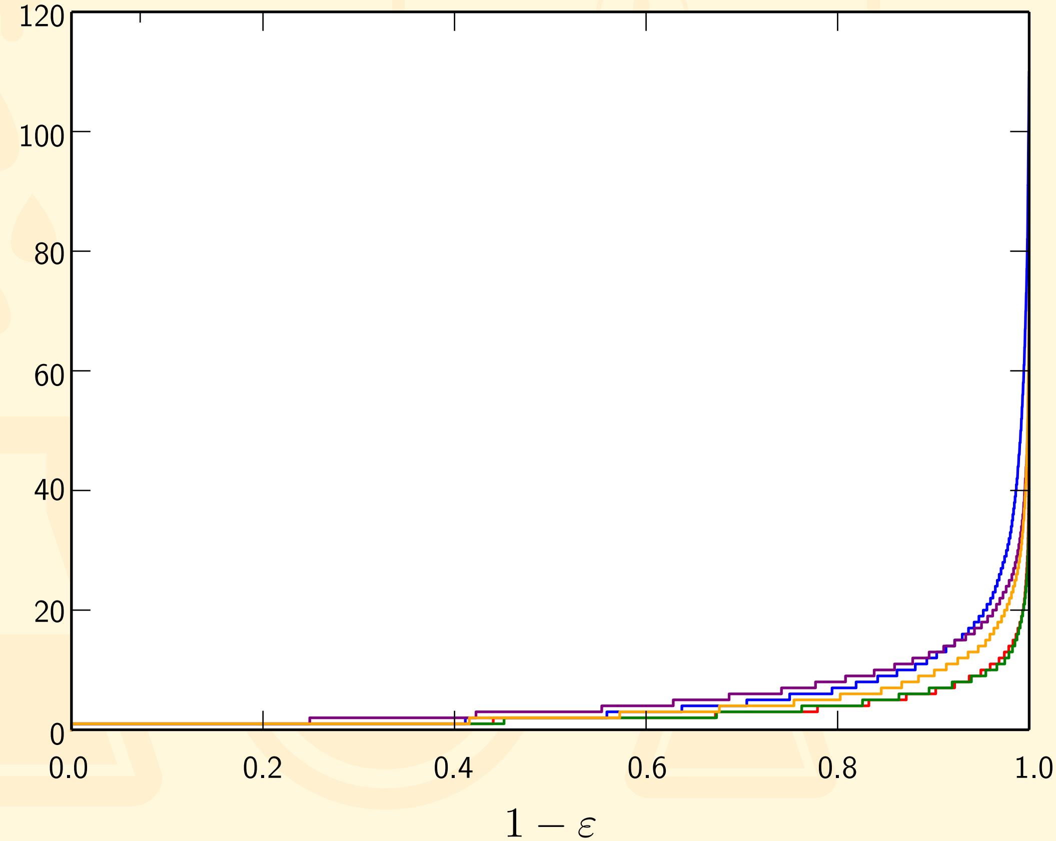
A simple Greedy Algorithm

Hot Springs Quinn Pk Sphinx Lakes Split Mountain Wren Peak

coarse



fine



A simple Greedy Algorithm

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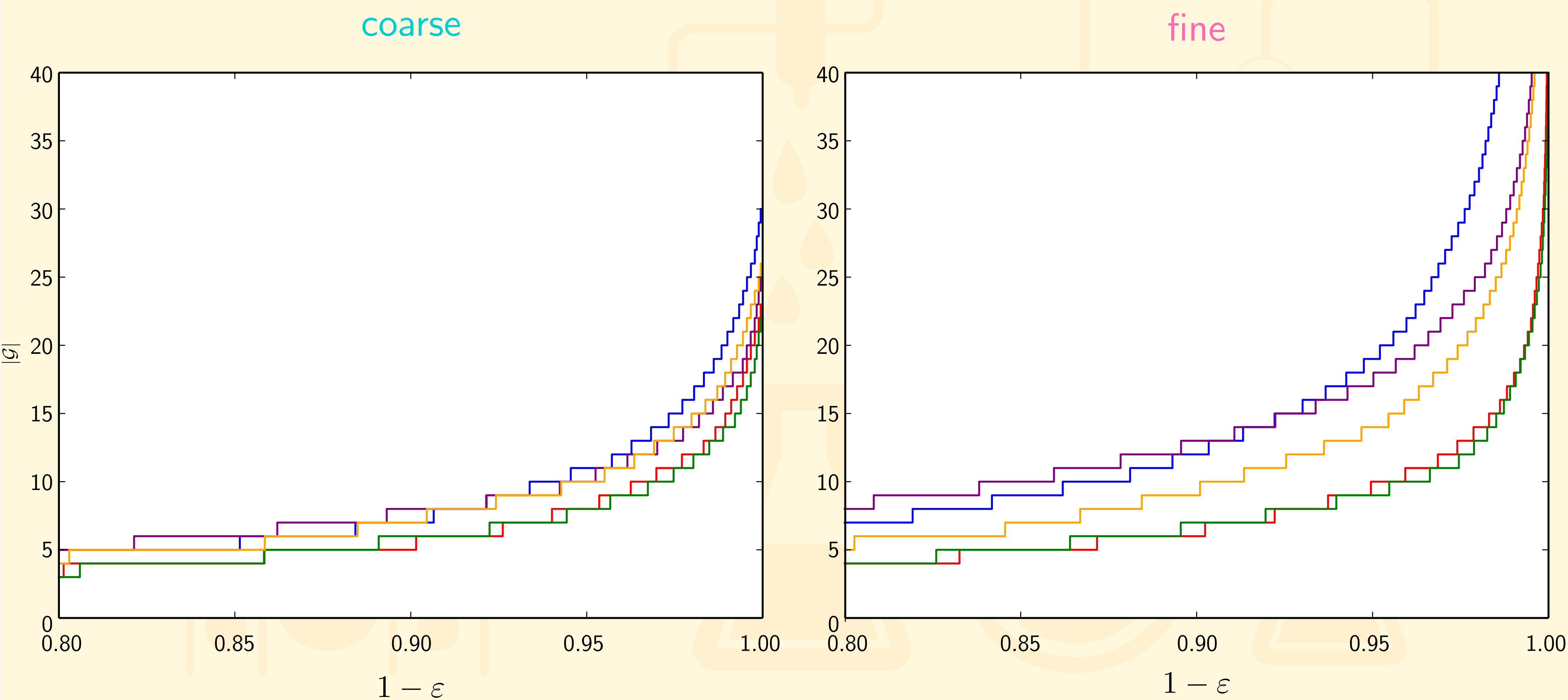

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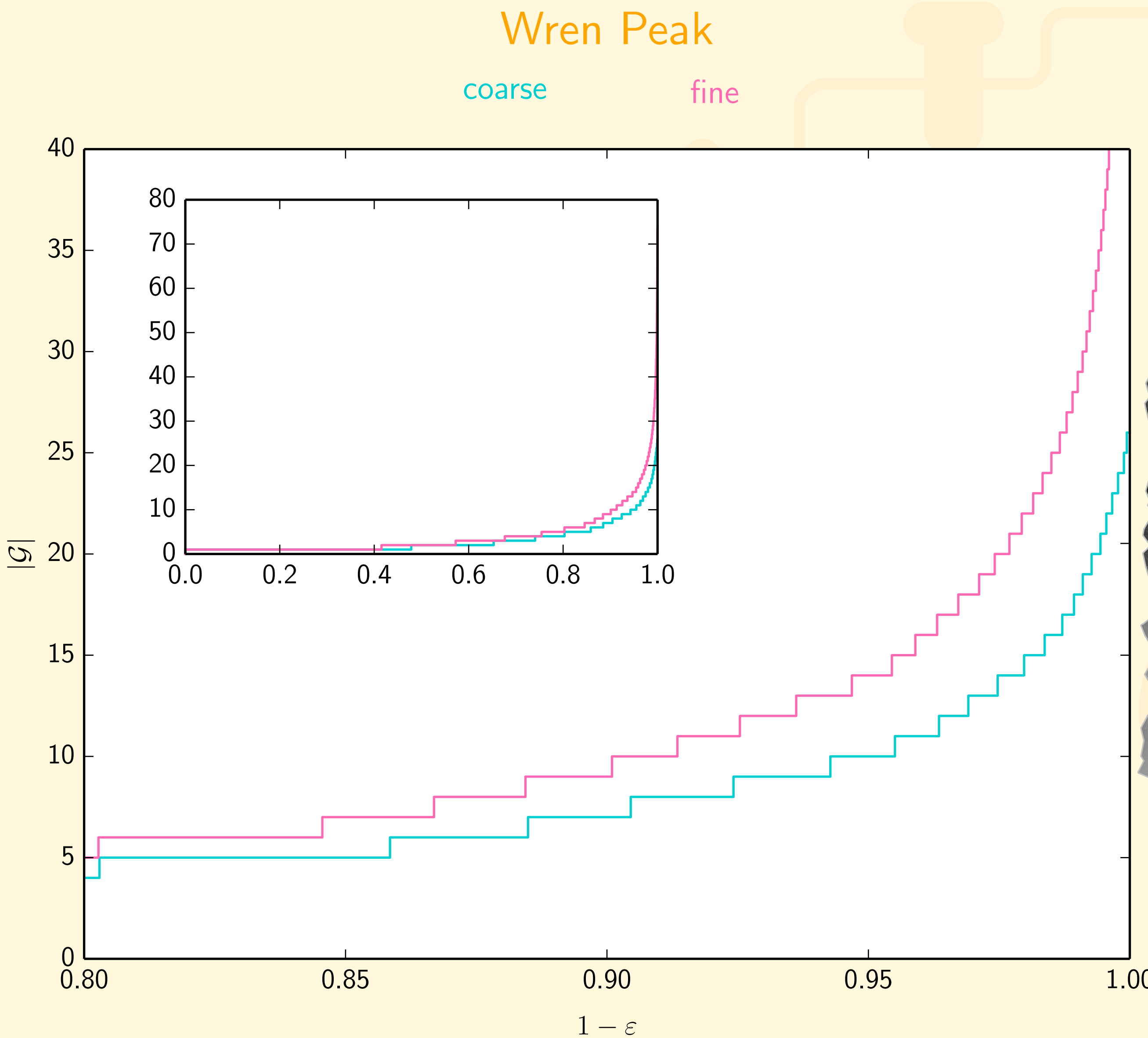
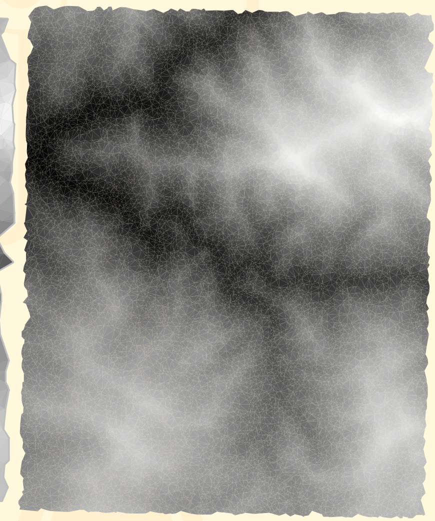
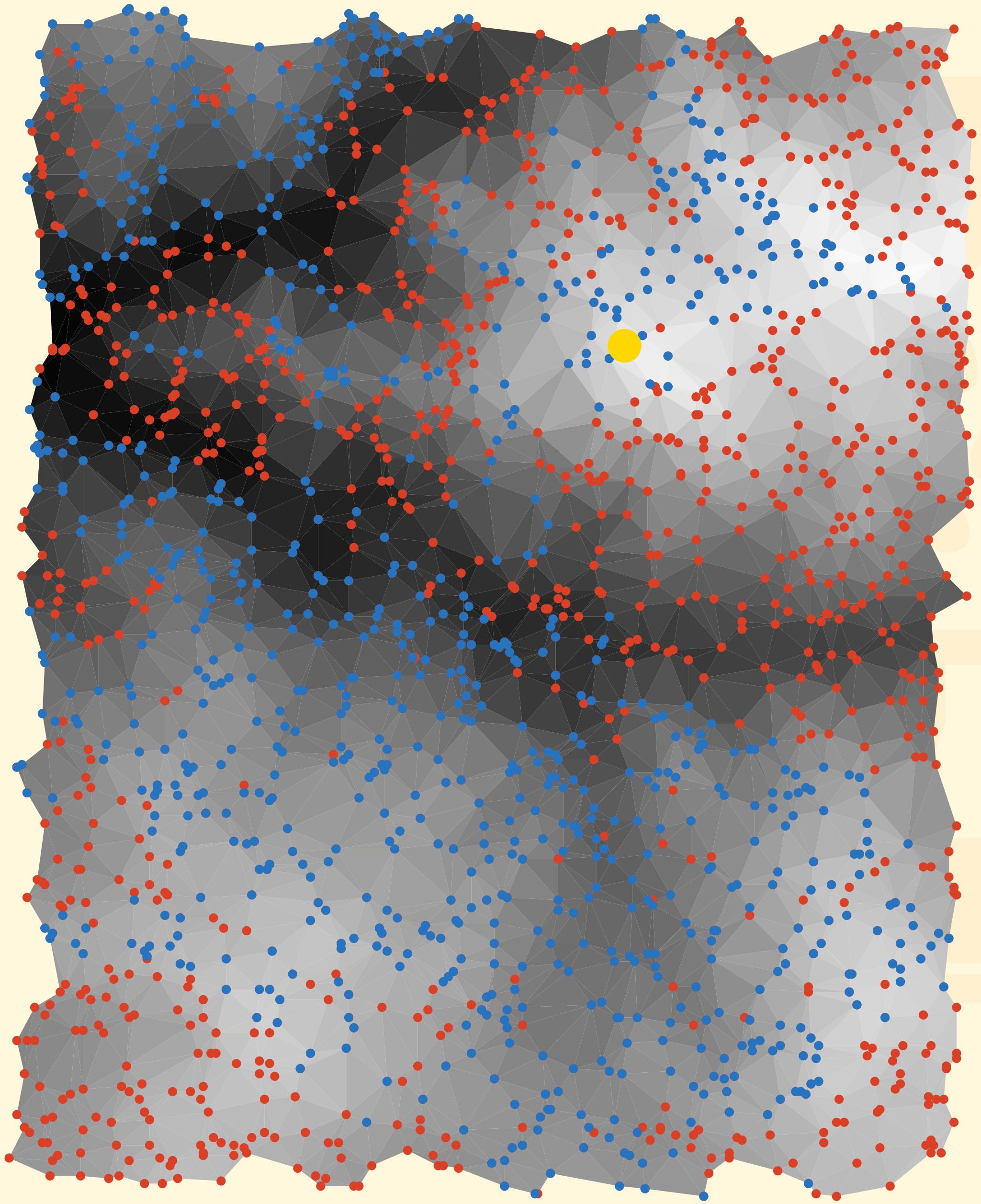




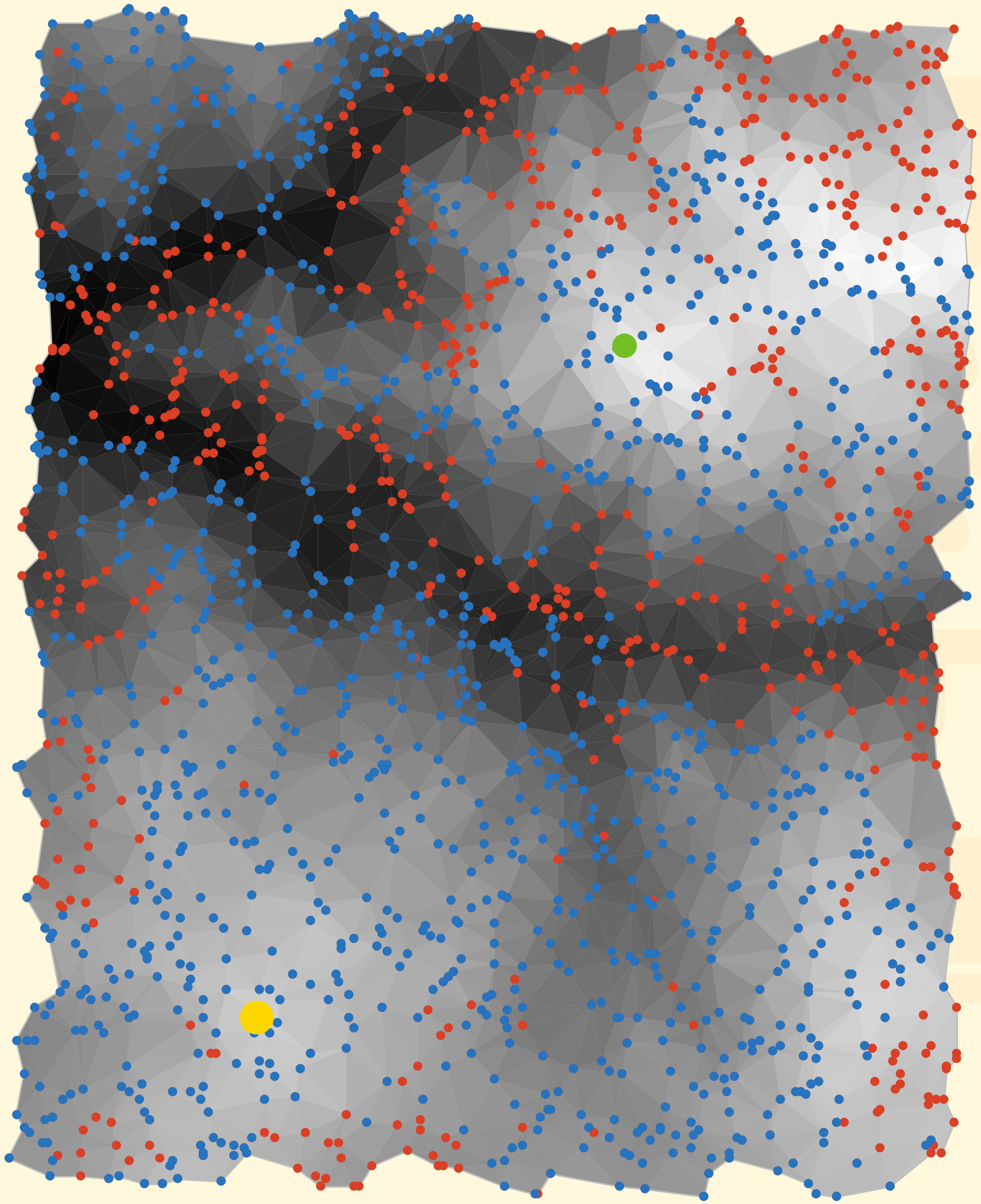
A simple Greedy Algorithm



Computing a 0.05-cover on a coarse Wren Peak using GreedyGuard

$$|\mathcal{G}| = 1$$

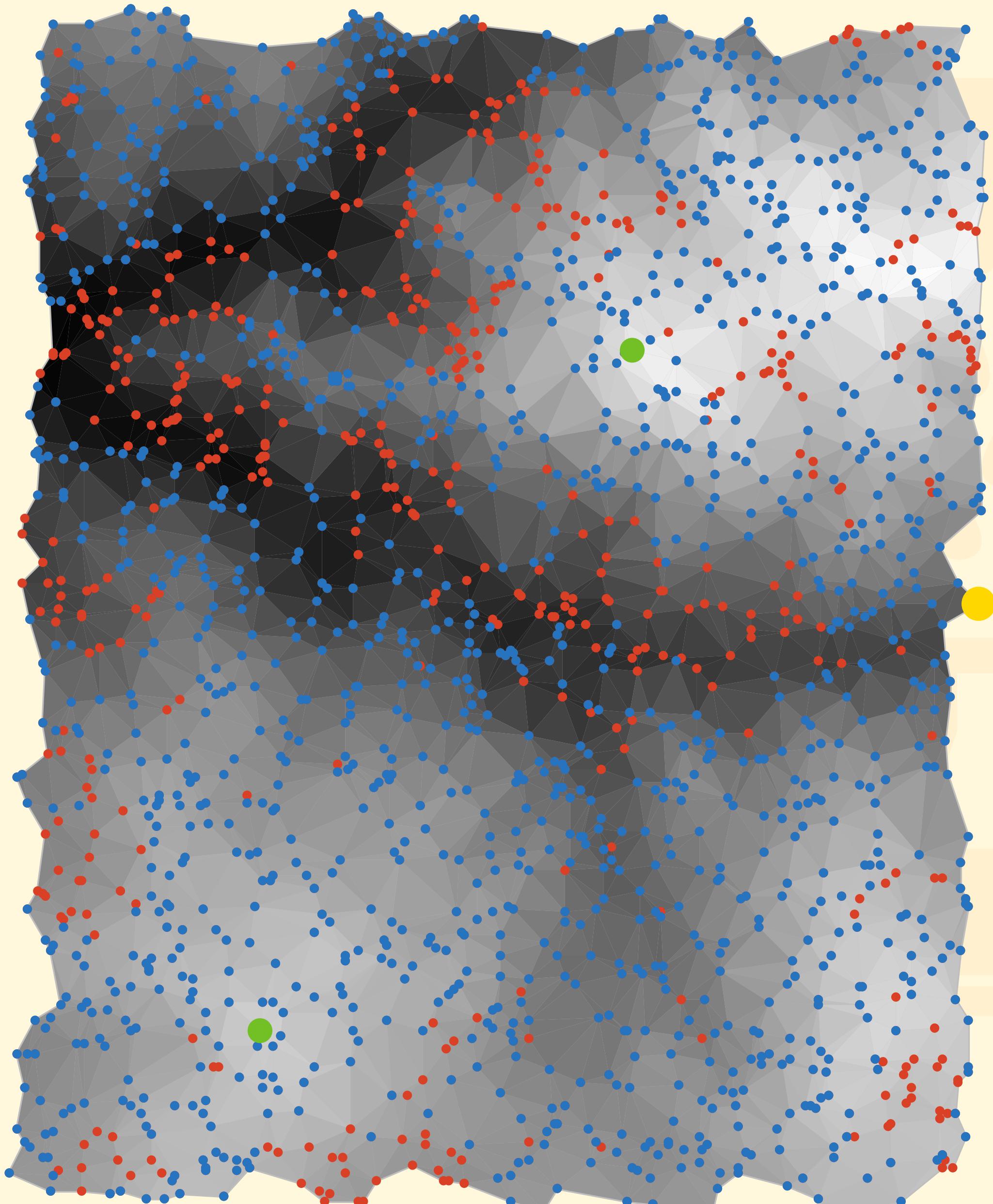
A simple Greedy Algorithm



Computing a 0.05-cover on a coarse Wren Peak using GreedyGuard

$$|\mathcal{G}| = 2$$

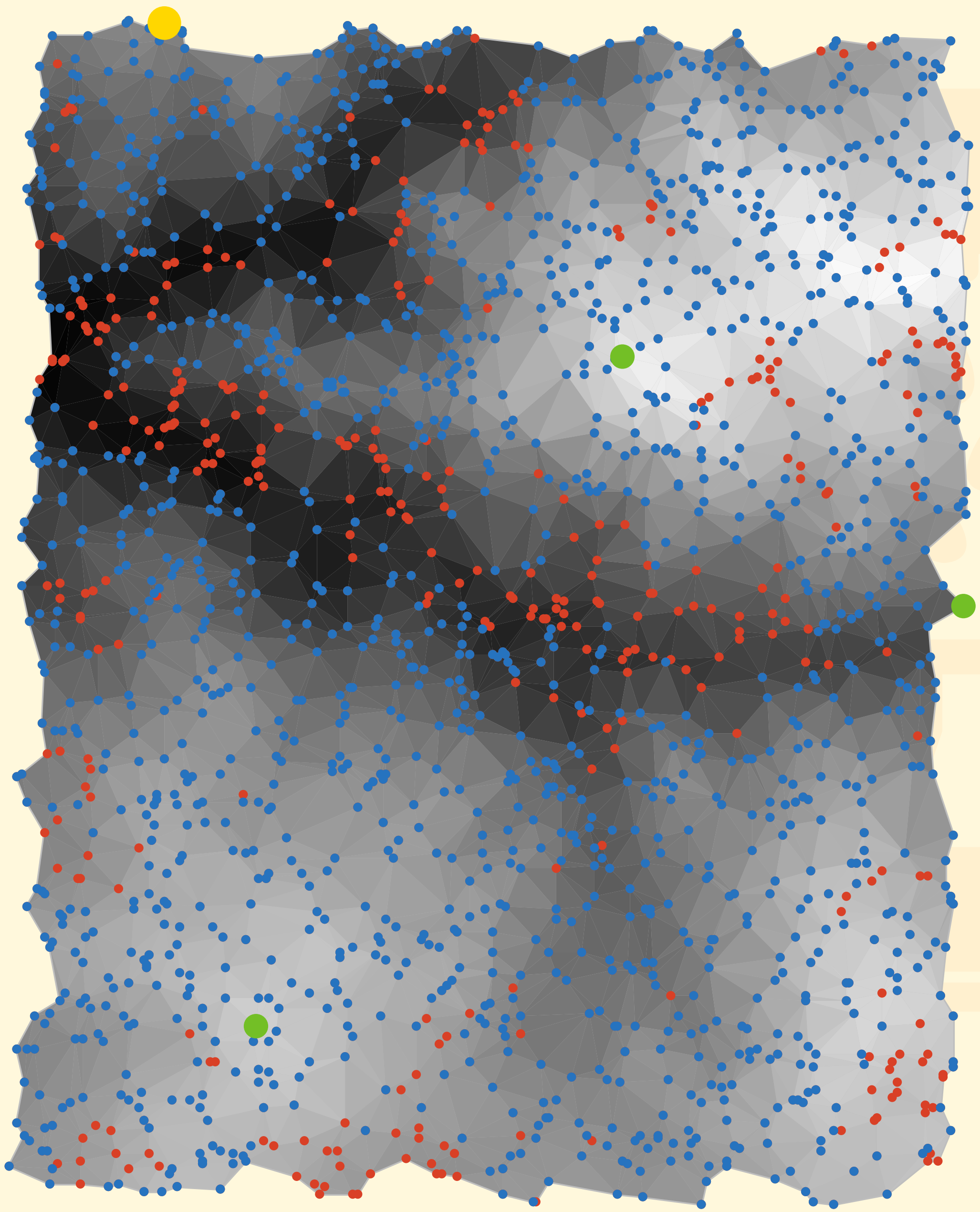
A simple Greedy Algorithm



Computing a 0.05-cover on a coarse Wren Peak using GreedyGuard

$|\mathcal{G}| = 3$

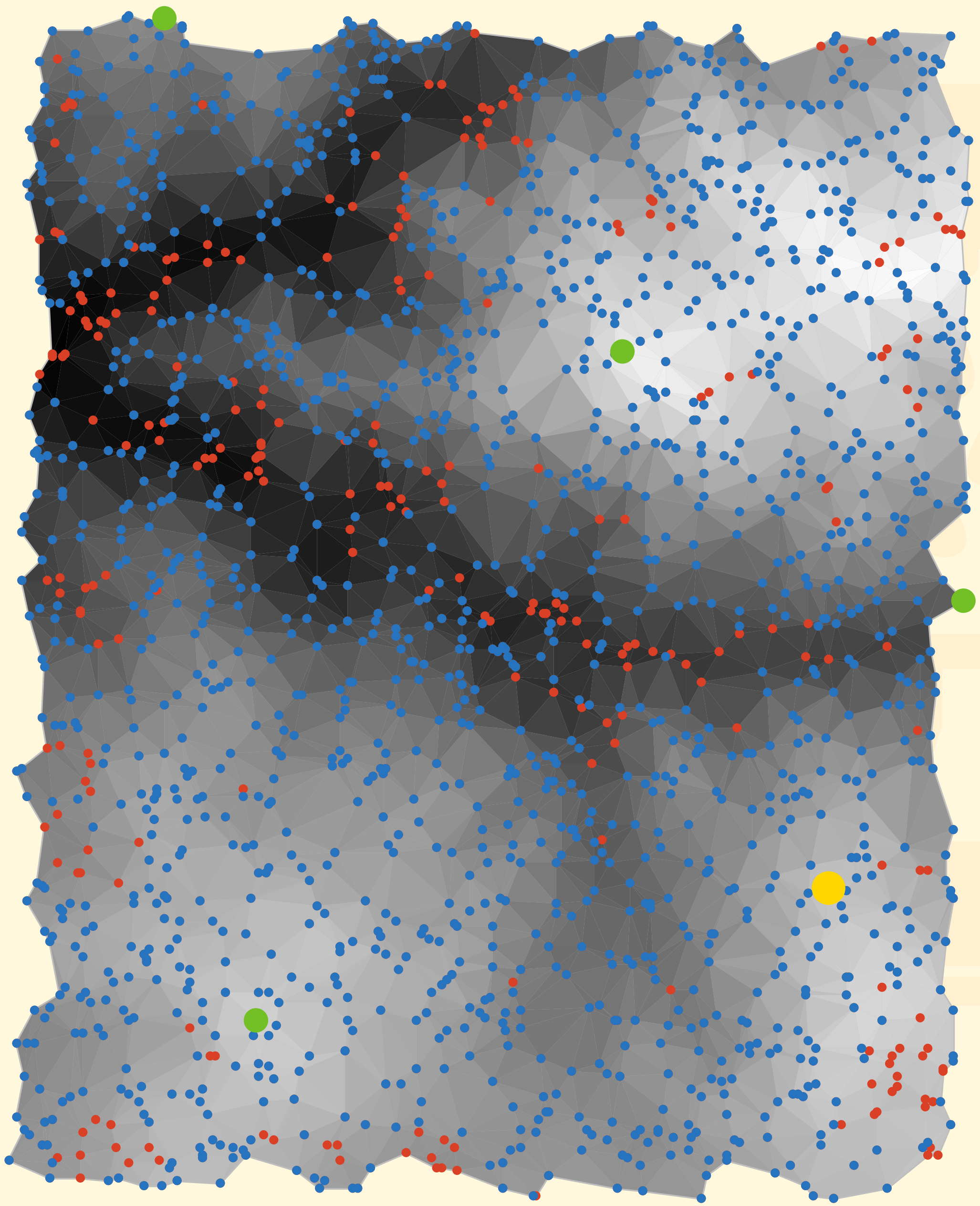
A simple Greedy Algorithm



Computing a 0.05-cover on a coarse Wren Peak using GreedyGuard

$|\mathcal{G}| = 4$

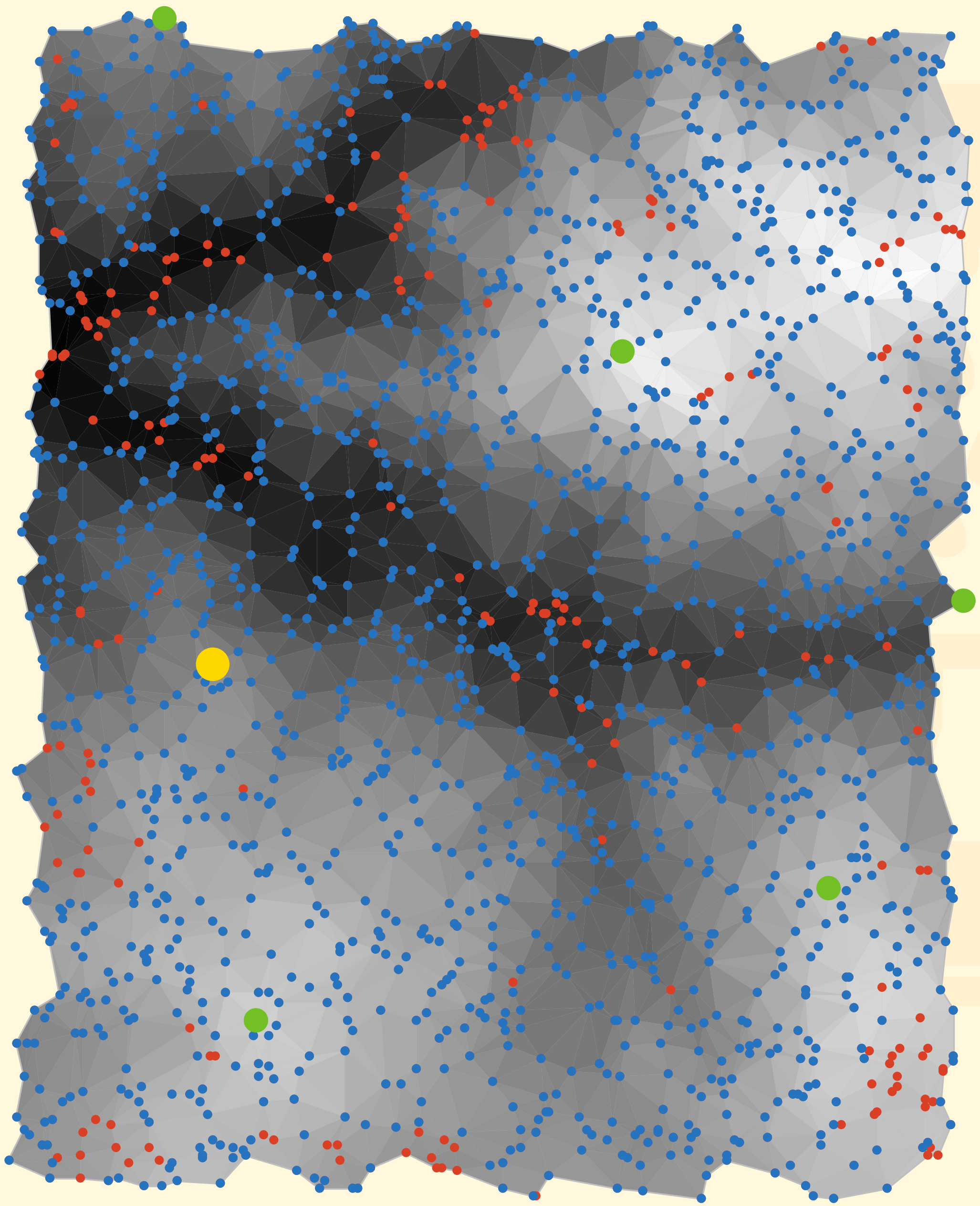
A simple Greedy Algorithm



Computing a 0.05-cover on a coarse Wren Peak using GreedyGuard

$|\mathcal{G}| = 5$

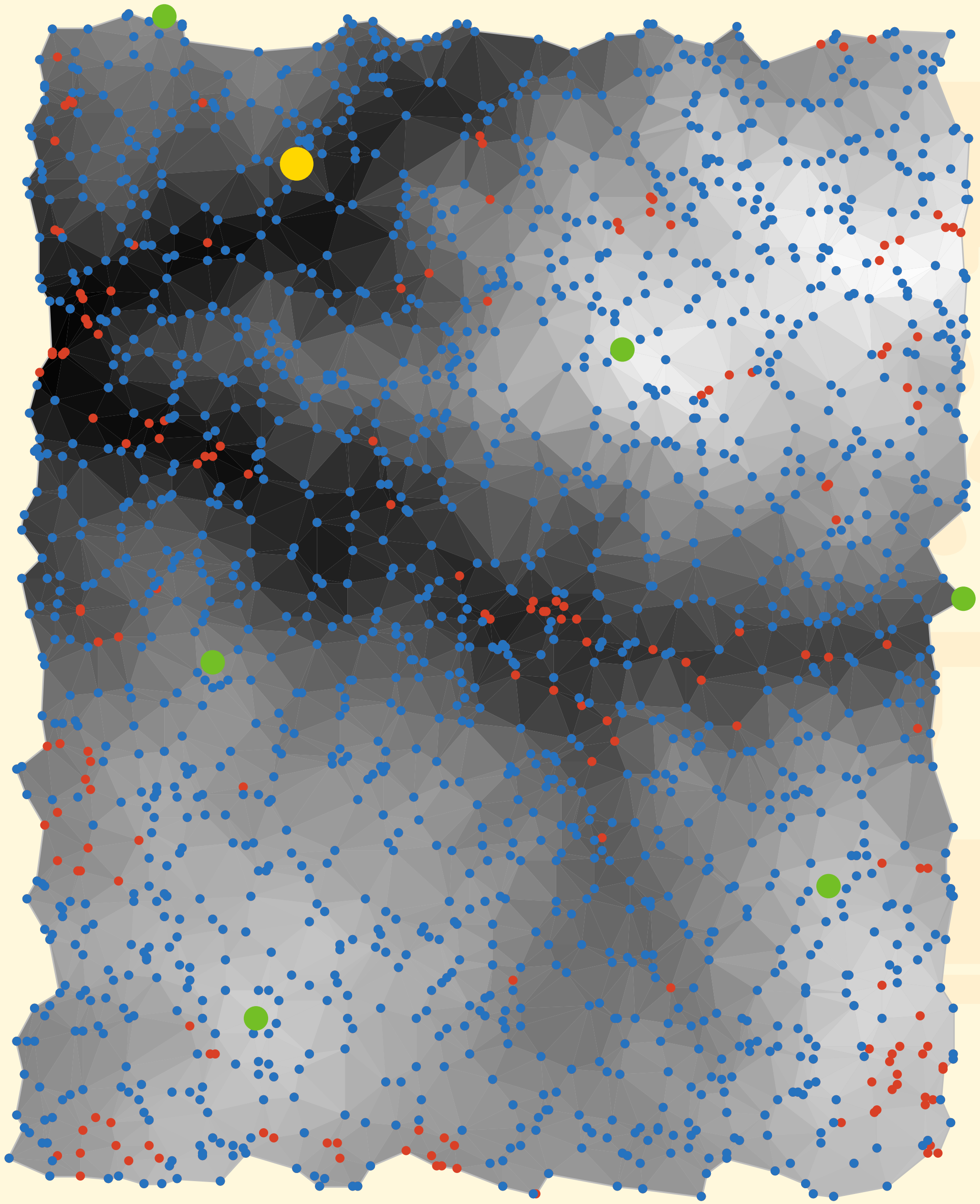
A simple Greedy Algorithm



Computing a 0.05-cover on a coarse Wren Peak using GreedyGuard

$|\mathcal{G}| = 6$

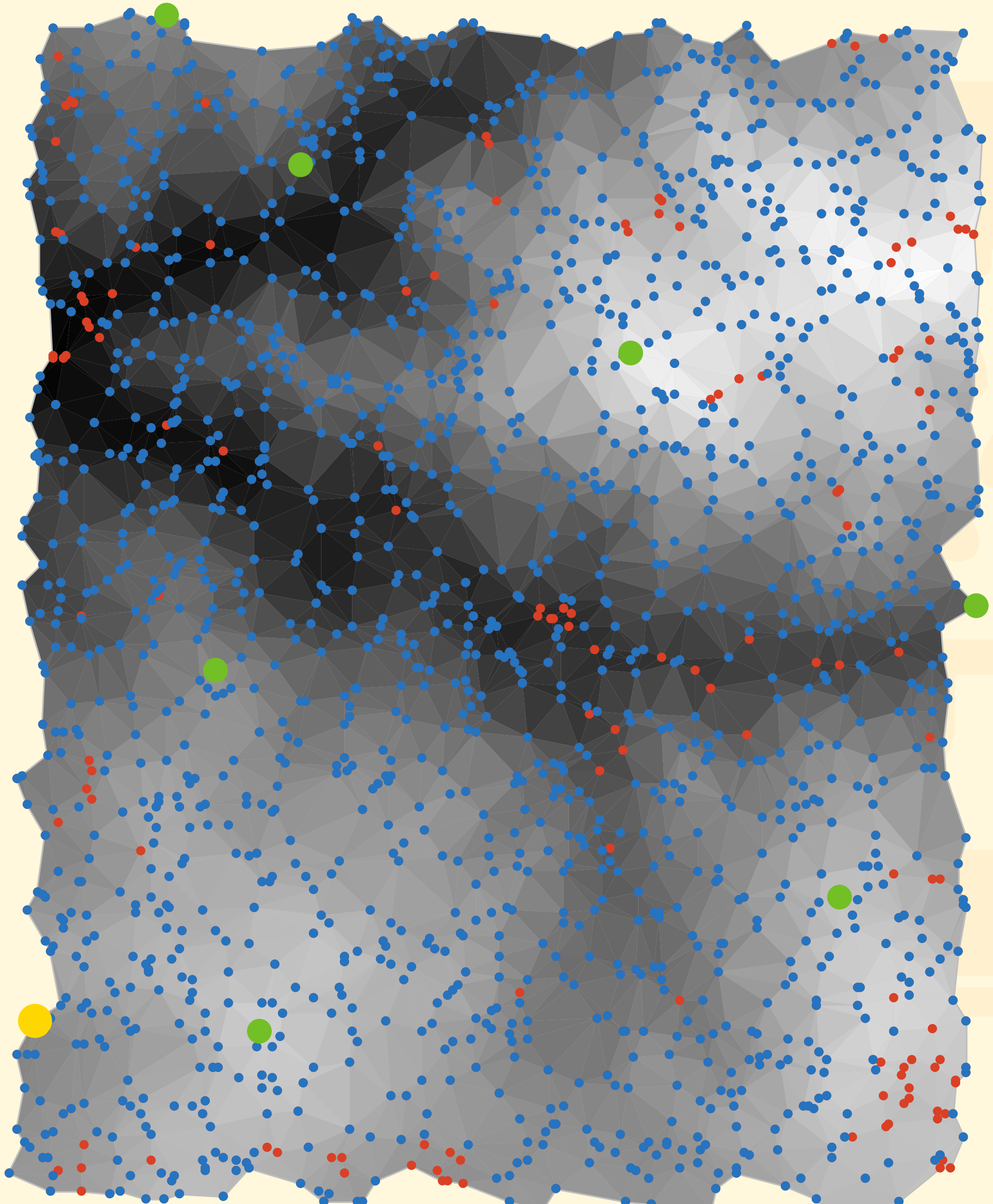
A simple Greedy Algorithm



Computing a 0.05-cover on a coarse Wren Peak using GreedyGuard

$|\mathcal{G}| = 7$

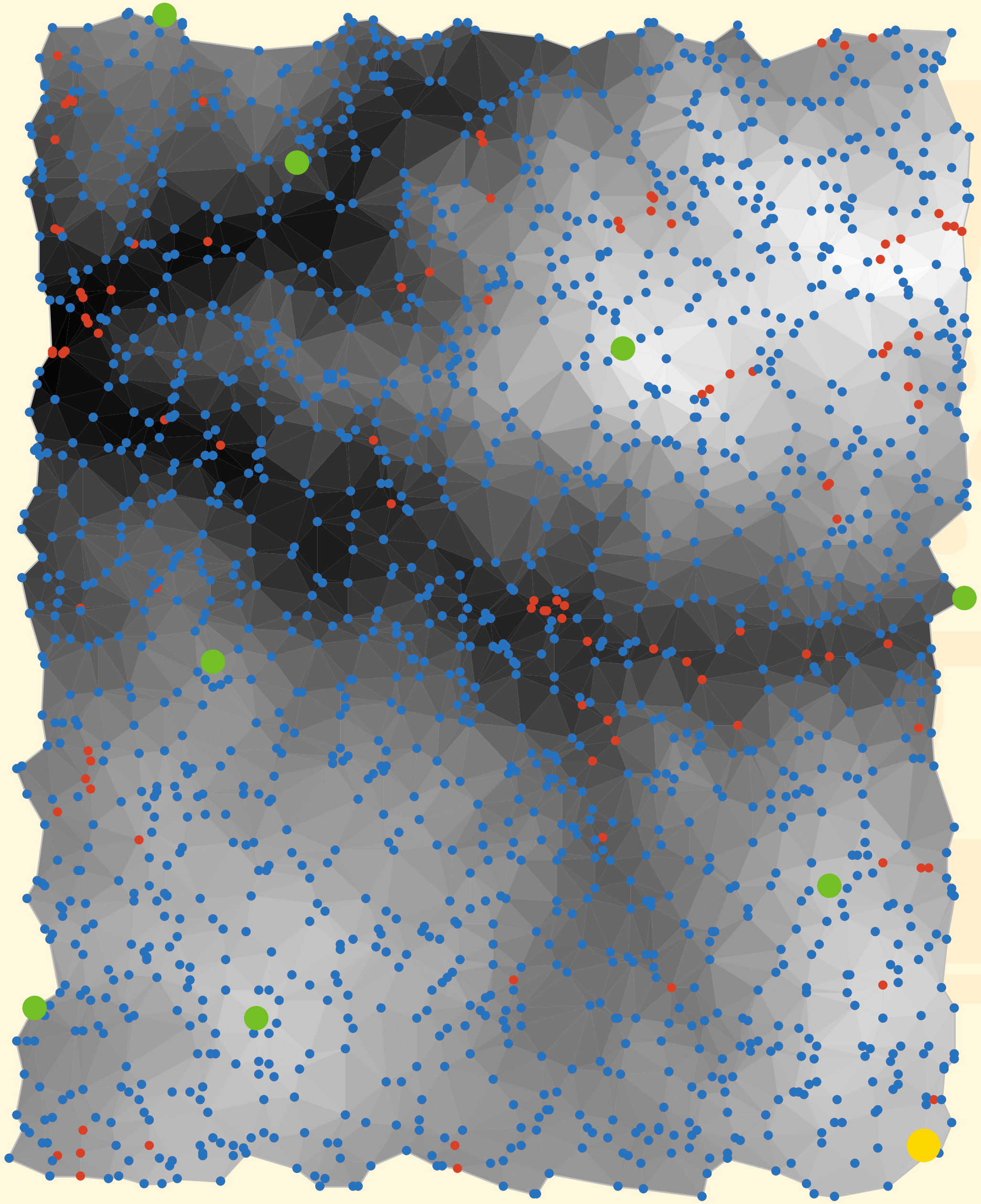
A simple Greedy Algorithm



Computing a 0.05-cover on a coarse Wren Peak using GreedyGuard

$|\mathcal{G}| = 8$

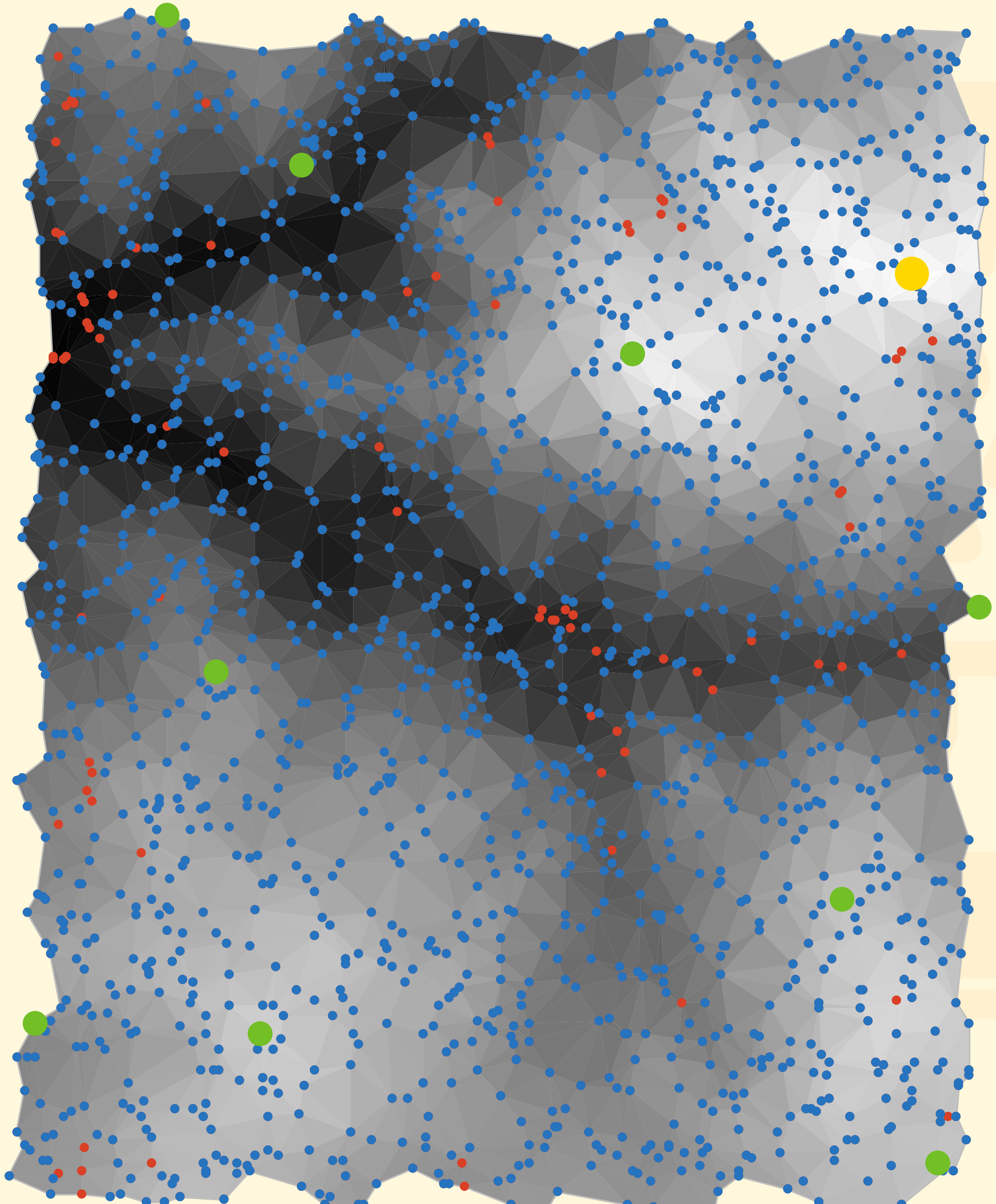
A simple Greedy Algorithm



Computing a 0.05-cover on a coarse Wren Peak using GreedyGuard

$$|\mathcal{G}| = 9$$

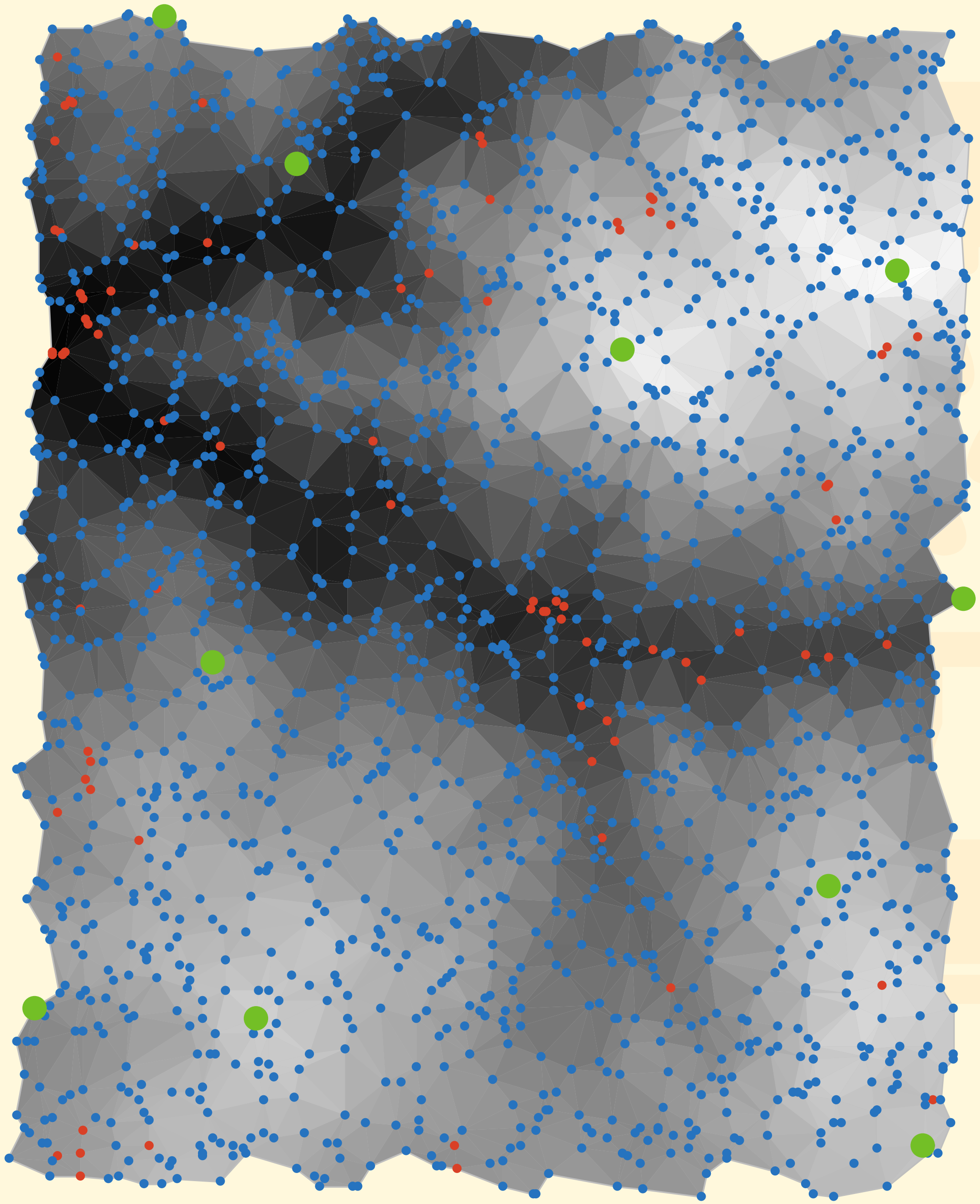
A simple Greedy Algorithm



Computing a 0.05-cover on a coarse Wren Peak using GreedyGuard

$|\mathcal{G}| = 10$

A simple Greedy Algorithm



Computing a 0.05-cover on a coarse Wren Peak using GreedyGuard

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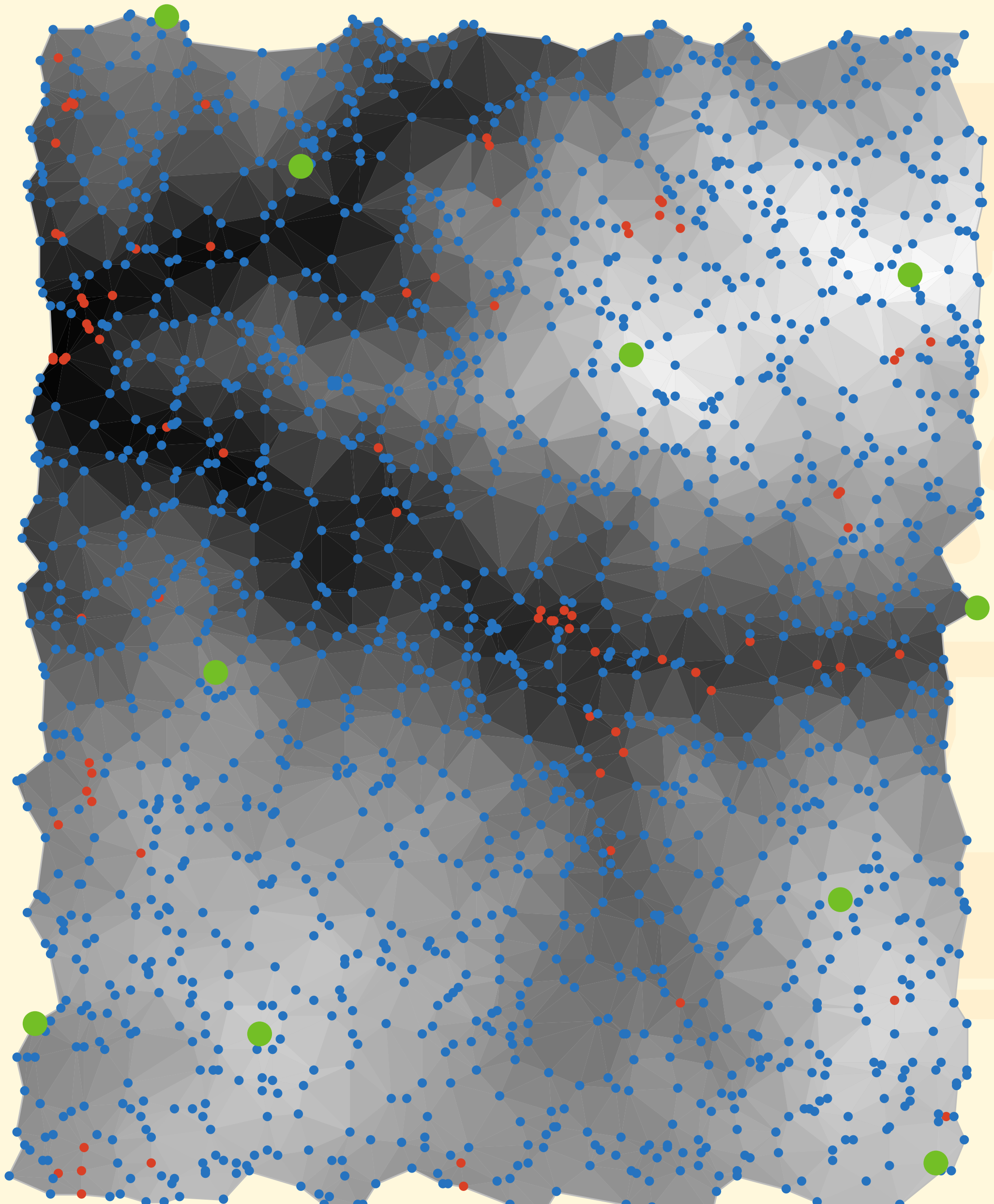


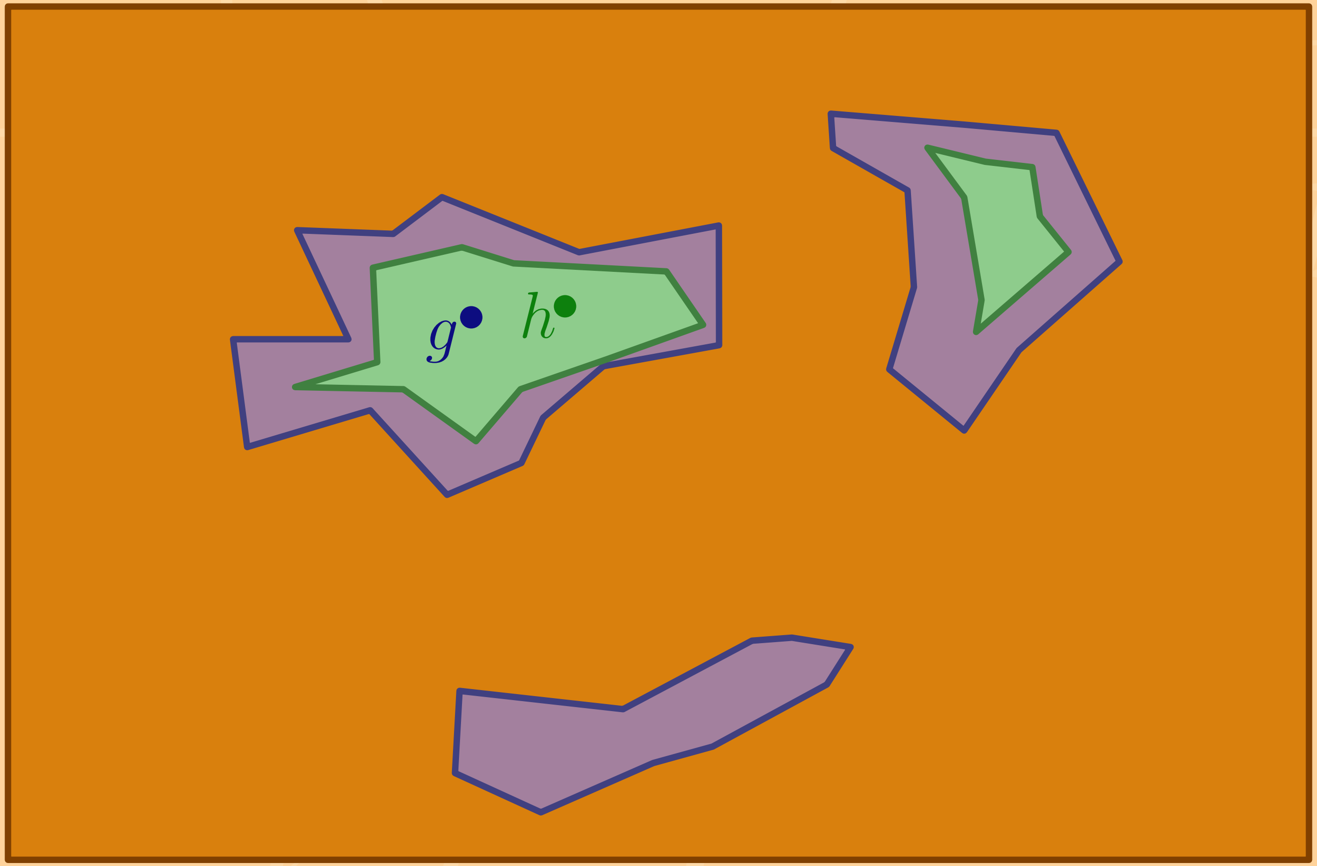
Computing a 0.05-cover on a coarse Wren Peak using GreedyGuard

$|\mathcal{G}| = 10$

We need another 15 guards to view all remaining vertices!

Dominating Guards

g dominates h $\equiv$ $\mathcal{V}(h) \subseteq \mathcal{V}(g)$



Dominating Guards

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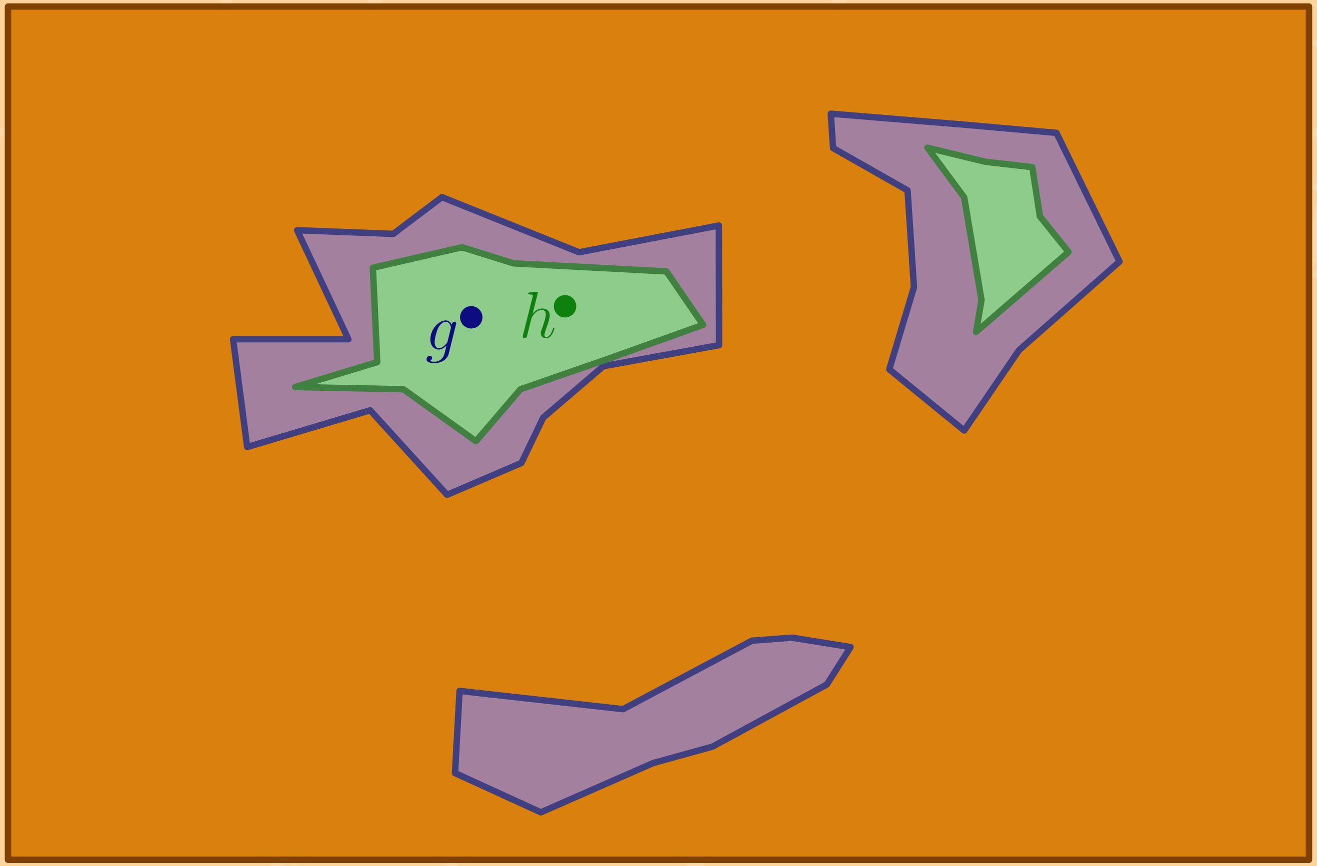

Dominating Guards

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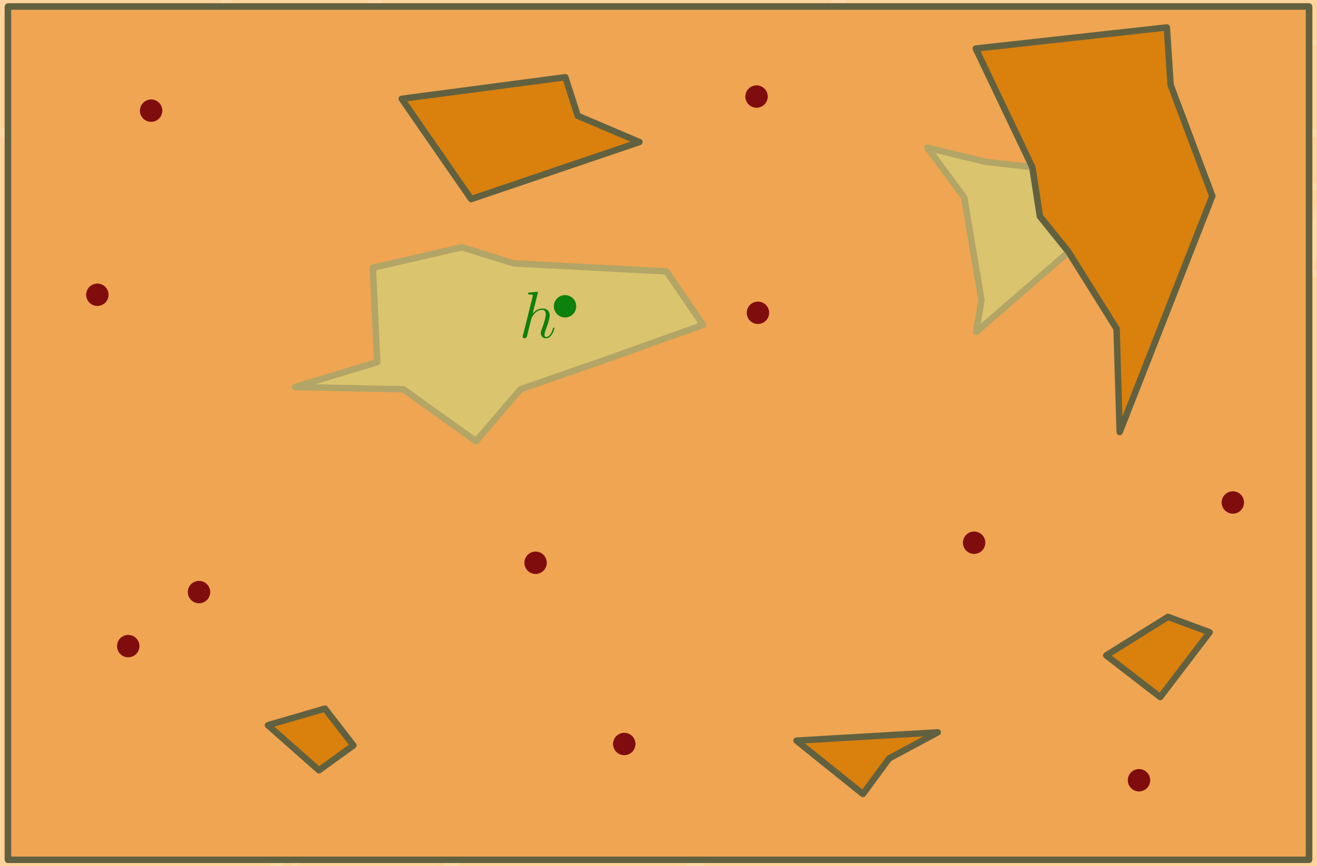
Let $\mathcal{H} = \{p_1, .., p_k, h\}$ be an ε-cover.



Dominating Guards

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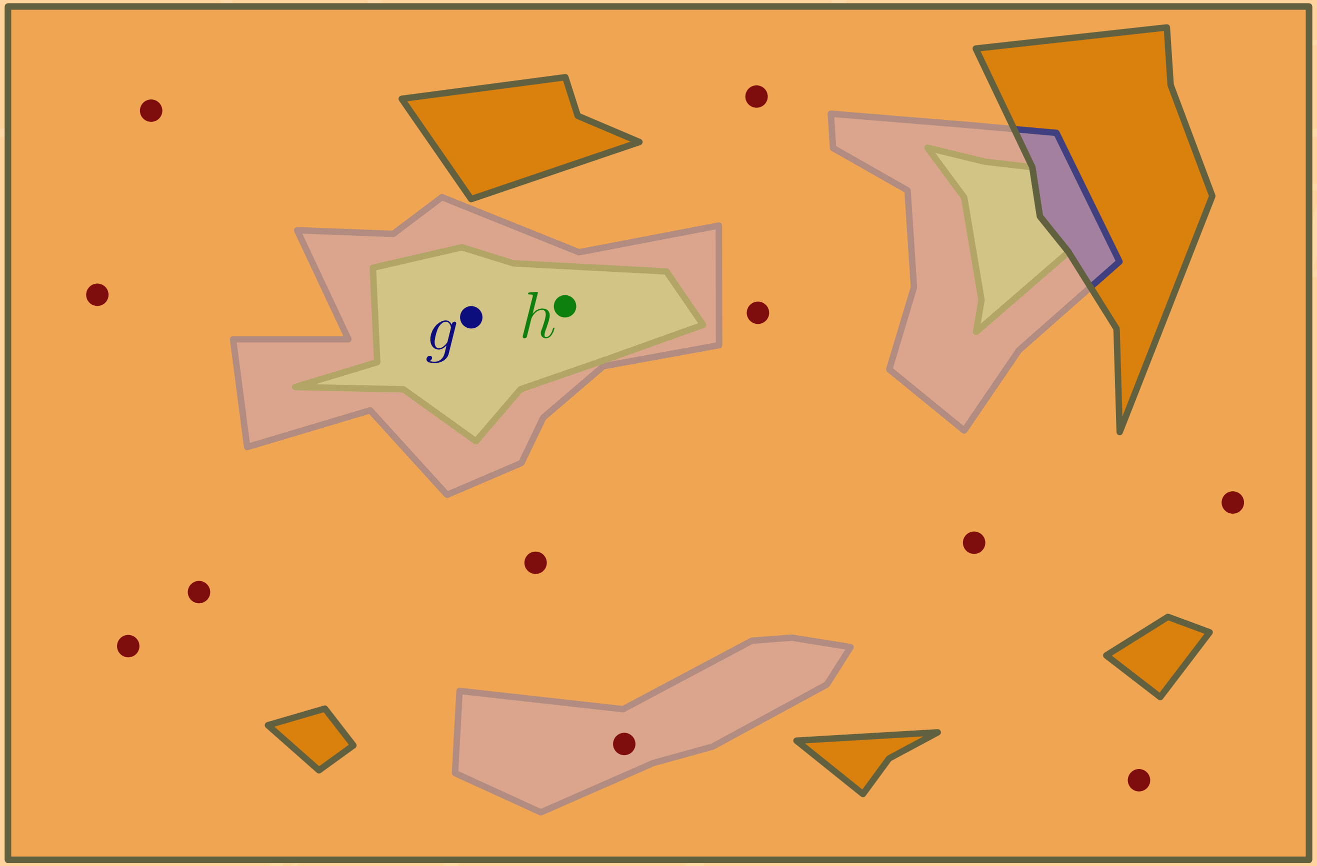
$\implies \mathcal{G} = \{p_1, .., p_k, g\}$ is an ε-cover.



Dominating Guards

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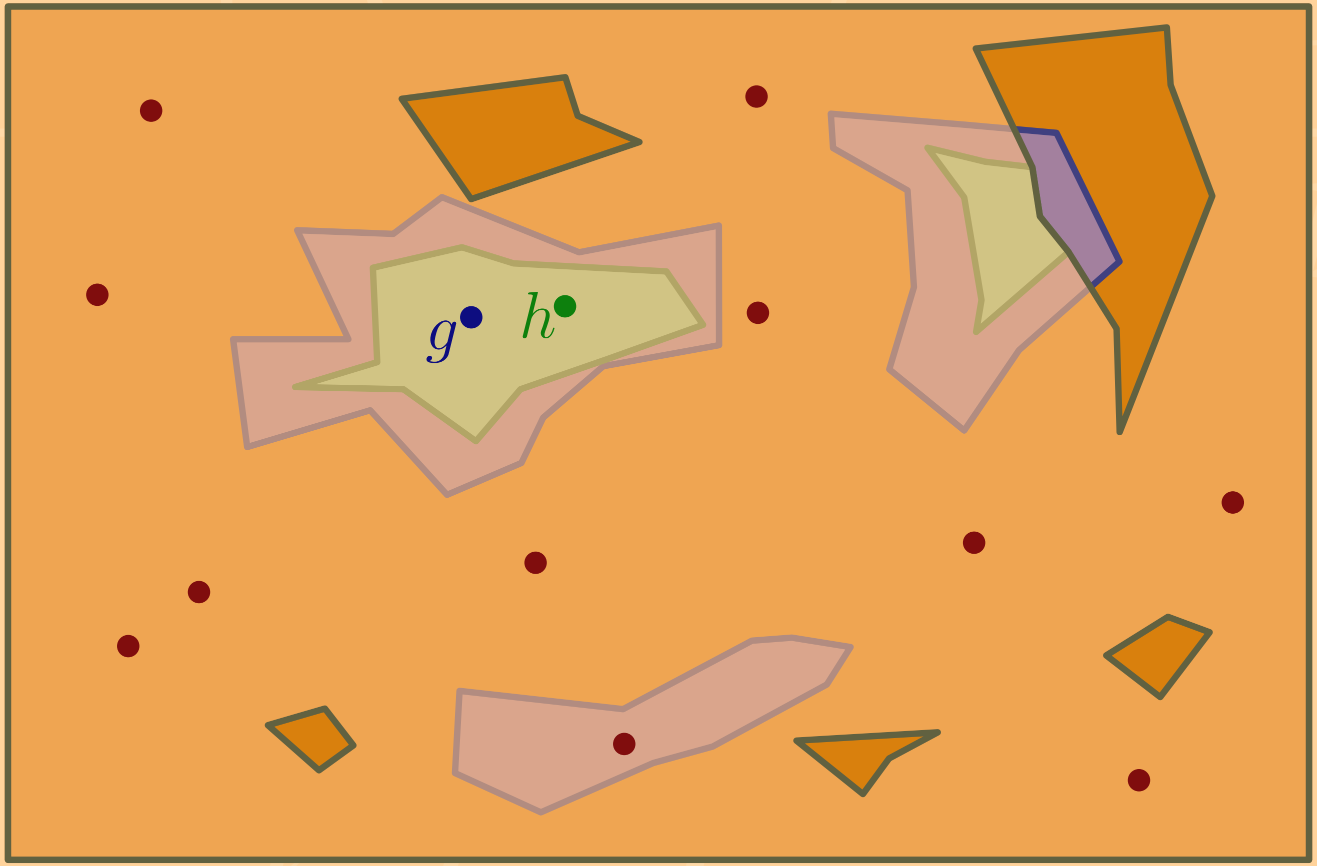
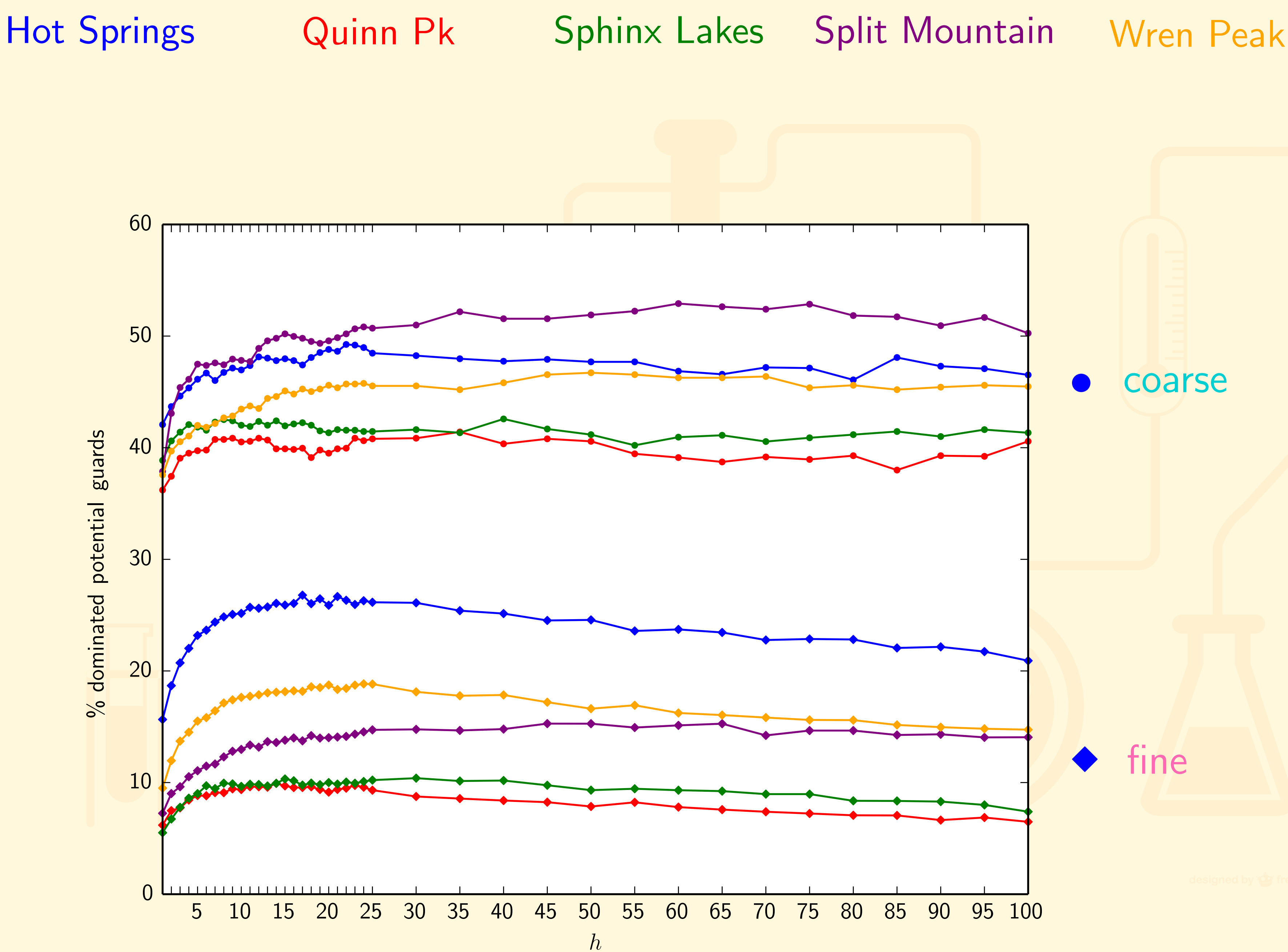
$\Longrightarrow \mathcal{G} = \{p_1, .., p_k, g\}$ is an ε-cover.



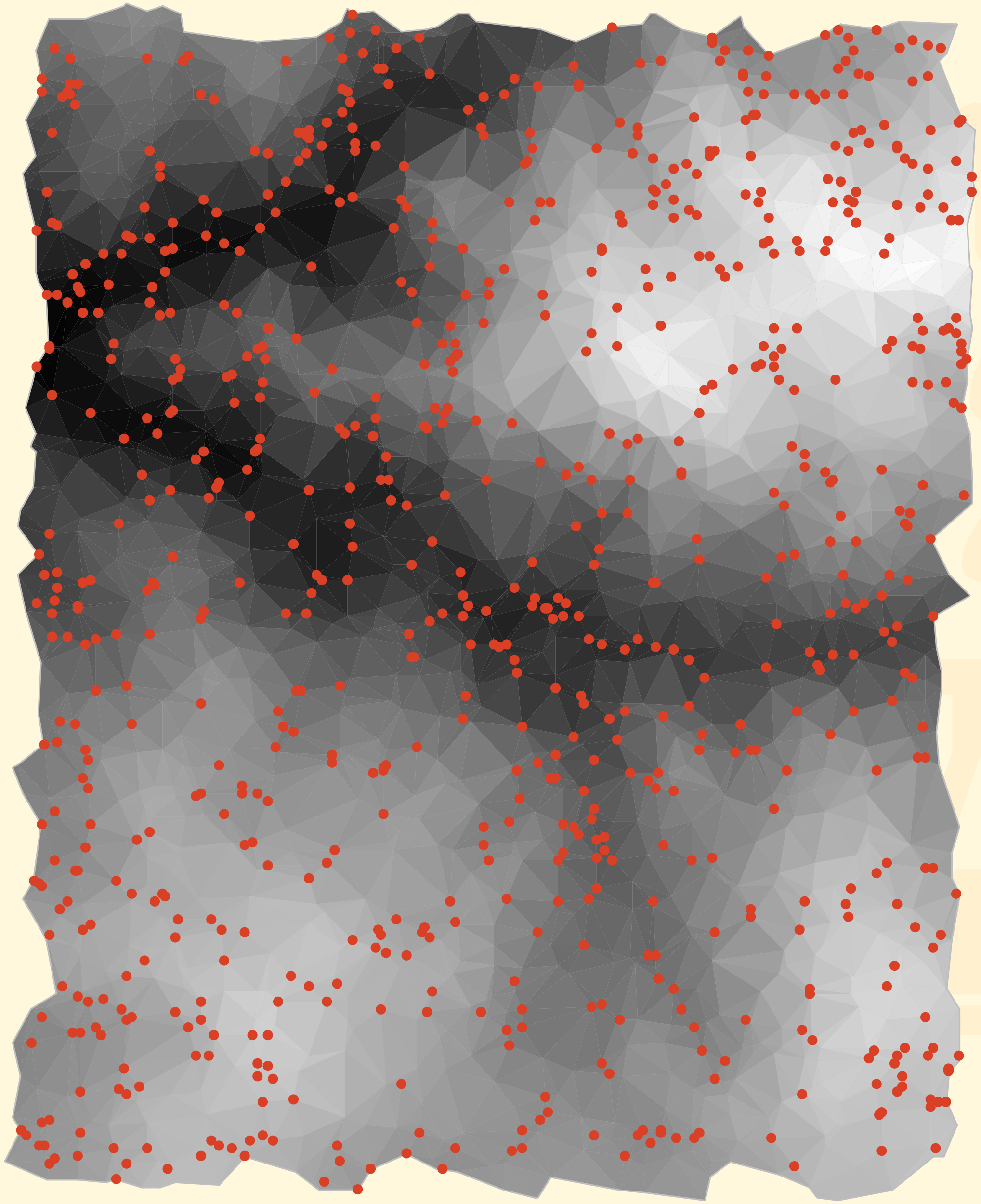
Observation 2. *Let $\mathcal{P}$ be a set of potential guards. There is an optimal (minimum size) ε-cover $\mathcal{G}$ of $\mathcal{V}(\mathcal{P})$ such that no guard in $\mathcal{G}$ is strictly dominated by any guard in $\mathcal{P}$.*

Dominating Guards

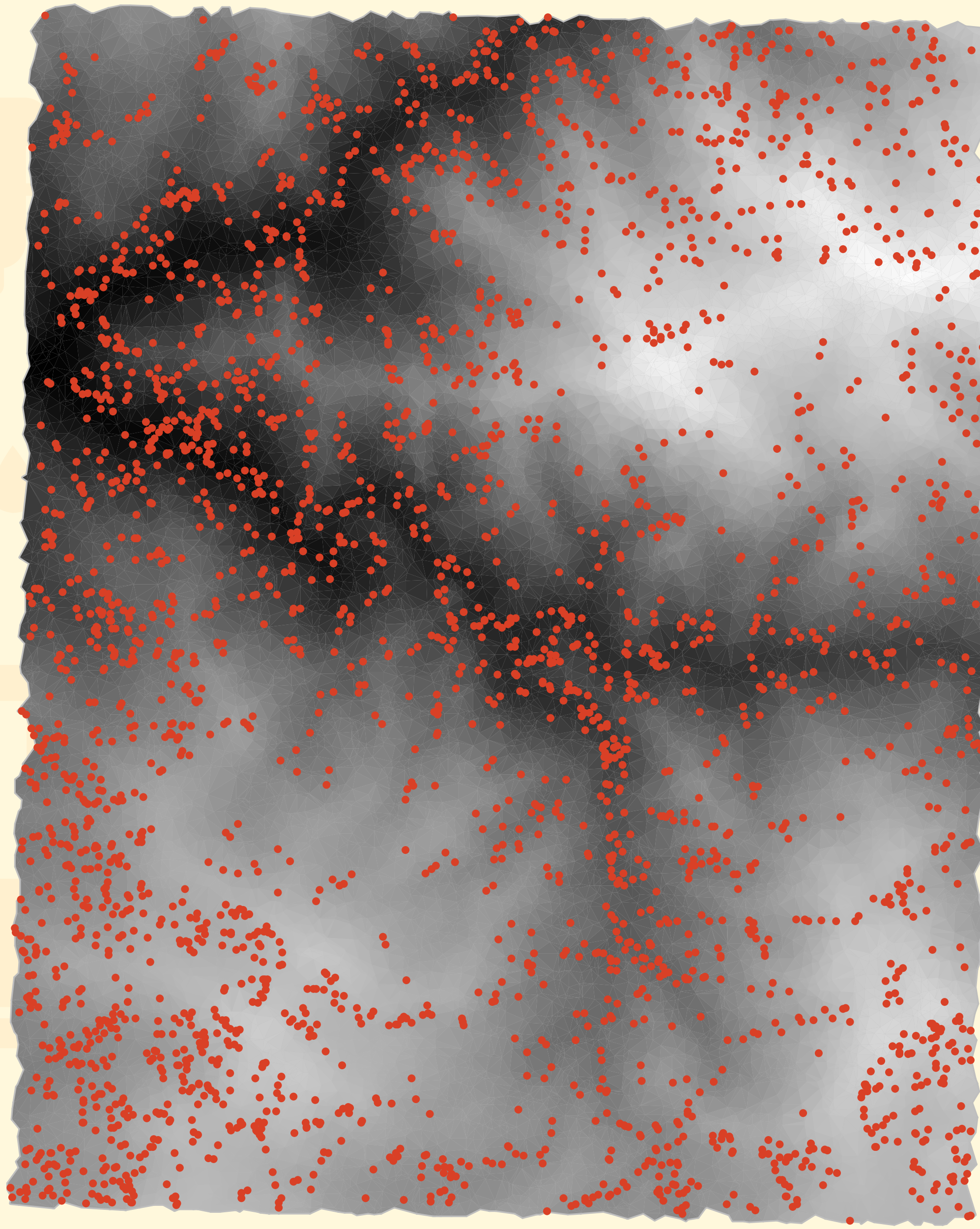


Dominating Guards

Wren Peak



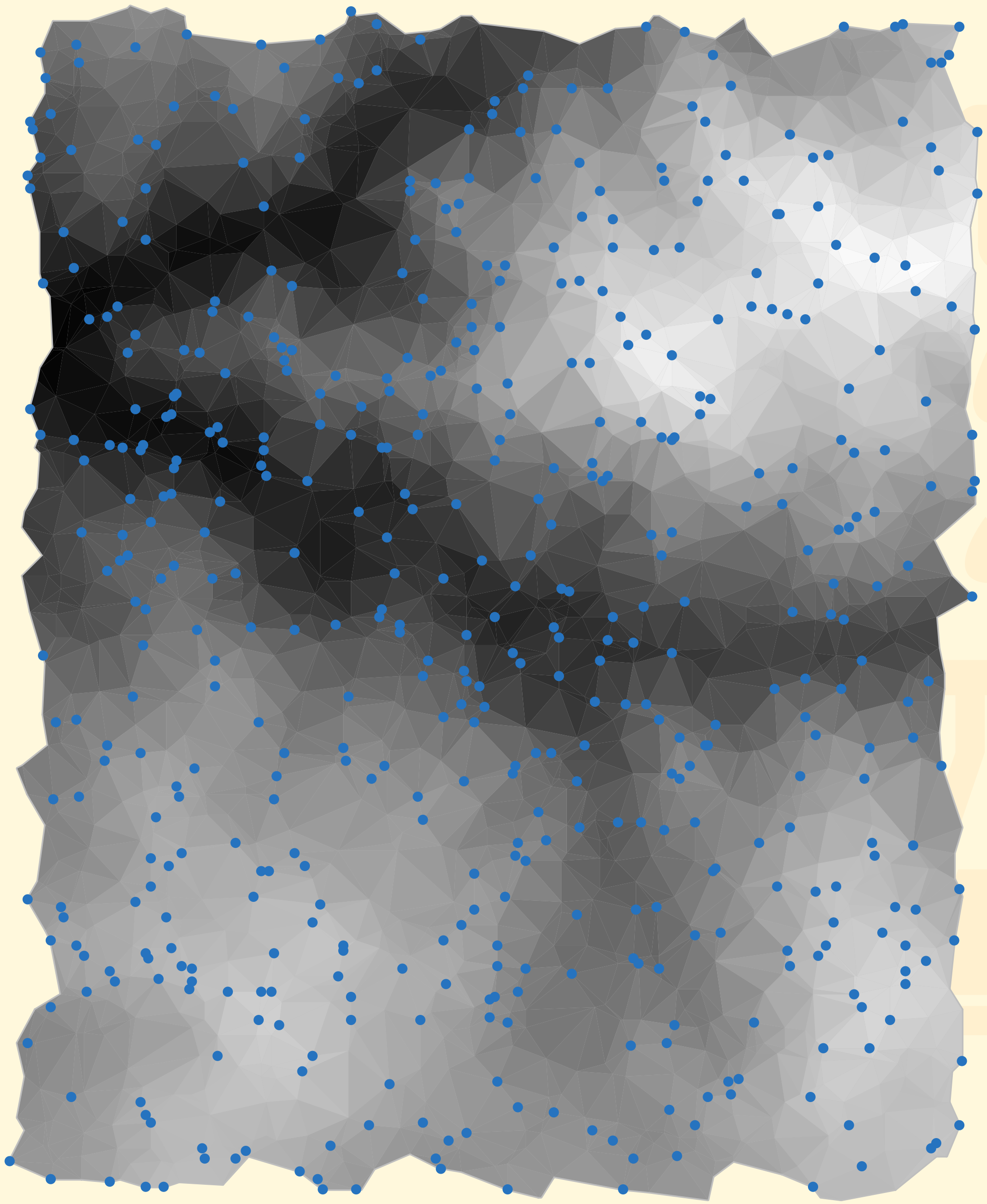
≈ 45%

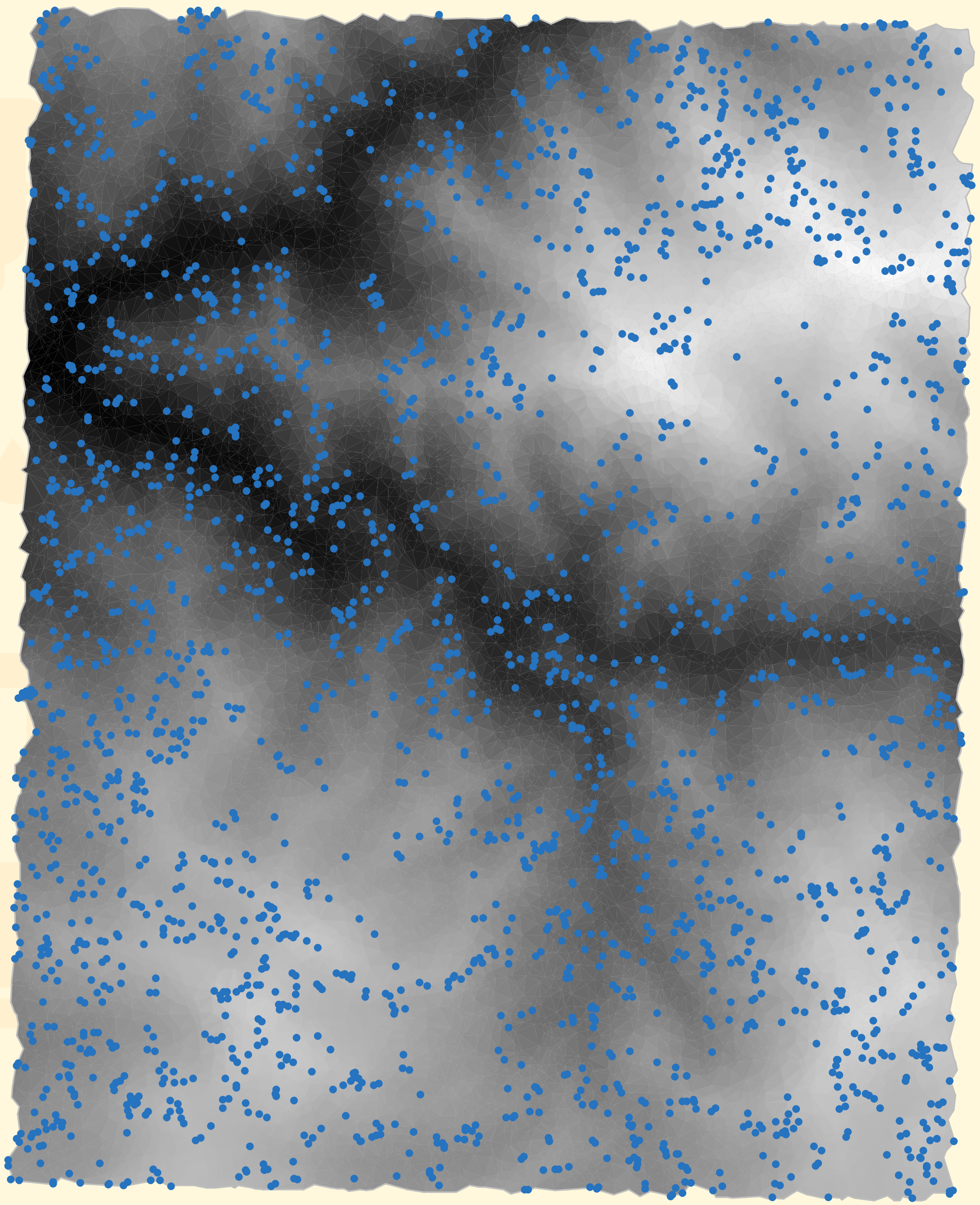


≈ 20%

Dominating Guards

Wren Peak





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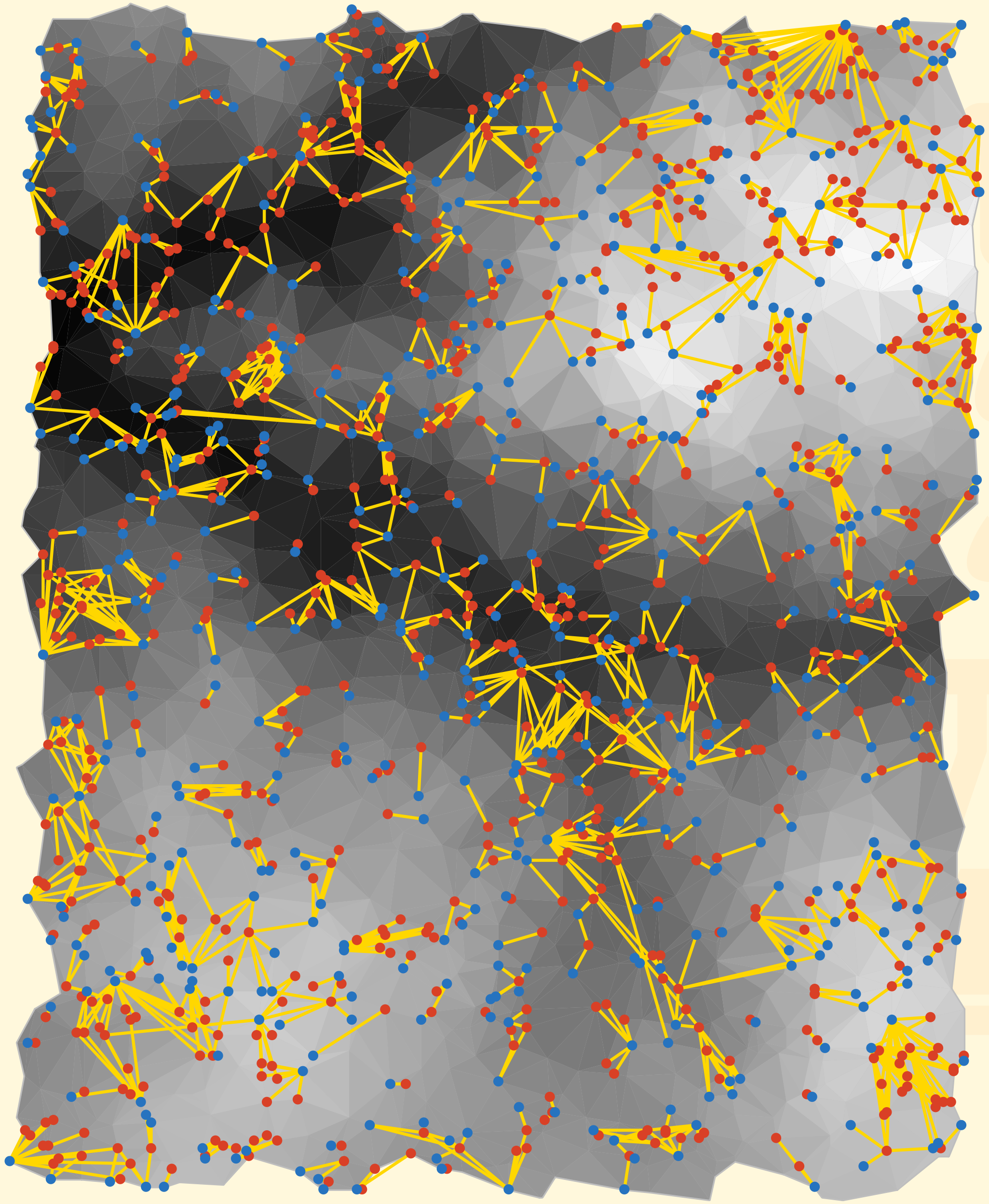
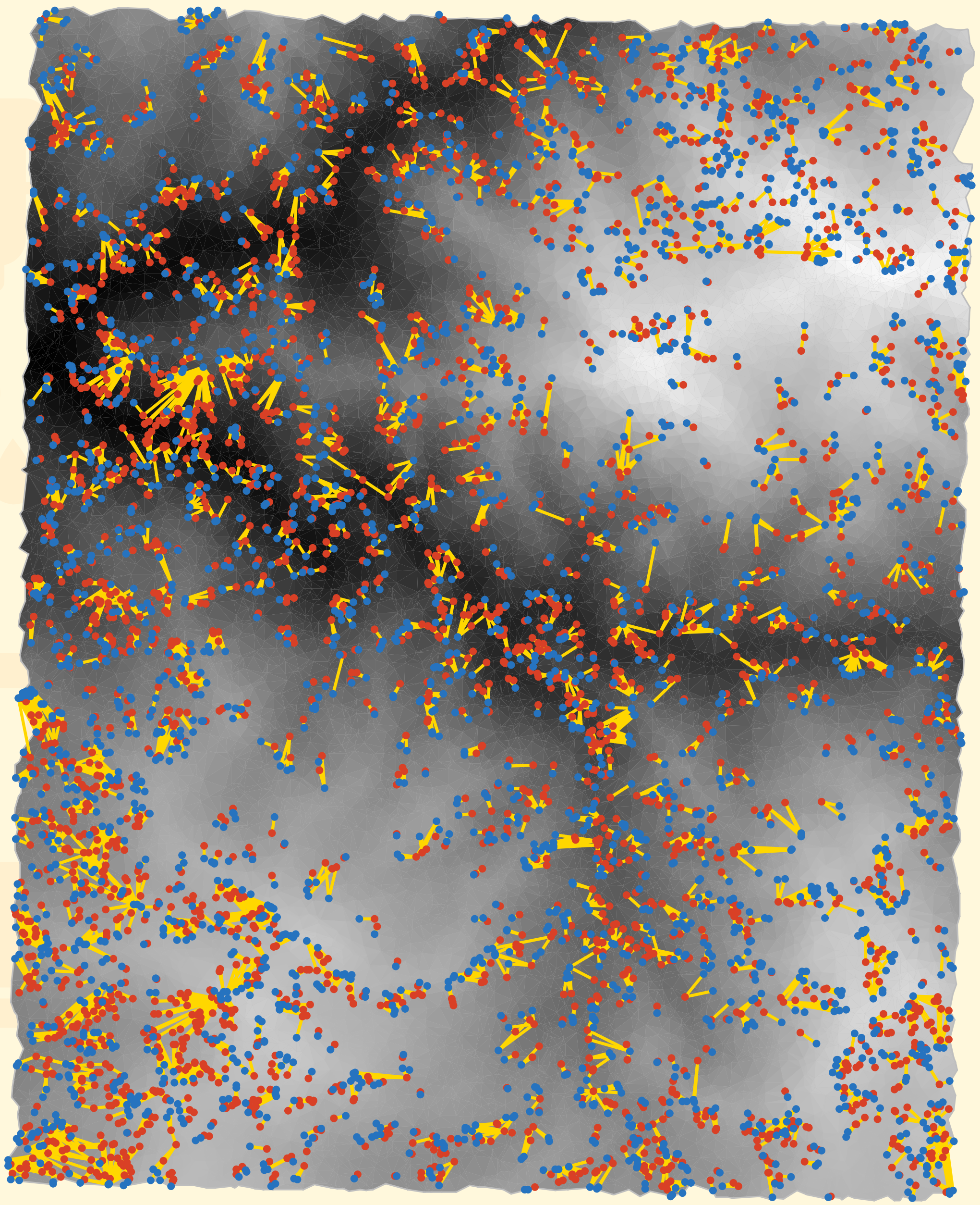
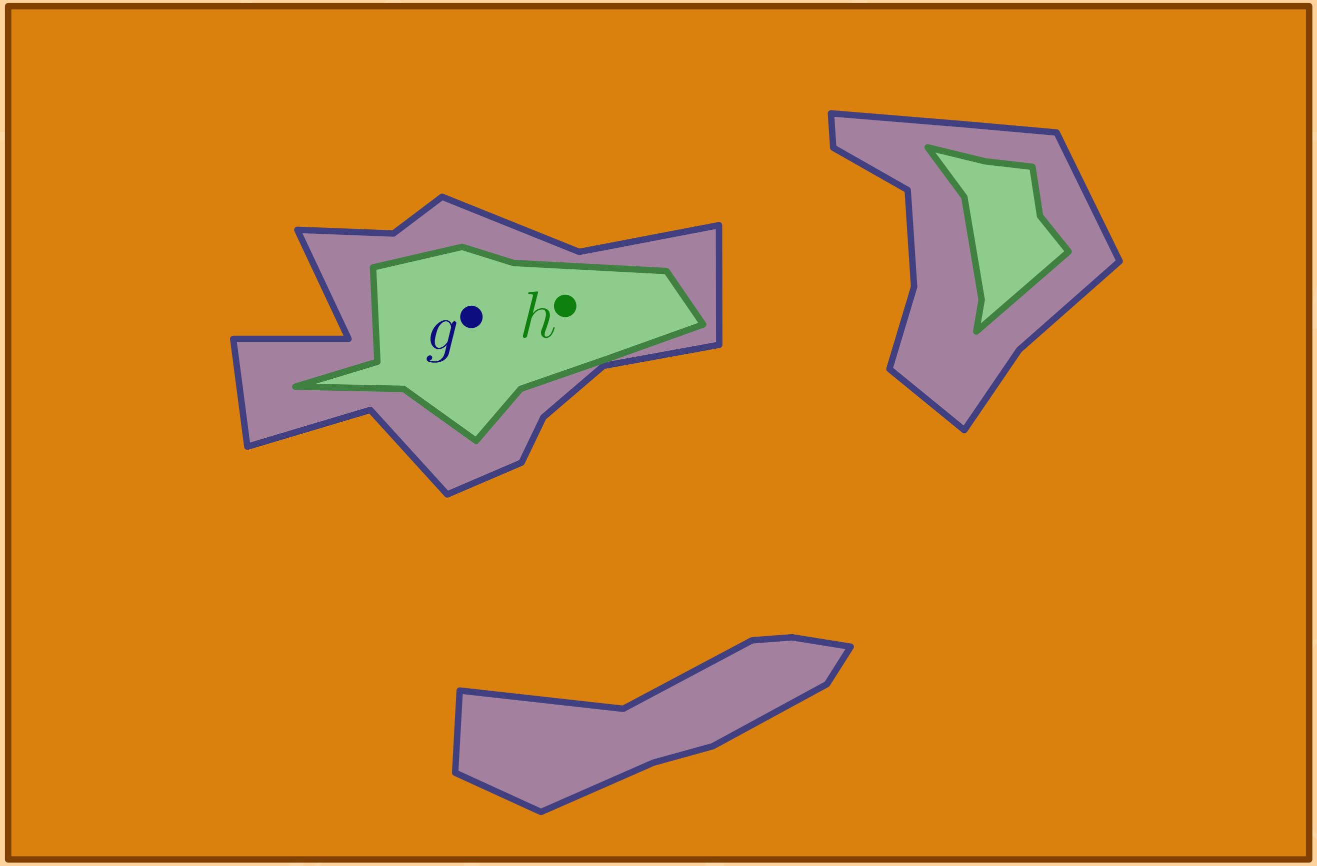




Dominating Guards

g dominates h $\equiv$ $\mathcal{V}(h) \subseteq \mathcal{V}(g)$



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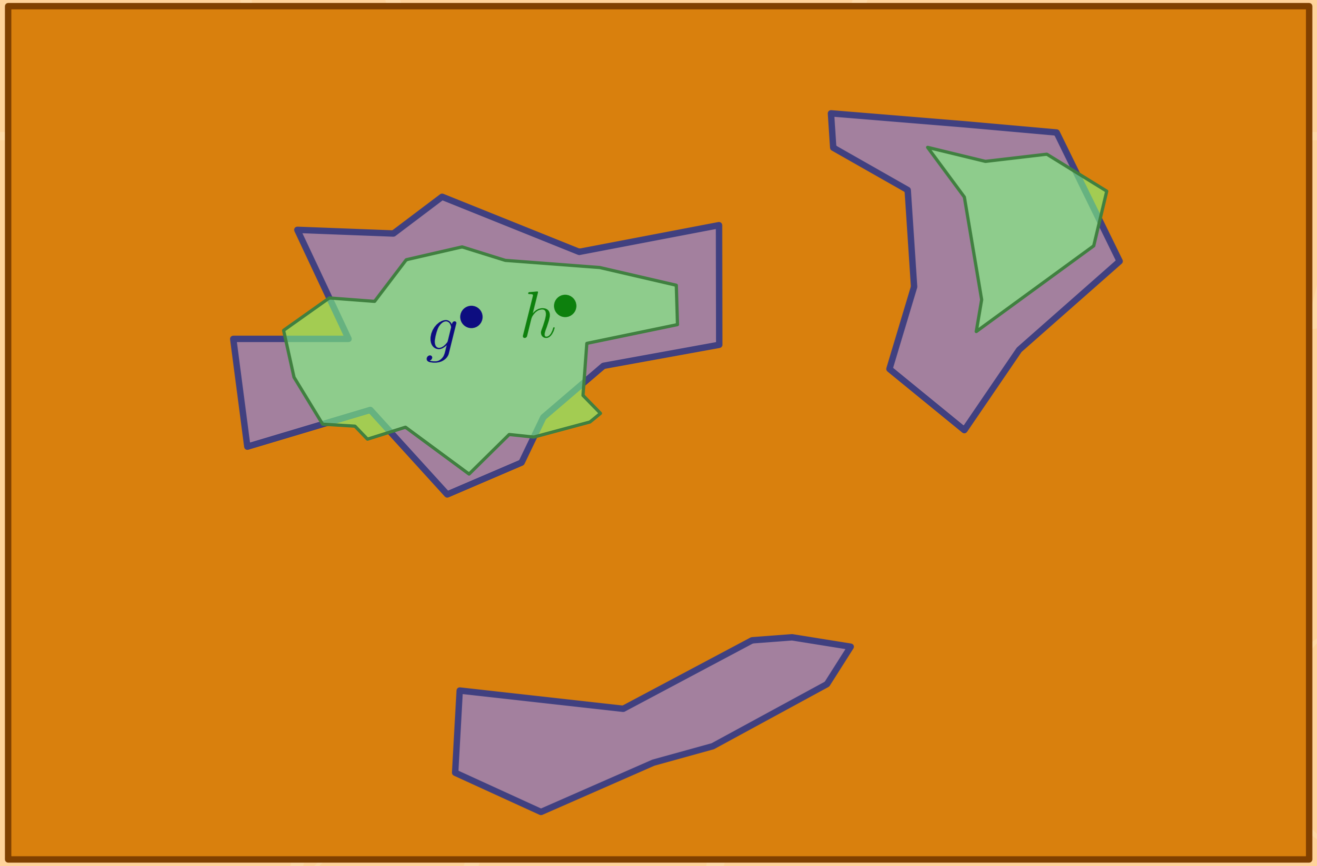
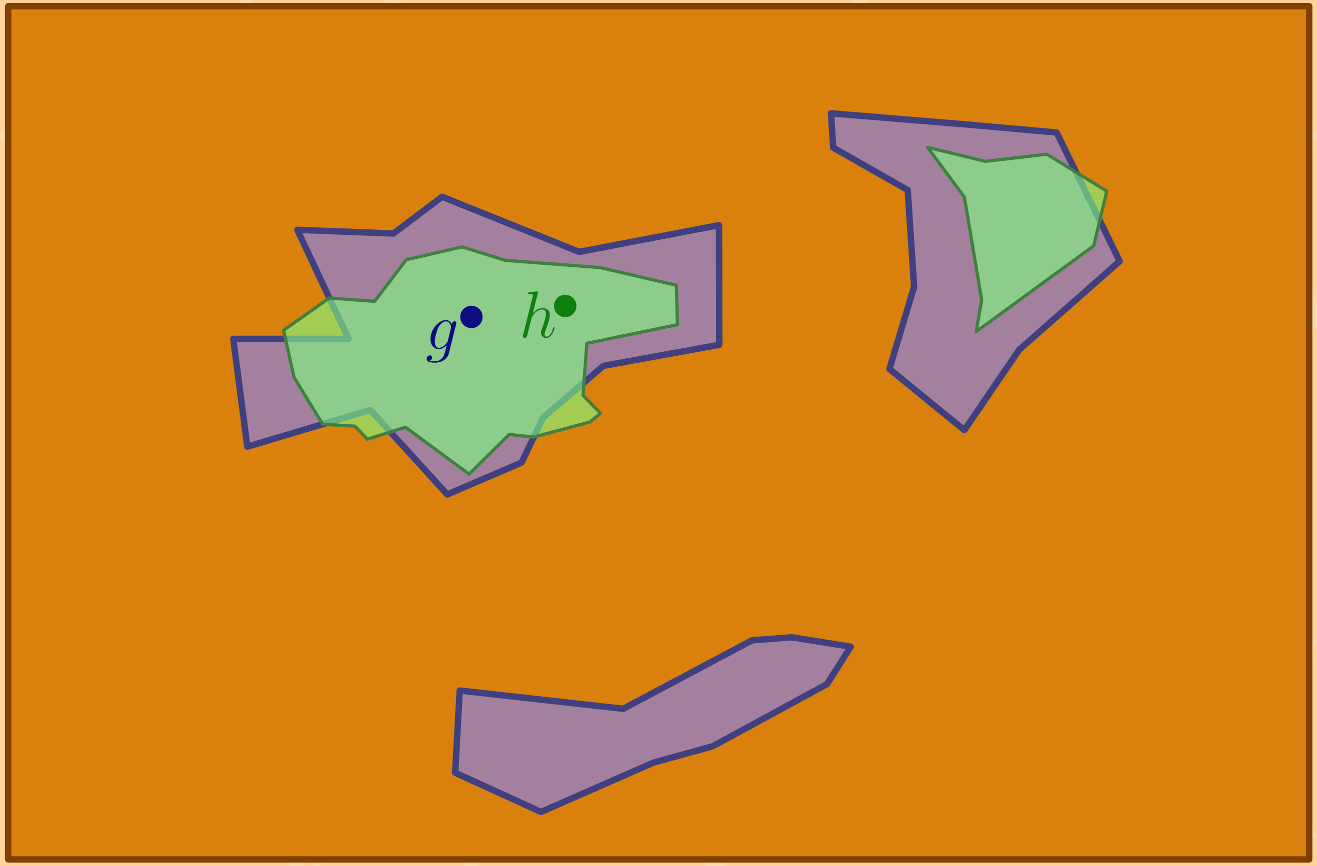


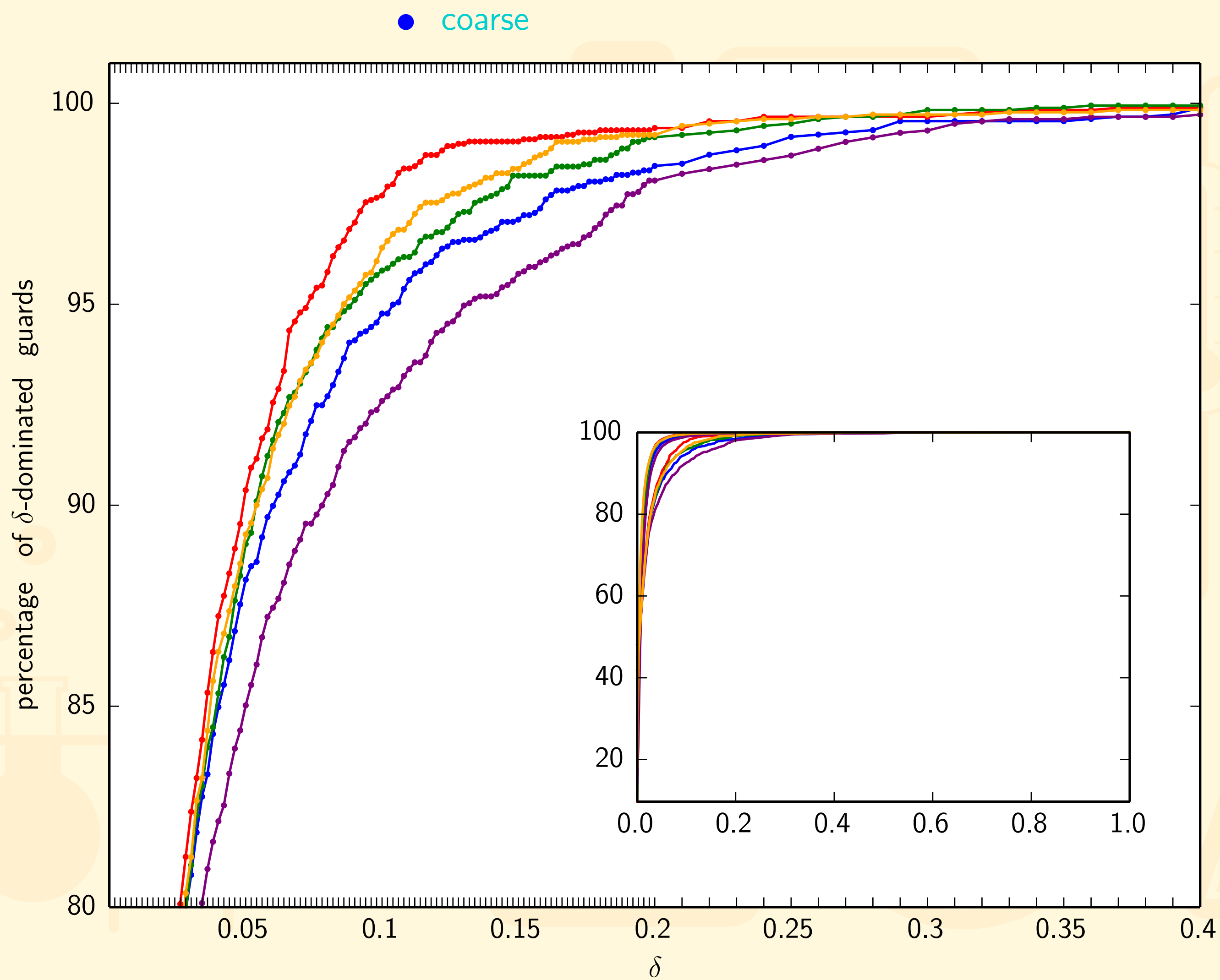
δ-Dominating Guards

g δ-dominates $h \quad \equiv \quad [\![\mathcal{V}(h) \setminus \mathcal{V}(g)]\!] \,/\, [\![\mathcal{V}(h)]\!] \le \delta$



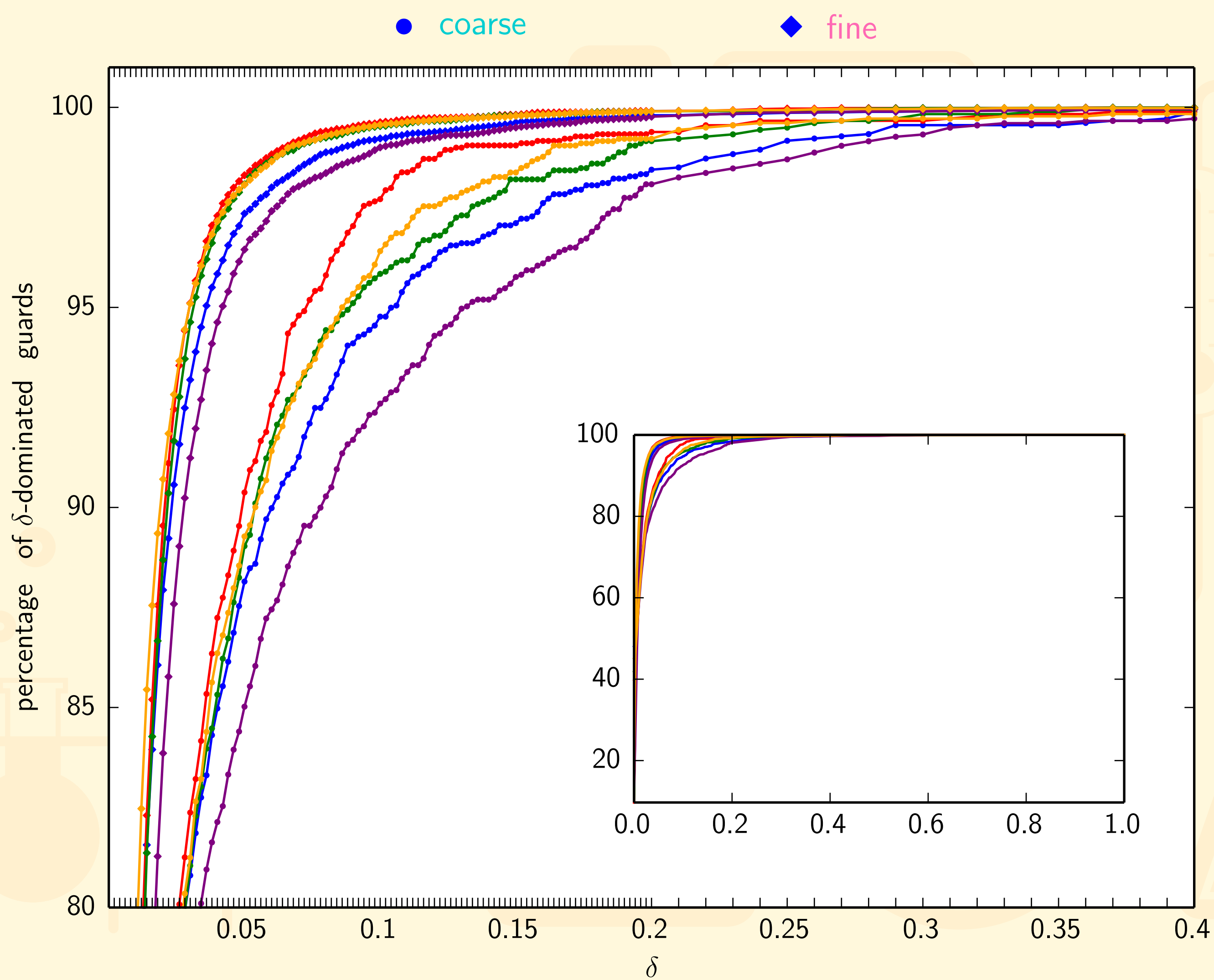
δ-Dominating Guards

Hot Springs Quinn Pk Sphinx Lakes Split Mountain Wren Peak



δ-Dominating Guards

Hot Springs Quinn Pk Sphinx Lakes Split Mountain Wren Peak



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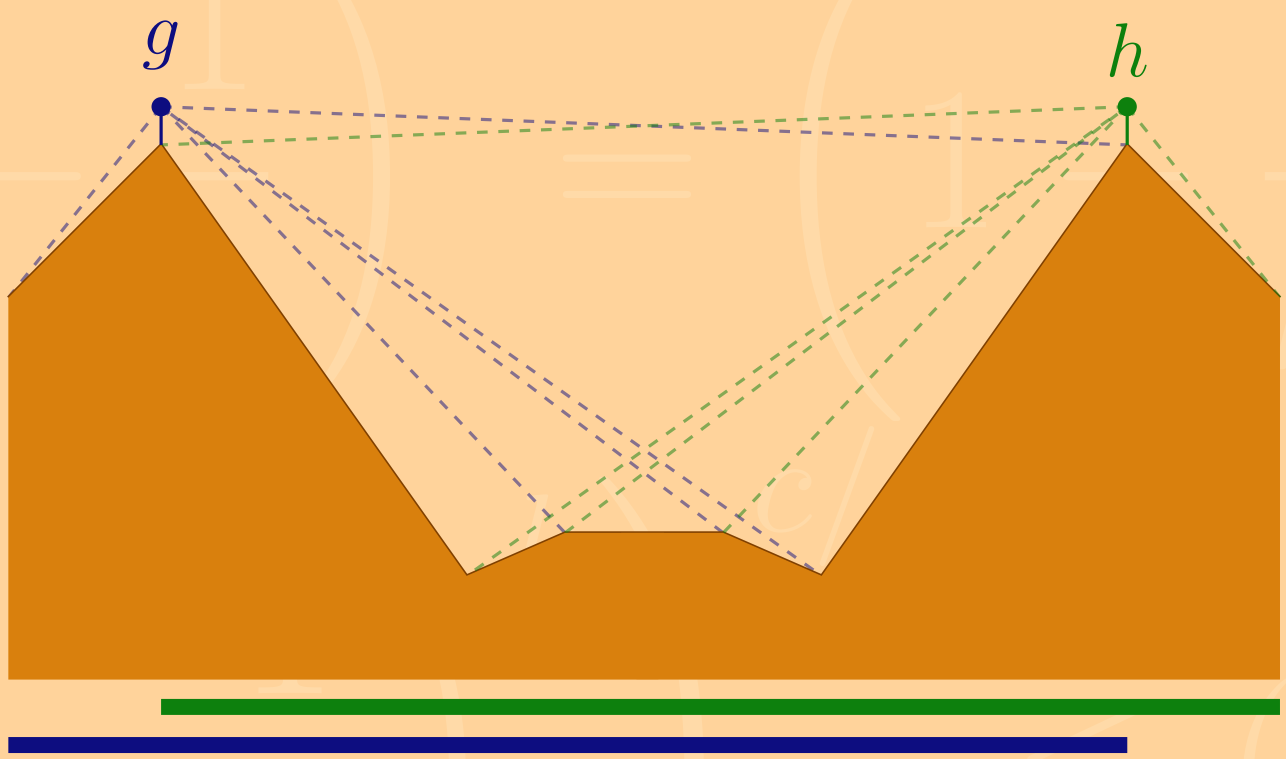

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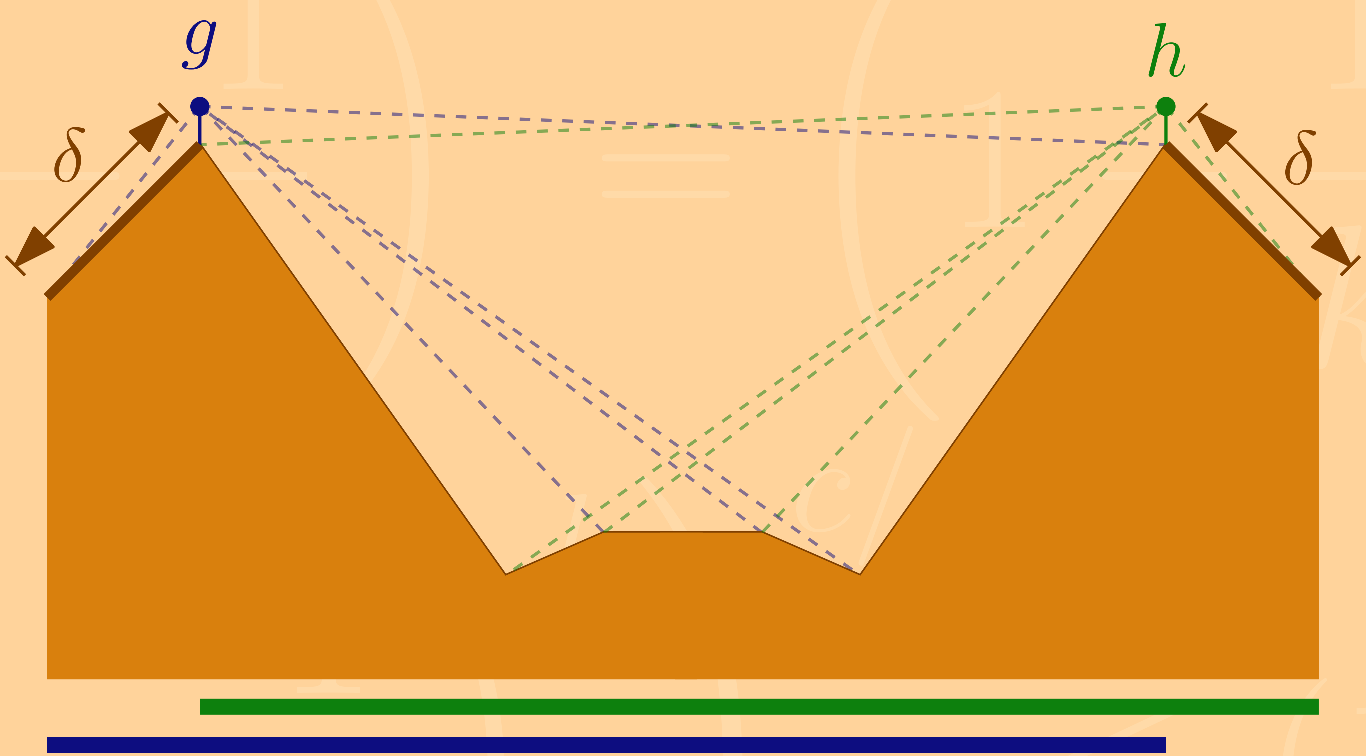

g δ-dominates h

h δ-dominates g

δ-Dominating Guards

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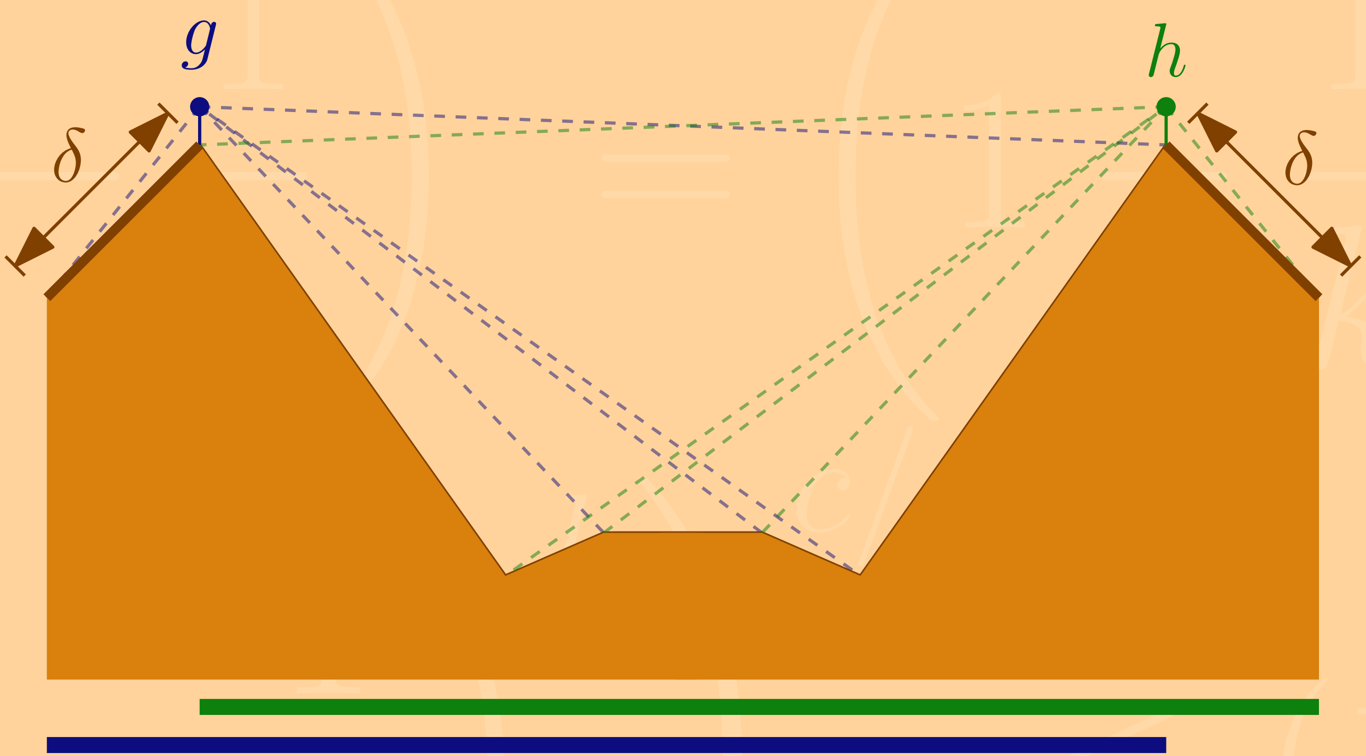

g δ-dominates h

h δ-dominates g

We cannot throw away all δ-dominated guards!

δ-Dominating Guards

g δ-dominates $h \quad \equiv \quad [\![\mathcal{V}(h) \setminus \mathcal{V}(g)]\!] \,/\, [\![\mathcal{V}(h)]\!] \leq \delta$

extend to sets of guards $\mathcal{G}$ and $\mathcal{H}$:

$\mathcal{G}$ δ-dominates $\mathcal{H} \quad \equiv \quad [\![\mathcal{V}(\mathcal{H}) \setminus \mathcal{V}(\mathcal{G})]\!] \,/\, [\![\mathcal{V}(\mathcal{H})]\!] \leq \delta$

Find a minimum size set of guards $\mathcal{D}$ that δ-dominate $\mathcal{P}$.

δ-Dominating Guards

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Find a minimum size set of guards $\mathcal{D}$ that δ-dominate $\mathcal{P}$.

Computing $\mathcal{D}$ is NP-hard

δ-Dominating Guards

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extend to sets of guards $\mathcal{G}$ and $\mathcal{H}$:

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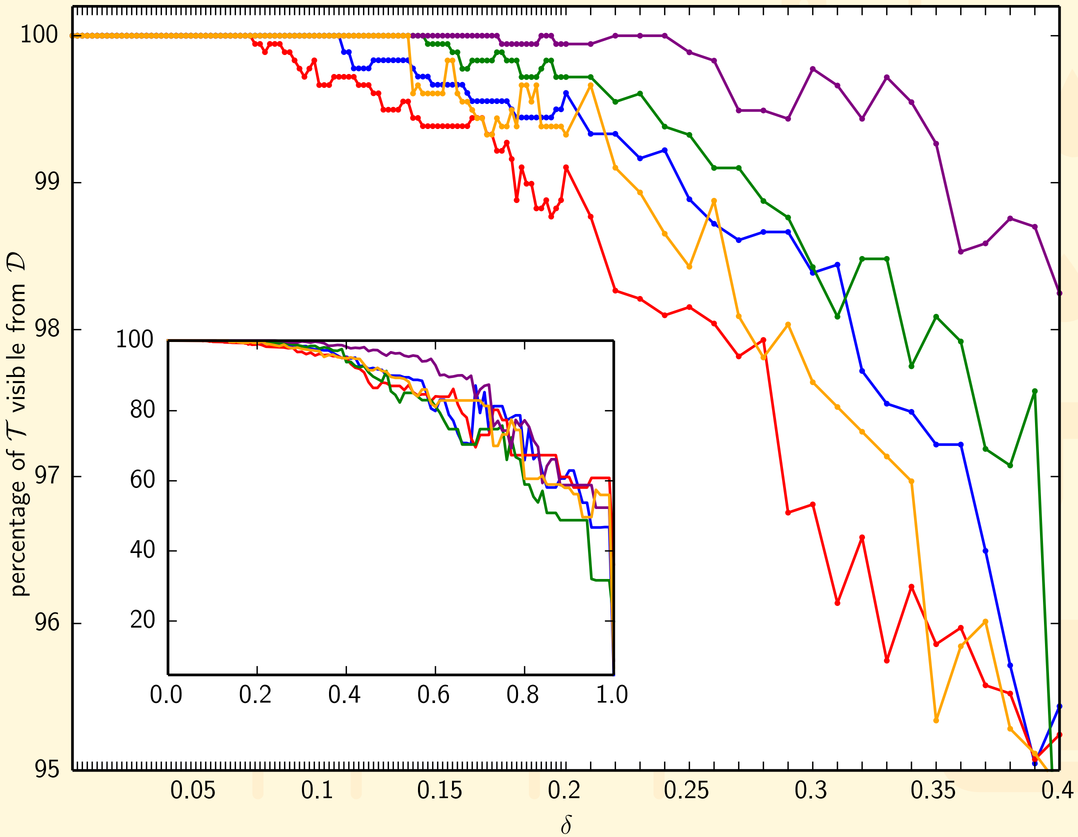
Find a minimal size set of guards $\mathcal{D}$ that δ-dominate $\mathcal{P}$.

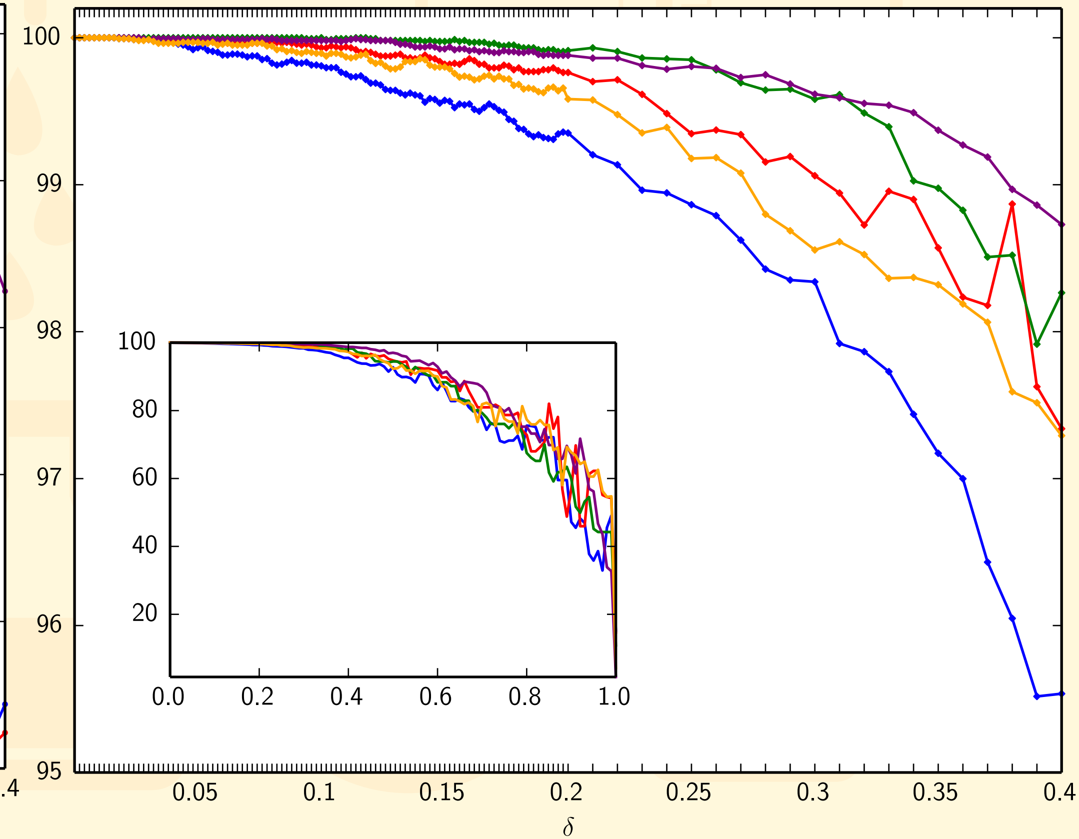
δ-Dominating Guards

Hot Springs Quinn Pk Sphinx Lakes Split Mountain Wren Peak

coarse



fine



Using δ-Domination

Algorithm $\textsc{DominatingGuard}(\mathcal{T}, \varepsilon, \delta, \mathcal{P})$

1. Compute the viewsheds for all guards in $\mathcal{P}$.
2. Compute a minimal set of guards $\mathcal{D}$ that δ-dominates $\mathcal{P}$.
3. Let $\hat{\delta} = [\![\mathcal{V}(\mathcal{D})]\!] \,/\, [\![\mathcal{V}(\mathcal{P})]\!]$ be the fraction of $\mathcal{V}(\mathcal{P})$ covered by $\mathcal{D}$.
4. Let $\gamma = (\varepsilon - \delta)/(1 - \hat{\delta})$ and let $\hat{\mathcal{T}} = \mathcal{V}(\mathcal{D})$.
5. **return** $\textsc{GreedyGuard}(\hat{\mathcal{T}}, \gamma, \mathcal{D})$

Using δ-Domination

Algorithm $\textsc{DominatingGuard}(\mathcal{T}, \varepsilon, \delta, \mathcal{P})$

1. Compute the viewsheds for all guards in $\mathcal{P}$.
2. Compute a minimal set of guards $\mathcal{D}$ that δ-dominates $\mathcal{P}$.
3. Let $\hat{\delta} = [\![\mathcal{V}(\mathcal{D})]\!] \,/\, [\![\mathcal{V}(\mathcal{P})]\!]$ be the fraction of $\mathcal{V}(\mathcal{P})$ covered by $\mathcal{D}$.
4. Let $\gamma = (\varepsilon - \delta)/(1 - \hat{\delta})$ and let $\hat{\mathcal{T}} = \mathcal{V}(\mathcal{D})$.
5. **return** $\textsc{AnyAlgorithmToComputeAn}\varepsilon\textsc{-Cover}(\hat{\mathcal{T}}, \gamma, \mathcal{D})$

$$\left(1 - \frac{1}{k}\right)^{\ell} \left(1 - \frac{1}{k}\right)^{ck/\ldots}$$

$$\left(\left(1 - \frac{1}{k}\right)^{k}\right)^{c/\varepsilon} \leq (1/e)^{c\ldots}$$

Using δ-Domination

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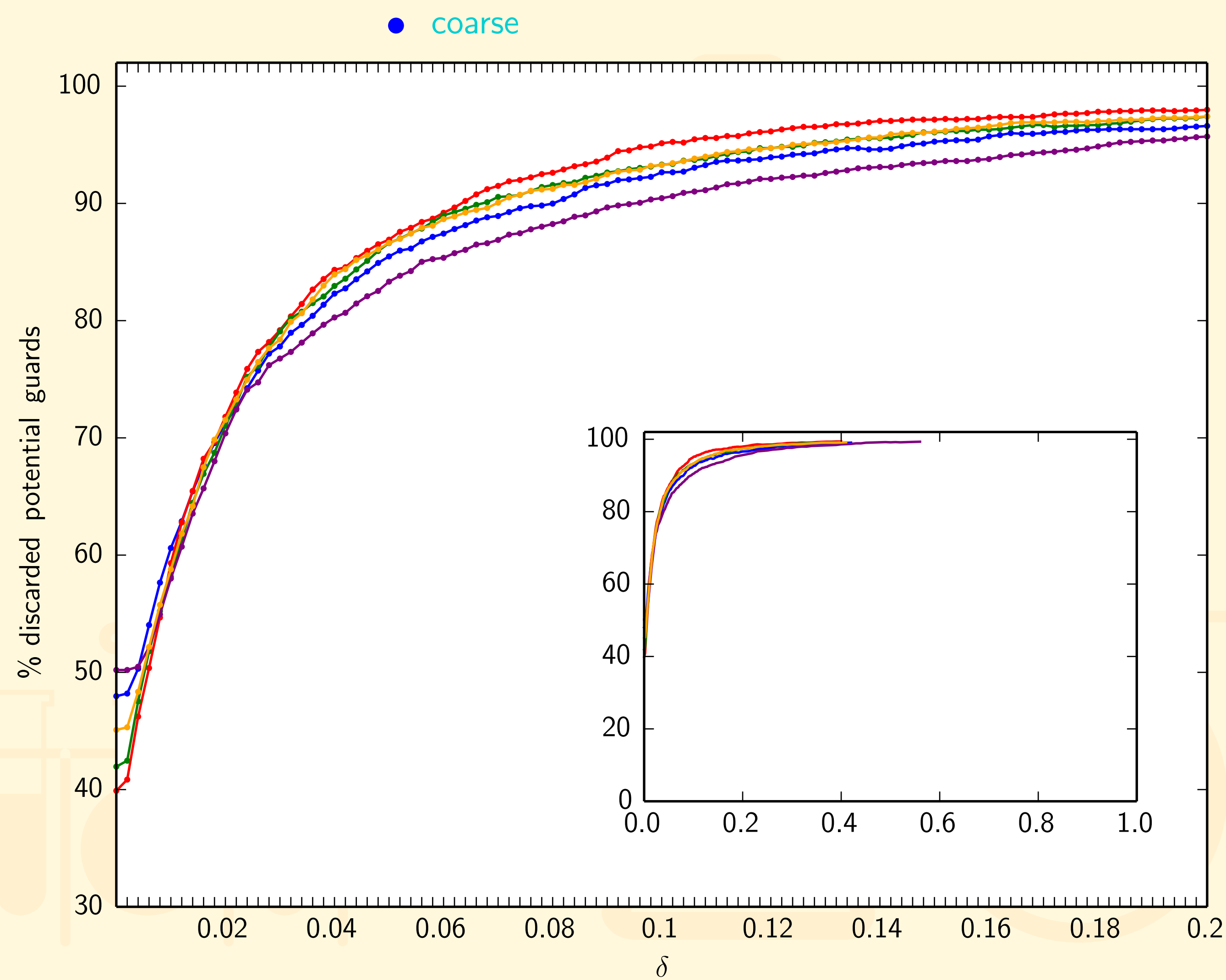
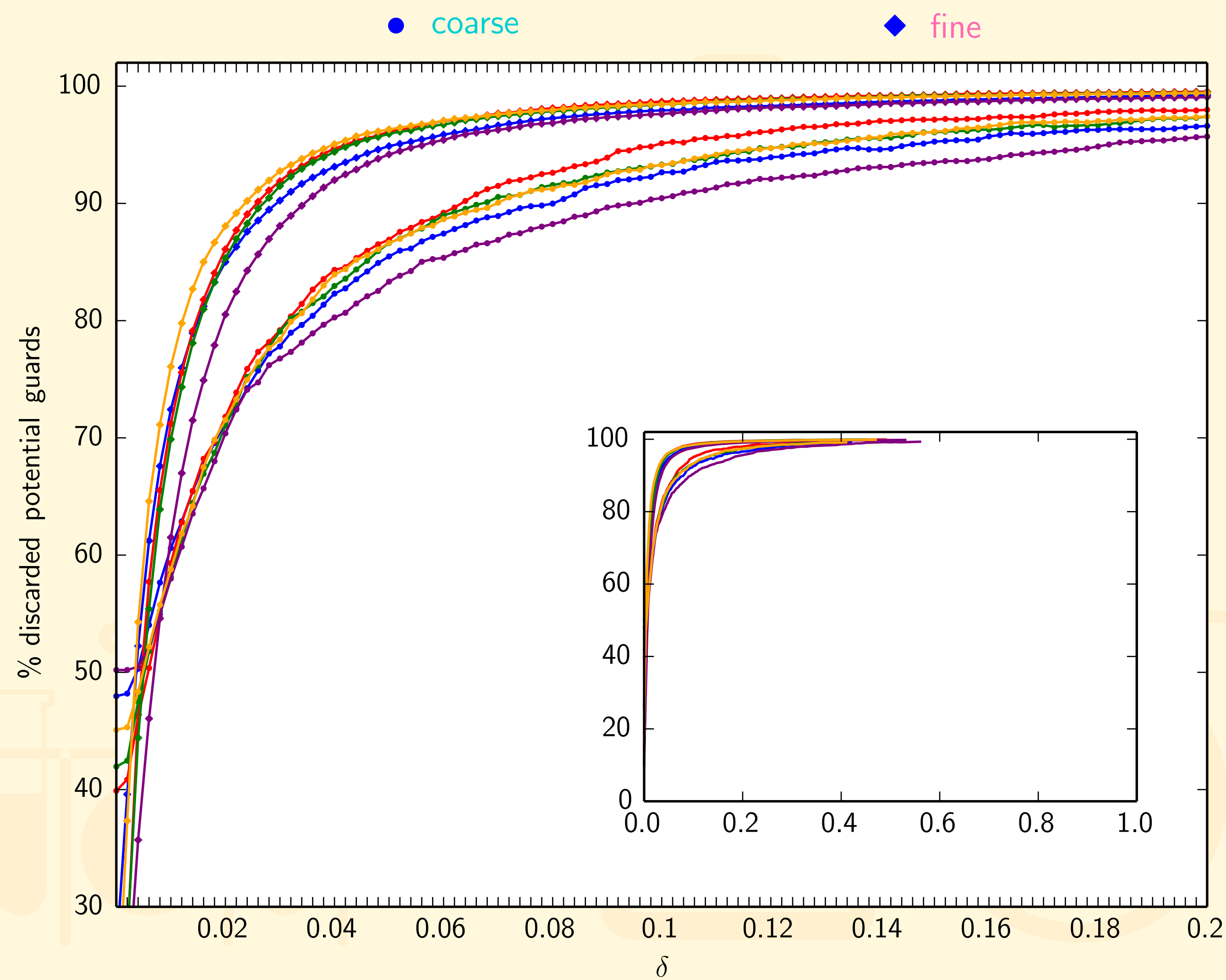


$\varepsilon = 0.05$

Using δ-Domination

Hot Springs Quinn Pk Sphinx Lakes Split Mountain Wren Peak



$\varepsilon = 0.05$

Using δ-Domination

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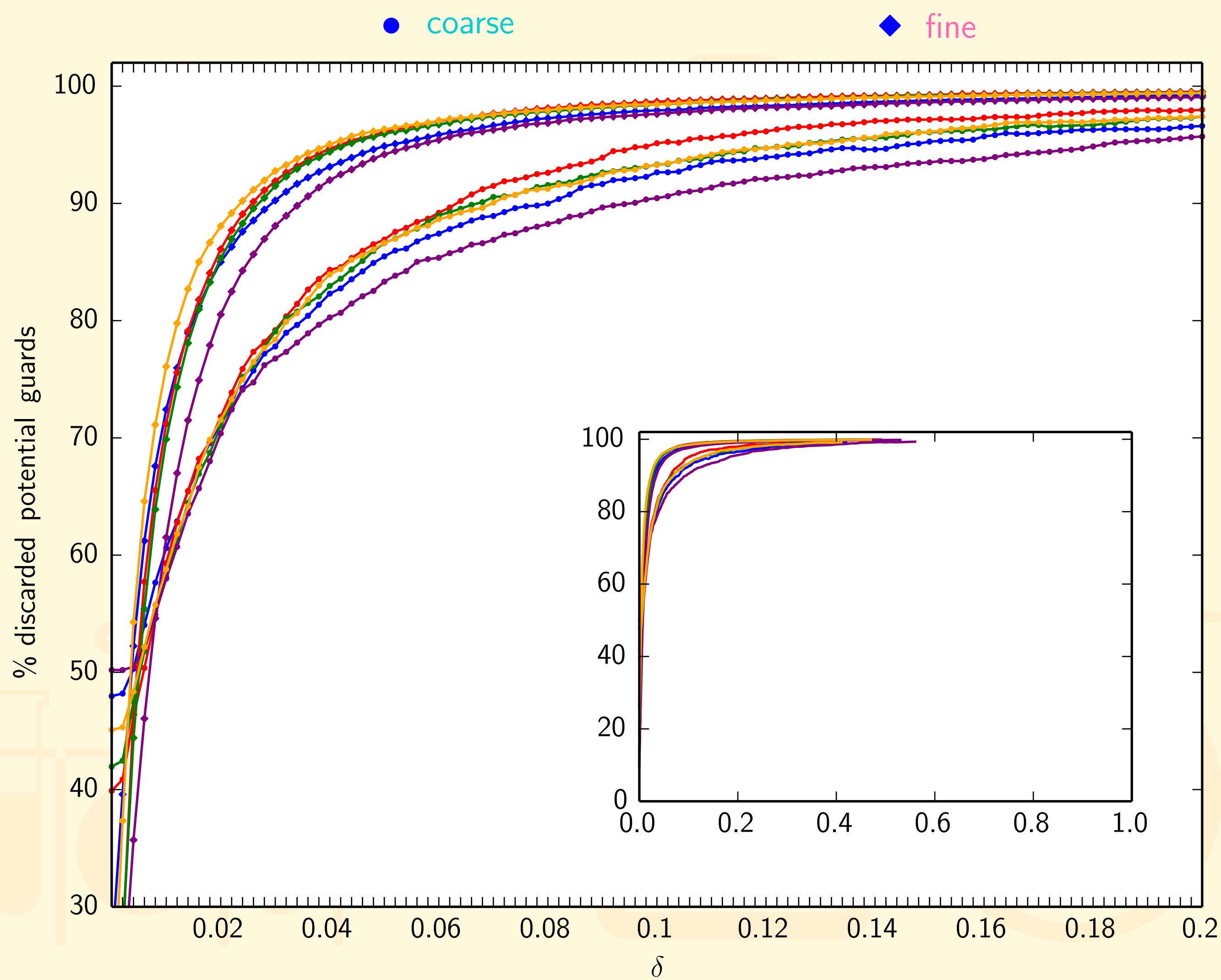

$\varepsilon = 0.05$

$\#$ of guards in $\mathcal{G}$ was the same for all δ.

Future Work

Quality guarantees on δ-domination.

Measure $[\![\mathcal{V}(g)]\!]$ by area instead of # vertices.

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Quality guarantees on δ-domination.

Measure $[\![\mathcal{V}(g)]\!]$ by area instead of # vertices.

Thank you!