On the complexity of minimum-link path problems

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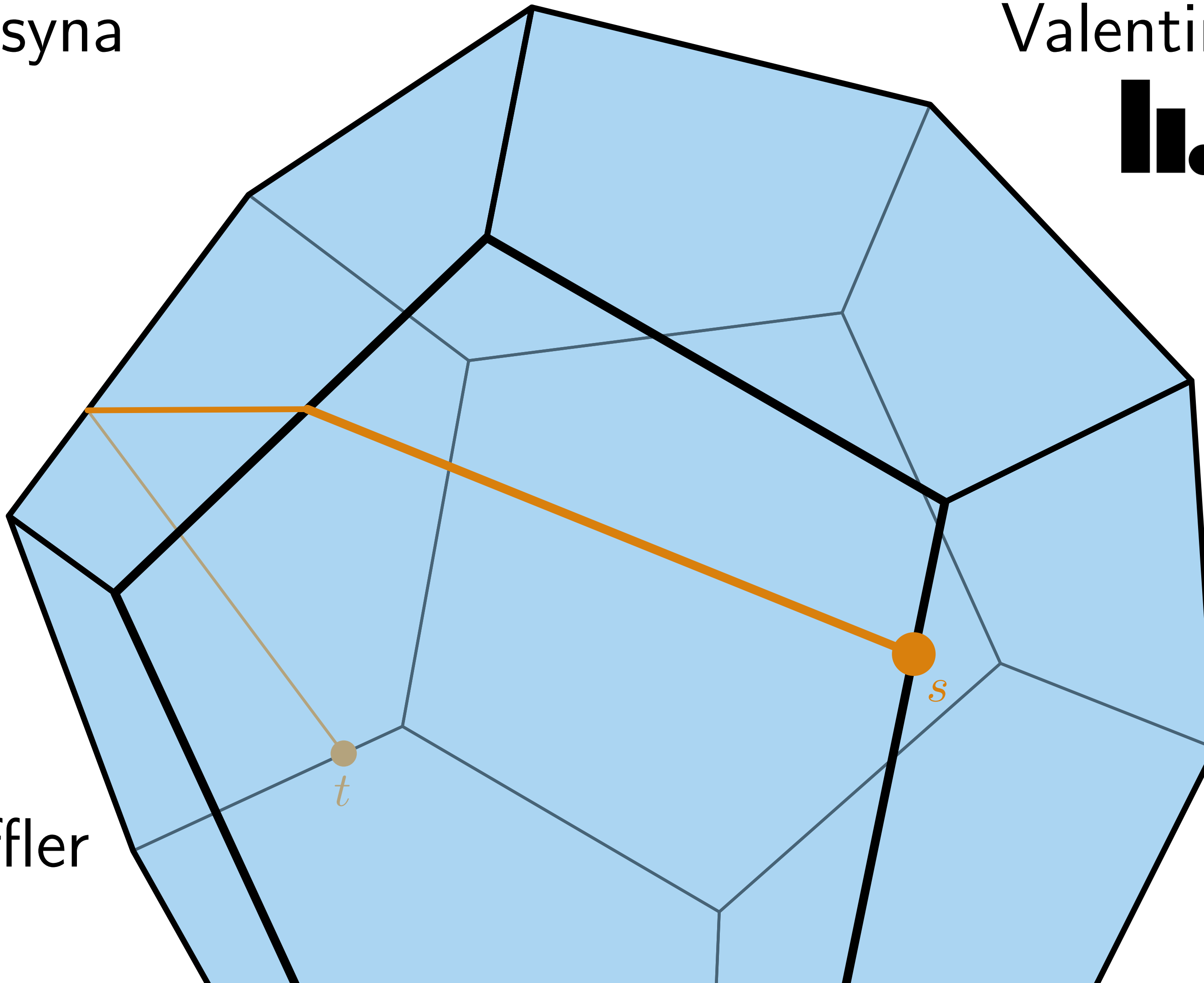
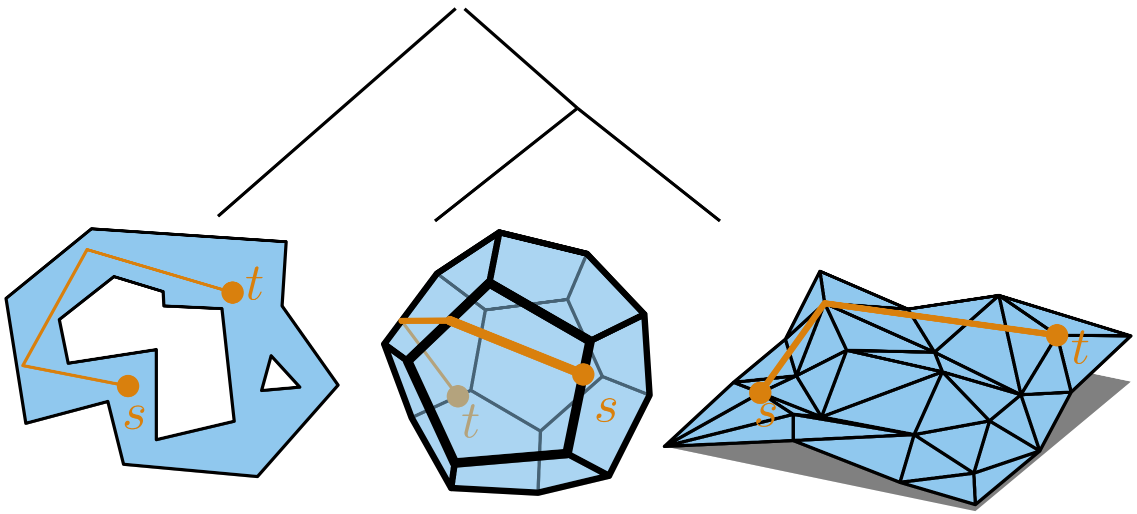


Maarten Löffler

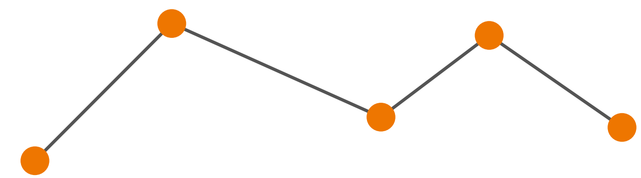
madalgo
CENTER FOR MASSIVE DATA ALGORITHMICS
Frank Staals

Minimum-link path problems

Given a domain D, and two points $s, t \in D$ find a minimum-link path P between s and t,



s.t. the bends of P lie in $D|^a$,



and the links of P lie in $D|^b$

Minimum-link path problems

a \ b	1	2 (faces)	3 (anywhere)
0 (vertices)			
1 (edges)			
2 (faces)			
3 (anywhere)			

Minimum-link path problems

a b	1	2 (faces)	3 (anywhere)
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Minimum-link path problems

a \ b	1	2 (faces)	3 (anywhere)
0 (vertices)	$O(n)$	$O^*(n^2)$	$O^*(n^2)$
1 (edges)			
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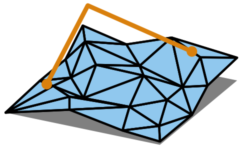
3 (anywhere)			 $O(1)$

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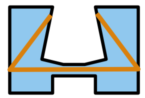
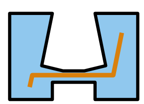
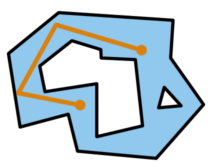
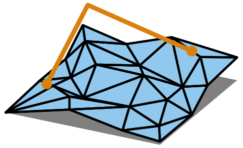
1 (edges)		 $O(n^9)$ [Aronov et al., 2006]	
2 (faces)		 $O(n)$ [Suri, 1986]  $O^*(n^2)$ [Mitchell et al., 1992]	
3 (anywhere)			 $O(1)$

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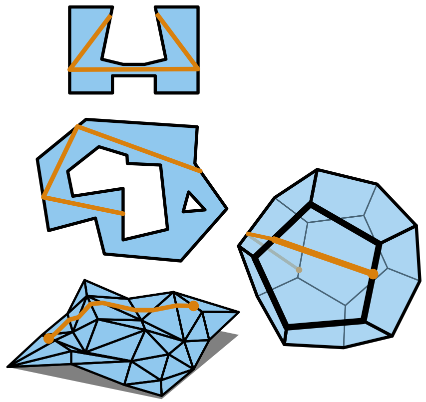
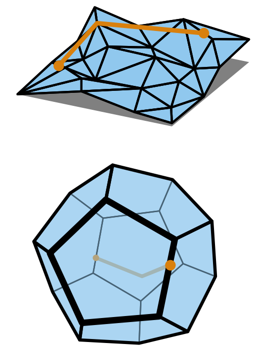
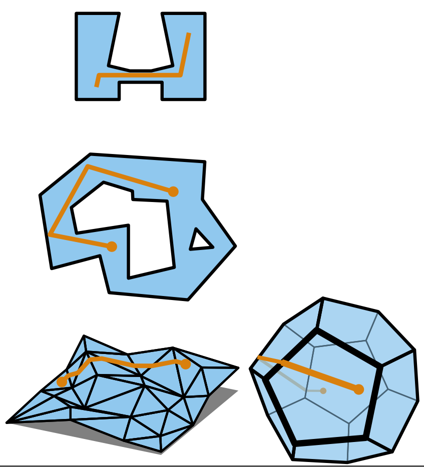
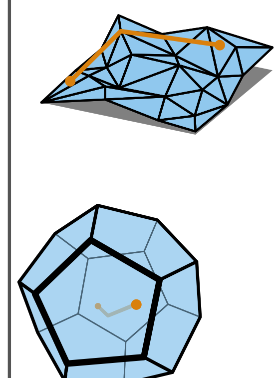
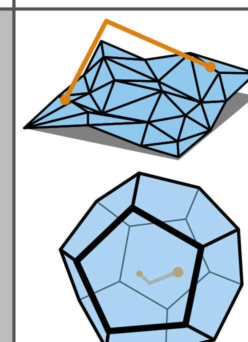
1 (edges)		 $O(n^9)$ [Aronov et al., 2006]	
2 (faces)		 $O(n)$ [Suri, 1986] $O^*(n^2)$ [Mitchell et al., 1992]	
3 (anywhere)			 $O(1)$

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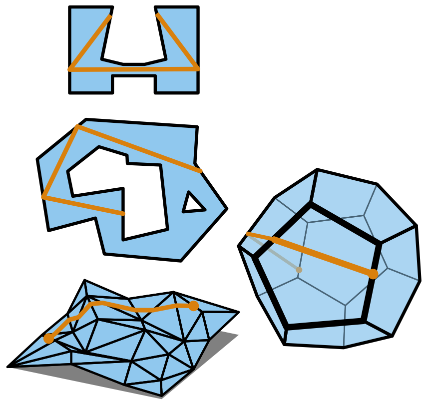
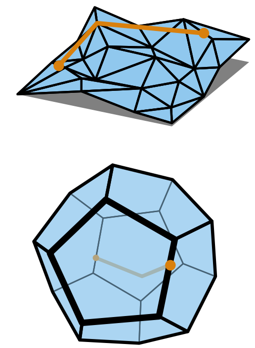
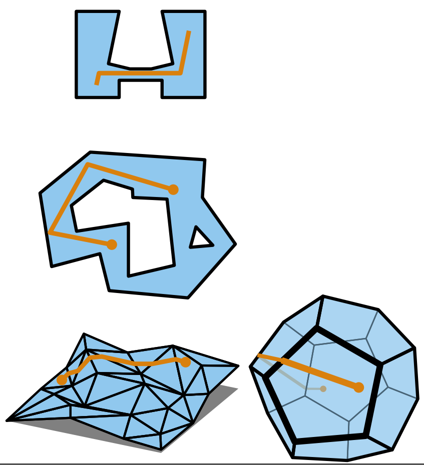
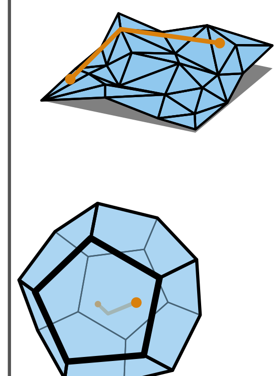
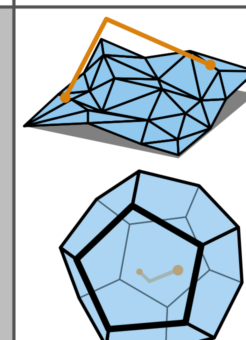
1 (edges)		 $O(n^9)$ [Aronov et al., 2006] Open	 Open
2 (faces)		 $O(n)$ [Suri, 1986] $O^*(n^2)$ [Mitchell et al., 1992] Open	 Open
3 (anywhere)			 $O(1)$ Open

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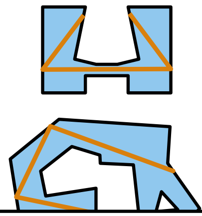
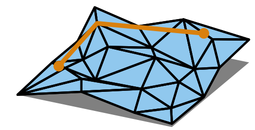
1 (edges)		 $O(n^9)$ [Aronov et al., 2006]	 Open
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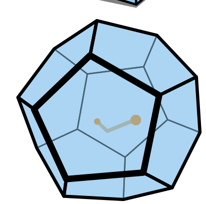
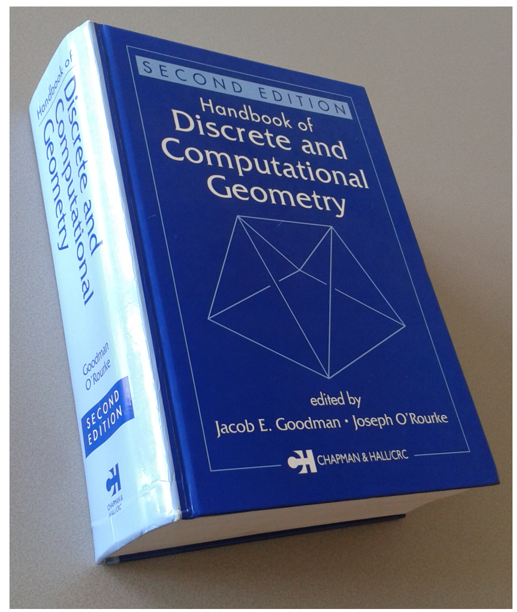
3 (anywhere)			)  Open



“What is the complexity of the minimum-link path problem in 3-space?”

Minimum-link path problems

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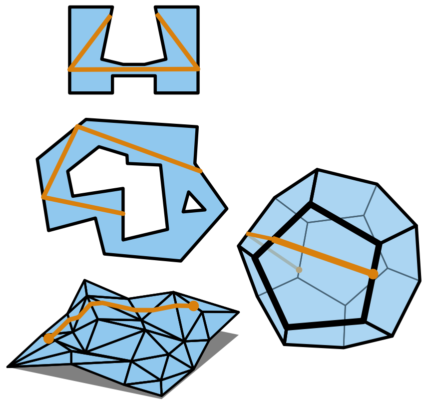
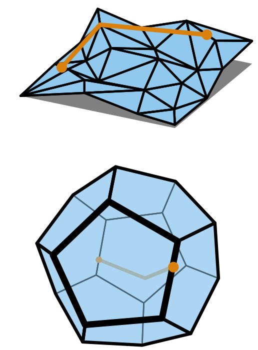
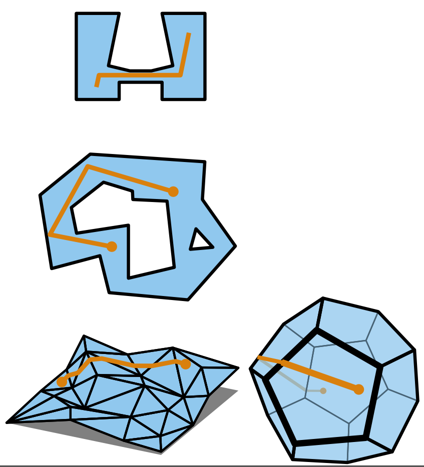
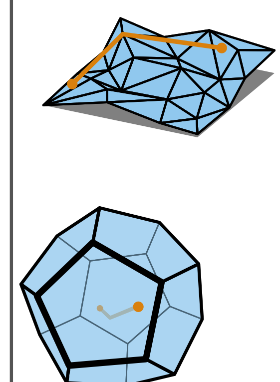
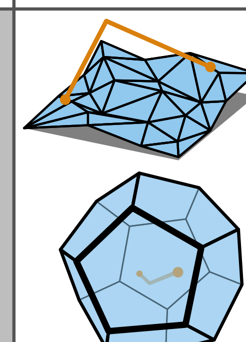
1 (edges)		 $O(n^9)$ [Aronov et al., 2006] NP-hard	 NP-hard
2 (faces)		 $O(n)$ [Suri, 1986] $O^*(n^2)$ [Mitchell et al., 1992] NP-hard	 NP-hard
3 (anywhere)			 $O(1)$ NP-hard

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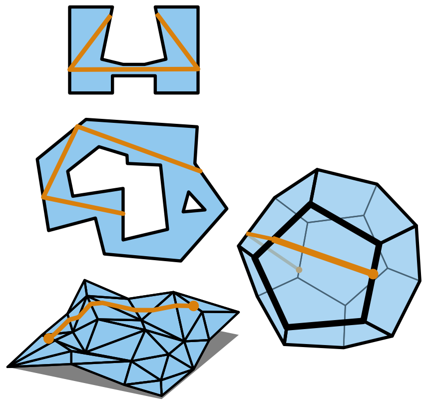
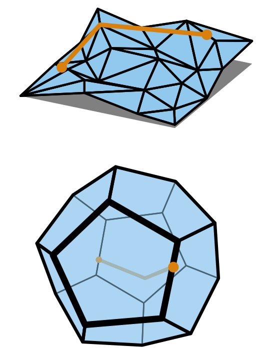
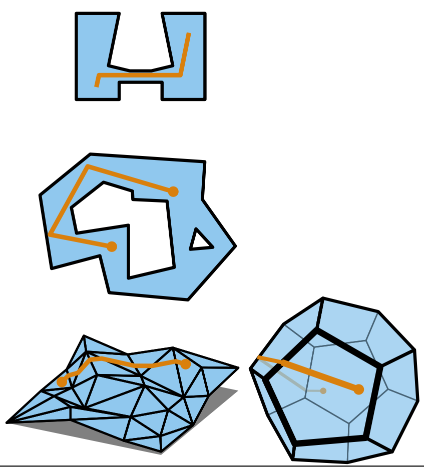
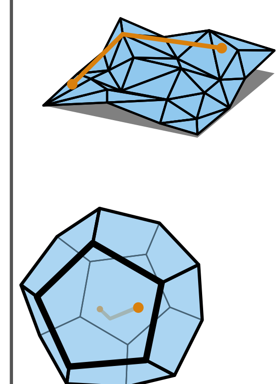
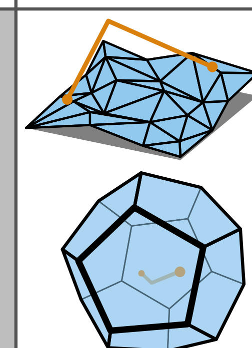
1 (edges)		 $O(n^9)$ [Aronov et al., 2006] NP-hard no FPTAS	 NP-hard no FPTAS
2 (faces)		 $O(n)$ [Suri, 1986] $O^*(n^2)$ [Mitchell et al., 1992] NP-hard no FPTAS	 NP-hard no FPTAS
3 (anywhere)			 $O(1)$ NP-hard no FPTAS

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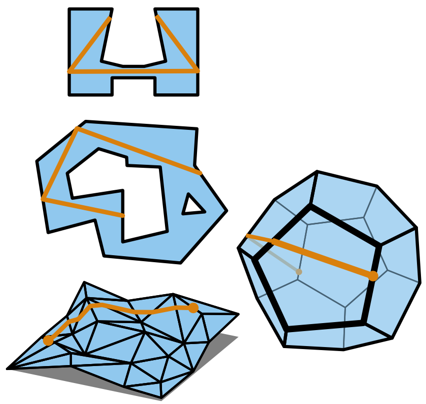
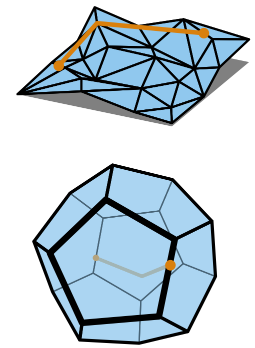
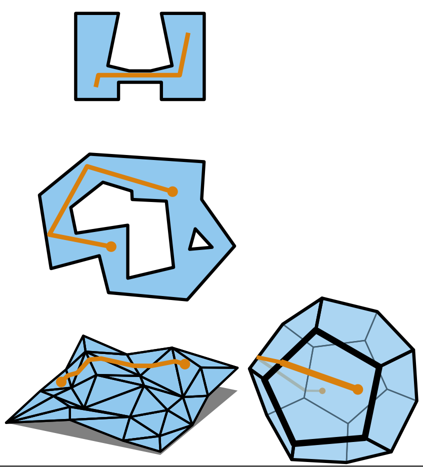
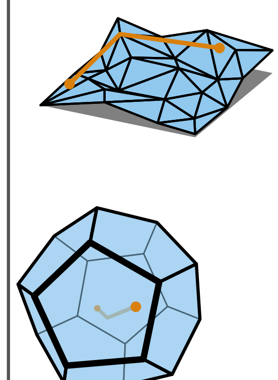
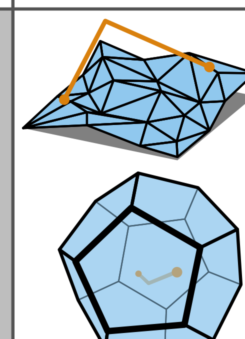
1 (edges)		 $O(n^9)$ [Aronov et al., 2006] NP-hard no FPTAS PTAS	 NP-hard no FPTAS PTAS
2 (faces)		 $O(n)$ [Suri, 1986] $O^*(n^2)$ [Mitchell et al., 1992] NP-hard no FPTAS PTAS	 NP-hard no FPTAS PTAS
3 (anywhere)			 $O(1)$ NP-hard no FPTAS PTAS

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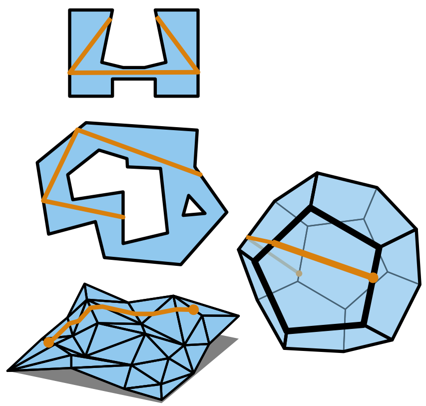
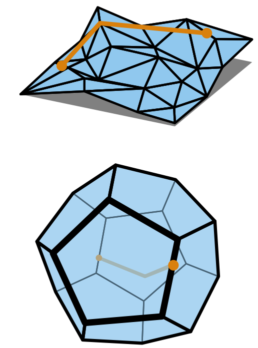
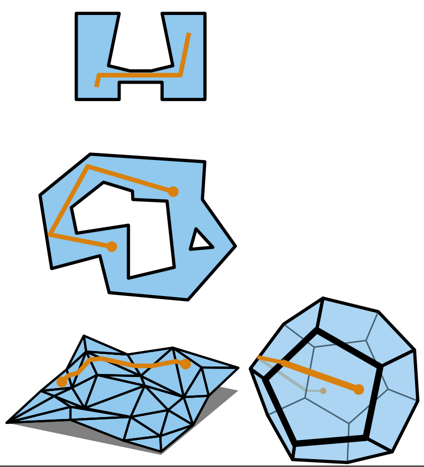
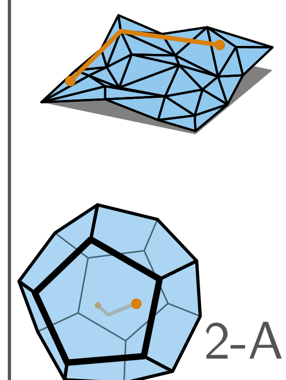
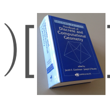
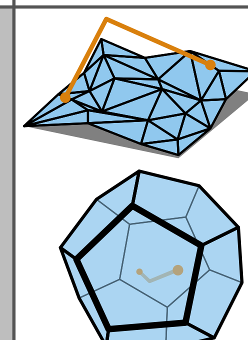
1 (edges)		 $O(n^9)$ [Aronov et al., 2006] NP-hard no FPTAS PTAS	 NP-hard no FPTAS PTAS
2 (faces)		 $O(n)$ [Suri, 1986] $O^*(n^2)$ [Mitchell et al., 1992] NP-hard no FPTAS PTAS	 NP-hard no FPTAS PTAS 2-Aprx: $O(n^4)$ 2-Aprx: $O(n^{7+\varepsilon})$[]
3 (anywhere)			 $O(1)$ NP-hard no FPTAS PTAS

Algebraic complexity in $\mathbb{R}^2$

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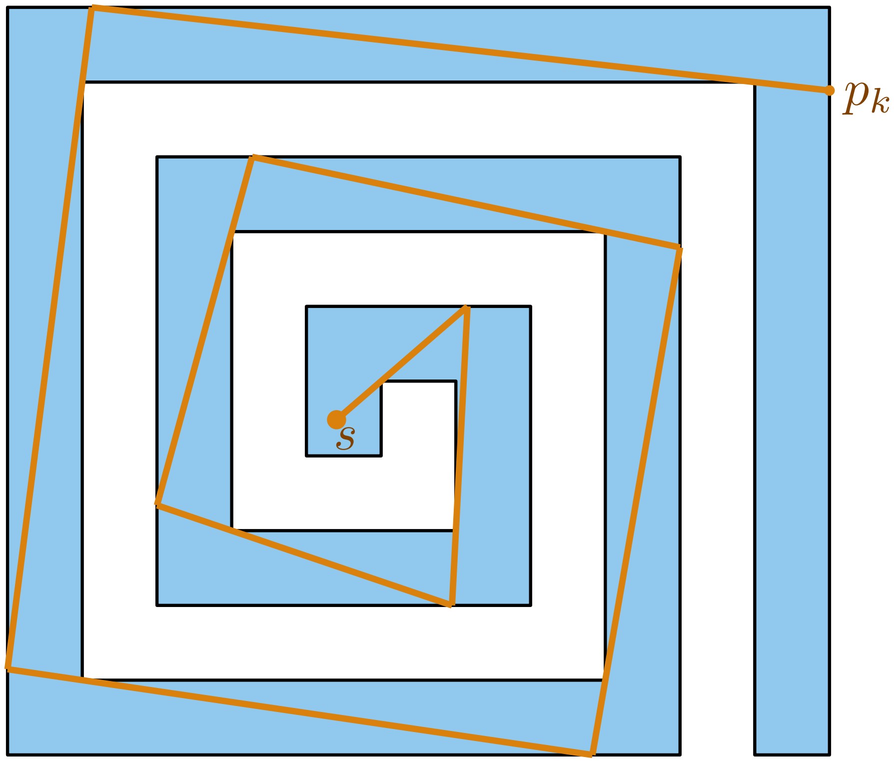
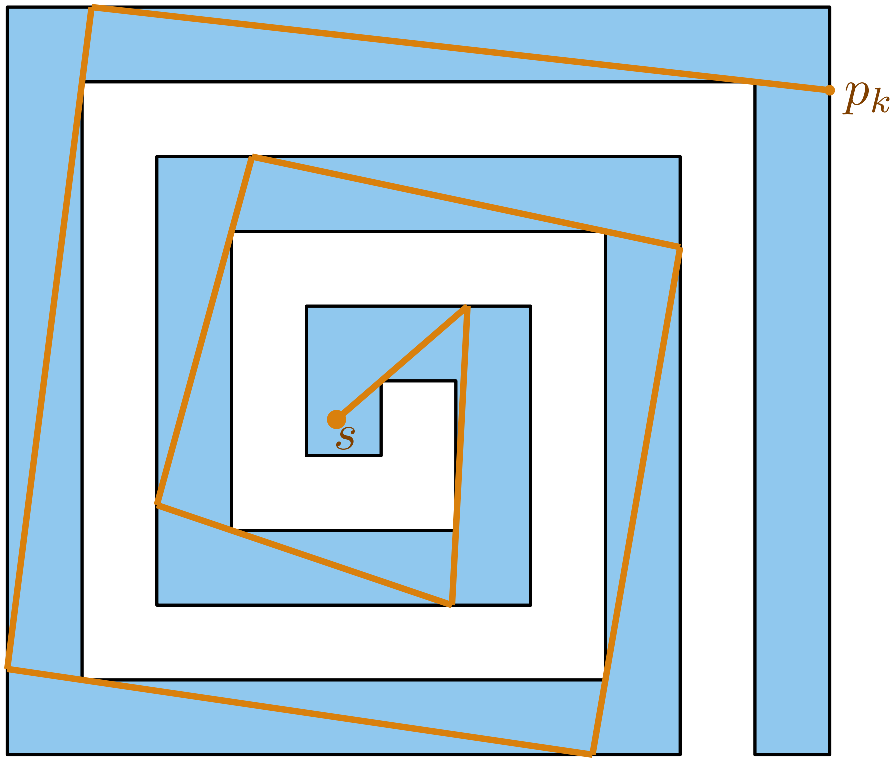


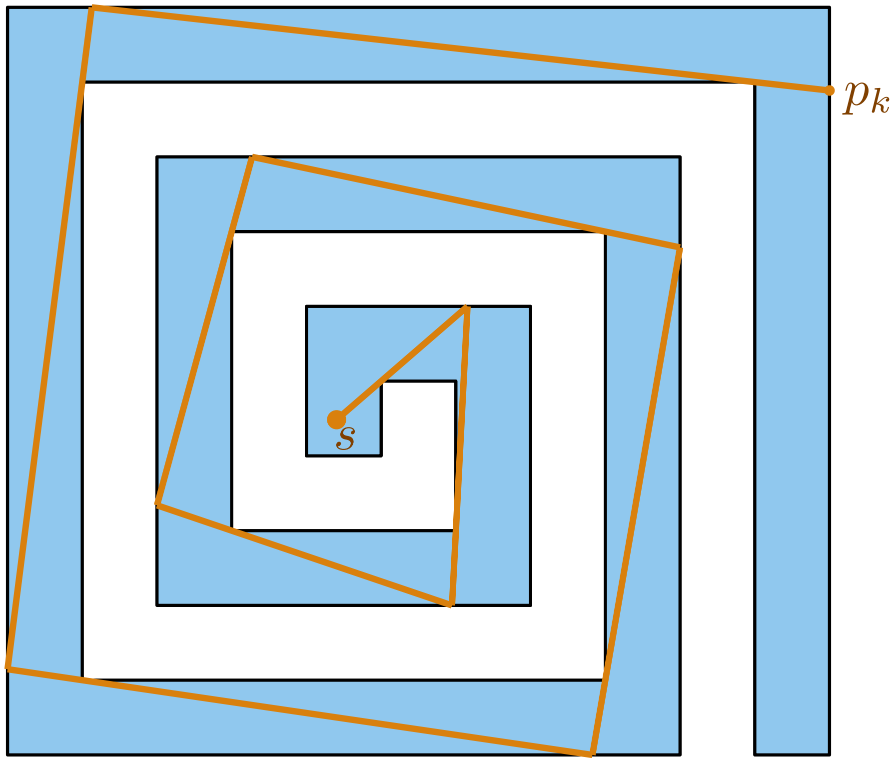
Lemma. [Kahan & Snoeyink, 1999]
There is a simple polygon with vertices of bit-complexity $\log n$ s.t. the boundary of the region reachable from s in k steps has vertices with bit-complexity $\Omega(k \log n)$.

Algebraic complexity in $\mathbb{R}^2$



Lemma.
A MinLinkPath$_{ab}$ of length k between s and t in a simple polygon whose vertices, as well as s and t, have bit-complexity $\log n$, may contain vertices of bit-complexity $\Omega(k \log n)$.

Algebraic complexity in $\mathbb{R}^2$



Lemma.
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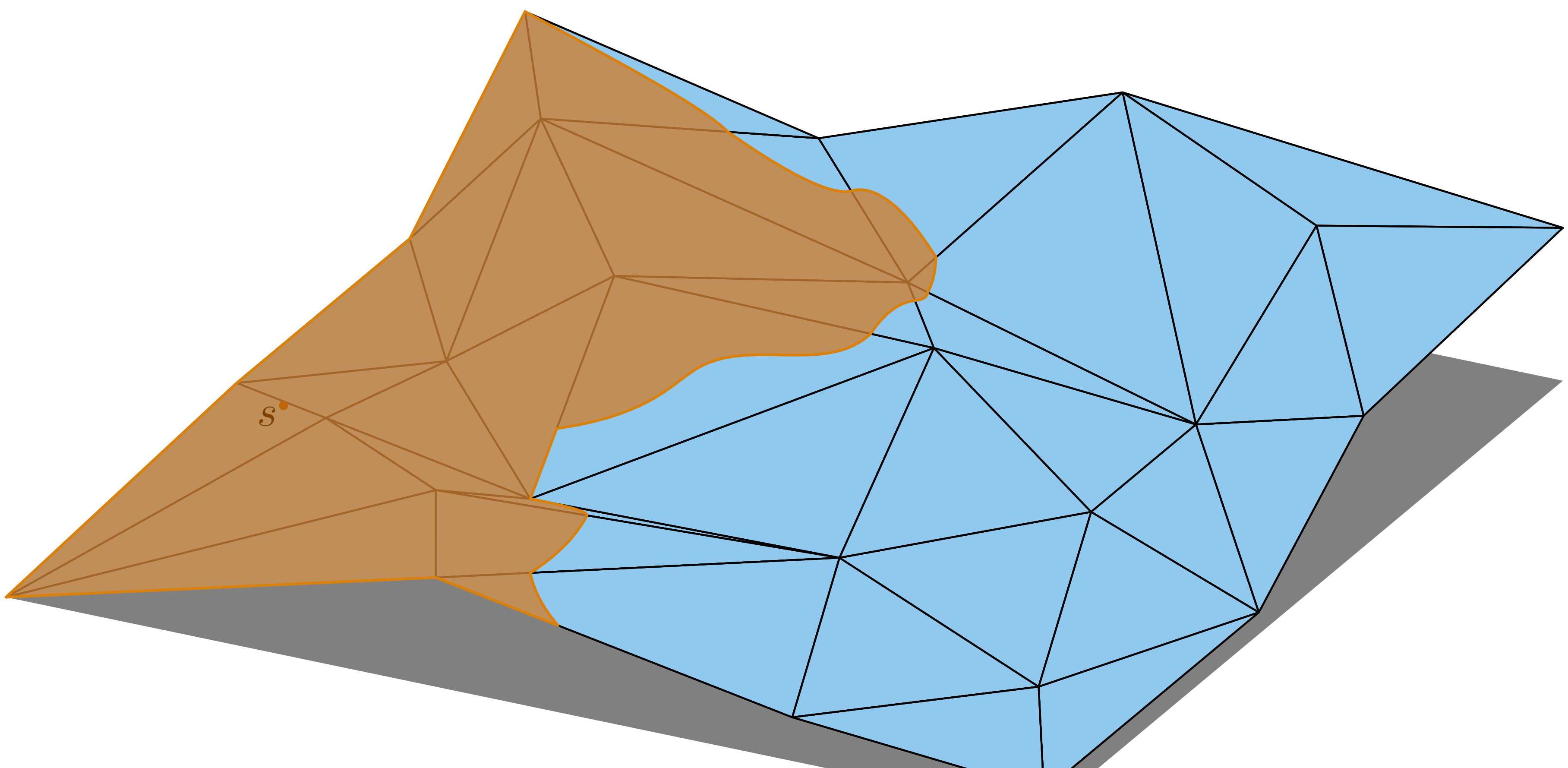
Lemma.
The k-reachable space has vertices with bit complexity $O(k \log n)$.

Algebraic complexity in $\mathbb{R}^3$

Lemma.
The boundary of the k-reachable space can be represented by curves of order $2k + 1$ (and order 2 when $k = 1$).



Algebraic complexity in $\mathbb{R}^3$

Lemma.
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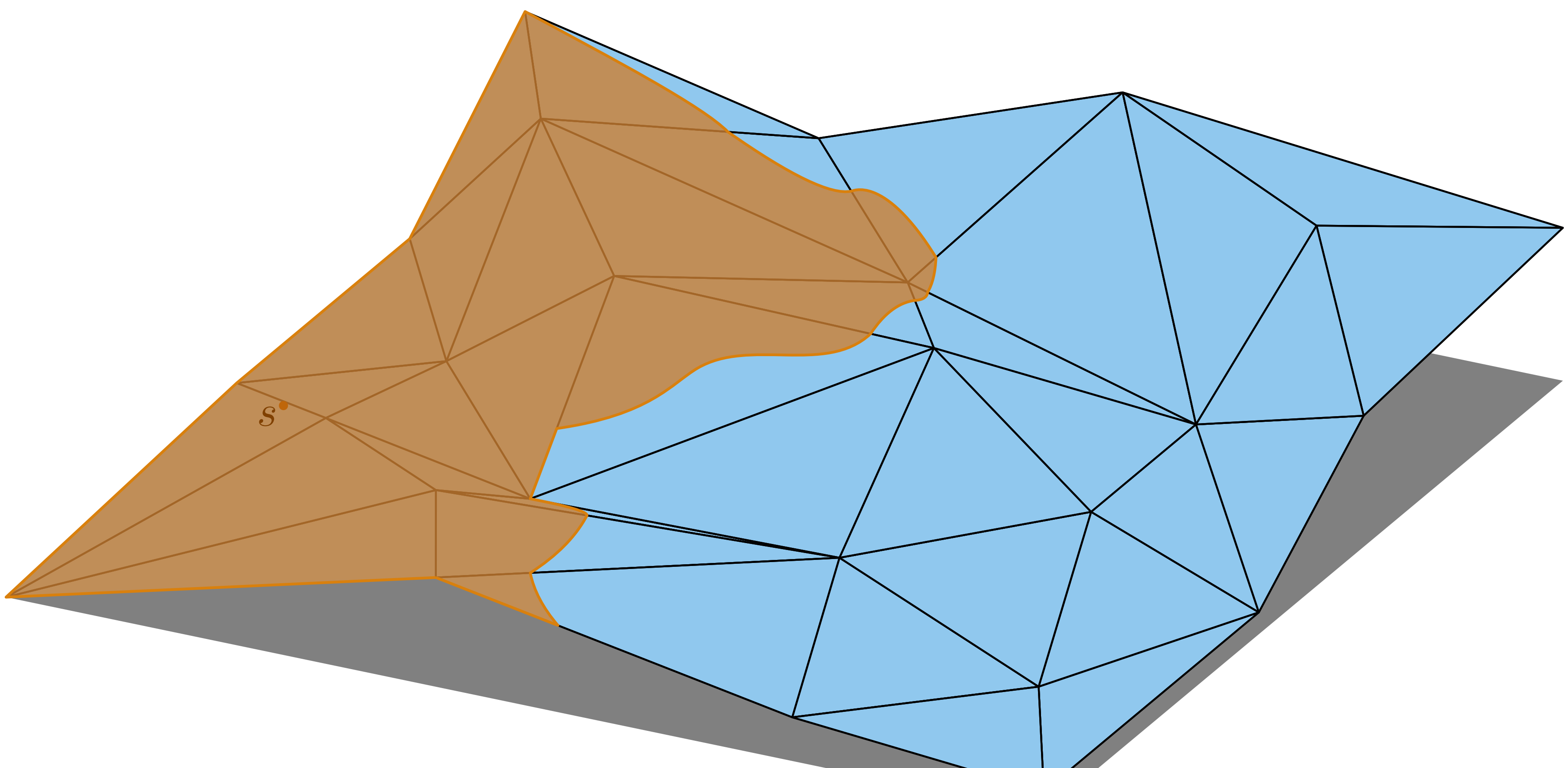
Lemma.
The k-reachable space has vertices with bit complexity $O(9^k)$.



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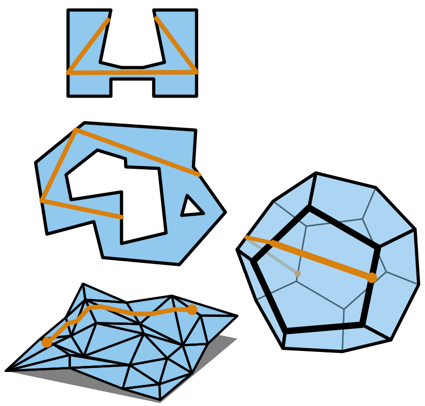
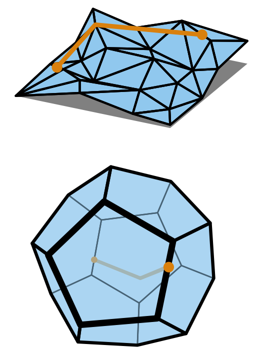
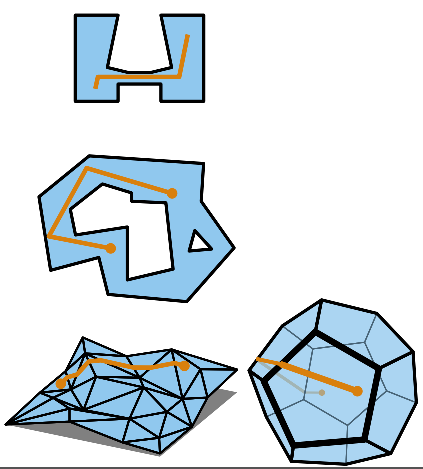
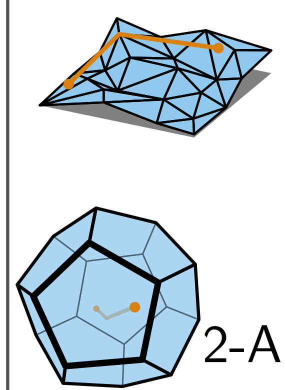
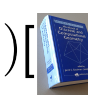
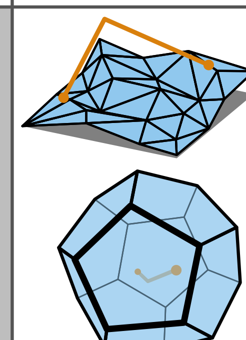
1 (edges)		 $O(n^9)$ [Aronov et al., 2006] NP-hard no FPTAS PTAS	 NP-hard no FPTAS PTAS
2 (faces)		 $O(n)$ [Suri, 1986] $O^*(n^2)$ [Mitchell et al., 1992] NP-hard no FPTAS PTAS	 NP-hard no FPTAS PTAS 2-Aprx: $O(n^4)$ 2-Aprx: $O(n^{7+\varepsilon})$[]
3 (anywhere)			 $O(1)$ NP-hard no FPTAS PTAS

Blueprint for the Reduction

2-Partition: n integers $a_1, .., a_n$ with $\sum a_i = 2W$. Is there a subset S that sums to W?

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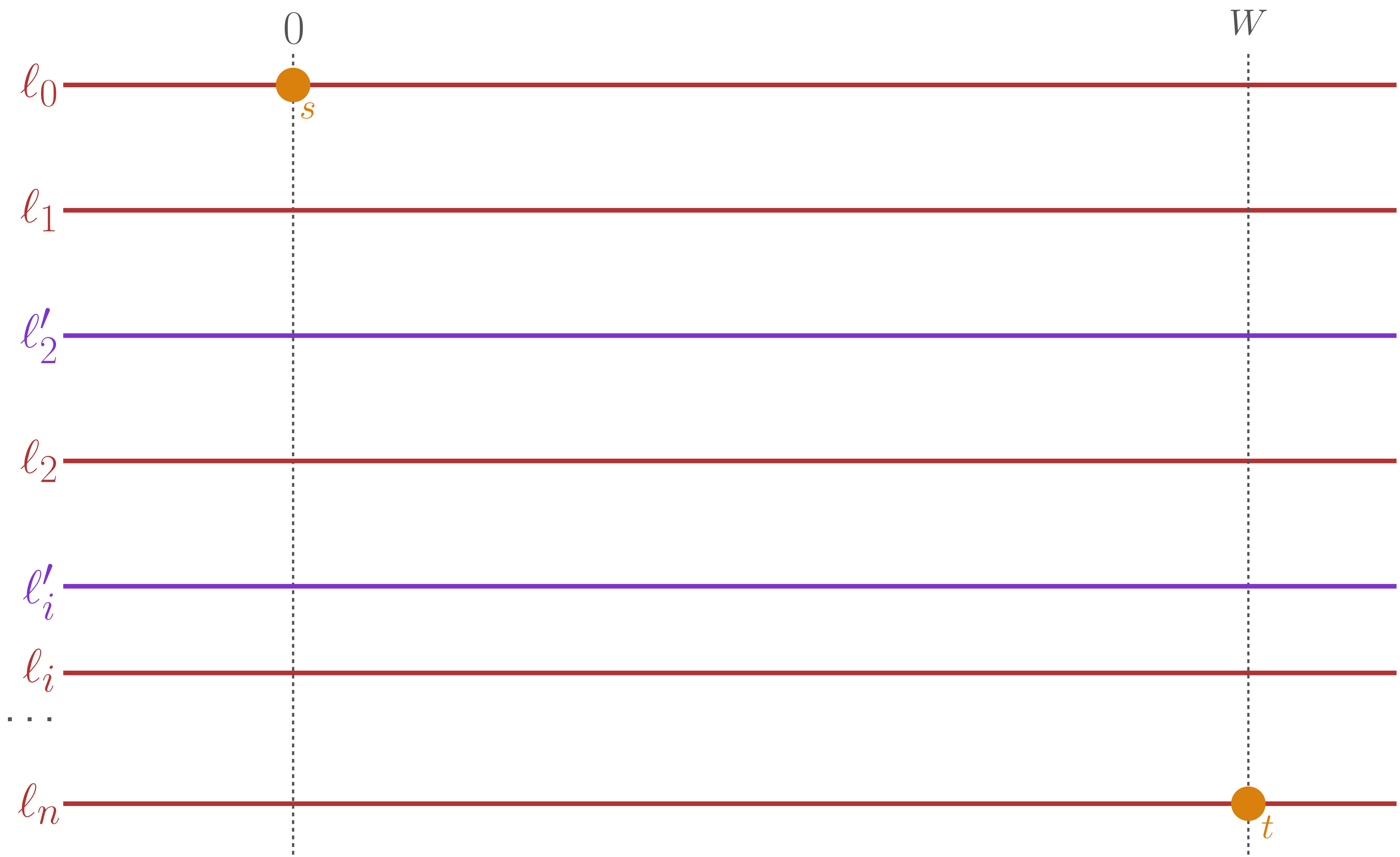
Min link path* with bends on lines, from s to t with $2n-1$ links $\iff$ $\exists$ subset S that sums to W



Blueprint for the Reduction

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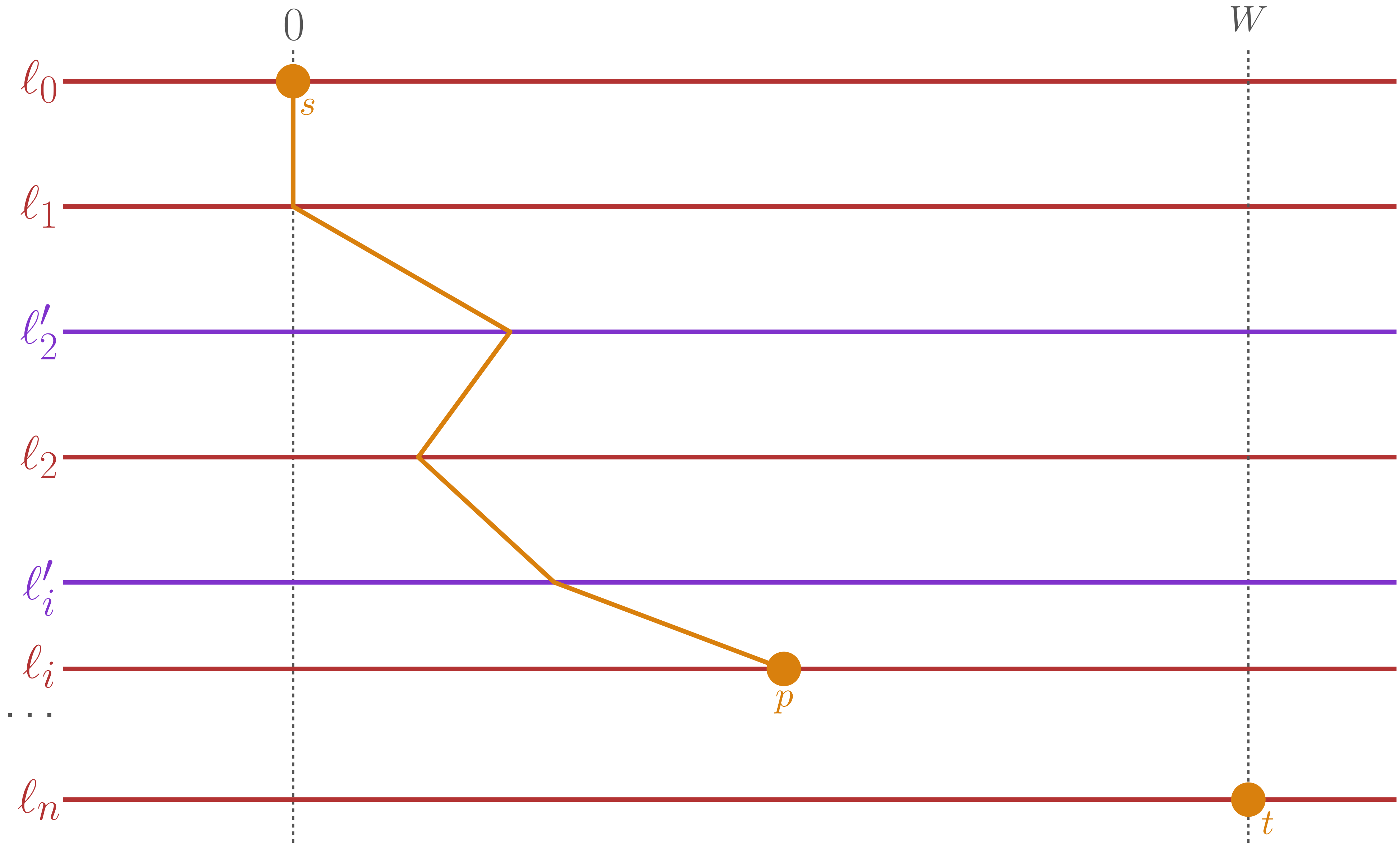
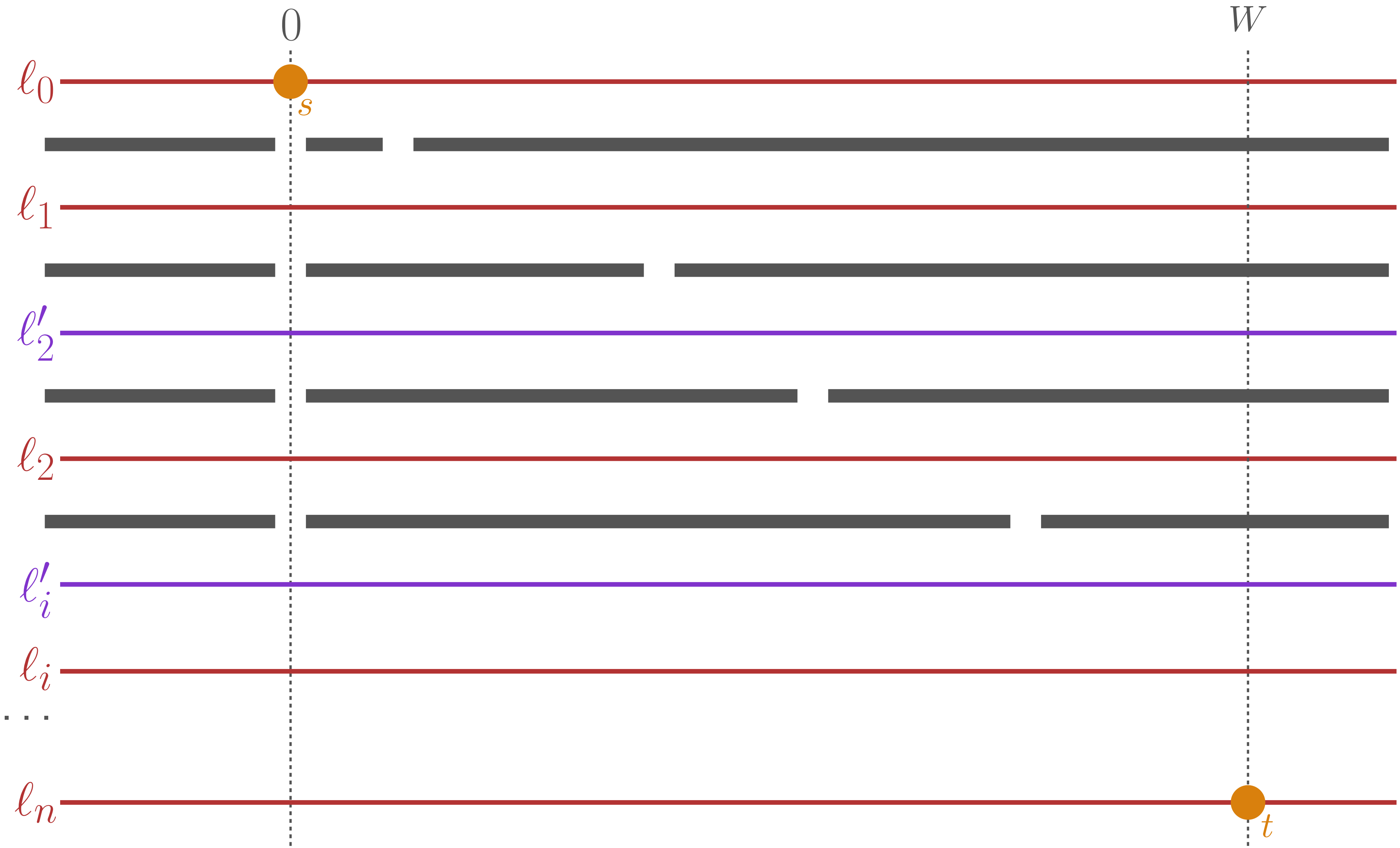
p on ℓ_i reachable with $2i - 1$ links $\iff$ p corrresponds to the sum of a subset of $a_1, .., a_i$



Blueprint for the Reduction

2-Partition: n integers $a_1, .., a_n$ with $\sum a_i = 2W$. Is there a subset S that sums to W?

p on ℓ_i reachable with $2i - 1$ links $\iff$ p corrresponds to the sum of a subset of $a_1, .., a_i$



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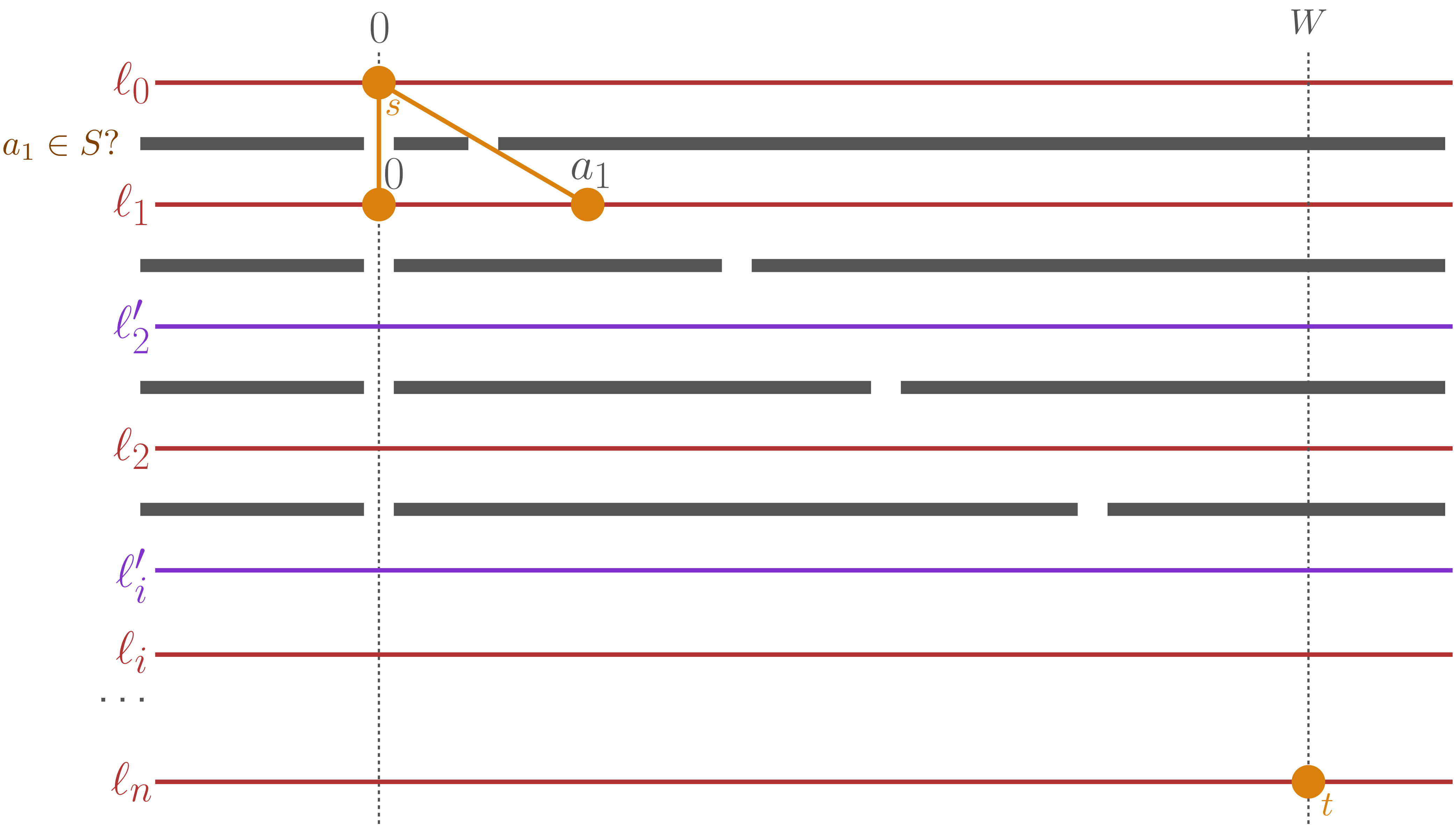

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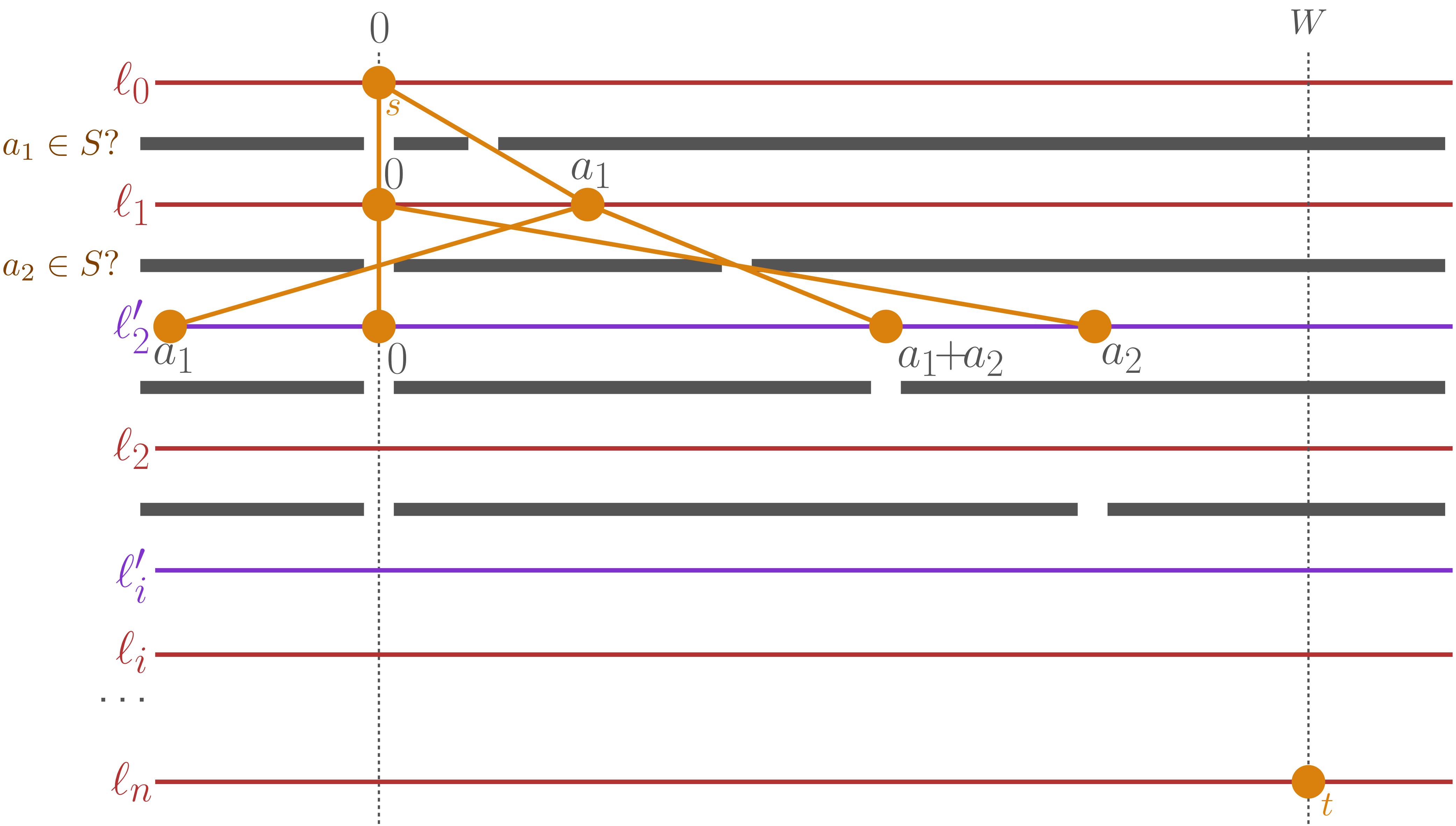

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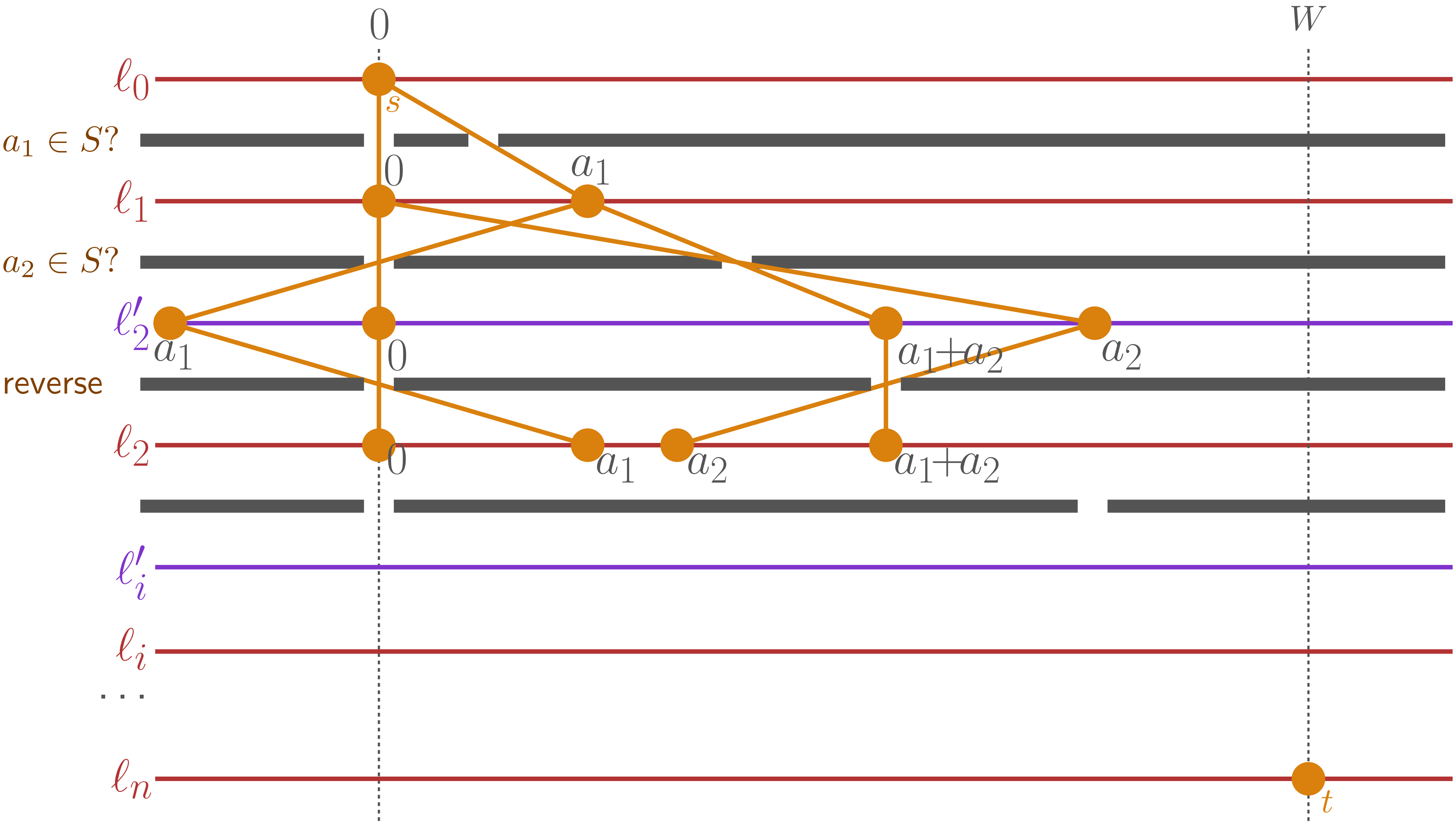

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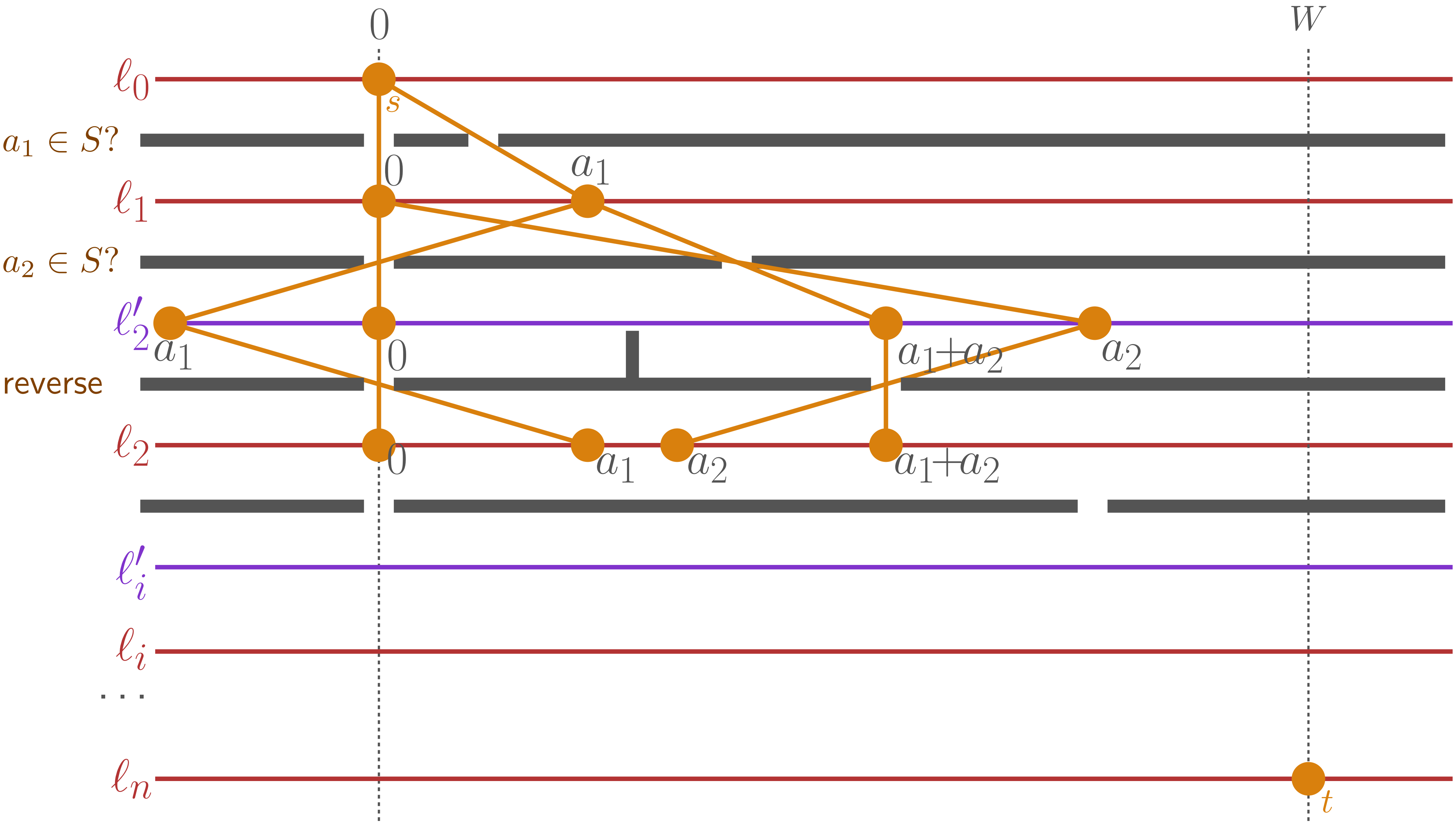
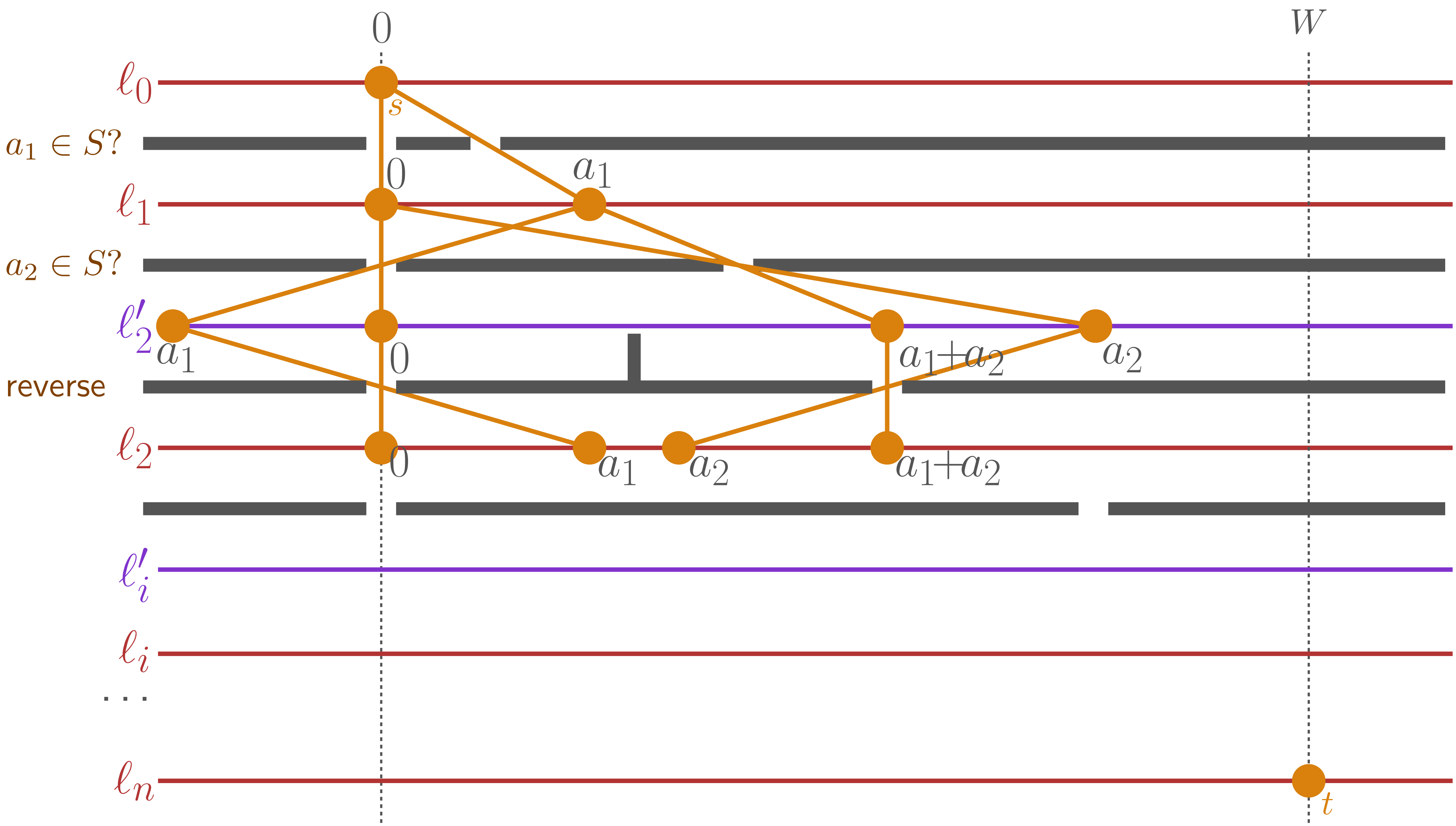


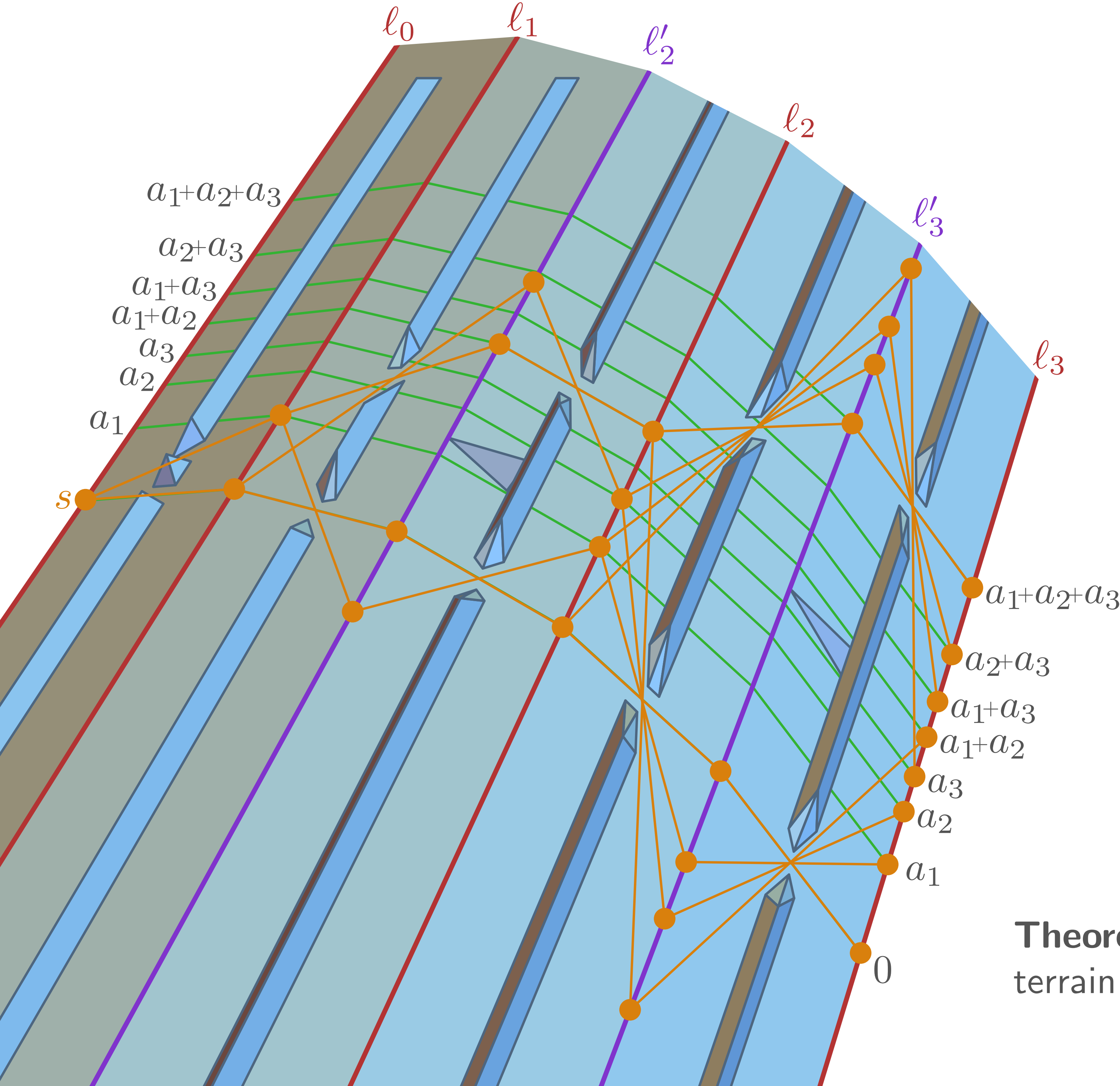
Blueprint for the Reduction

2-Partition: n integers $a_1, .., a_n$ with $\sum a_i = 2W$. Is there a subset S that sums to W?

Min link path* with bends on lines, from s to t with $2n-1$ links $\iff$ $\exists$ subset S that sums to W

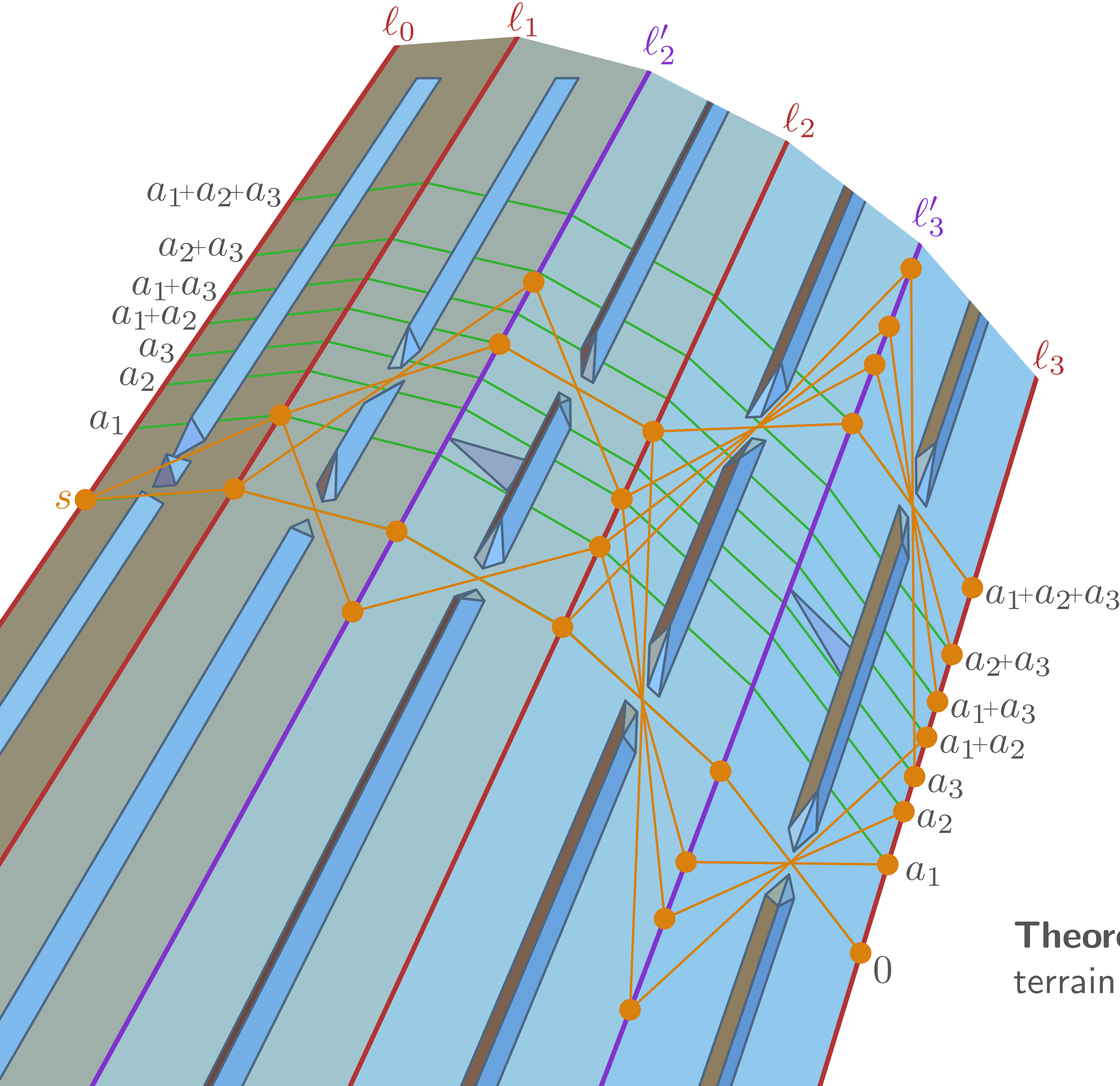


MinLinkPath in $\mathbb{R}^3$



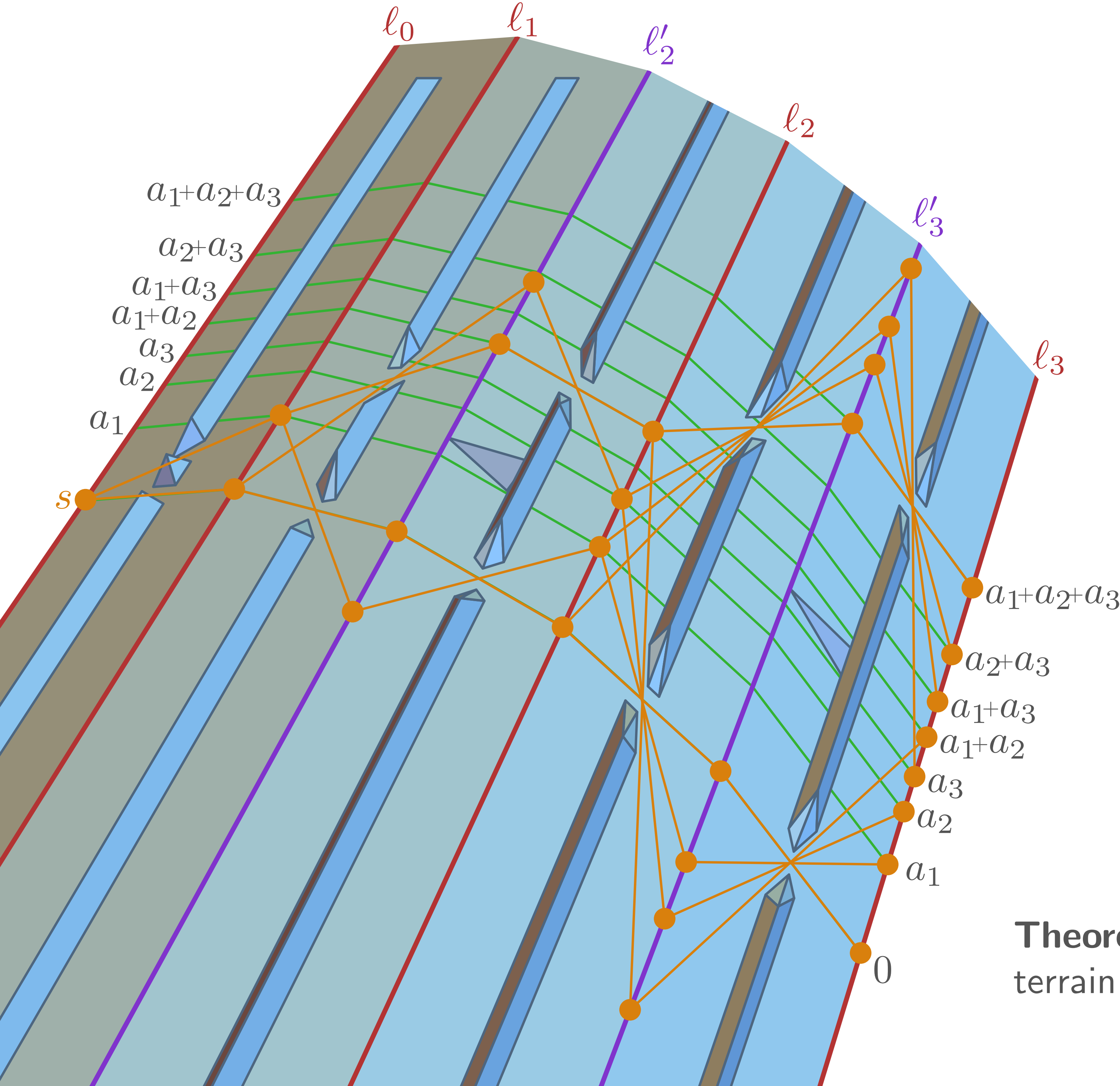
Theorem. MinLinkPath_{12} on a terrain is NP-hard

MinLinkPath in $\mathbb{R}^3$



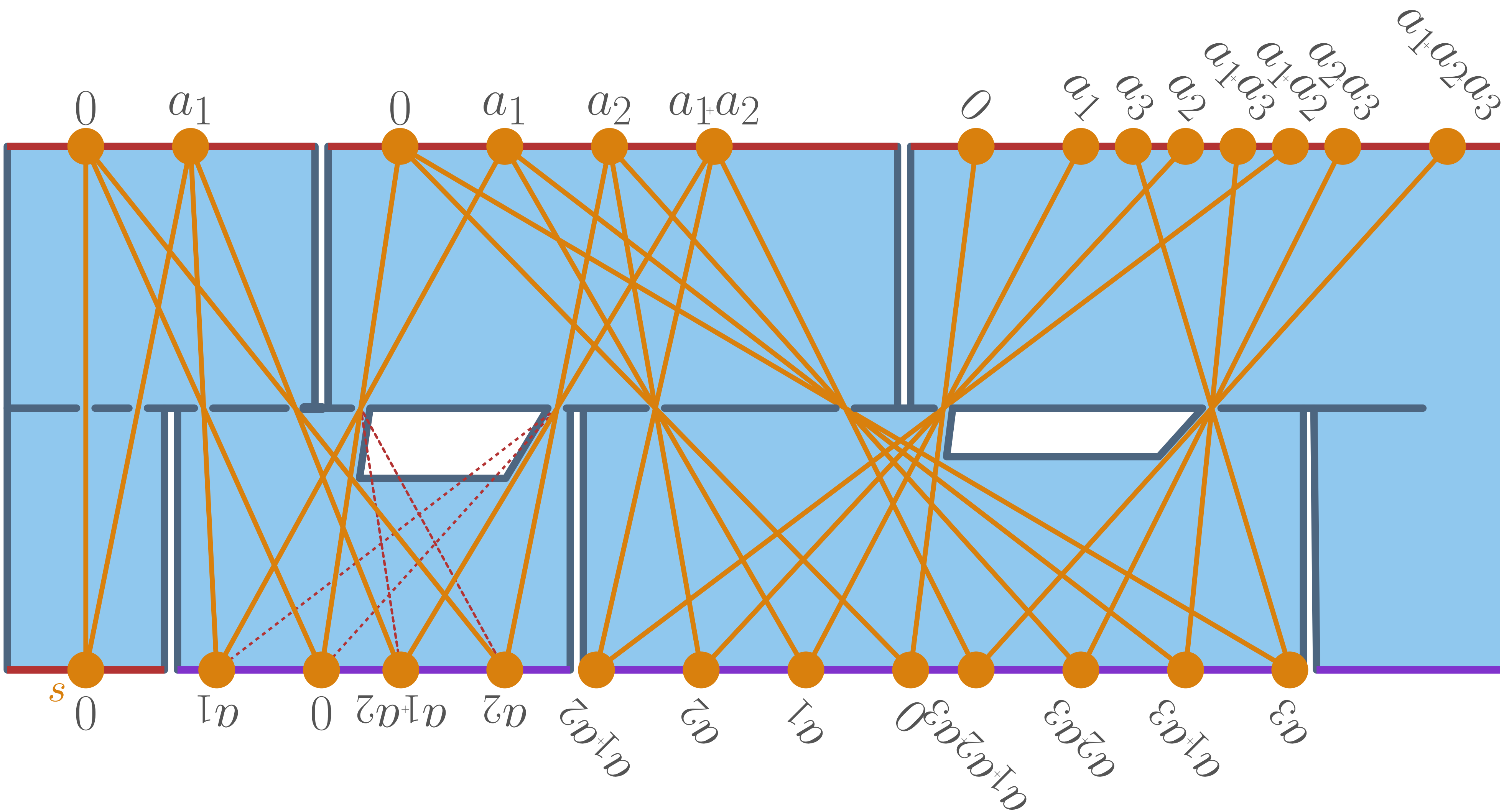
Theorem. MinLinkPath$_{a2}$ on a terrain is NP-hard.

MinLinkPath in $\mathbb{R}^3$



Theorem. MinLinkPath$_{a3}$ on a terrain is NP-hard.

MinLinkPath in $\mathbb{R}^2$



Theorem. MinLinkPath$_{a2}$ in a polygon with holes is NP-hard.

Future Work

- Is Minimum link path strongly NP-hard?

  or, can design a pseudo polynomial time algorithm?

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Future Work

- Is Minimum link path strongly NP-hard?

  or, can design a pseudo polynomial time algorithm?
- Is there a polynomial upper bound on the bit-complexity in $\mathbb{R}^3$?
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Thank you!