Clear Unit-Distance Graphs

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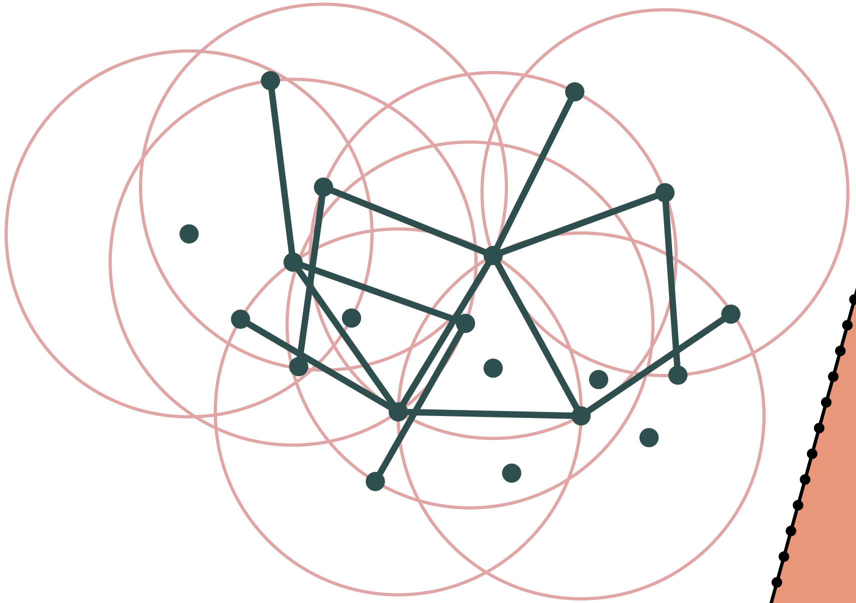
Utrecht University

Puzzles

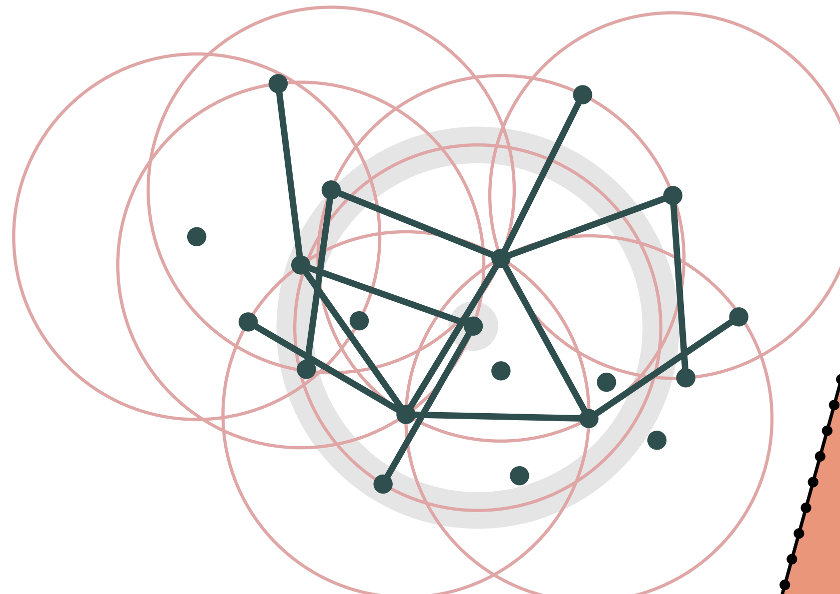
Puzzles



Unit Distance Graph:

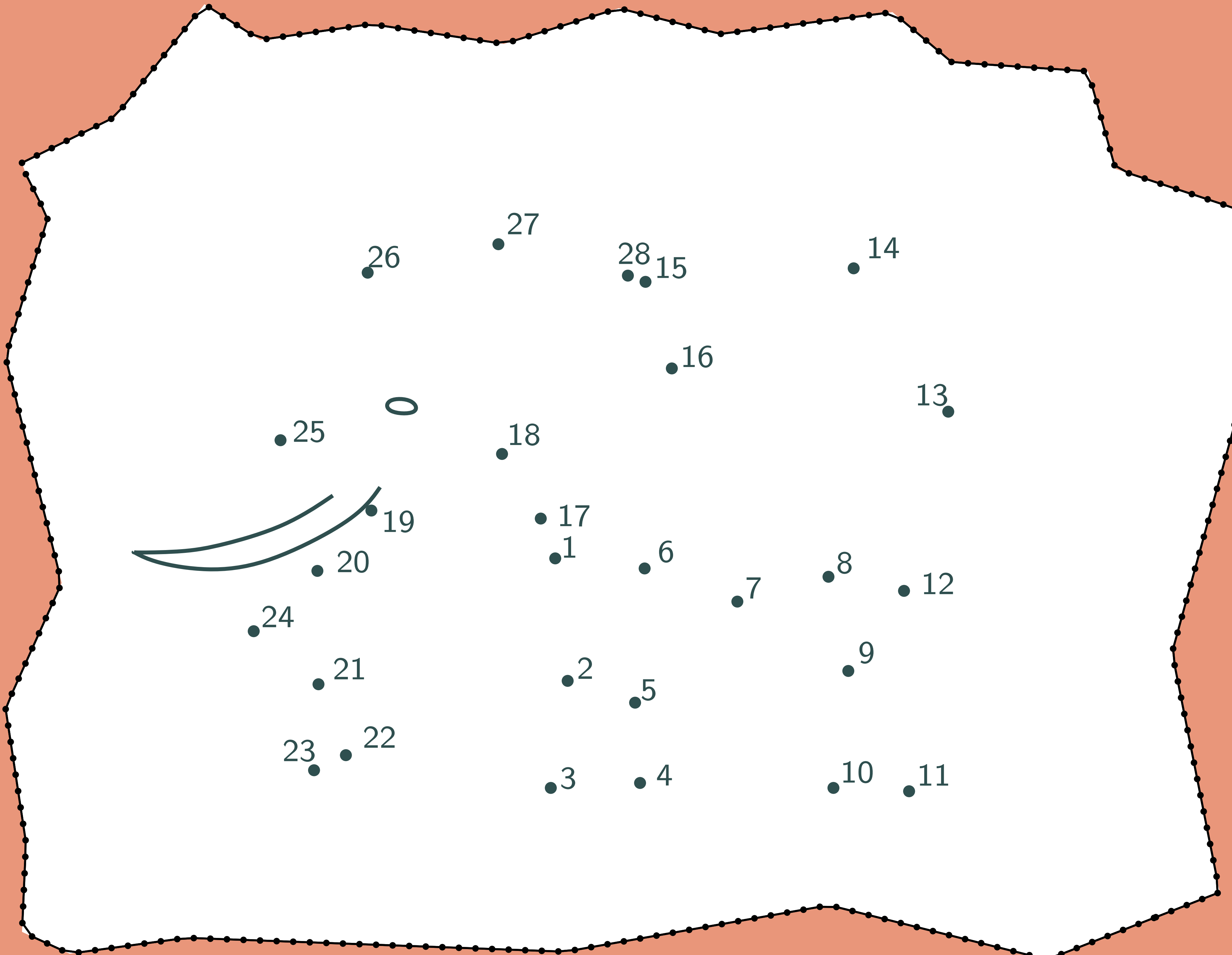
- all edges (p, q) have $d(p, q) = 1$
- all non-edges (p, q) have $d(p, q) \neq 1$

Puzzles



Clear Unit Distance Graph:

- all edges (p, q) have $d(p, q) = 1$
- all non-edges (p, q) have $d(p, q) \in [\varepsilon, 1 - \varepsilon] \cup [1 + \varepsilon, \infty)$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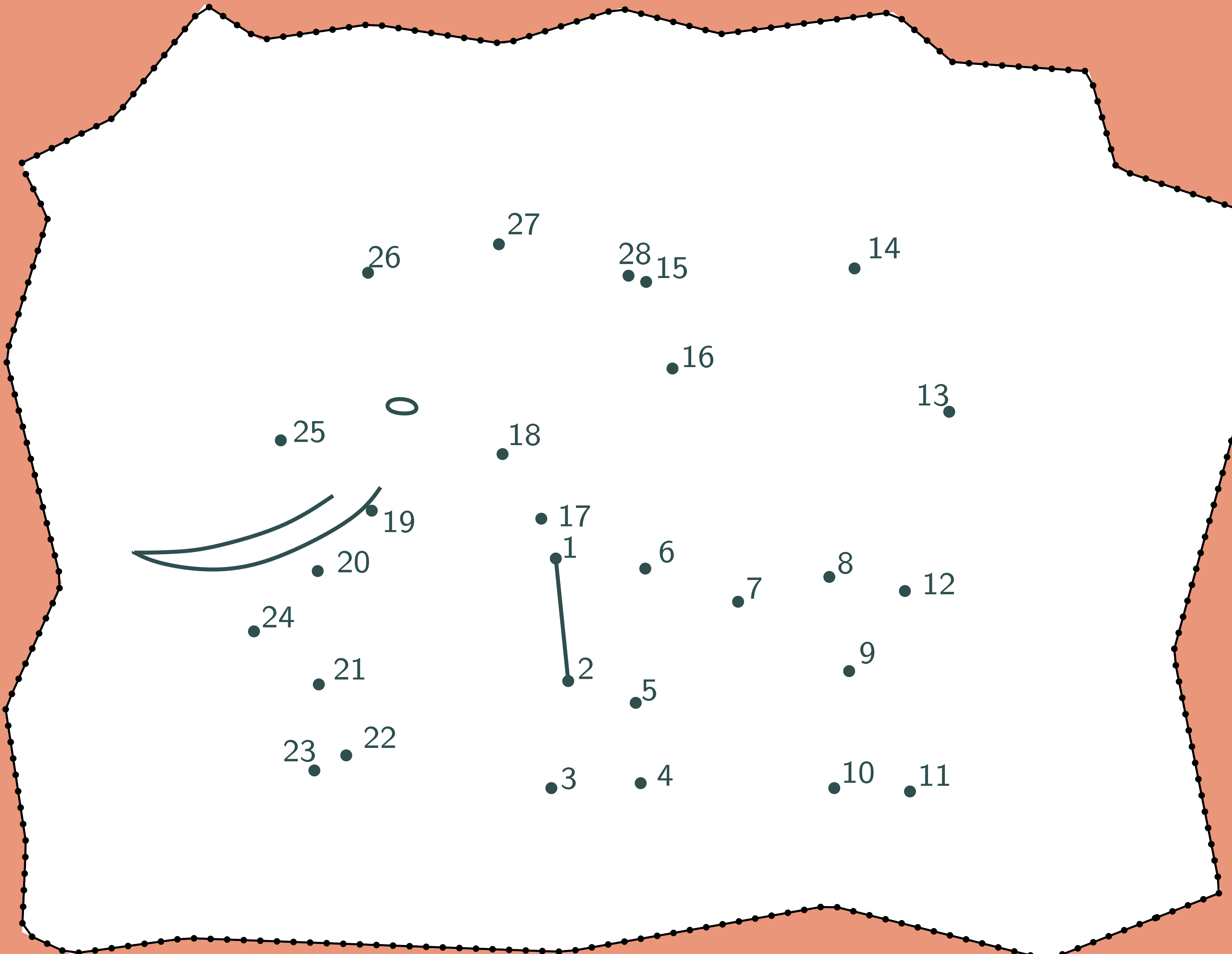
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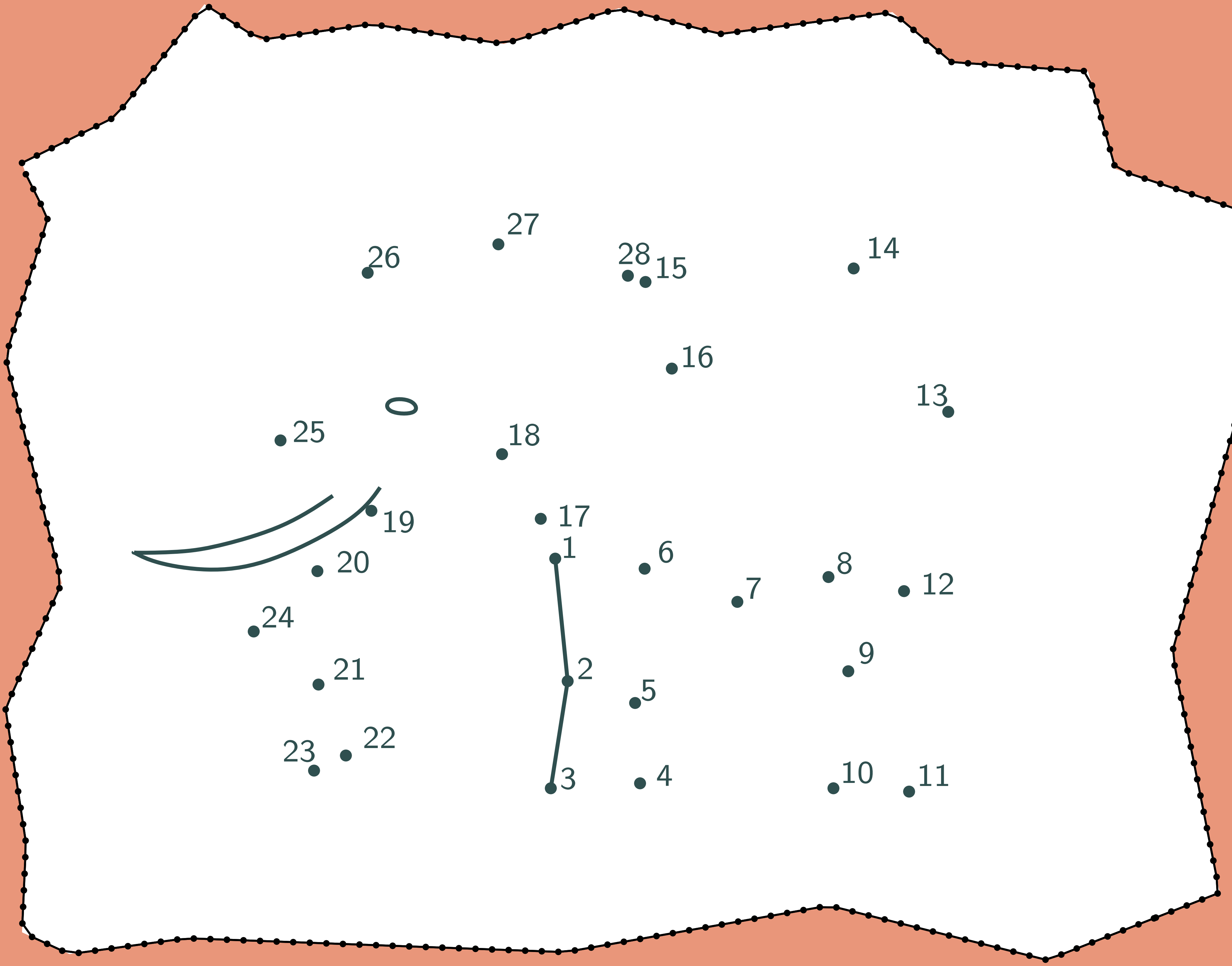
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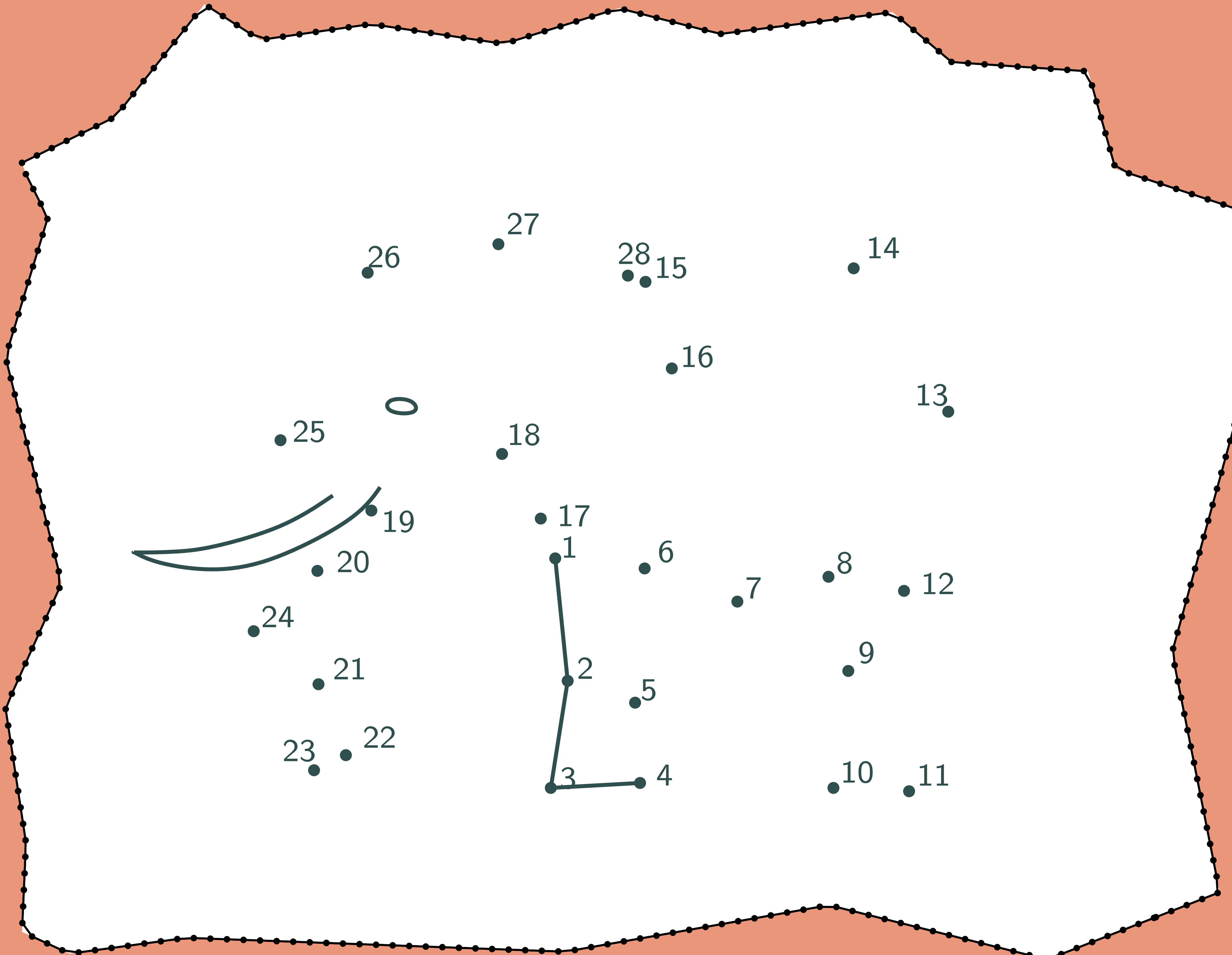

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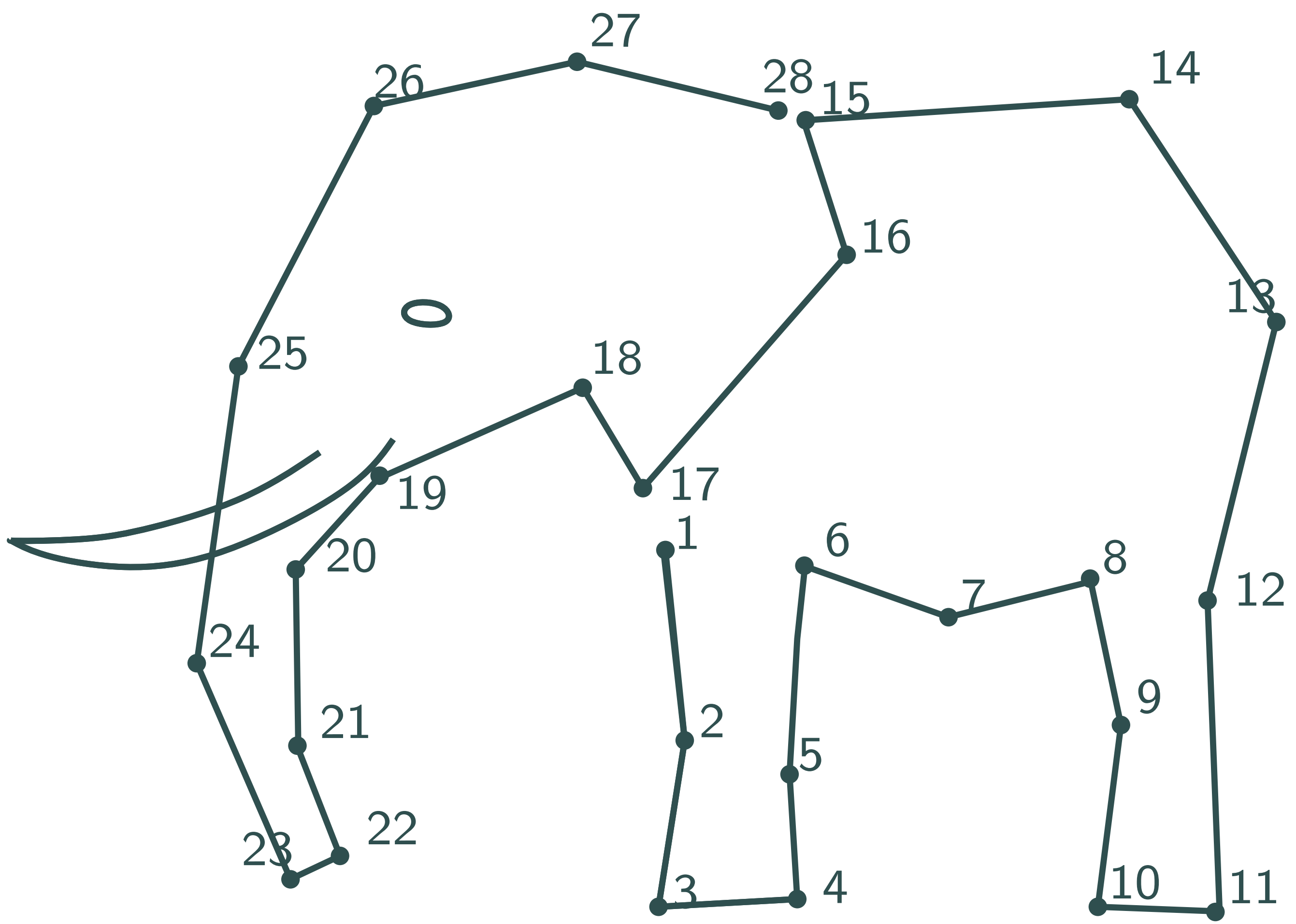
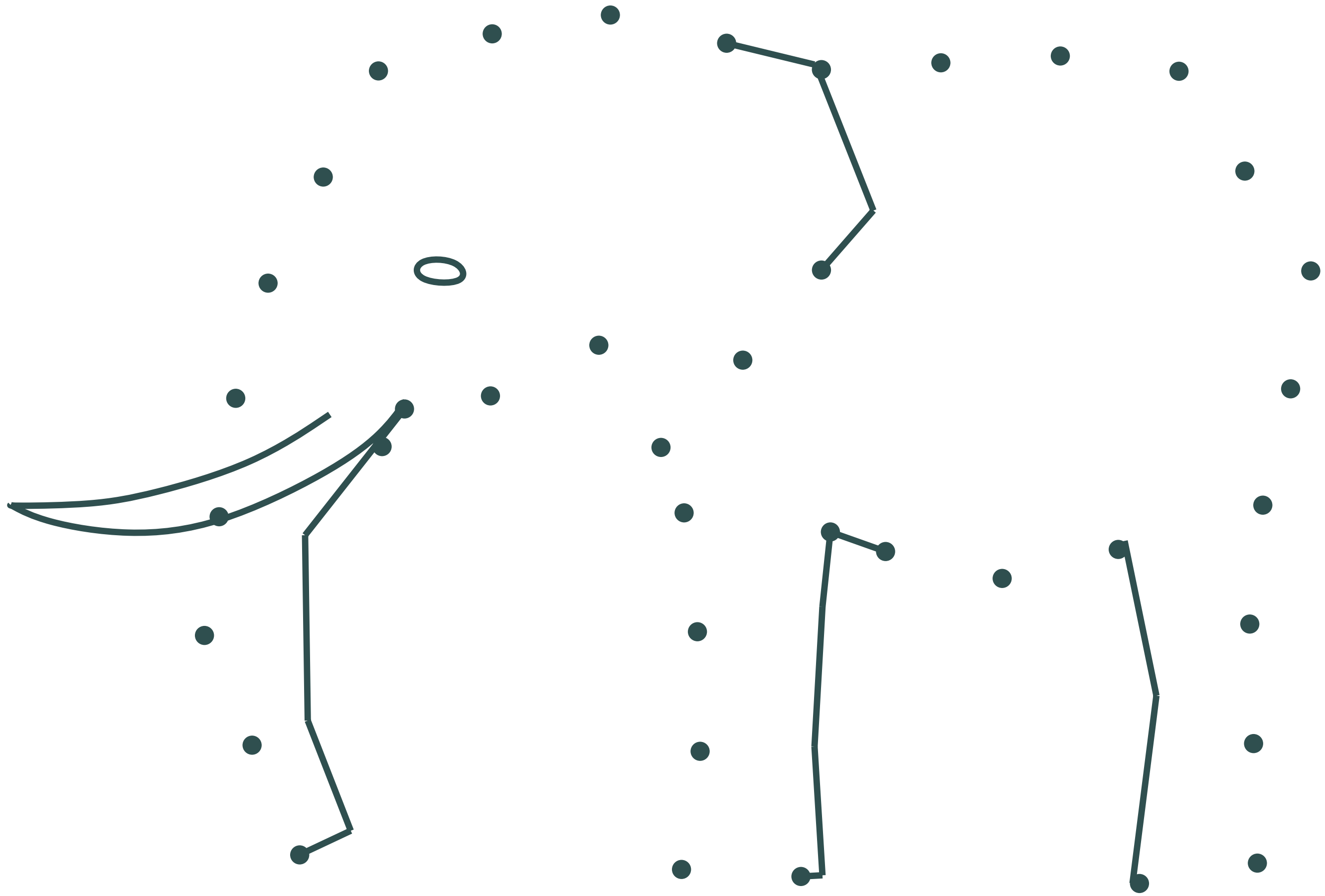
Connect-the-dots



Unite-the-dots

Connect all pairs p, q with $d(p, q) = 1$



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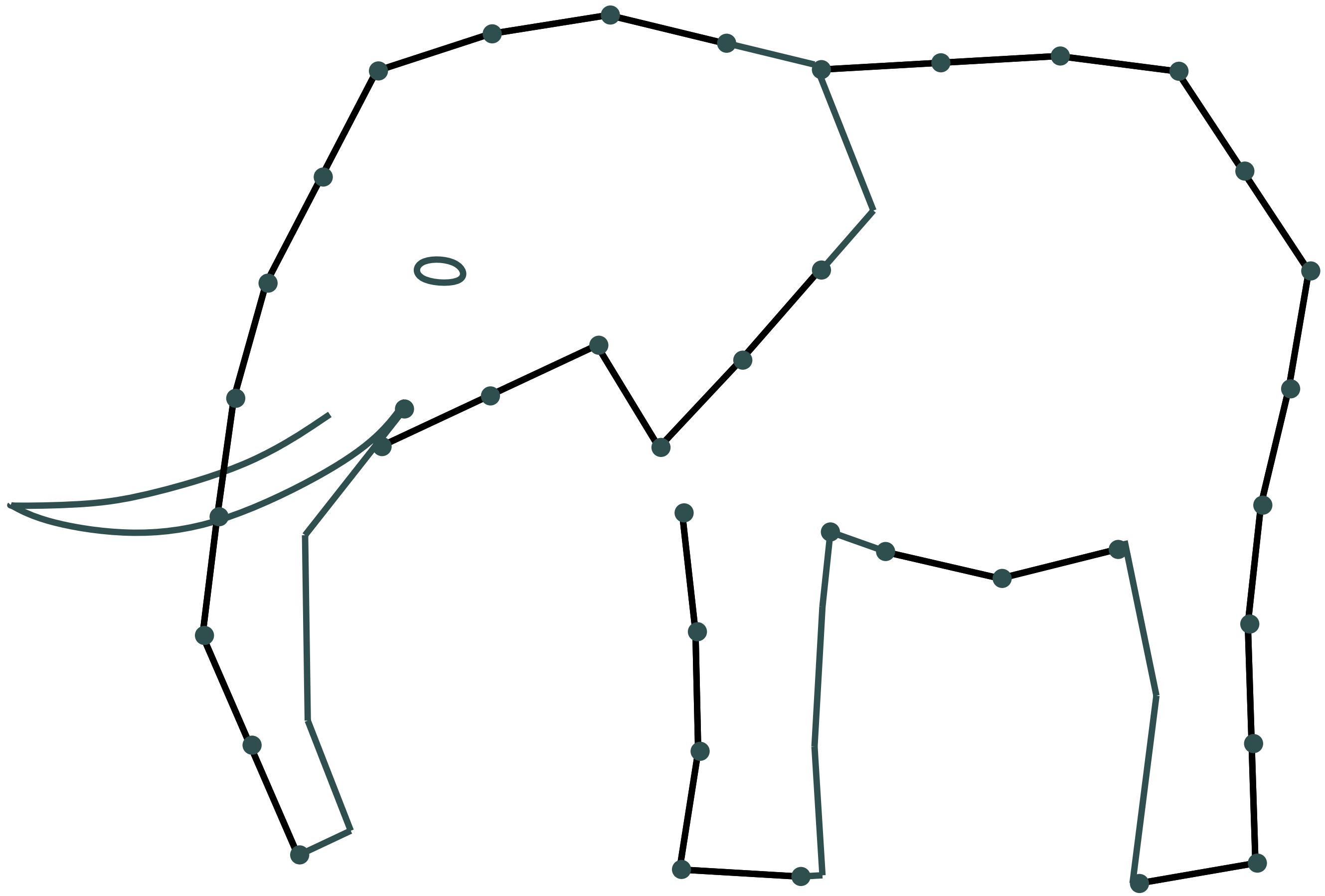
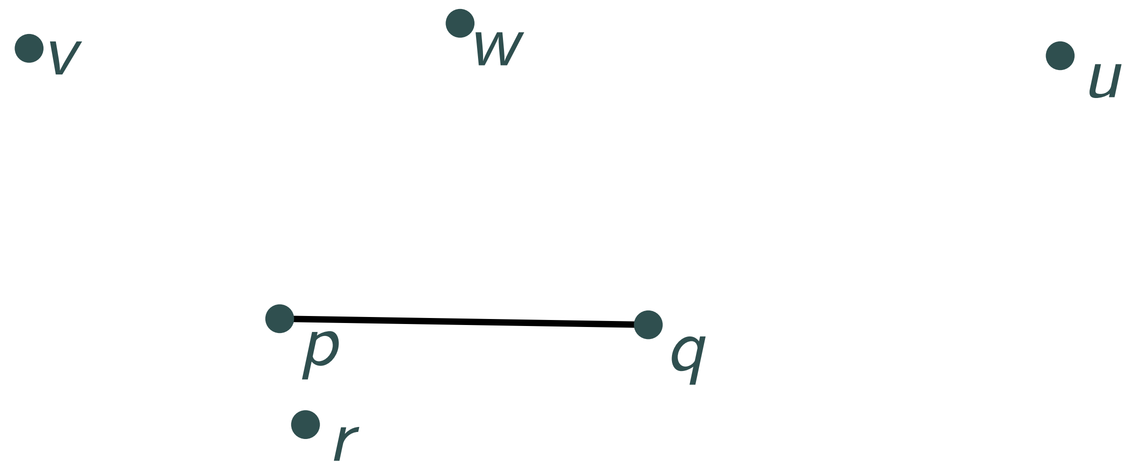


Unite-the-dots

Which points are at distance 1 from p?



Unite-the-dots

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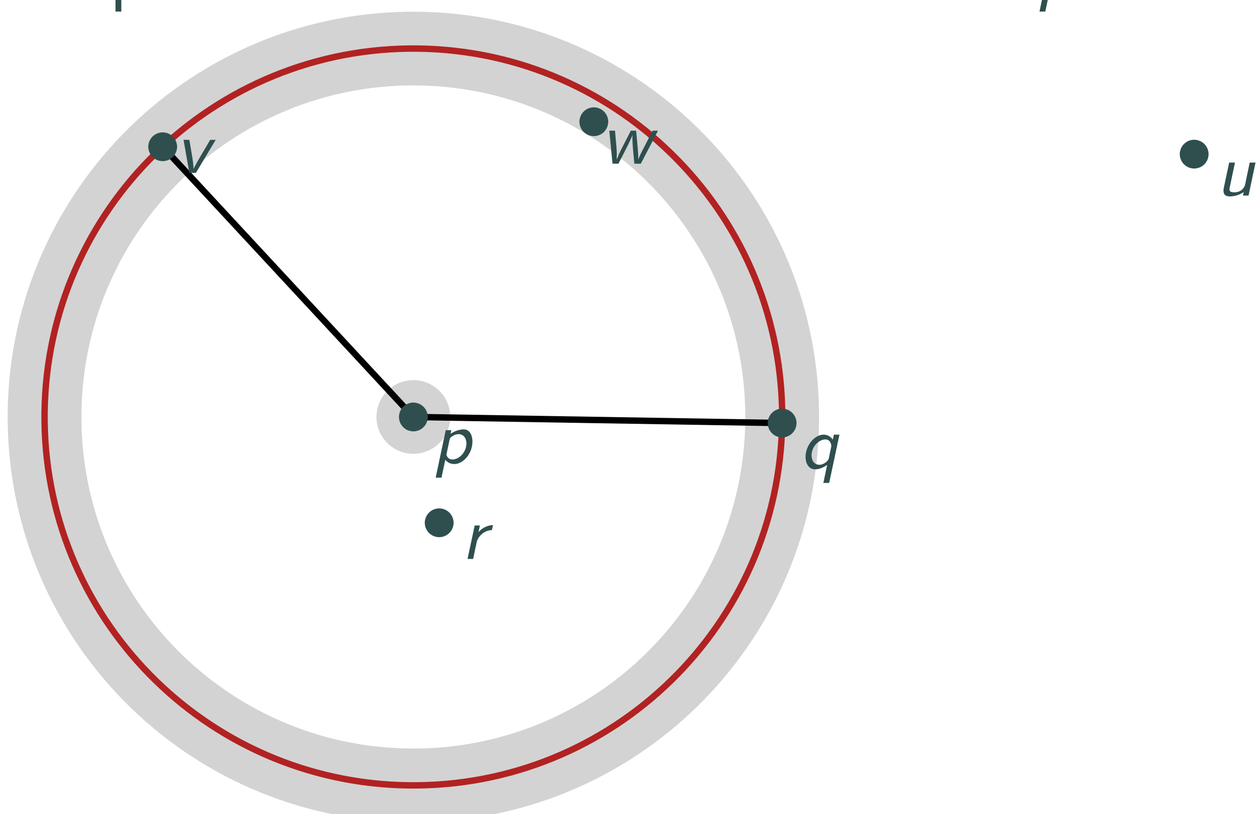


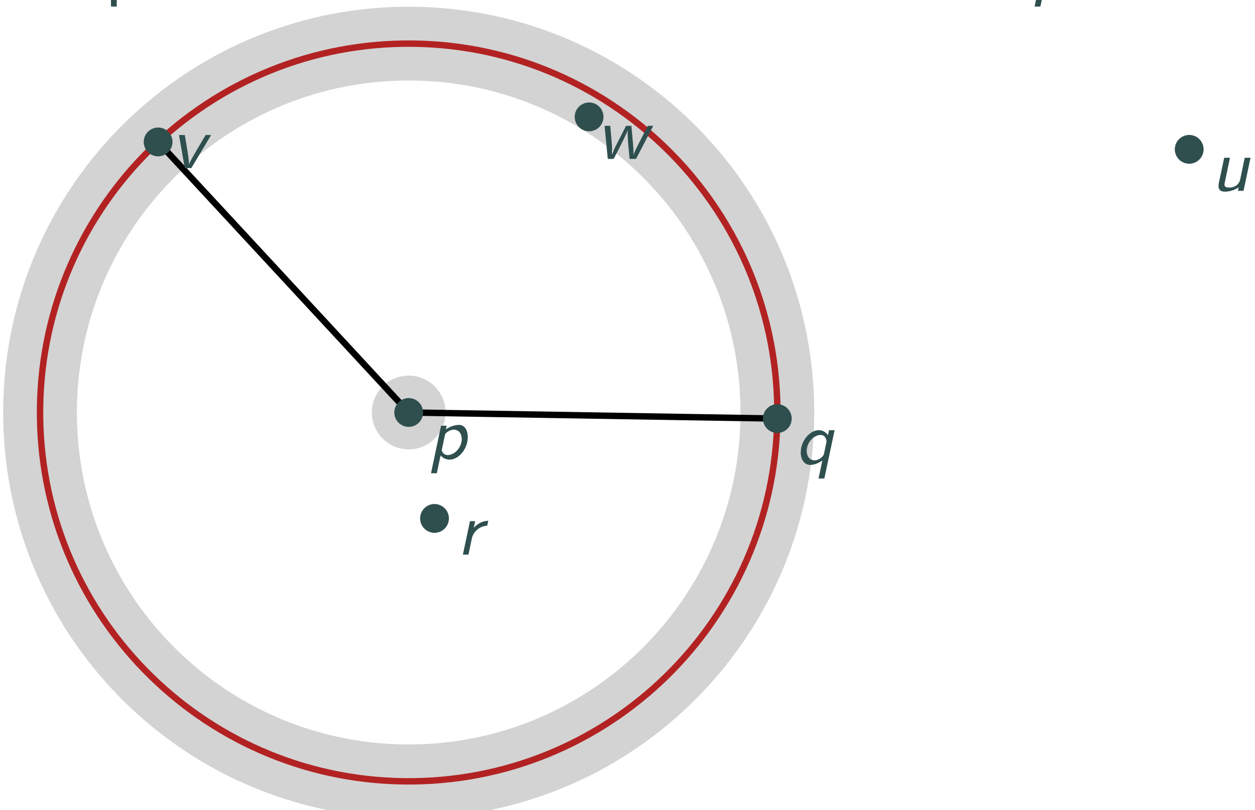
It should be clear which pairs to connect

for all pairs p, q, we require

$d(p, q) \in [\varepsilon, 1 - \varepsilon] \cup [1] \cup [1 + \varepsilon, \infty)$

Unite-the-dots

Which points are at distance 1 from p?



It should be clear which pairs to connect

the points should be the vertices of a clear unit distance graph

Properties of Clear UD Graphs

Density

#points in region of constant diameter?

(Geometric) Diameter

Size of the paper required to draw the graph?

Number of Crossings

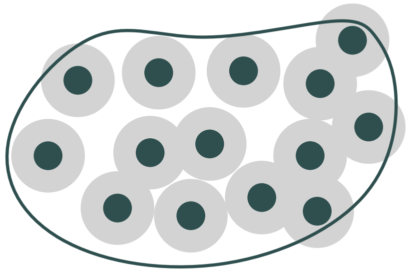
Properties of Clear UD Graphs

Density

Given a $(1, \varepsilon)$-graph.

#points in region of constant diameter?

Upperbound: $O(1/\varepsilon^2)$



Properties of Clear UD Graphs

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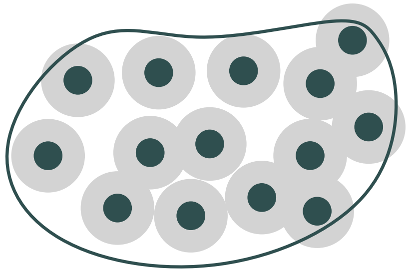

Witness: $\Omega(1/\varepsilon^2)$

Properties of Clear UD Graphs

Density

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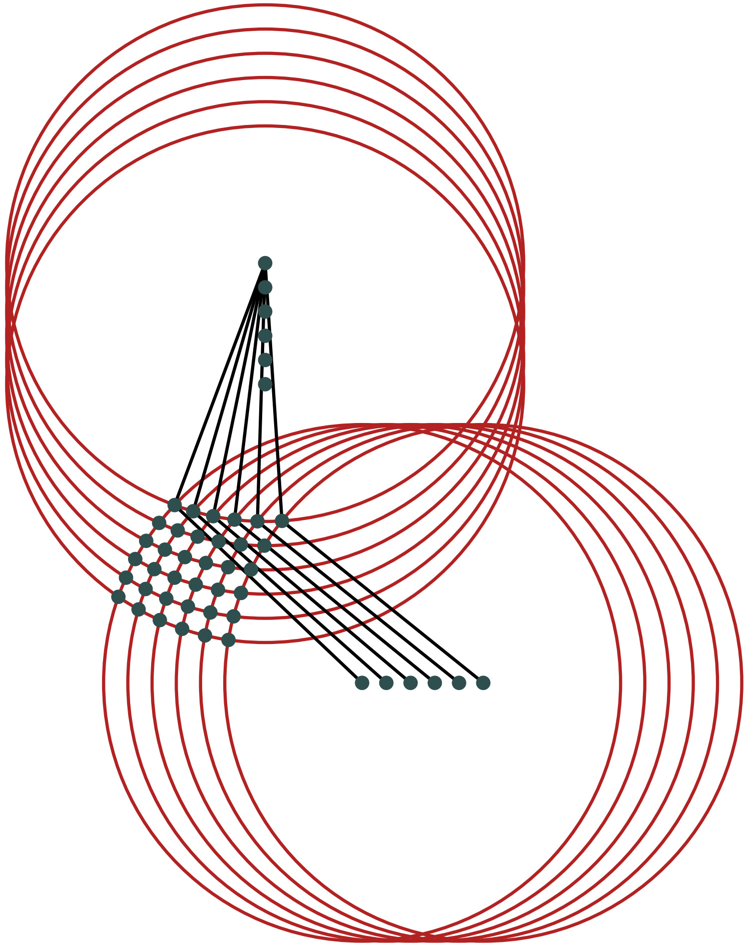


Properties of Clear UD Graphs

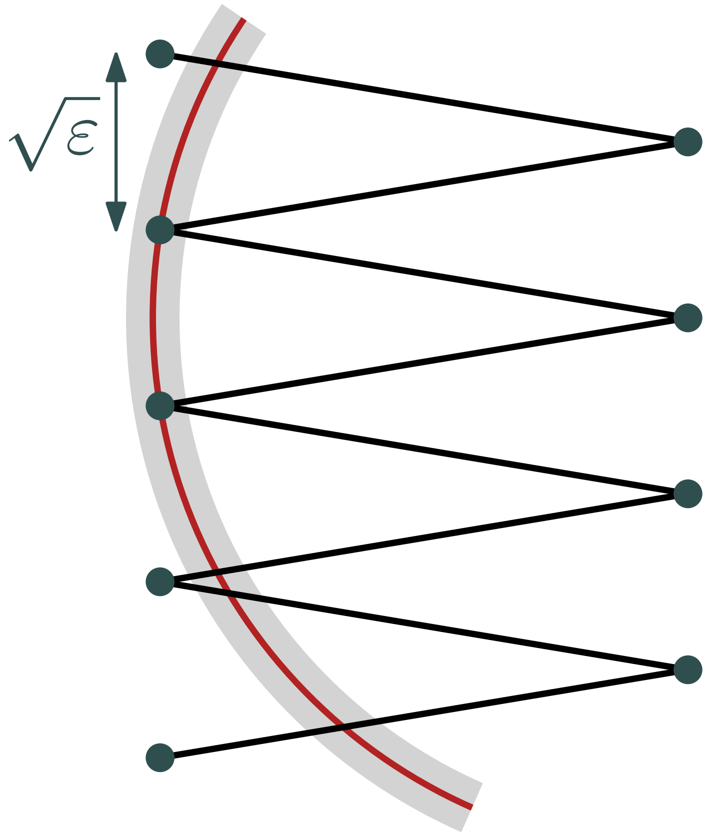
Density

Given a $(1, \varepsilon)$-path.

#points in region of constant diameter?

Upperbound: $O(1/\varepsilon^2)$

Witness: $\Omega(1/\sqrt{\varepsilon})$



Properties of Clear UD Graphs

Diameter

Given a connected $(1, \varepsilon)$-graph.
What is the (geometric) diameter?

Upperbound: $O(n)$ trivial

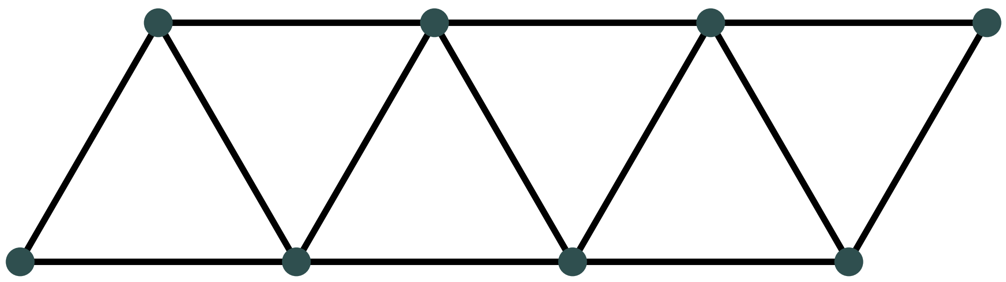
Properties of Clear UD Graphs

Diameter

Given a connected $(1, \varepsilon)$-graph, with $0 < \varepsilon \leq \sqrt{3} - 1$.
What is the (geometric) diameter?

Upperbound: $O(n)$ trivial

Witness: $\Omega(n)$



Properties of Clear UD Graphs

Diameter

Given a connected $(1, \varepsilon)$-graph.
What is the (geometric) diameter?

Upperbound: $O(n)$ trivial

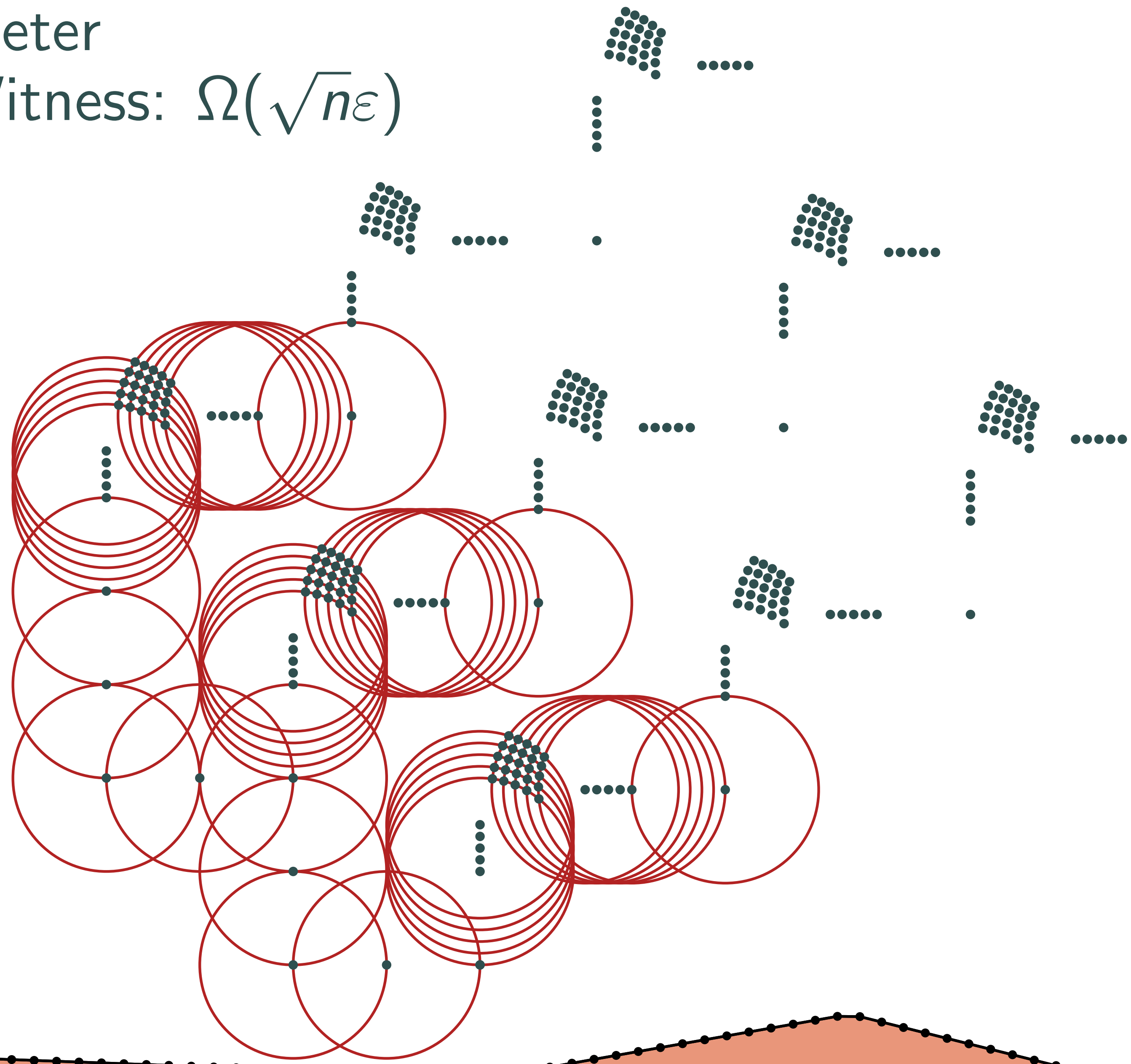
Lowerbound: $\Omega(\sqrt{n}\varepsilon)$

Witness: $\Omega(\sqrt{n}\varepsilon)$

Properties of Clear UD Graphs

Diameter

Witness: $\Omega(\sqrt{n}\varepsilon)$



Unite-the-dots

Input:

Output:

Unite-the-dots

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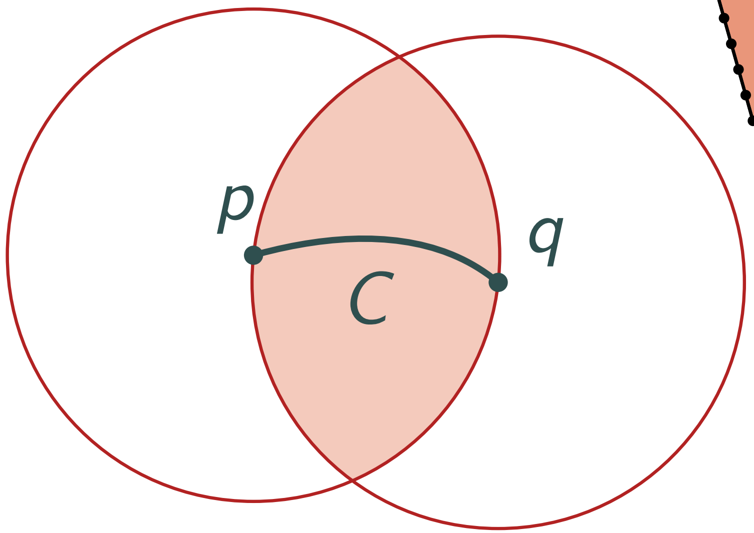
Output:

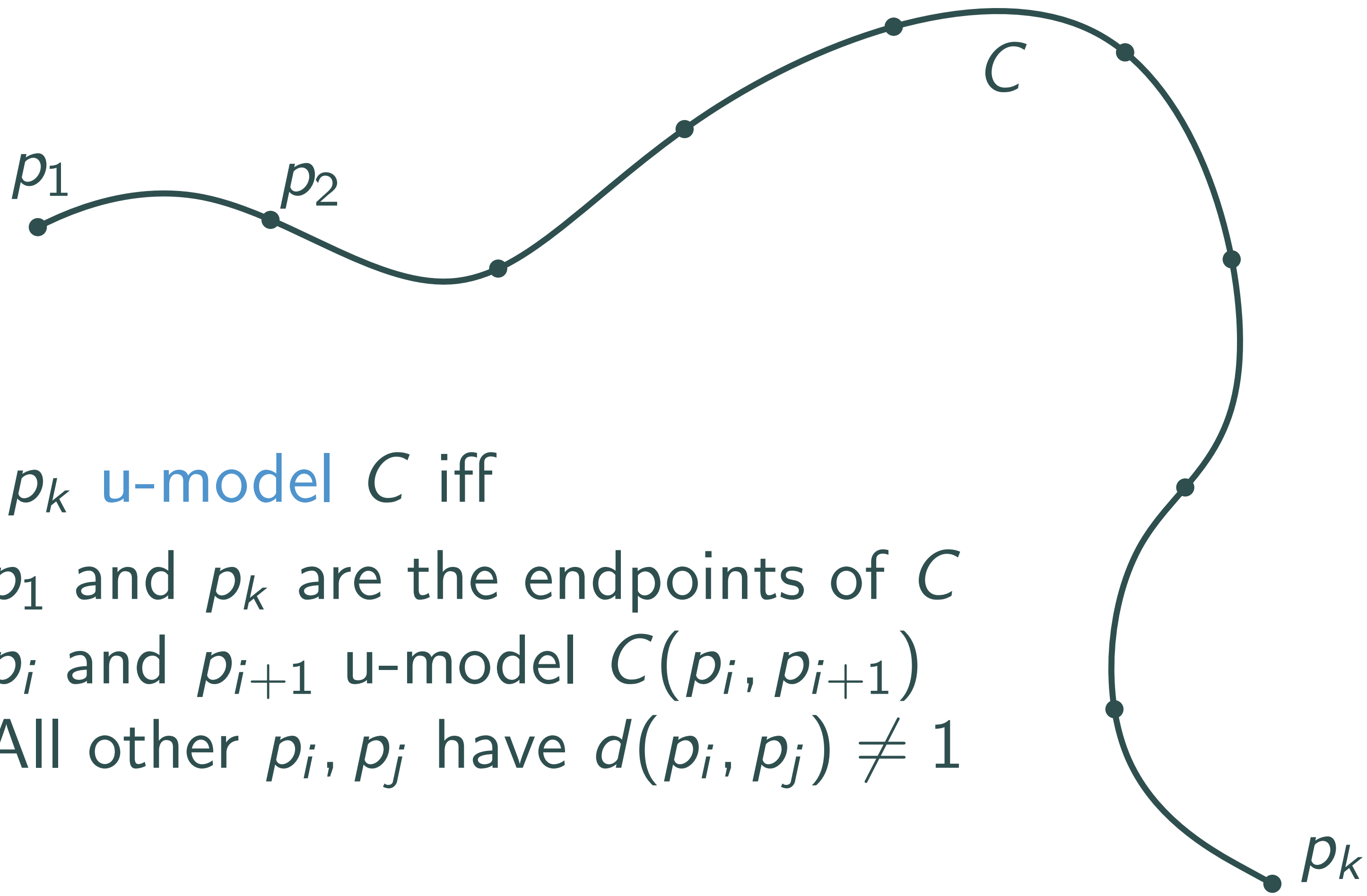
Unite-the-dots



p and q u-model C iff

- $d(p, q) = 1$
- $\|C\| \leq 1 + \delta$
- C inside both unit discs centered at p and q

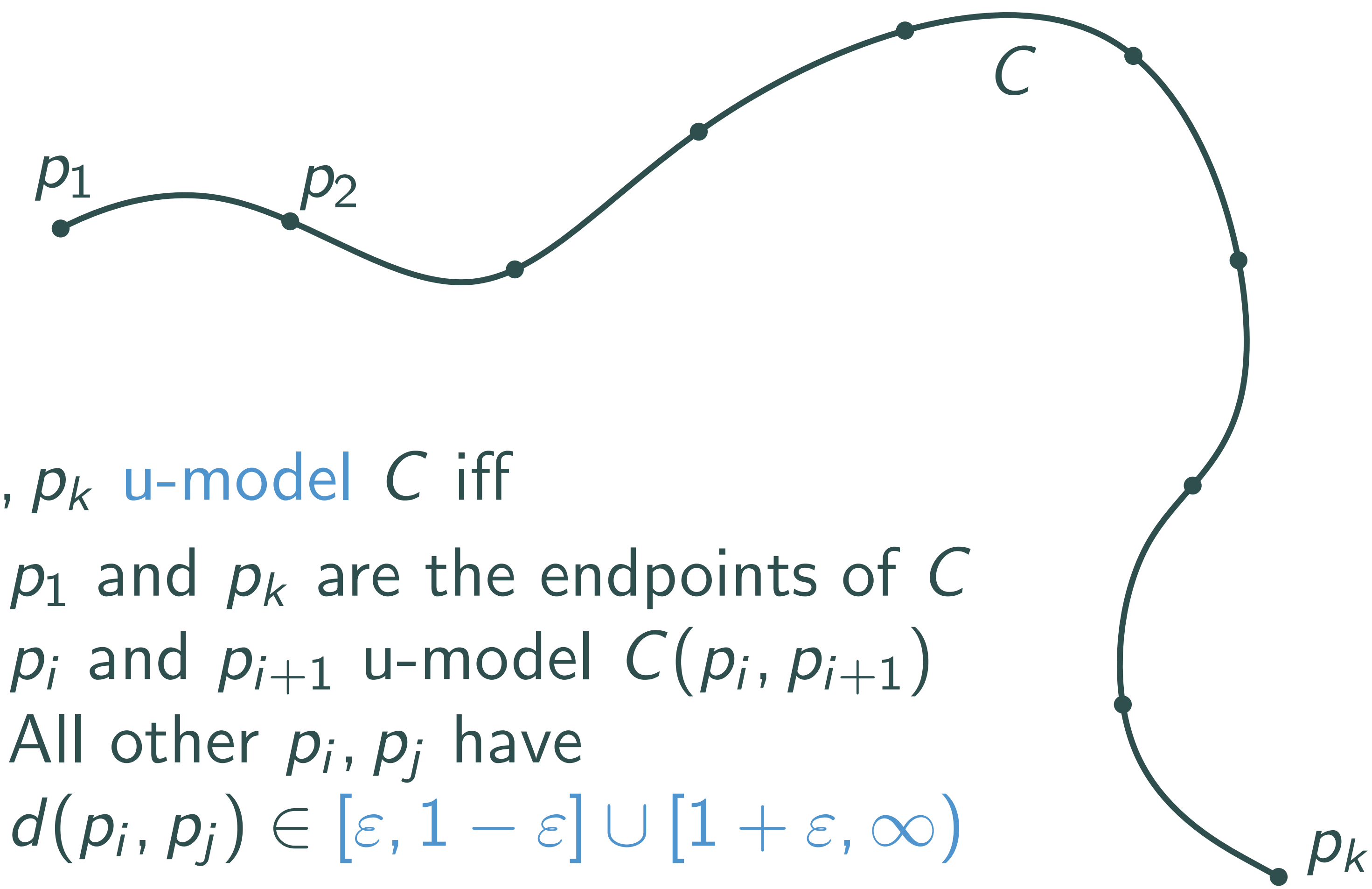
Unite-the-dots



$p_1, .., p_k$ u-model C iff

- p_1 and p_k are the endpoints of C
- p_i and p_{i+1} u-model $C(p_i, p_{i+1})$
- All other p_i, p_j have $d(p_i, p_j) \neq 1$

Unite-the-dots



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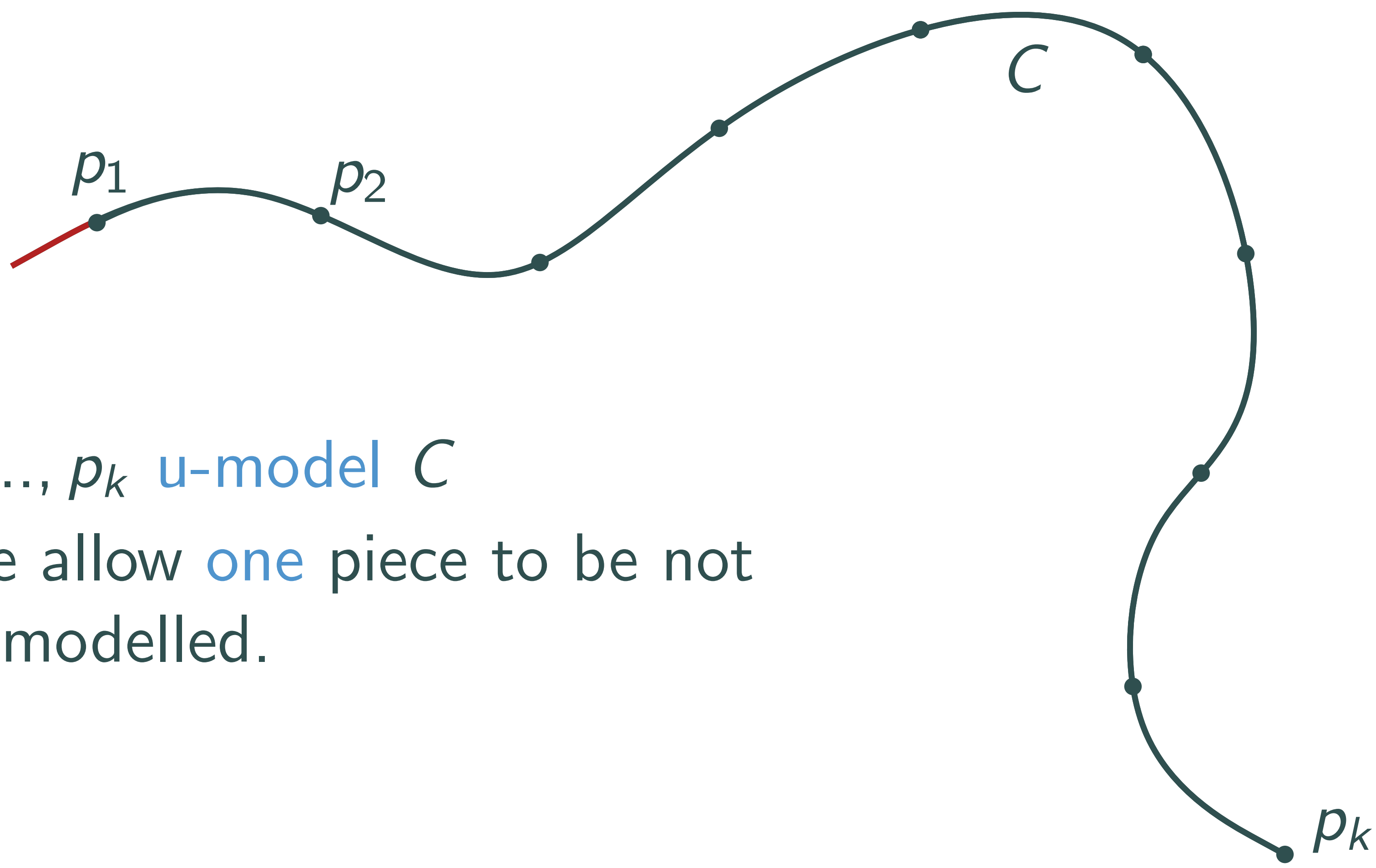
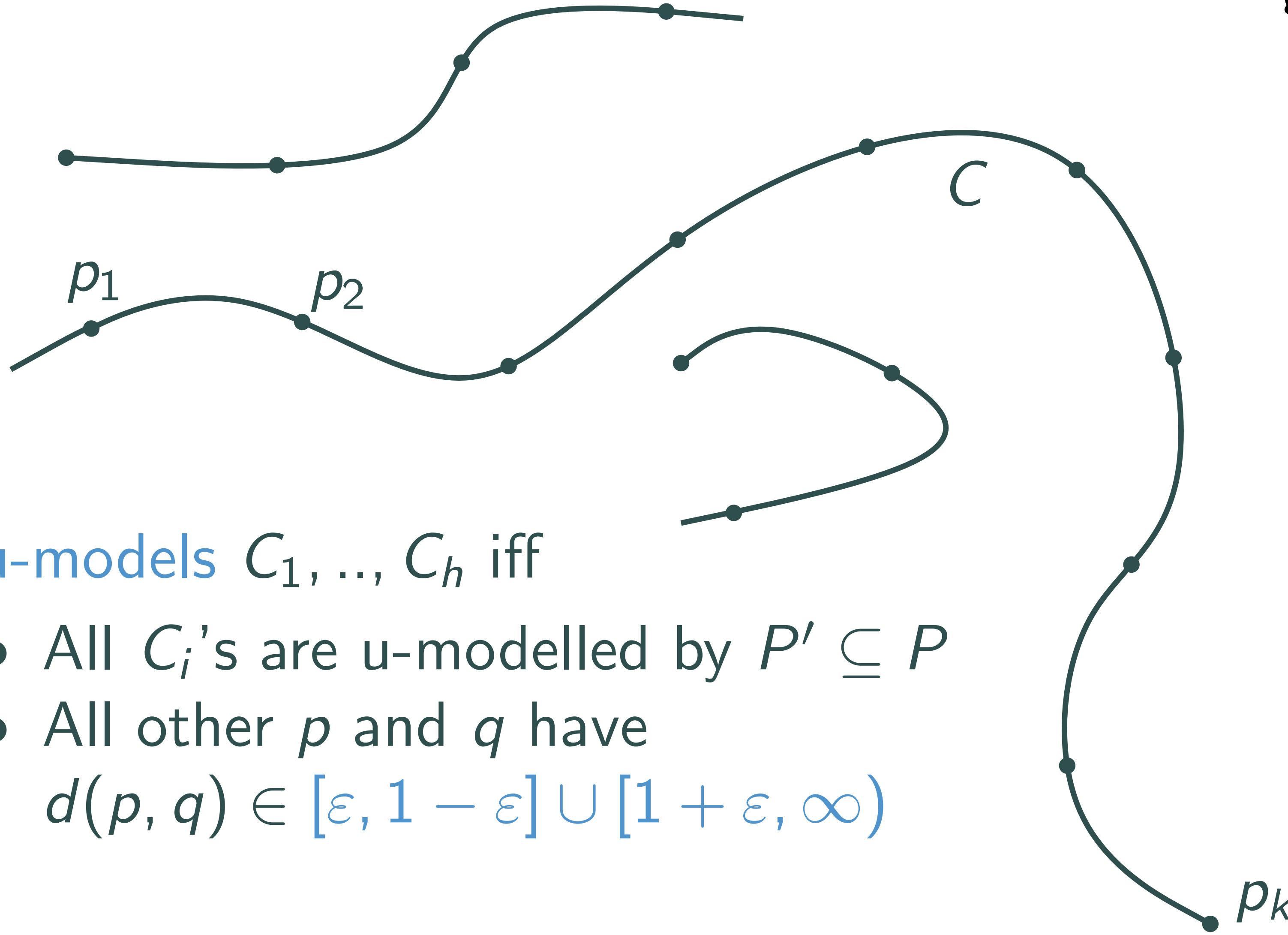
Unite-the-dots



$p_1, .., p_k$ u-model C

we allow one piece to be not u-modelled.

Unite-the-dots



P u-models $C_1, .., C_h$ iff

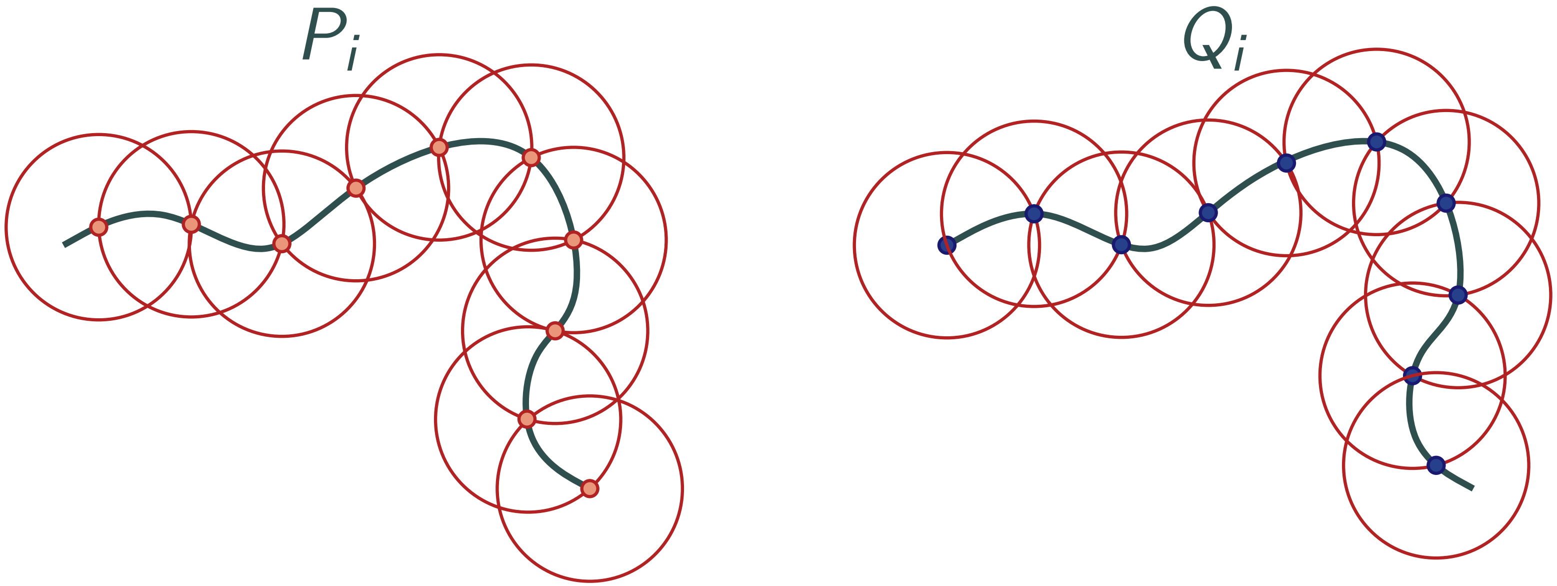
- All C_i's are u-modelled by $P' \subseteq P$
- All other p and q have $d(p, q) \in [\varepsilon, 1 - \varepsilon] \cup [1 + \varepsilon, \infty)$

Algorithm

Pre-drawn piece at the end

Algorithm

Pre-drawn piece at the end



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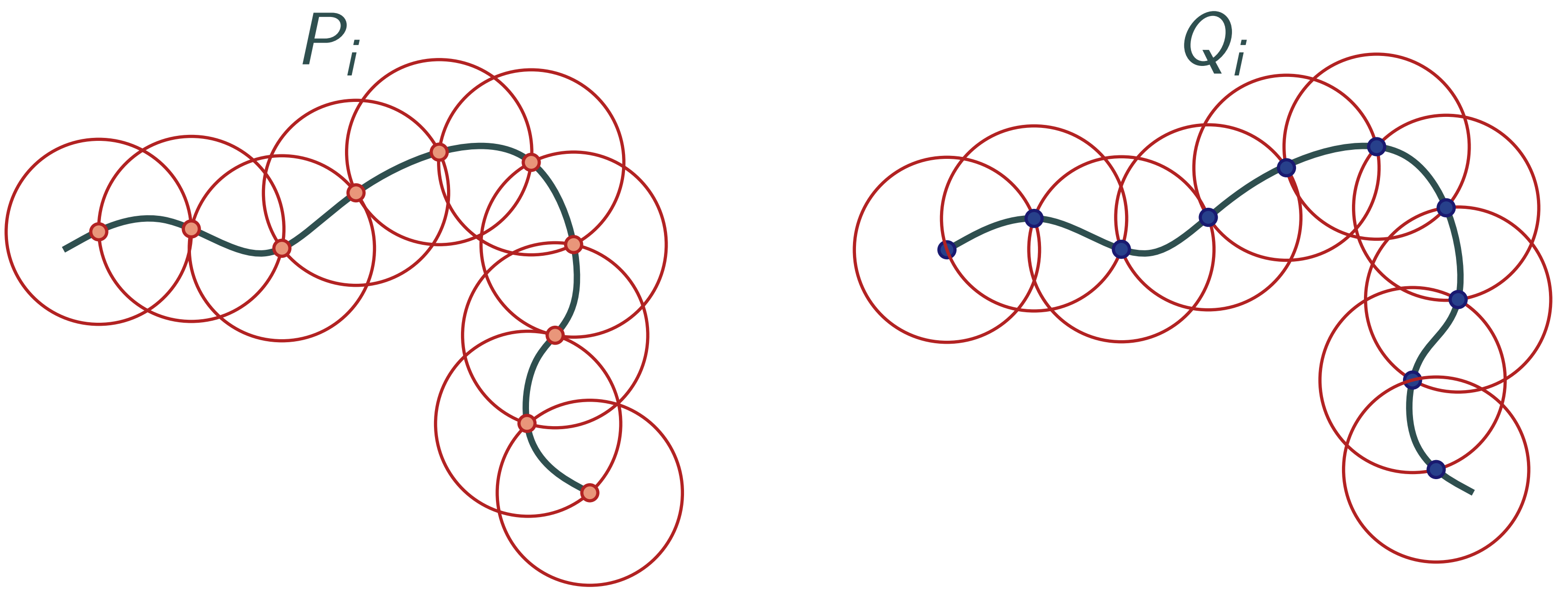


Each curve C_i 2 choices P_i or Q_i

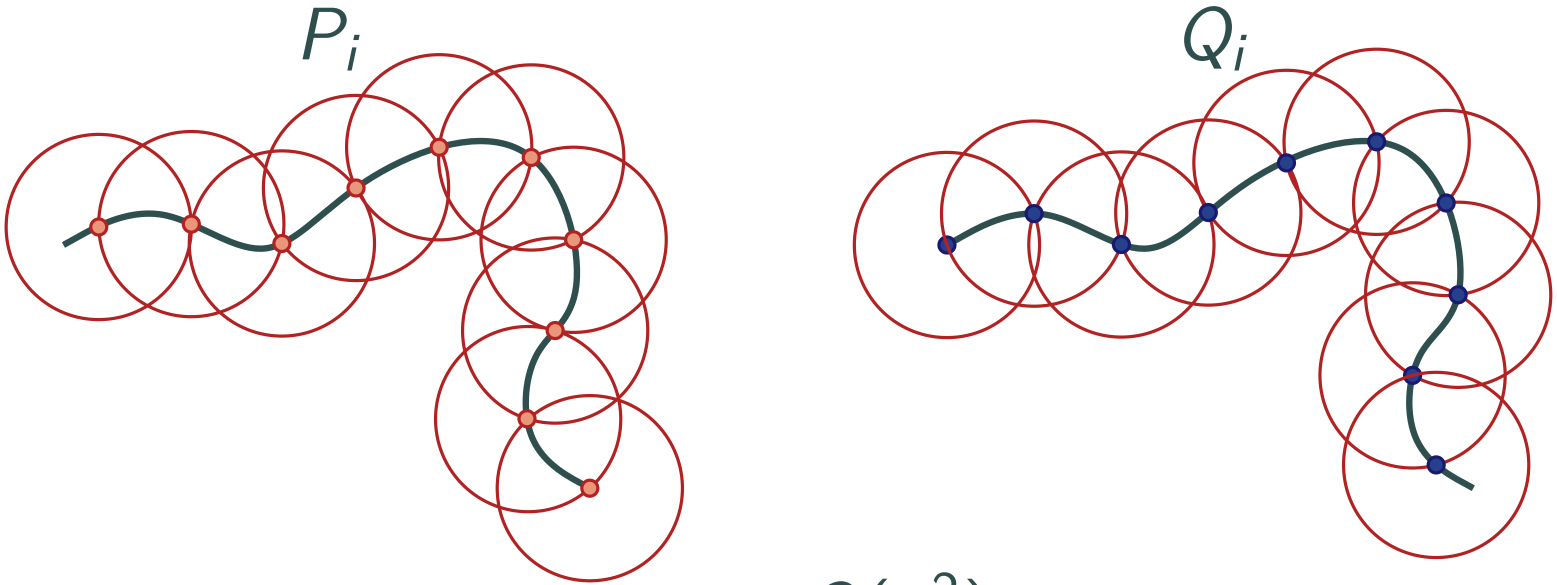
$\Updownarrow$

$x_i = \textsc{True}$ and $x_i = \textsc{False}$

Build a 2-SAT formula:
if $Q_i \cup P_j$ not a $(1, \varepsilon)$-point set then add $x_i \vee \overline{x_j}$

Algorithm

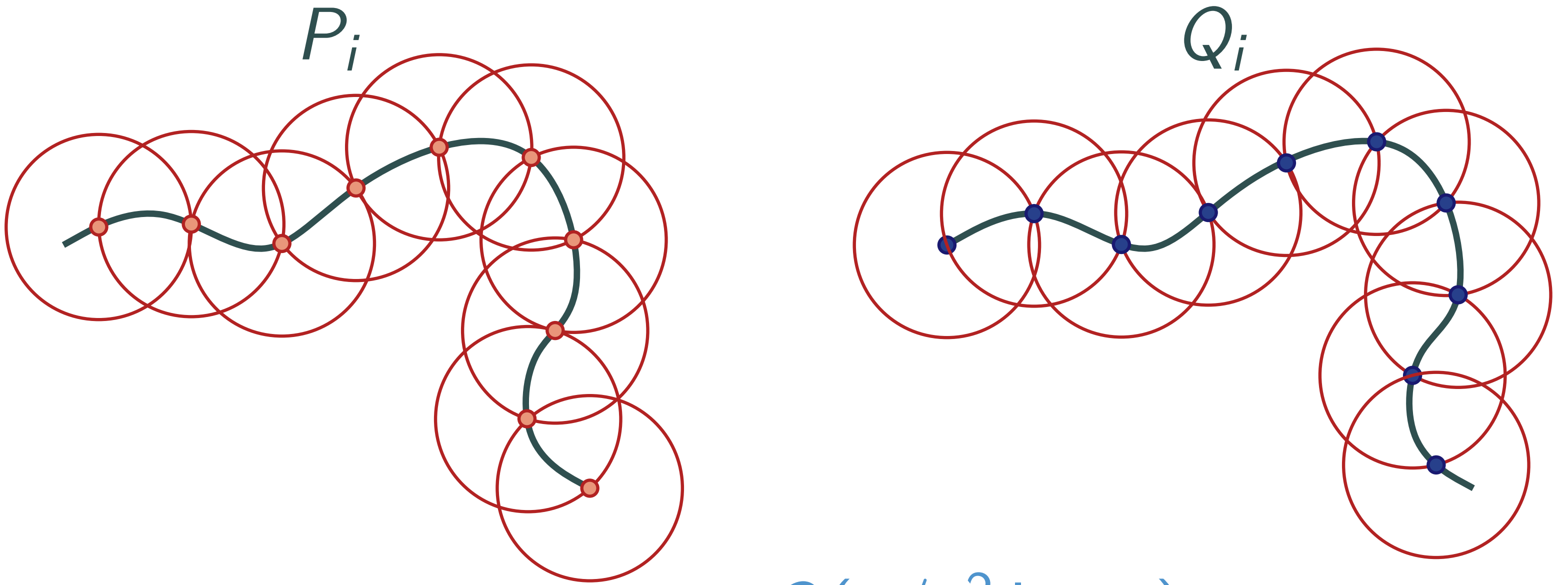
Pre-drawn piece at the end



Running time: $O(n^2)$

Algorithm

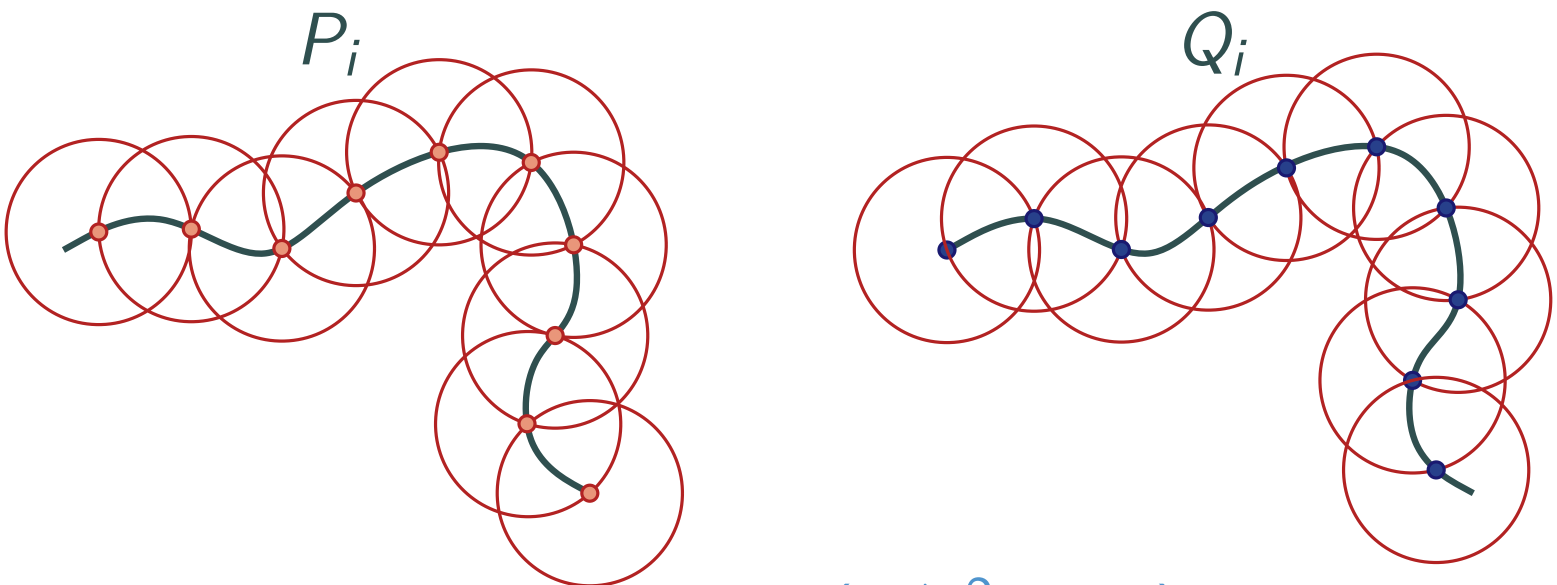
Pre-drawn piece at the end



Running time: $O(n/\varepsilon^2 \log n)$

Algorithm

Pre-drawn piece at the end



Running time: $O(n/\varepsilon^2 \log n)$

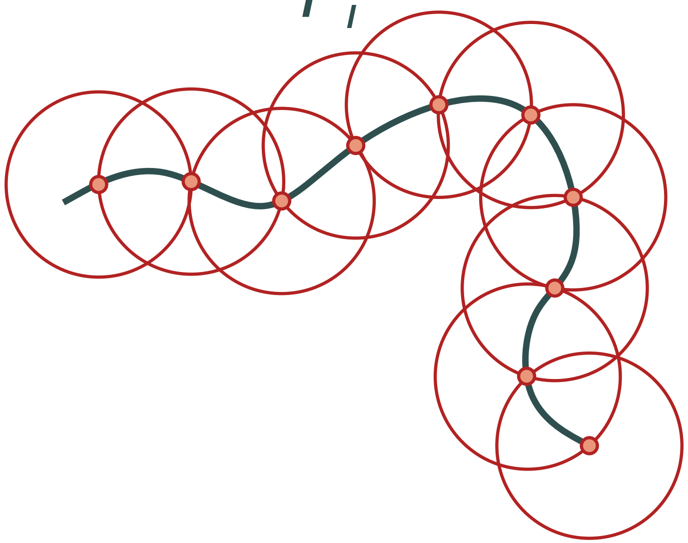
Depends on density bound for $(1, \varepsilon)$-paths

Future Work: Improve to ...?

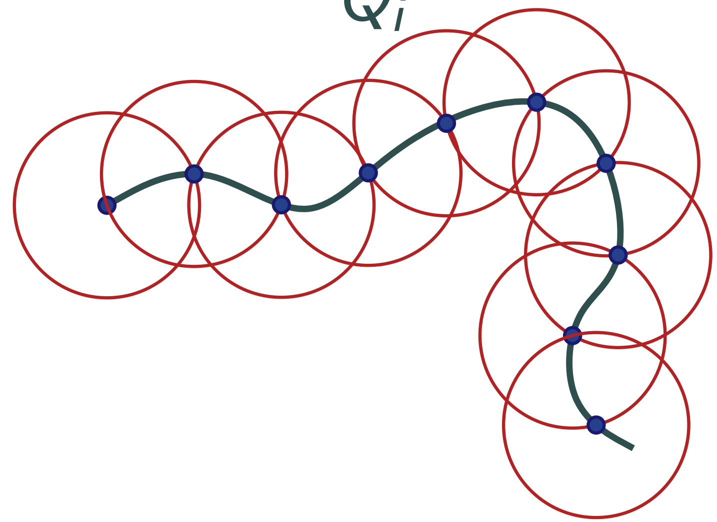
Algorithm

Pre-drawn piece at the end

P_i



Q_i

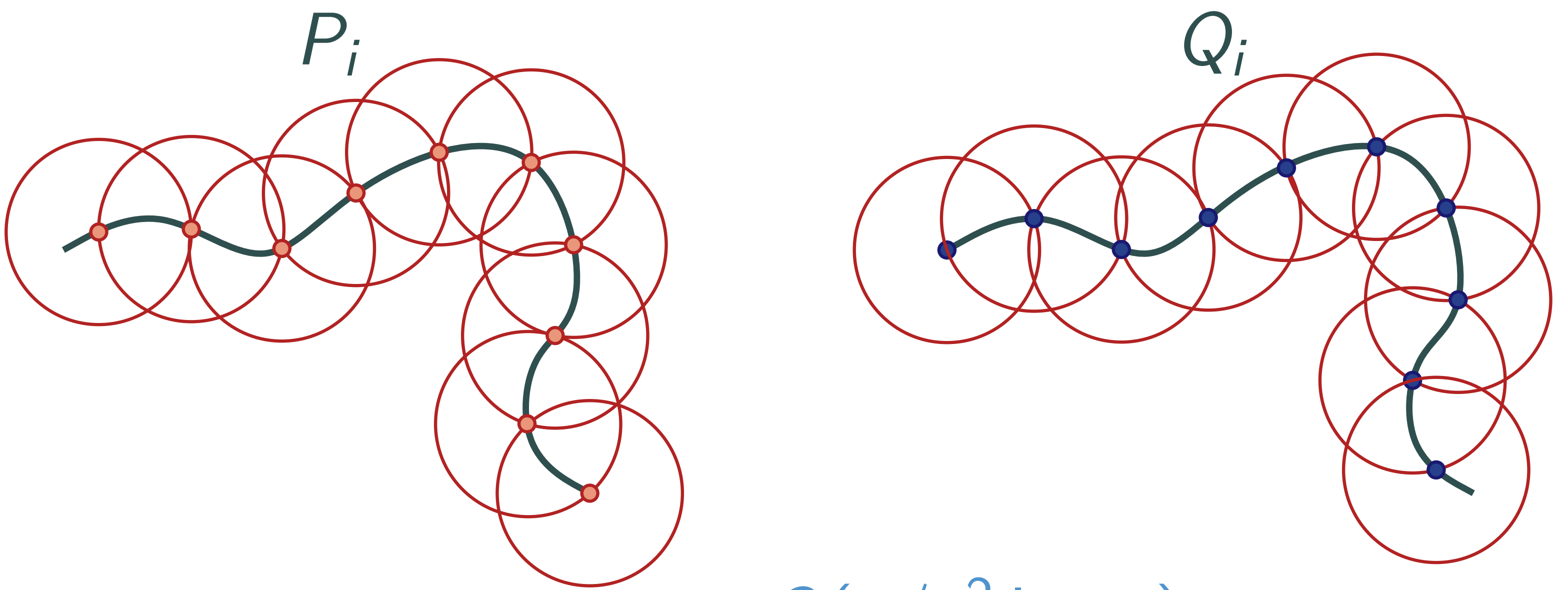


Running time: $O(n/\varepsilon^2 \log n)$

Interior piece: Similar approach

Algorithm

Pre-drawn piece at the end



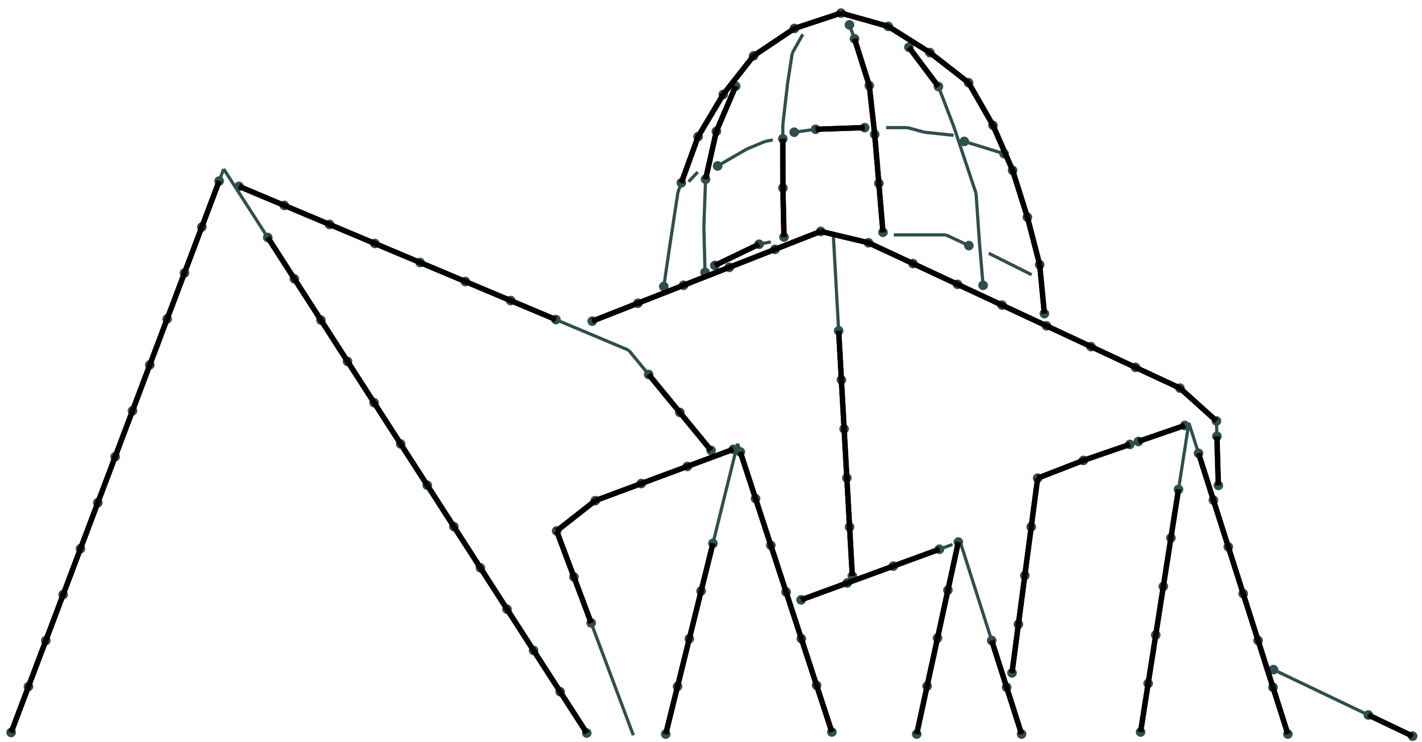
Running time: $O(n/\varepsilon^2 \log n)$

Interior piece: Similar approach

Minimize piece length: NP hard

>1 piece per curve: NP hard

Thank you!



Thank you!