A splitting line model for directional relations

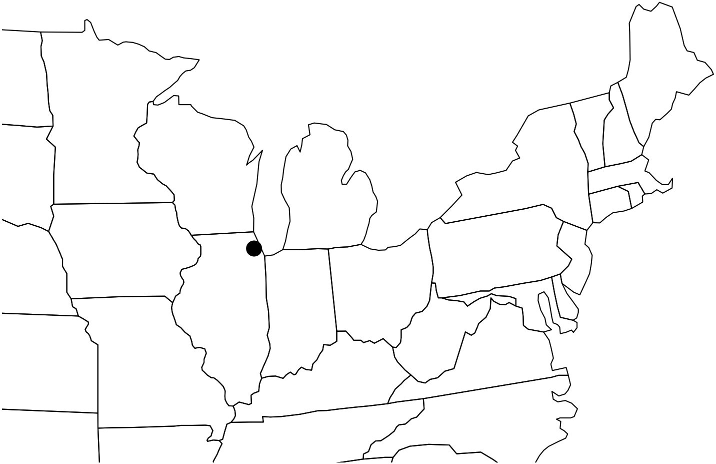
Kevin Buchin[1] Vincent Kusters[2] Bettina Speckmann[1]
Frank Staals[3] Bogdan Vasilescu[1]

[1]Eindhoven University of Technology

[2]ETH Zürich [3]Utrecht University

November 3, 2011

Problem: direction between regions



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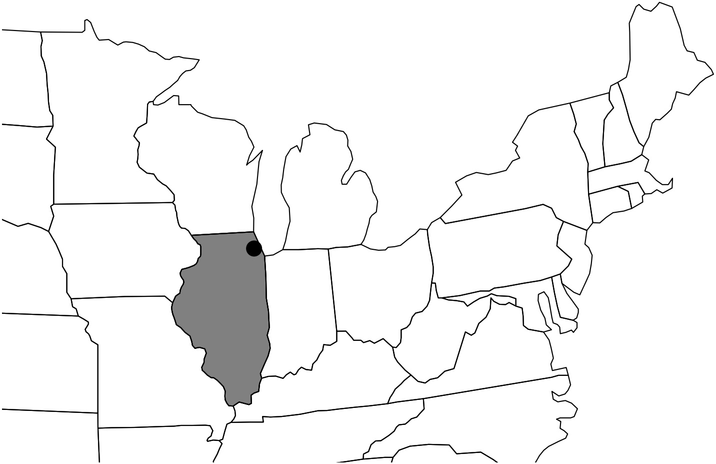

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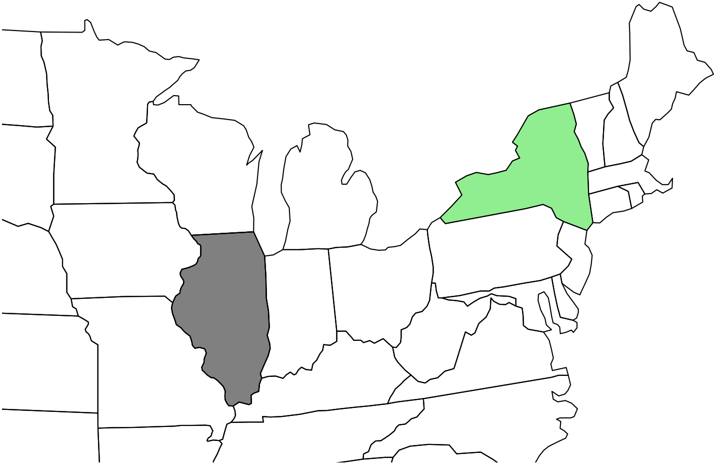
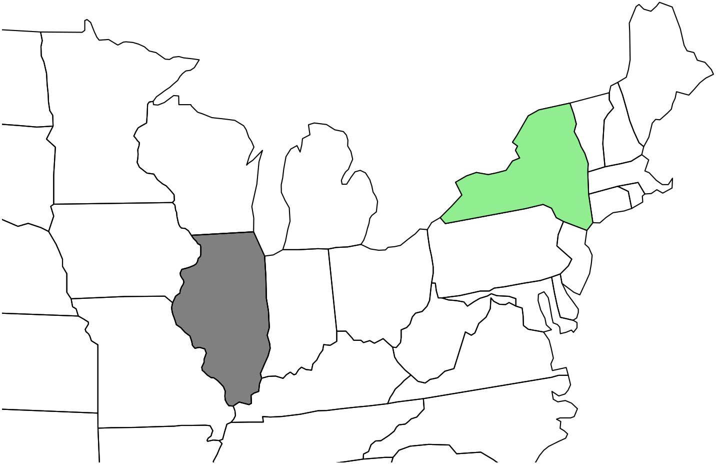


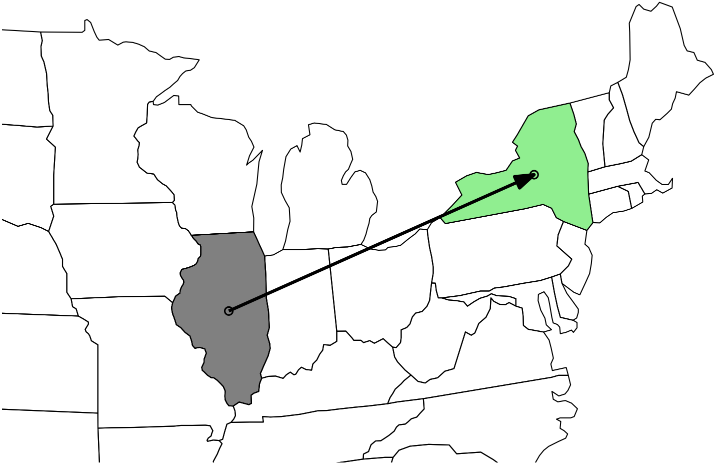
Where is New York with respect to Illinois?

Problem: direction between regions



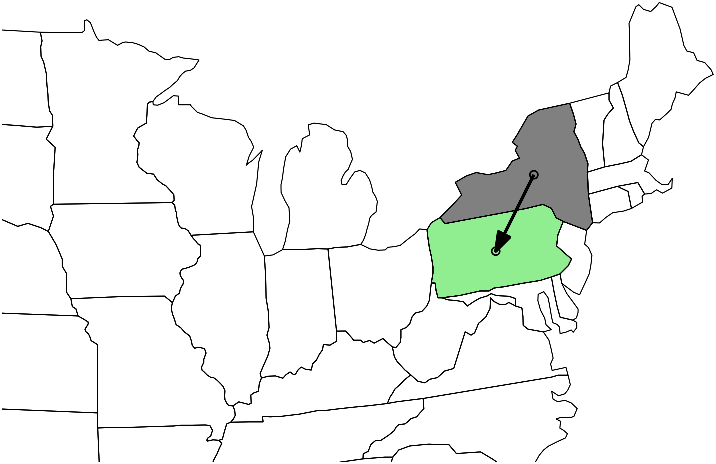
Where is target polygon B with respect to reference polygon A?

Problem: direction between regions



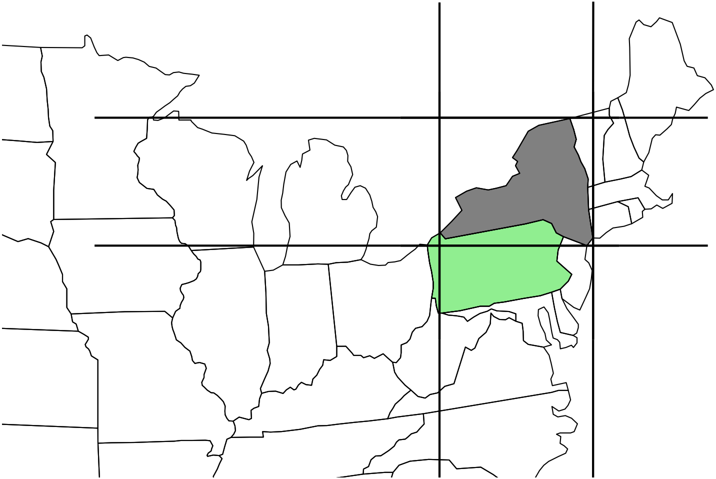
Centroids: B is northeast of A.

Problem: direction between regions



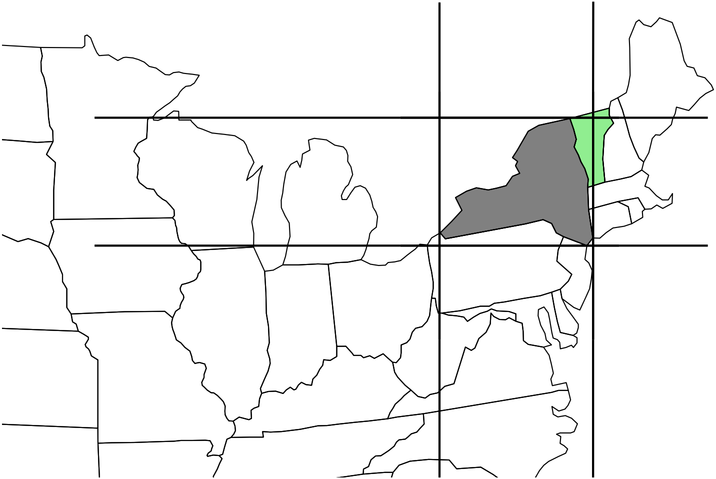
Centroids: B is southwest of A.

Problem: direction between regions



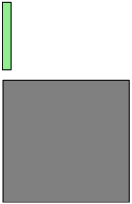
Matrix: *B* is south of *A*.

Problem: direction between regions



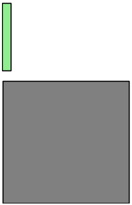
Matrix: ????????

Directional relations are subjective



North or northwest?

Directional relations are subjective



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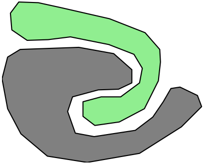
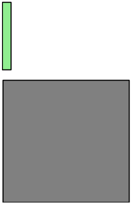
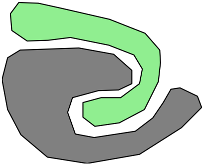


North, northeast, or east?

Directional relations are subjective



North or northwest?



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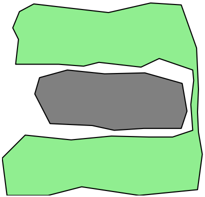
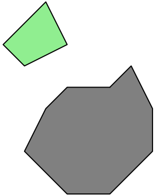


????

Criteria for directional relations

Area



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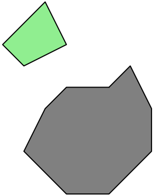
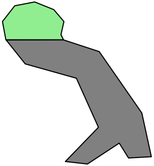


Alignment



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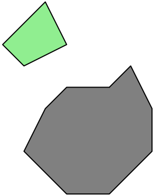

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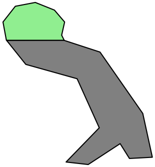
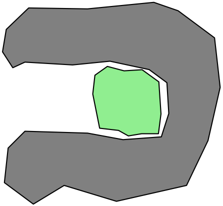


Removal direction



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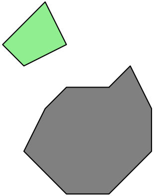

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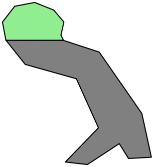

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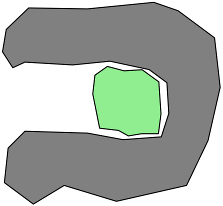
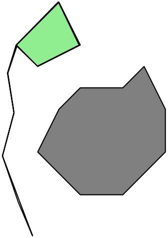


Robustness



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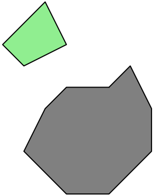

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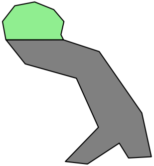

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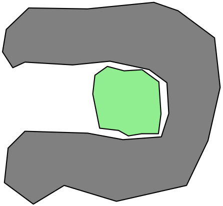

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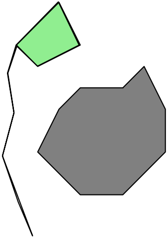
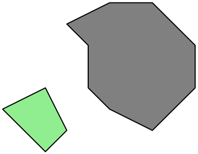
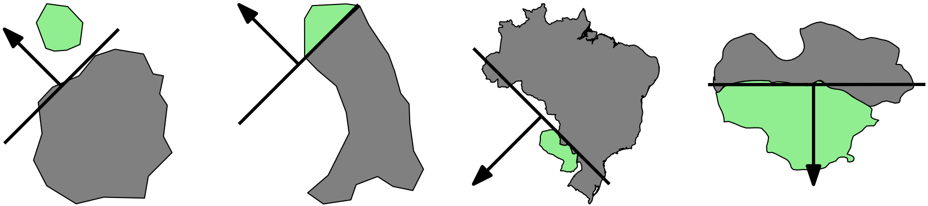


Affine transformation



A splitting line for directional relations



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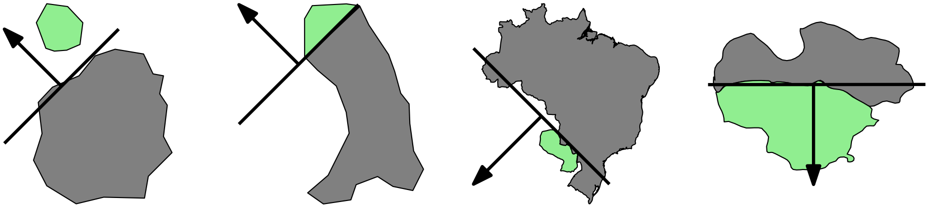
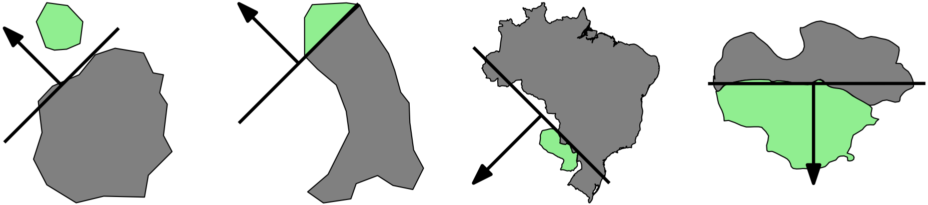


Our Approach: Compute the best splitting line that separates A and B.

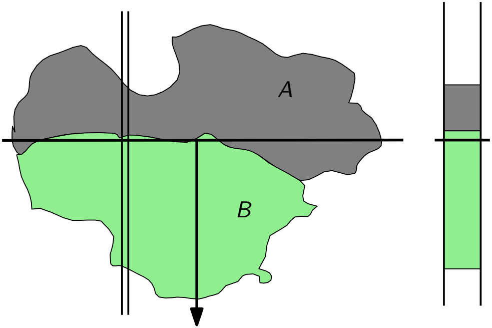
A splitting line for directional relations



Our Approach: Compute the best splitting line that separates A and B.

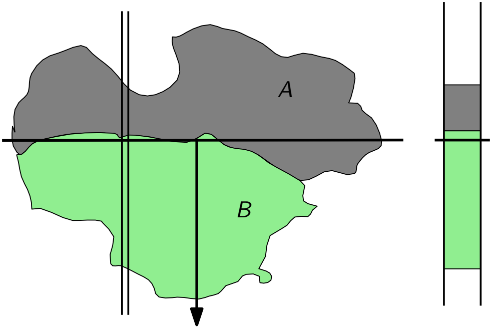
Question: What does it mean to be the best splitting line?

Measuring the quality of a splitting line



1. Divide the scene in slabs
2. Compute the quality of each slab.

Measuring the quality of a splitting line



1. Divide the scene in slabs
2. Compute the quality of each slab.

$$M_{line}(y) = \int_{-\infty}^{\infty} M(x,y)dx$$

The slab measure M

$$
\begin{aligned}
M = {} & \rho_1 \cdot f(B) \cdot \mathit{GoodA} + \\
& \rho_1 \cdot g(A) \cdot \mathit{GoodB} - \\
& \rho_2 \cdot h(B) \cdot \mathit{AlignmentA} - \\
& \rho_3 \cdot \mathit{ObstructA} - \rho_3 \cdot \mathit{ObstructB}
\end{aligned}
$$

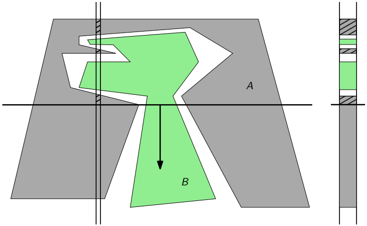
User definable parameters.

The slab measure M

$$M = \rho_1 \cdot f(B) \cdot GoodA + \rho_1 \cdot g(A) \cdot GoodB - \rho_2 \cdot h(B) \cdot AlignmentA - \rho_3 \cdot ObstructA - \rho_3 \cdot ObstructB$$

criterion: Area

Measures the amount of A on the A-side.



$$GoodA = \sum_{a \in TopA} \frac{height(a)}{area(A)}$$

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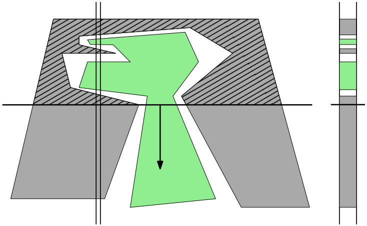

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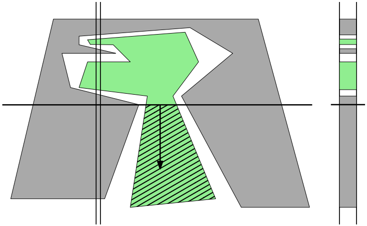
\rho_3 \cdot ObstructA - \rho_3 \cdot ObstructB
\end{aligned}
$$

criterion: Area

Measures the amount of B on the B-side.



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& \rho_3 \cdot \mathit{ObstructA} - \rho_3 \cdot \mathit{ObstructB}
\end{aligned}
$$

criterion: Alignment

Measures the alignment of ℓ with A.

The slab measure M

$$
\begin{aligned}
M = {} & \rho_1 \cdot f(B) \cdot GoodA \; + \\
& \rho_1 \cdot g(A) \cdot GoodB \; - \\
& \rho_2 \cdot h(B) \cdot AlignmentA \; - \\
& \rho_3 \cdot {\color{red} ObstructA} - \rho_3 \cdot ObstructB
\end{aligned}
$$

Measures the ease with which A can be moved away from B.

criterion: Removal direction.

The slab measure M

$$
\begin{aligned}
M = {} & \rho_1 \cdot f(B) \cdot GoodA + \\
& \rho_1 \cdot g(A) \cdot GoodB - \\
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& \rho_3 \cdot ObstructA - \rho_3 \cdot {\color{red} ObstructB}
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Measures the ease with which B can be moved away from A.

criterion: Removal direction.

The slab measure M

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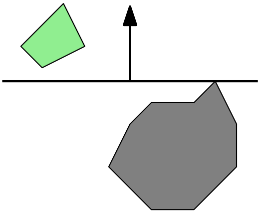
\end{aligned}
$$

criterion: Area

Weighing functions:



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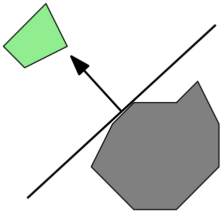

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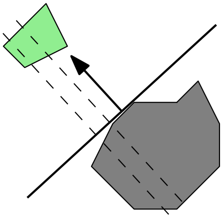

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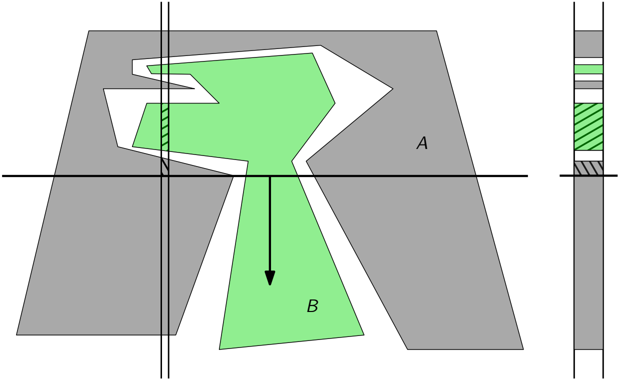
\rho_3 \cdot ObstructA - \rho_3 \cdot ObstructB
\end{aligned}
$$

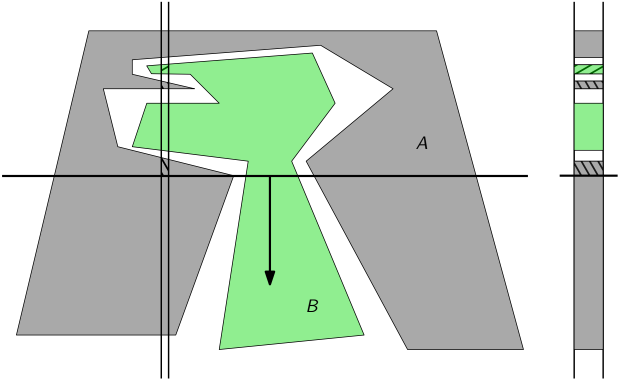
How about Robustness and Affine transformation?

ObstructB



$$ObstructB = \sum_{b \in TopB} \left(\frac{height(b)}{area(B)} \cdot \frac{\sum_{a \in \{s \mid s \in TopA \wedge b > s\}} height(a)}{\sum_{a \in TopA} height(a)} \right)$$

ObstructB



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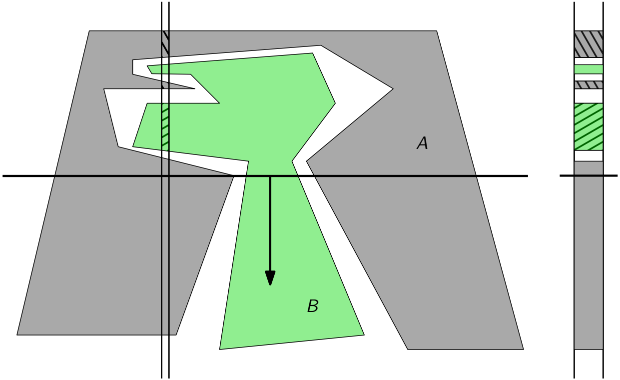
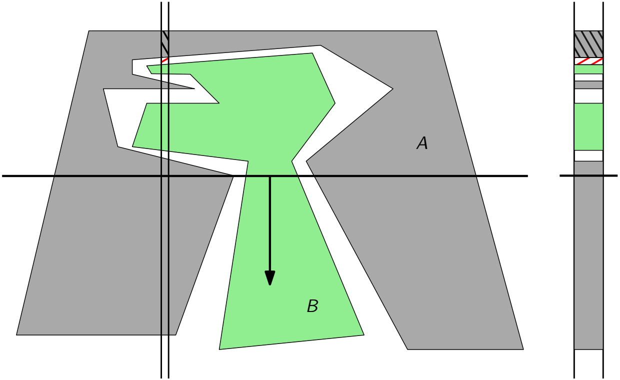
AlignmentA



$$AlignmentA = \sum_{w \in TopW \cup TopB} \left(\frac{height(w)}{\sum_{s \in Strip} height(s)} \cdot \frac{\sum_{a \in \{s | s \in TopA \wedge w < s\}} height(a)}{area(A)} \right)$$

AlignmentA



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An algorithm for M_{line}

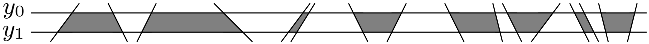
Observation: $M_{line}(y)$ maximal if $\frac{d}{dy} M_{line}(y) = 0$.

ALGORITHM

Sweep ℓ downwards and compute a description of M_{line} and its derivative.

Computing *GoodA*

$\int GoodA(y, x)dx$ is a piecewise quadratic function in y:



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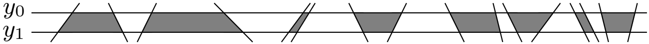
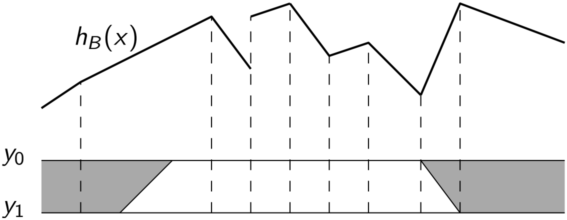


Maintain set P of trapezoids intersected by ℓ.

P changes at most n times $\Longrightarrow O(n \log n)$ to compute *GoodA*.

Computing *ObstructB*

$\int ObstructB(y, x)dx$ is the sum of rational functions in y:



Maintain $h_B(x)$: the amount of B above the sweep line.

Computing *ObstructB*

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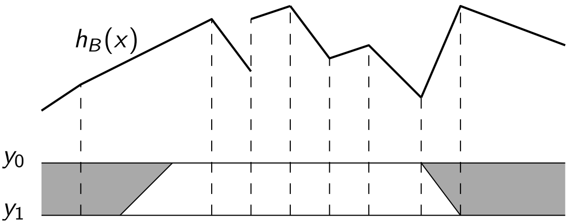

Maintain $h_B(x)$: the amount of B above the sweep line.

$O(n^2)$ events $\Longrightarrow O(n^2 \log n)$ to compute *ObstructB*.

An algorithm for directional relations

Compute the optimal splitting line for each direction and pick the best one.

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Question: Which directions should we use?

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- Use $k > 8$ directions (for example $k = 360$)

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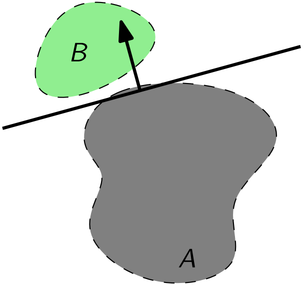

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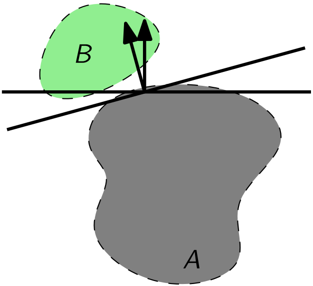

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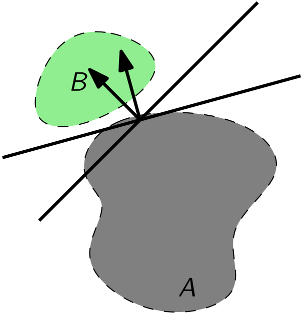
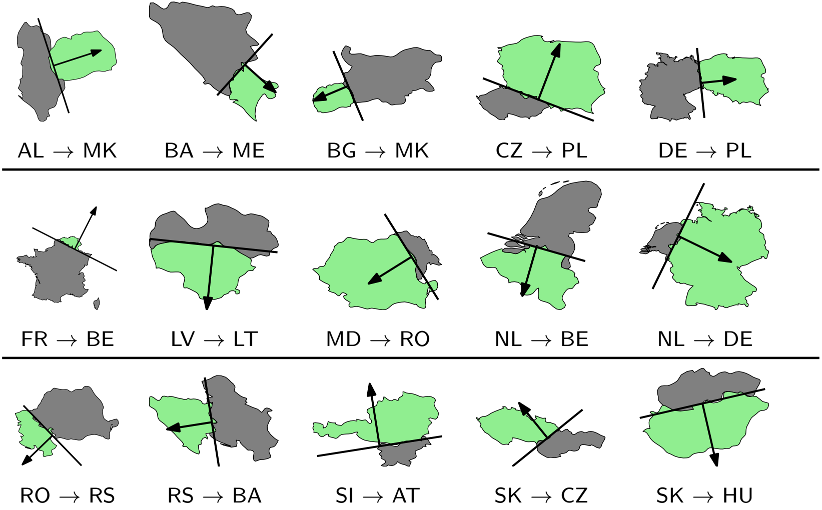
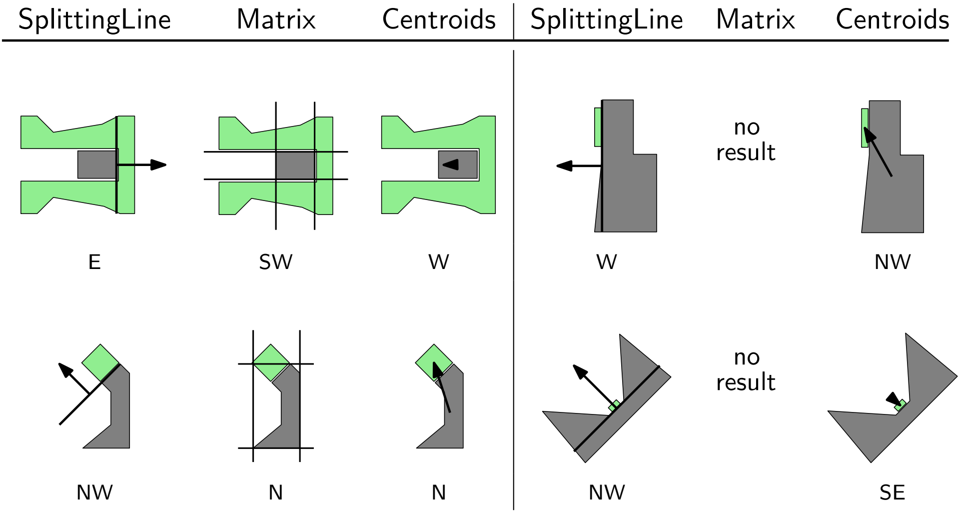
- Use the 8 compass directions N,NE,E,..,NW
- Use $k > 8$ directions (for example $k = 360$)



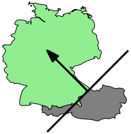
Optimal splitting lines using 360 directions



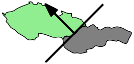
Splitting lines for directional relations



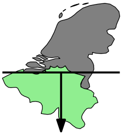
Splitting lines for directional relations



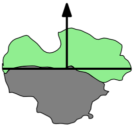
AT → DE



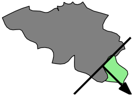
SK → CZ



NL → BE

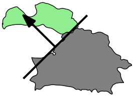


LT → LV

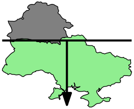


BE → LU

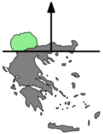
	AT → DE	SK → CZ	NL → BE	LT → LV	BE → LU
SplittingLine	NW	NW	S	N	SE
Centroids	NW	W	SW	NE	SE
Matrix	N	NW	S	N	-



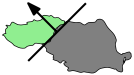
BY → LV



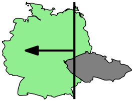
BY → UA



GR → MK



RO → HU

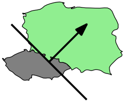


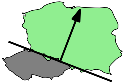
CZ → DE

	BY → LV	BY → UA	GR → MK	RO → HU	CZ → DE
SplittingLine	NW	S	N	NW	W
Centroids	NW	SE	NW	W	W
Matrix	N	SE	-	W	NW

Future Work

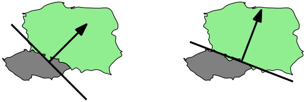
- 8 directions vs many directions



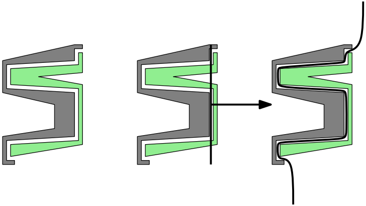


Future Work

- 8 directions vs many directions

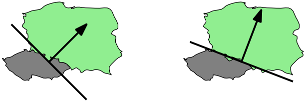


- Non-linear separator (e.g. curve or polyline)



Future Work

- 8 directions vs many directions



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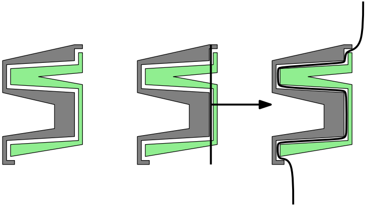
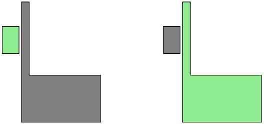
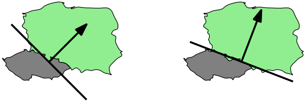


- Are directional relations (a)symmetric?

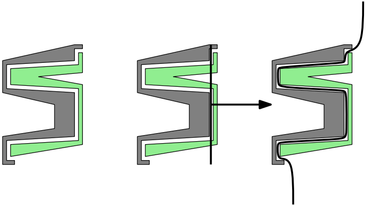


Future Work

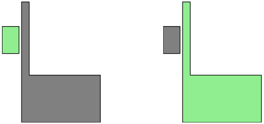
- 8 directions vs many directions



- Non-linear separator (e.g. curve or polyline)



- Are directional relations (a)symmetric?



Thank you! Questions?