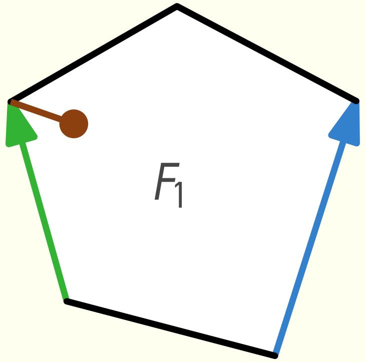
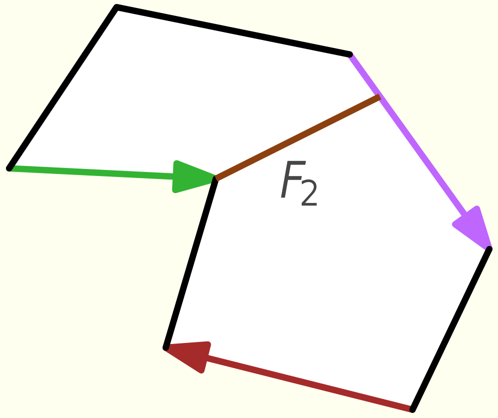
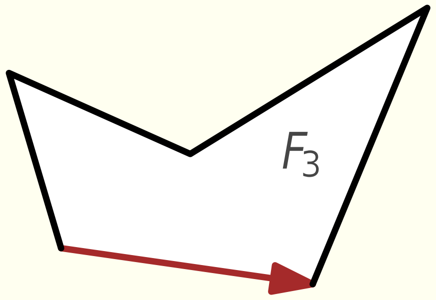
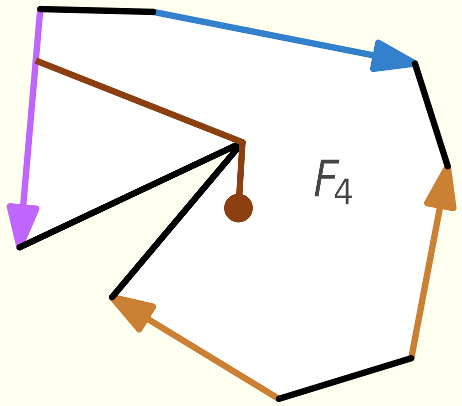
Shortest Paths in Portalgons







Maarten Löffler
Rodrigo Silveira
Tim Ophelders
Frank Staals



Portalgons?

Portalgon $\mathcal{P} = (\mathcal{F},\mathcal{E})$

$\mathcal{F}$: set of simple polygons

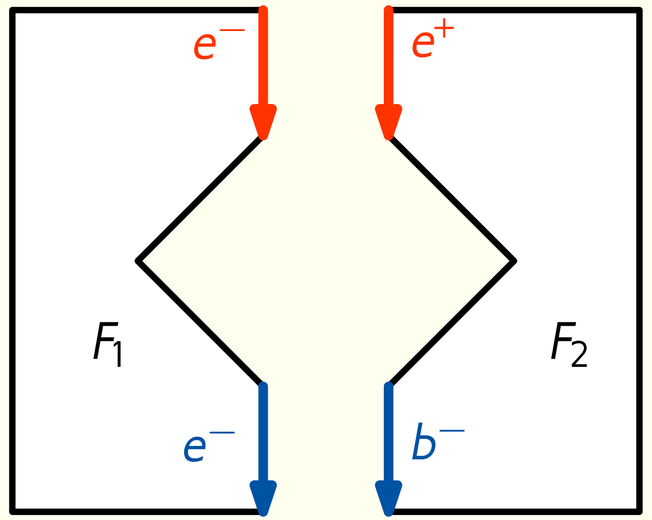
$\mathcal{E}$: set of portals

portal (e^-, e^+) : pair of identified edges

$\mathcal{P}$

$\mathcal{F} = \{F_1, F_2\}$

$\mathcal{E} = \{(e^-, e^+), (b^-, b^+)\}$



Portalgons?

Portalgon $\mathcal{P} = (\mathcal{F}, \mathcal{E})$

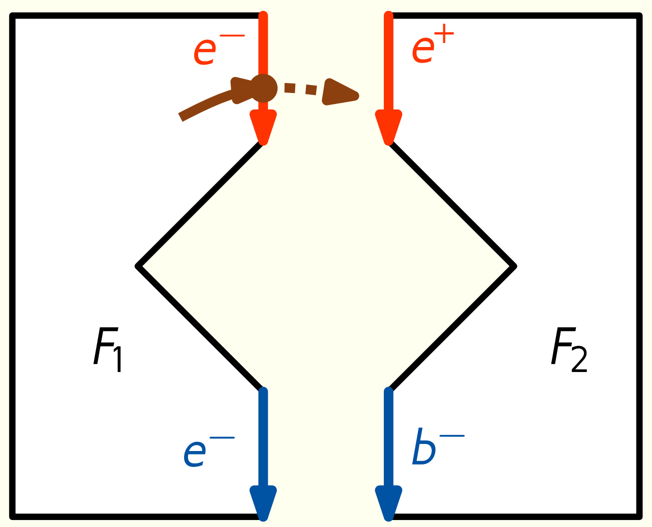
- $\mathcal{F}$: set of simple polygons
- $\mathcal{E}$: set of portals

portal (e^-, e^+) : pair of identified edges

$\mathcal{P}$

$\mathcal{F} = \{F_1, F_2\}$

$\mathcal{E} = \{(e^-, e^+), (b^-, b^+)\}$



Portalgons?

Portalgon $\mathcal{P} = (\mathcal{F}, \mathcal{E})$

$\mathcal{F}$: set of simple polygons

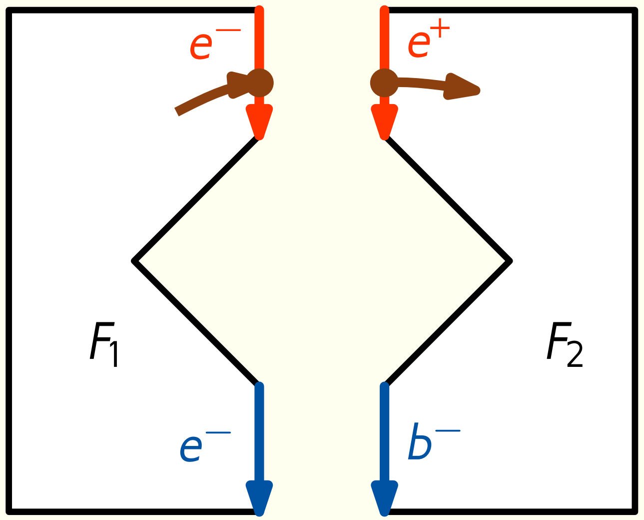
$\mathcal{E}$: set of portals

portal (e^-, e^+) : pair of identified edges

$\mathcal{P}$

$\mathcal{F} = \{F_1, F_2\}$

$\mathcal{E} = \{(e^-, e^+), (b^-, b^+)\}$



Portalgons?

Portalgon $\mathcal{P} = (\mathcal{F}, \mathcal{E})$

$\mathcal{F}$: set of simple polygons

$\mathcal{E}$: set of portals

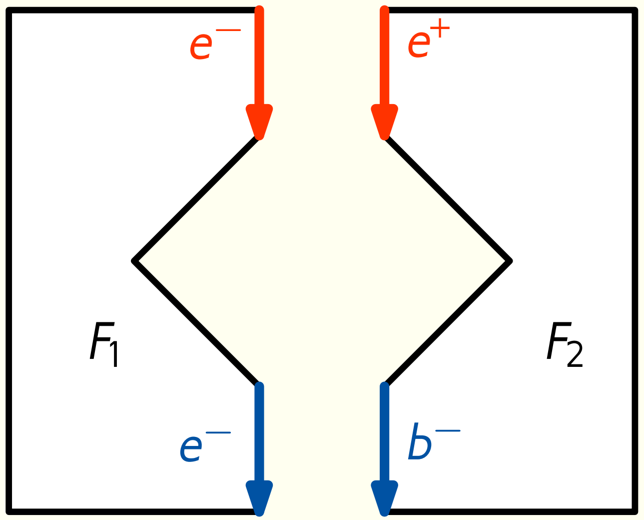
portal (e^-, e^+) : pair of identified edges

Generalization of a polygon, polyhedron, etc.

$\mathcal{P}$

$\mathcal{F} = \{F_1, F_2\}$

$\mathcal{E} = \{(e^-, e^+), (b^-, b^+)\}$



Portalgons?

Portalgon $\mathcal{P} = (\mathcal{F},\mathcal{E})$

$\mathcal{F}$: set of simple polygons

$\mathcal{E}$: set of portals

portal (e^-, e^+) : pair of identified edges

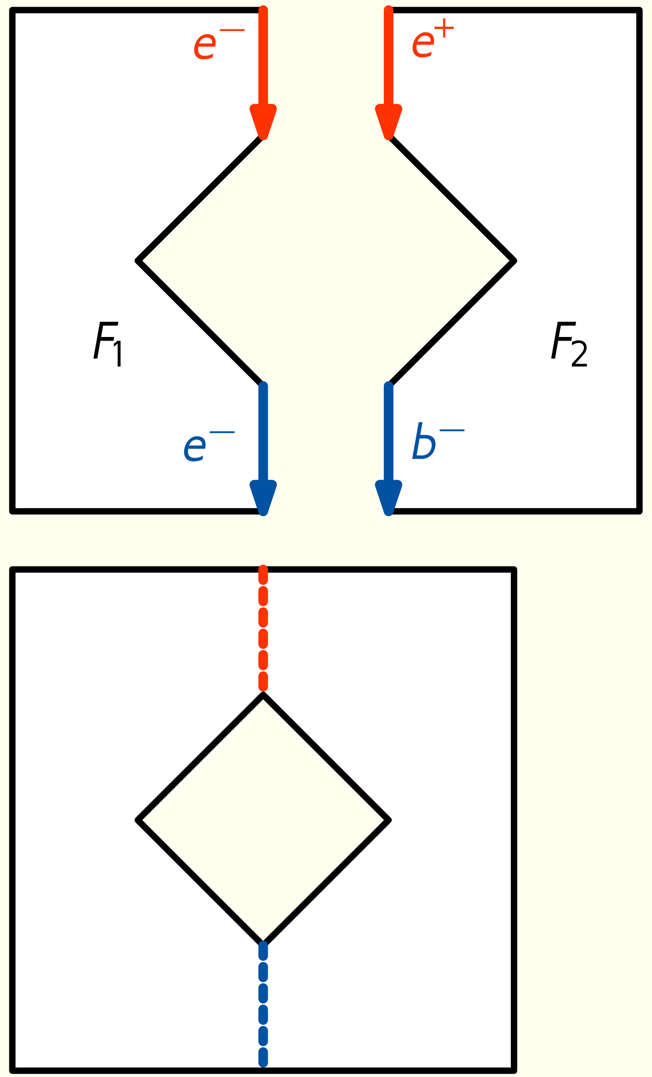
Generalization of a polygon, polyhedron, etc.

A portalgon is a **representation** of a surface Σ

$\mathcal{P}$

$\mathcal{F} = \{F_1, F_2\}$

$\mathcal{E} = \{(e^-, e^+), (b^-, b^+)\}$



Σ

Portalgons?

Portalgon $\mathcal{P} = (\mathcal{F}, \mathcal{E})$

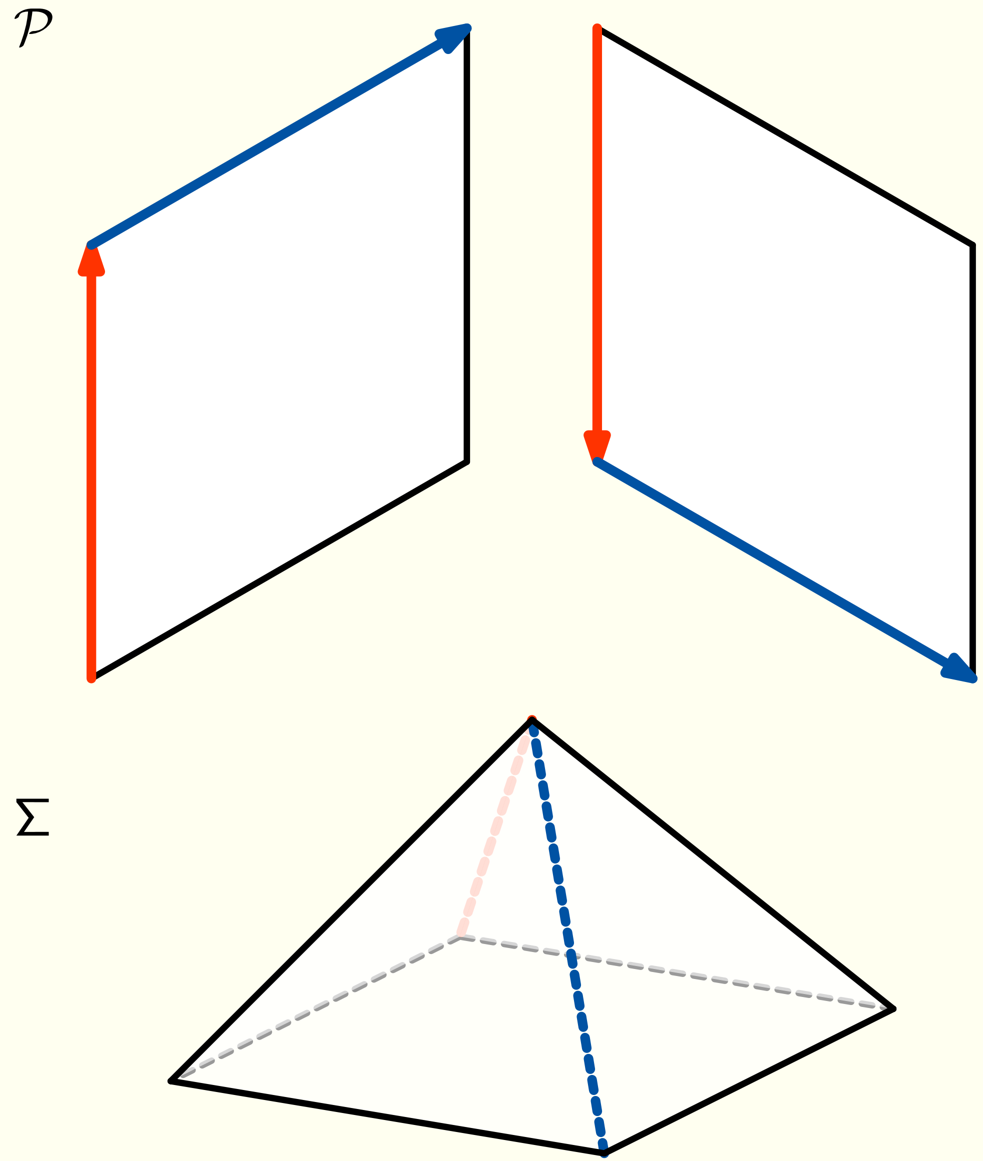
$\mathcal{F}$: set of simple polygons

$\mathcal{E}$: set of portals

portal (e^-, e^+) : pair of identified edges

Generalization of a polygon, polyhedron, etc.

A portalgon is a **representation** of a surface Σ



Portalgons?

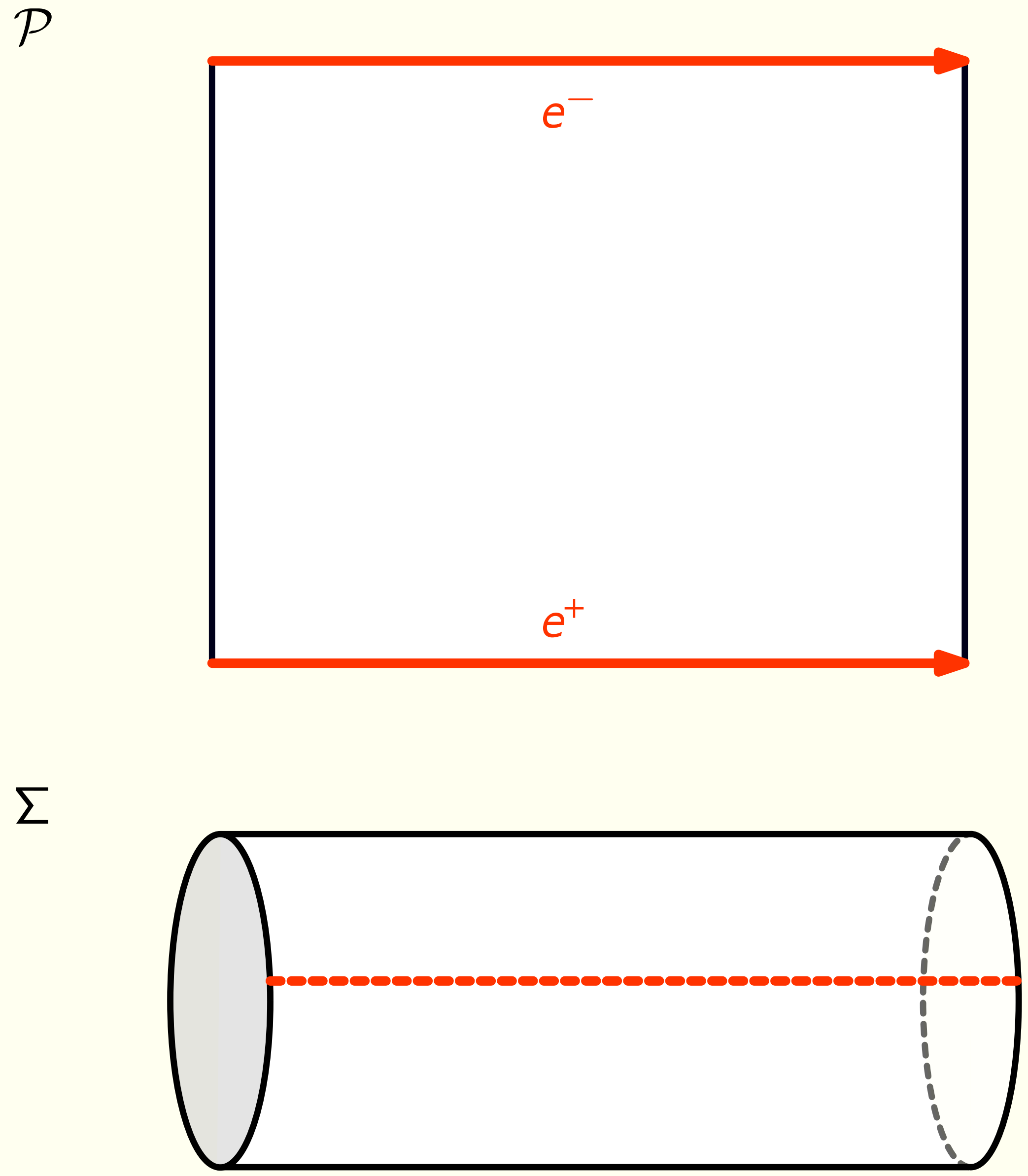
Portalgon $\mathcal{P} = (\mathcal{F}, \mathcal{E})$

- $\mathcal{F}$: set of simple polygons
- $\mathcal{E}$: set of portals

portal (e^-, e^+) : pair of identified edges

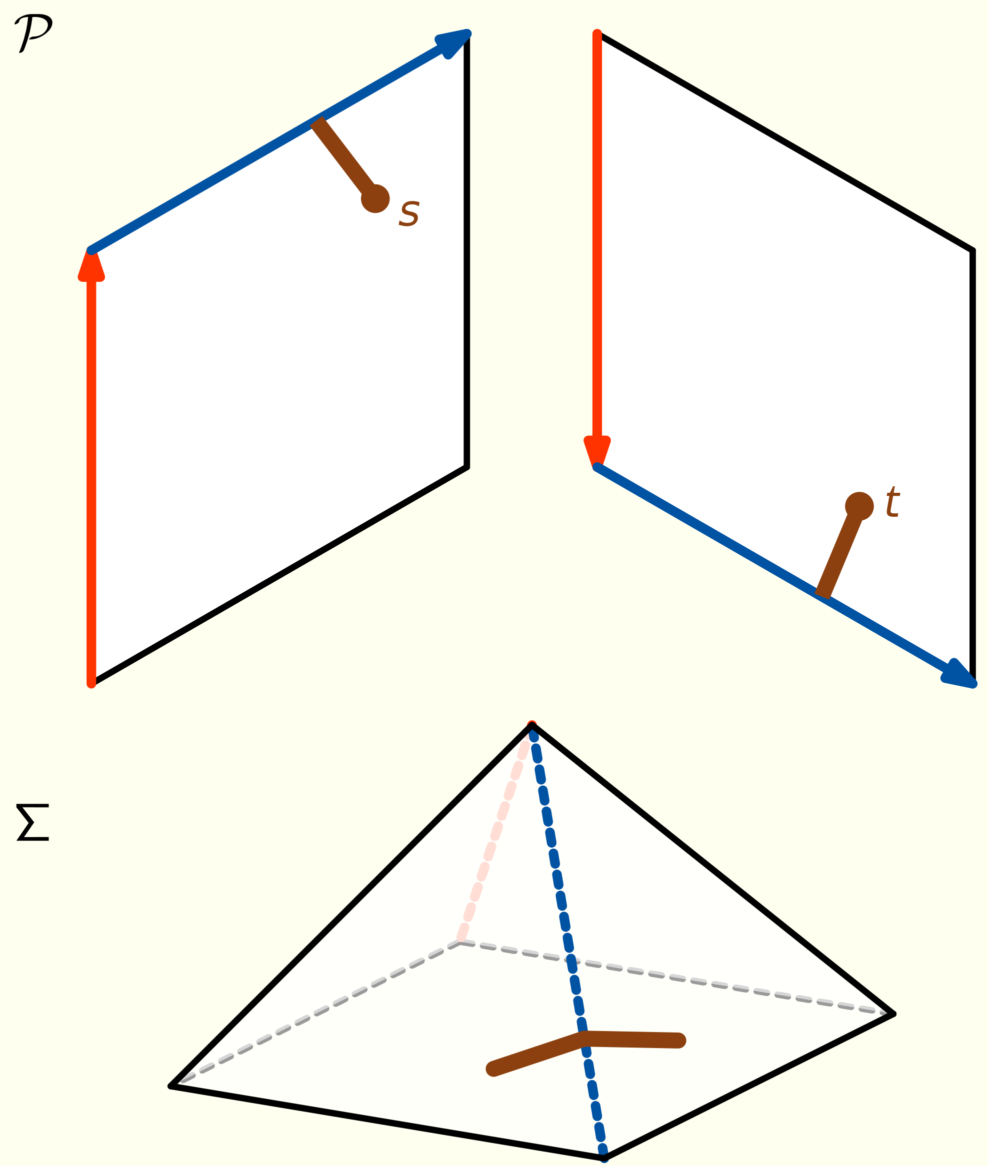
Generalization of a polygon, polyhedron, etc.

A portalgon is a **representation** of a surface Σ



Portalgons?

Q: Can we efficiently compute a **shortest path** $\pi(s, t)$ in $\mathcal{P}$?



Portalgons?

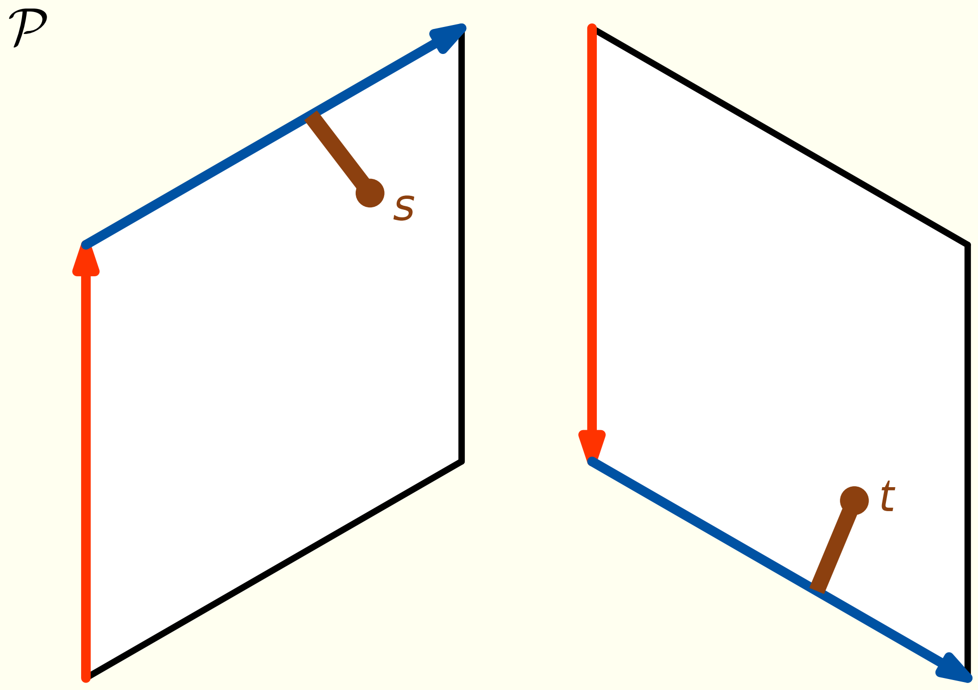
Q: Can we efficiently compute a **shortest path** $\pi(s,t)$ in $\mathcal{P}$?

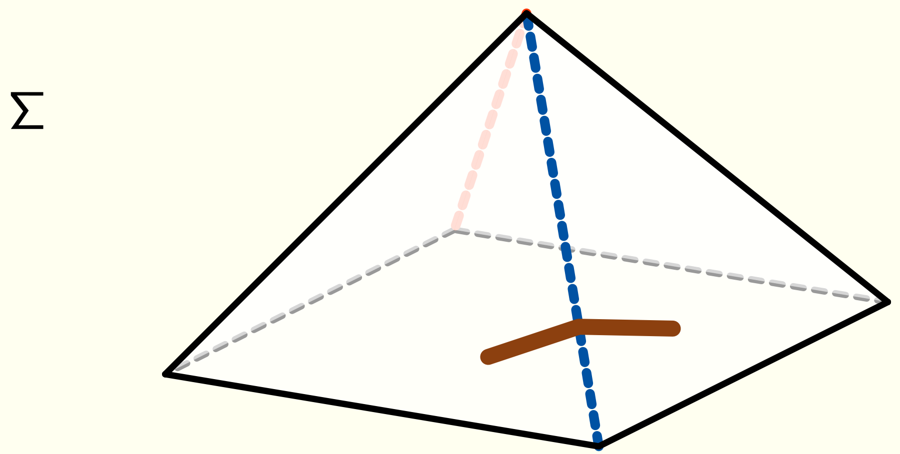
computing $\pi(s,t)$ in/on a

simple polygon	$O(n)$	[GHLST, 1987]
polygonal domain	$O(n + k \log k)$	[Wang, 2021]
convex polyhedron	$O(n \log n)$	[Schreiber, 2007]
polyhedron	$O(n^2)$	[Chen & Han, 1996]

n = #vertices

k = #holes

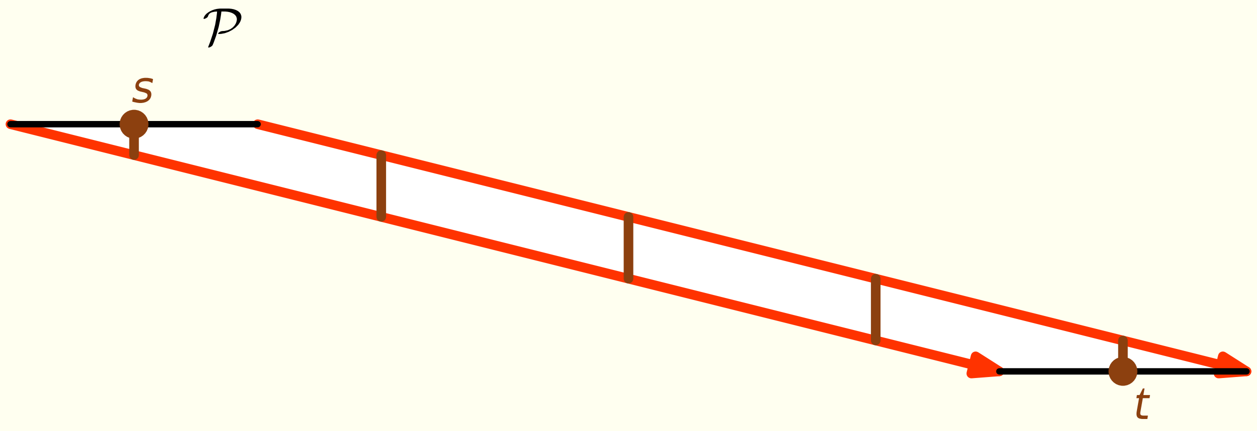




Portalgons?

Q: Can we efficiently compute a **shortest path** $\pi(s, t)$ in $\mathcal{P}$?

Obs. Complexity $\pi(s, t)$ unbounded in terms of n.

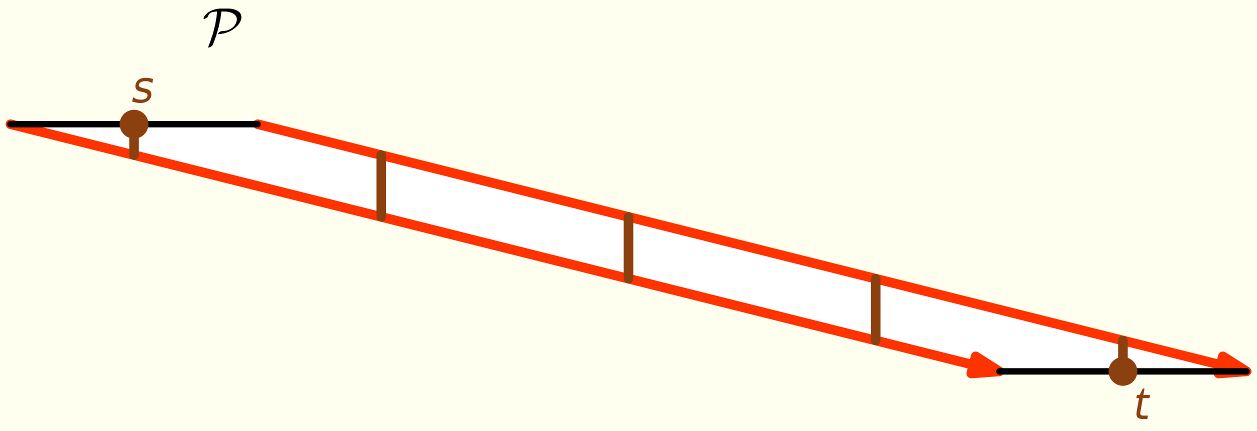


Σ

Portalgons?

Q: Can we efficiently compute a **shortest path** $\pi(s,t)$ in $\mathcal{P}$?

Obs. Complexity $\pi(s,t)$ unbounded in terms of n.



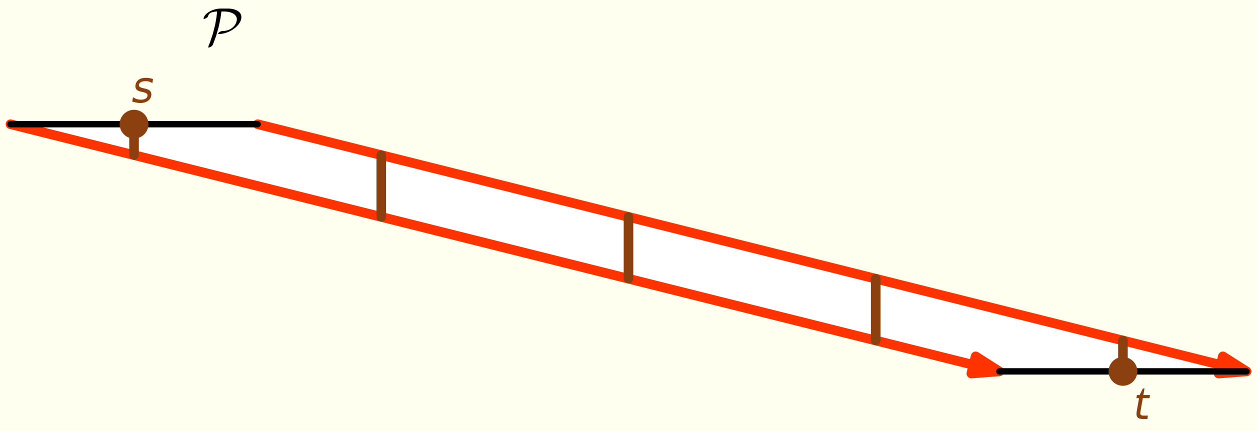
Define the **happiness** h of $\mathcal{P}$ as

$$h = \max_{s,t \in \mathcal{P}} \max_{F \in \mathcal{F}} \max_{\pi(s,t)} \#\text{components in } \pi \cap F$$

Σ

Portalgons?

Q: Can we efficiently compute a **shortest path** $\pi(s,t)$ in $\mathcal{P}$?



> **Obs.** Complexity $\pi(s,t)$ unbounded in terms of n.

Define the **happiness** h of $\mathcal{P}$ as

$$h = \max_{s,t \in \mathcal{P}} \max_{F \in \mathcal{F}} \max_{\pi(s,t)} \#\text{components in } \pi \cap F$$

> **Thm.** Complexity of any shortest path $\pi(s,t)$ in a h-happy $\mathcal{P}$ is $\Theta(n + hm)$

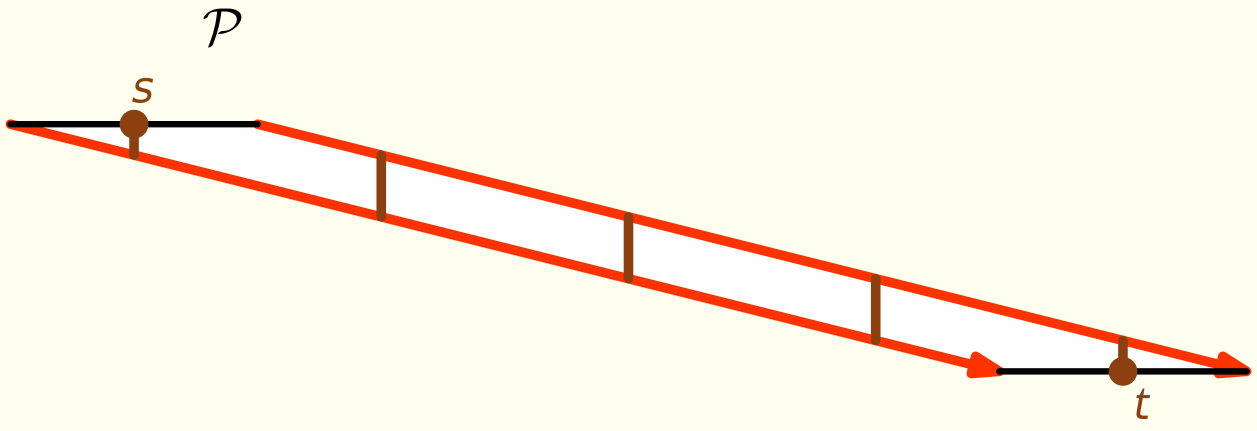
m = #portal edges $\qquad (m \leq n)$

Σ

Portalgons?

Q: Can we efficiently compute a **shortest path** $\pi(s, t)$ in $\mathcal{P}$?

Obs. Complexity $\pi(s, t)$ unbounded in terms of n.



Define the **happiness** h of $\mathcal{P}$ as

$$h = \max_{s,t \in \mathcal{P}} \max_{F \in \mathcal{F}} \max_{\pi(s,t)} \#\text{components in } \pi \cap F$$

Thm. Complexity of any shortest path $\pi(s, t)$ in a h-happy $\mathcal{P}$ is $\Theta(n + hm)$

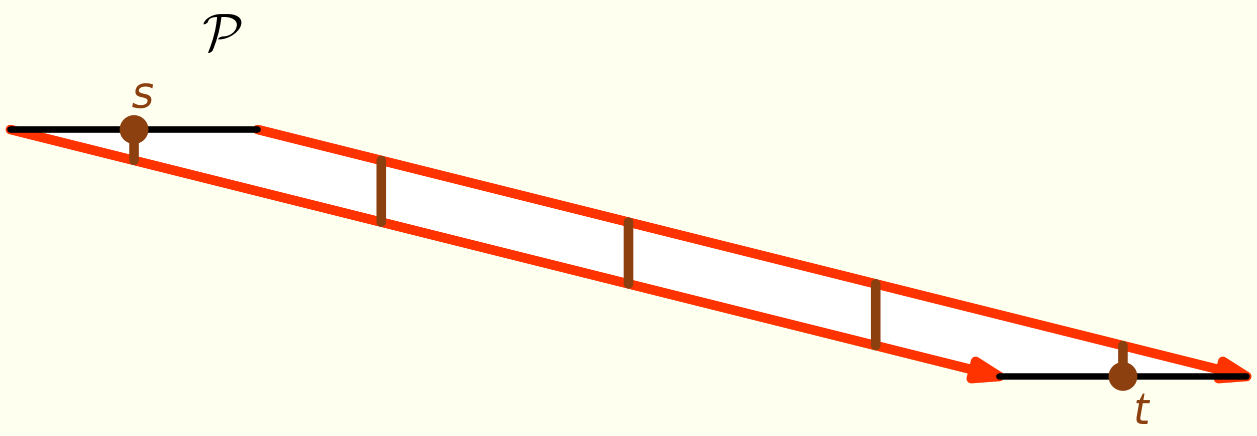
$m = \#$portal edges $\qquad (m \leq n)$

Σ

Thm. Complexity of the **shortest path map** SPM(s) in a h-happy $\mathcal{P}$ is $O(n^2 h)$,

Portalgons?

Q: Can we efficiently compute a **shortest path** $\pi(s,t)$ in $\mathcal{P}$?



> **Obs.** Complexity $\pi(s,t)$ unbounded in terms of n.

Define the **happiness** h of $\mathcal{P}$ as

$$h = \max_{s,t \in \mathcal{P}} \max_{F \in \mathcal{F}} \max_{\pi(s,t)} \#\text{components in } \pi \cap F$$

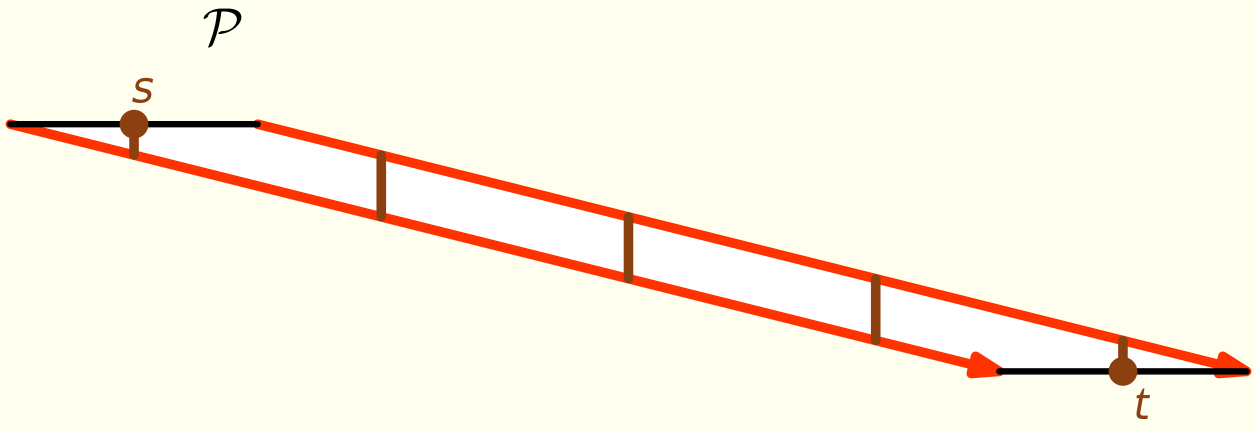
> **Thm.** Complexity of any shortest path $\pi(s,t)$ in a h-happy $\mathcal{P}$ is $\Theta(n + hm)$

Σ

$m = \#\text{portal edges} \qquad (m \leq n)$

> **Thm.** Complexity of the **shortest path map** SPM(s) in a h-happy $\mathcal{P}$ is $O(n^2 h)$, and can be computed in $O(\lambda_4(n^2 h) \log^2(nh))$ time.

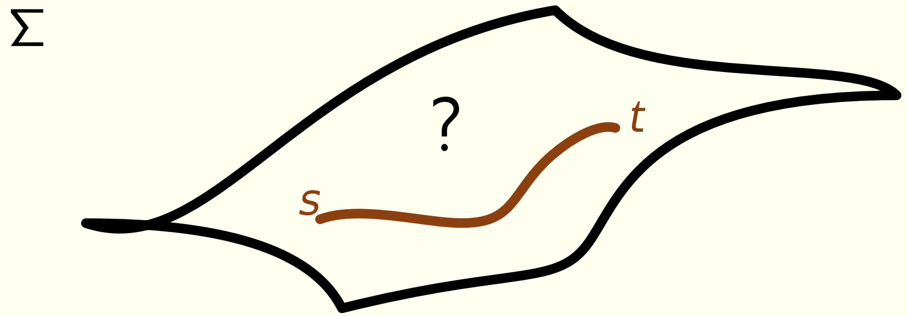
Portalgons?



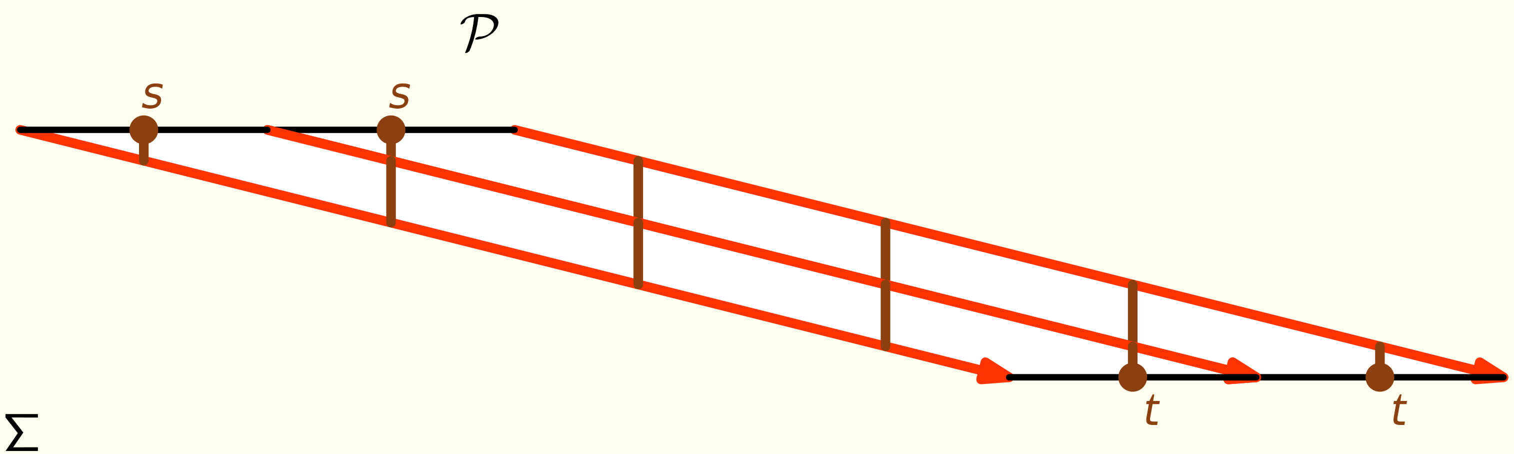
Main Question: Given a portalgon $\mathcal{P}$ representing Σ

a) is there a $O(1)$-happy equivalent portalgon $\mathcal{P}'$ representing Σ?

b) can we compute $\mathcal{P}'$ efficiently?



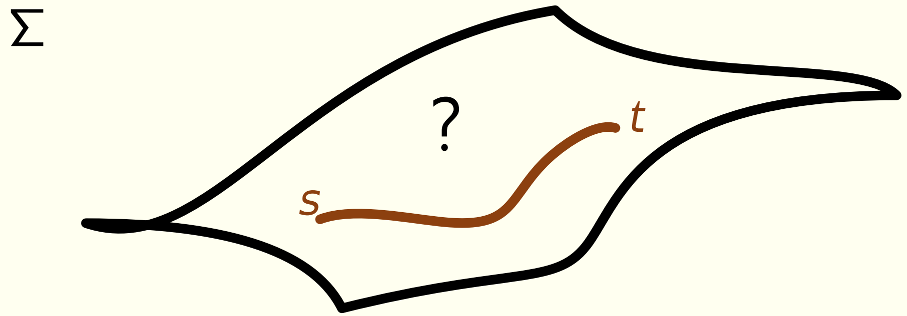
Portalgons?



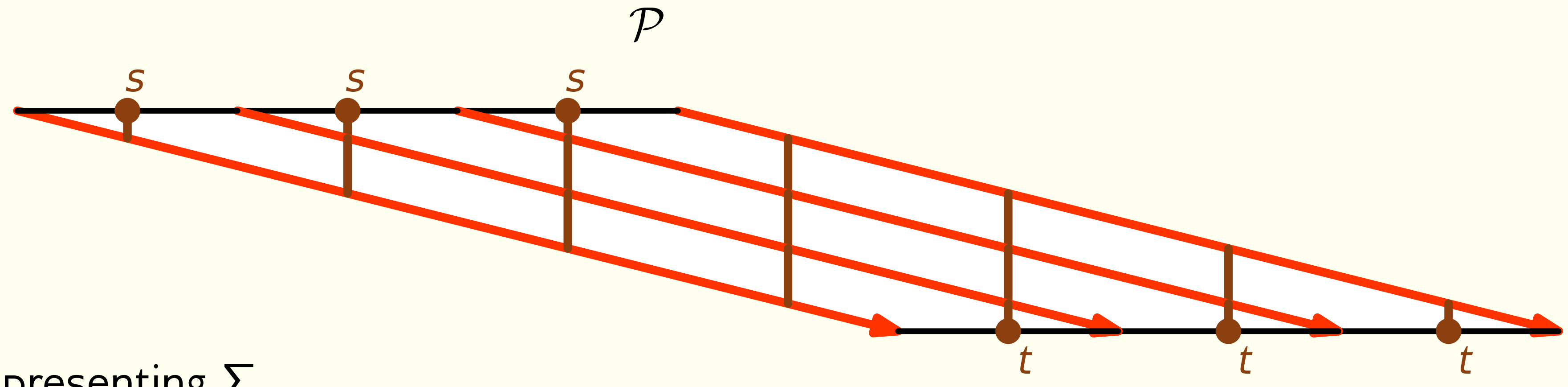
Main Question: Given a portalgon $\mathcal{P}$ representing Σ

a) is there a $O(1)$-happy equivalent portalgon $\mathcal{P}'$ representing Σ?

b) can we compute $\mathcal{P}'$ efficiently?



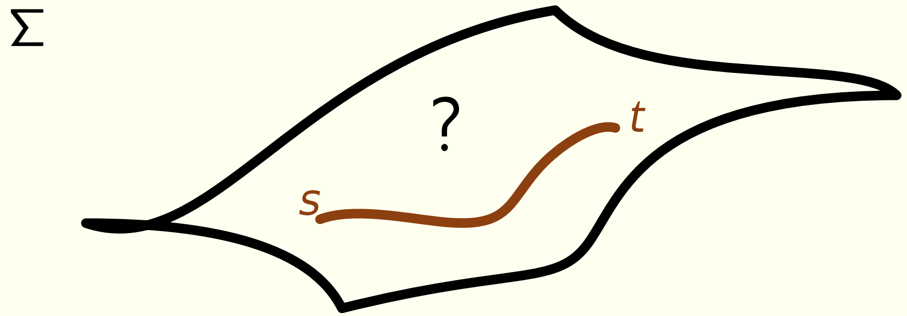
Portalgons?



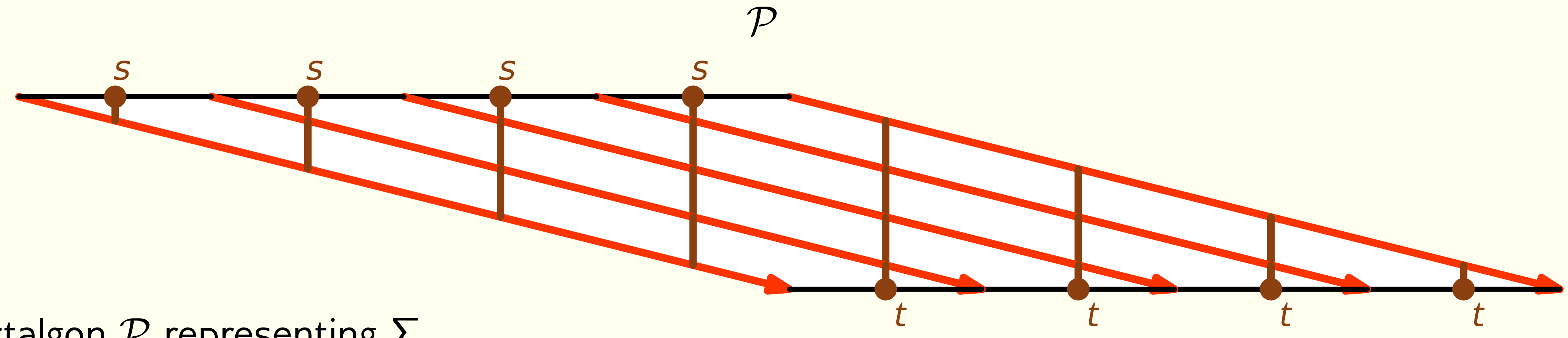
Main Question: Given a portalgon $\mathcal{P}$ representing Σ

a) is there a $O(1)$-happy equivalent portalgon $\mathcal{P}'$ representing Σ?

b) can we compute $\mathcal{P}'$ efficiently?



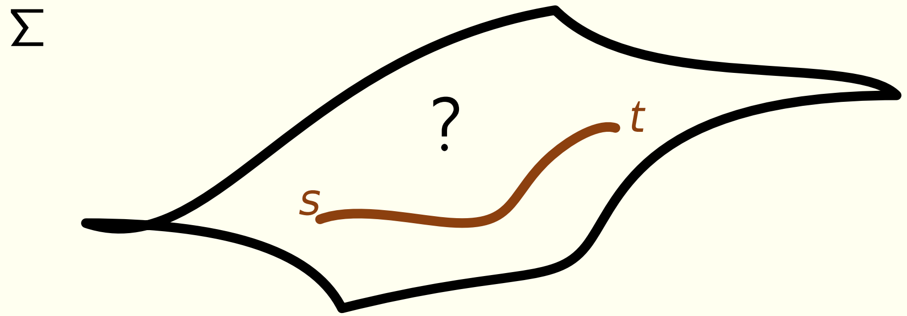
Portalgons?



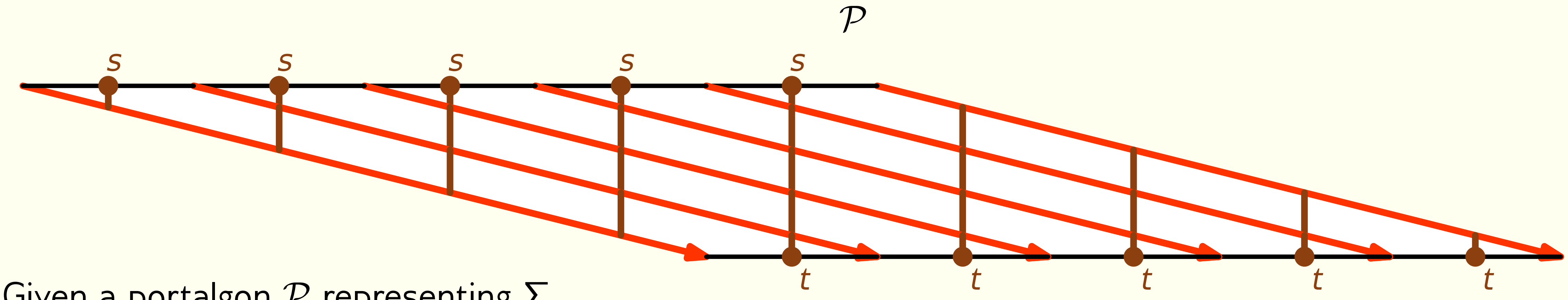
Main Question: Given a portalgon $\mathcal{P}$ representing Σ

a) is there a $O(1)$-happy equivalent portalgon $\mathcal{P}'$ representing Σ?

b) can we compute $\mathcal{P}'$ efficiently?



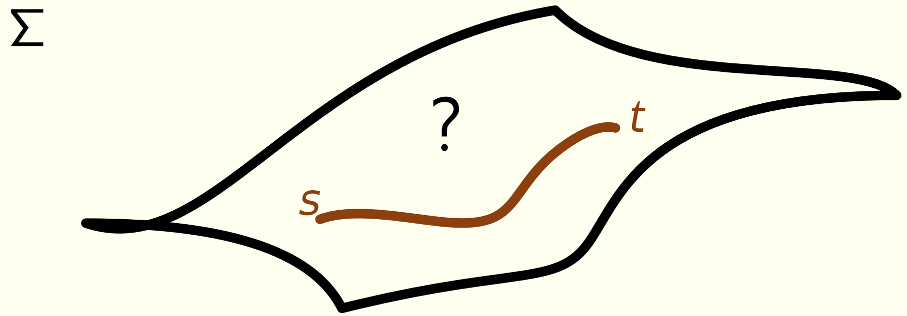
Portalgons?



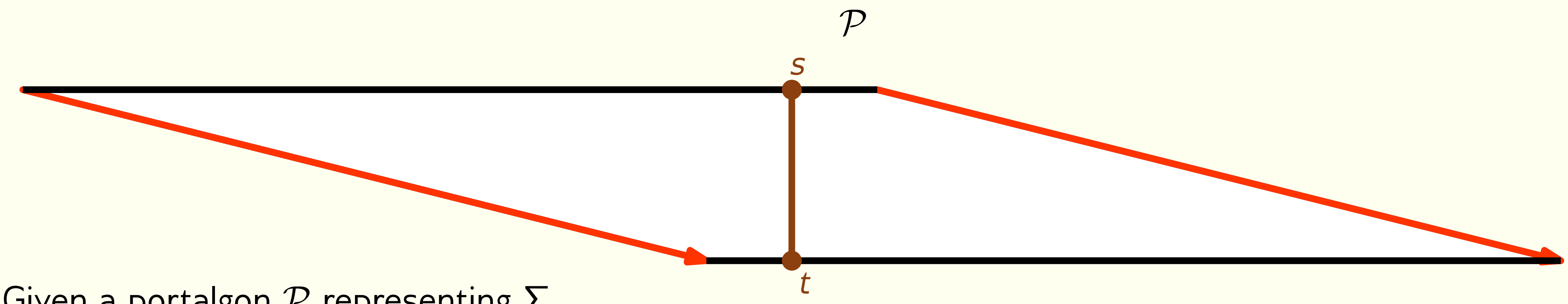
Main Question: Given a portalgon $\mathcal{P}$ representing Σ

a) is there a $O(1)$-happy equivalent portalgon $\mathcal{P}'$ representing Σ?

b) can we compute $\mathcal{P}'$ efficiently?



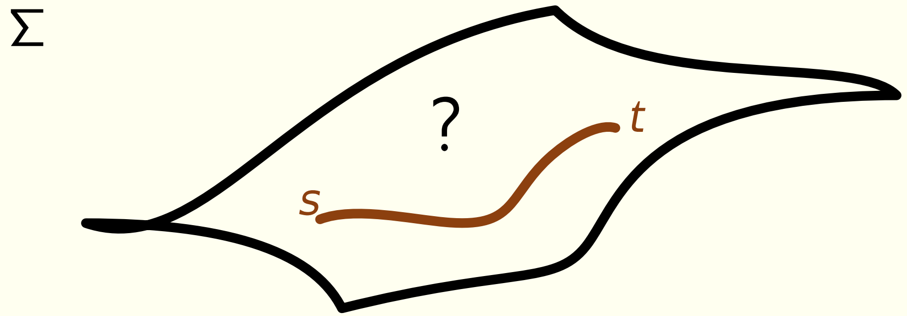
Portalgons?



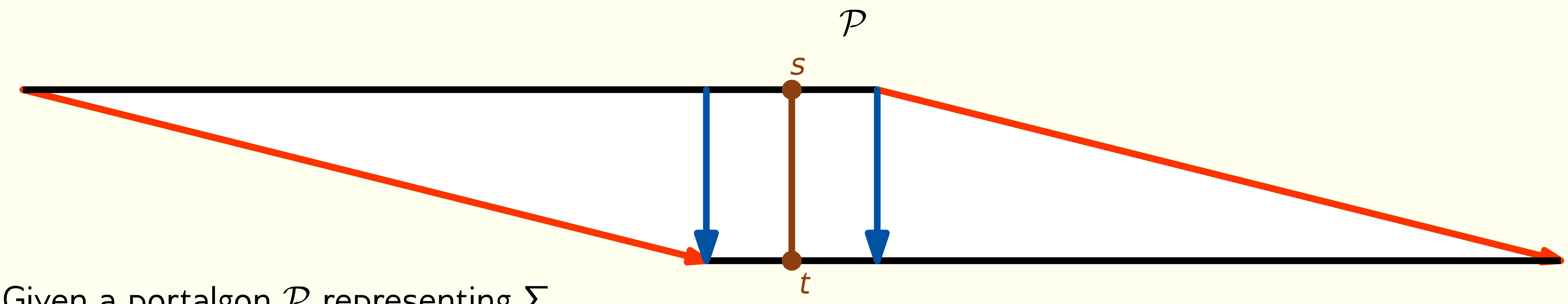
Main Question: Given a portalgon $\mathcal{P}$ representing Σ

a) is there a $O(1)$-happy equivalent portalgon $\mathcal{P}'$ representing Σ?

b) can we compute $\mathcal{P}'$ efficiently?



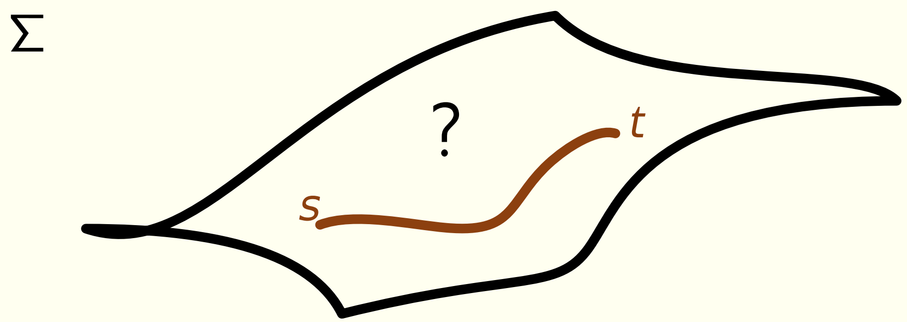
Portalgons?



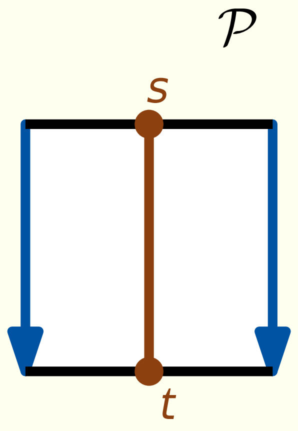
Main Question: Given a portalgon $\mathcal{P}$ representing Σ

a) is there a $O(1)$-happy equivalent portalgon $\mathcal{P}'$ representing Σ?

b) can we compute $\mathcal{P}'$ efficiently?



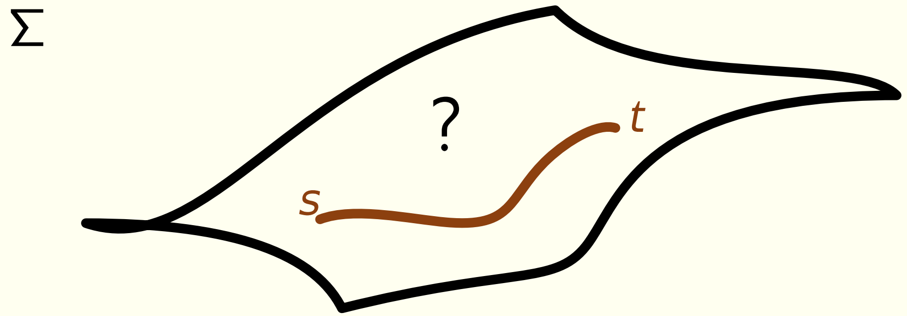
Portalgons?



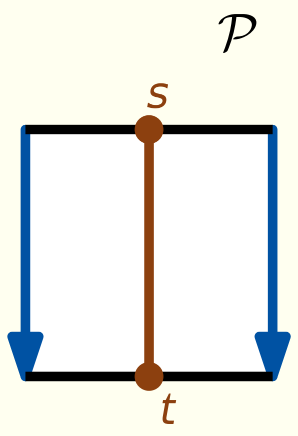
Main Question: Given a portalgon $\mathcal{P}$ representing Σ

a) is there a $O(1)$-happy equivalent portalgon $\mathcal{P}'$ representing Σ?

b) can we compute $\mathcal{P}'$ efficiently?



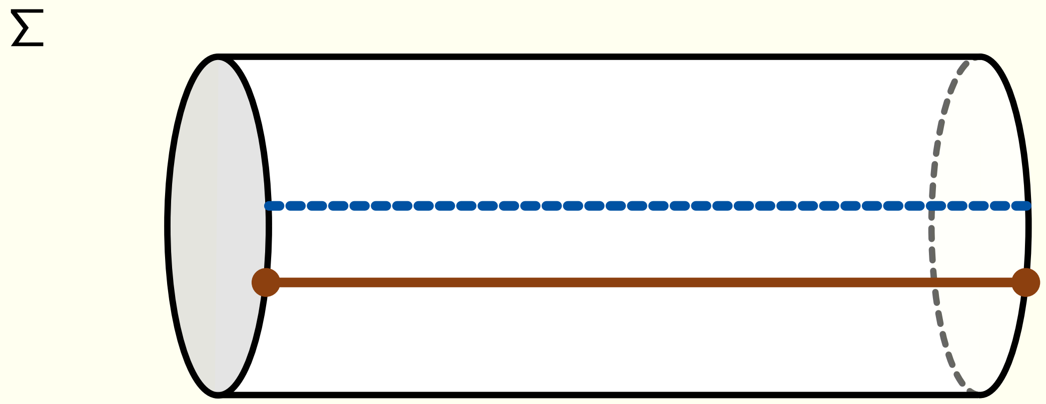
Portalgons?



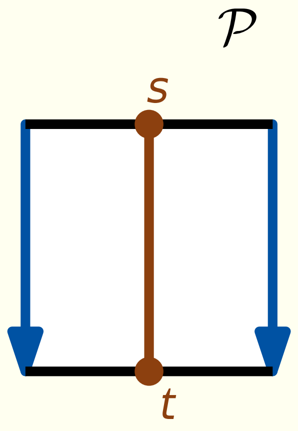
Main Question: Given a portalgon $\mathcal{P}$ representing Σ

a) is there a $O(1)$-happy equivalent portalgon $\mathcal{P}'$ representing Σ?

b) can we compute $\mathcal{P}'$ efficiently?



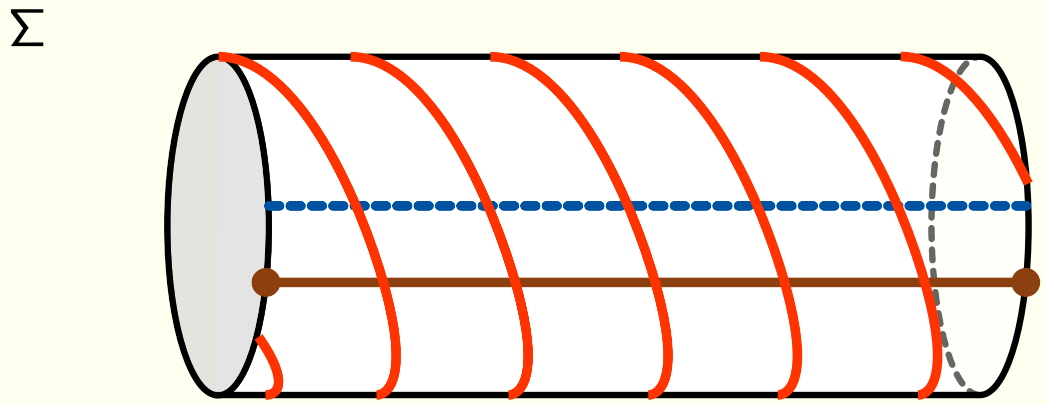
Portalgons?



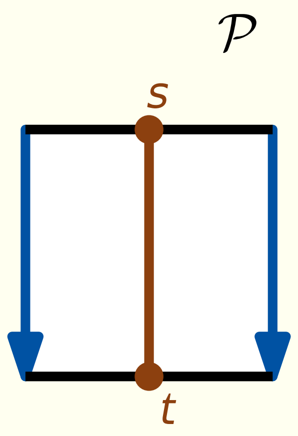
Main Question: Given a portalgon $\mathcal{P}$ representing Σ

a) is there a $O(1)$-happy equivalent portalgon $\mathcal{P}'$ representing Σ?

b) can we compute $\mathcal{P}'$ efficiently?



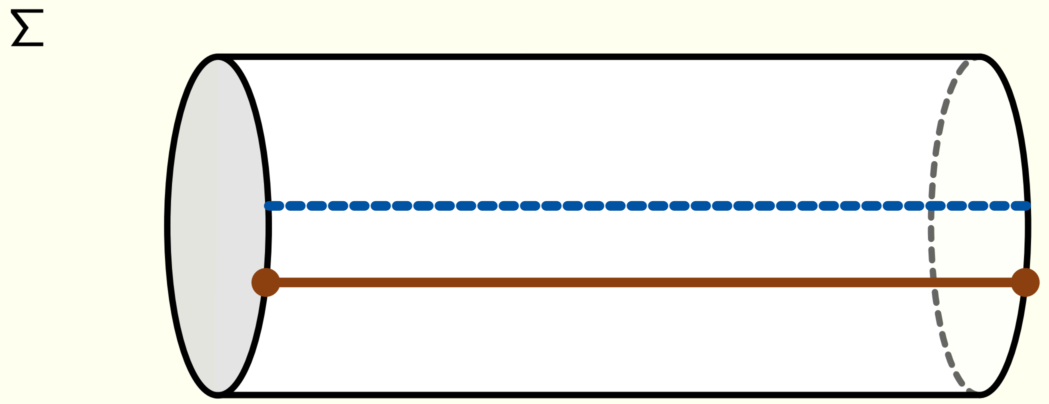
Portalgons?



Main Question: Given a portalgon $\mathcal{P}$ representing Σ

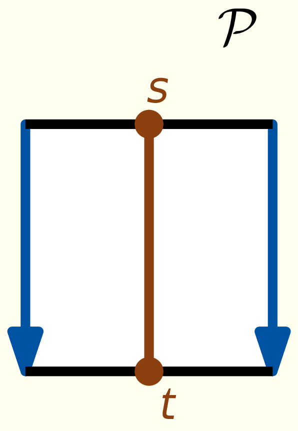
a) is there a $O(1)$-happy equivalent portalgon $\mathcal{P}'$ representing Σ?

b) can we compute $\mathcal{P}'$ efficiently?



Thm. There is an $O(1)$-happy $\mathcal{P}'$ equivalent to $\mathcal{P}$.

Portalgons?

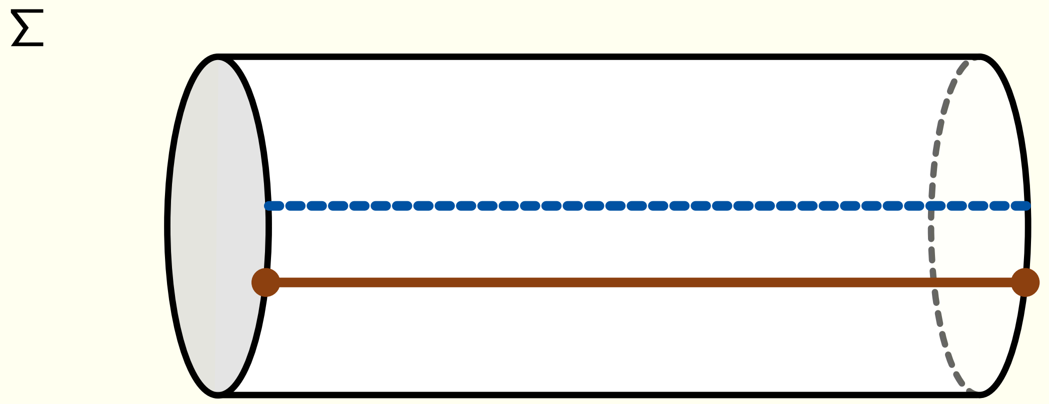


Main Question: Given a portalgon $\mathcal{P}$ representing Σ

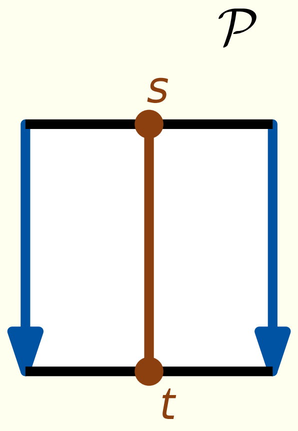
a) is there a $O(1)$-happy equivalent portalgon $\mathcal{P}'$ representing Σ?

b) can we compute $\mathcal{P}'$ efficiently?

Thm. The **intrinsic Delaunay Triangulation** $\mathcal{P}'$ of $\mathcal{P}$ is $O(1)$ happy (and equivalent to $\mathcal{P}$).



Portalgons?

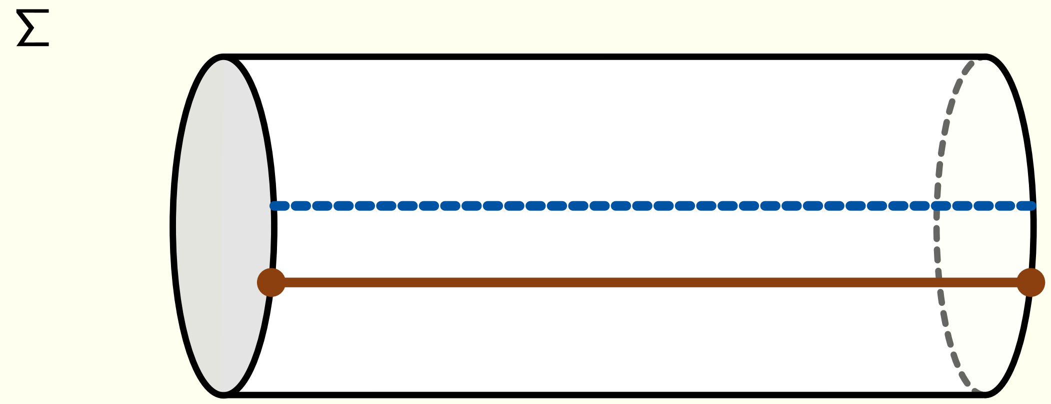


Main Question: Given a portalgon $\mathcal{P}$ representing Σ

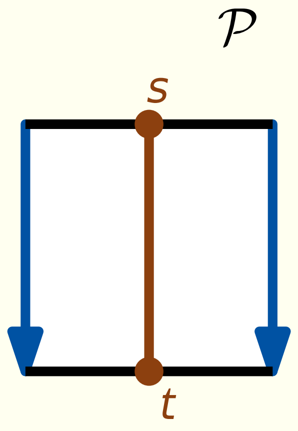
a) is there a $O(1)$-happy equivalent portalgon $\mathcal{P}'$ representing Σ?

b) can we compute $\mathcal{P}'$ efficiently? OPEN

Thm. The **intrinsic Delaunay Triangulation** $\mathcal{P}'$ of $\mathcal{P}$ is $O(1)$ happy (and equivalent to $\mathcal{P}$).



Portalgons?



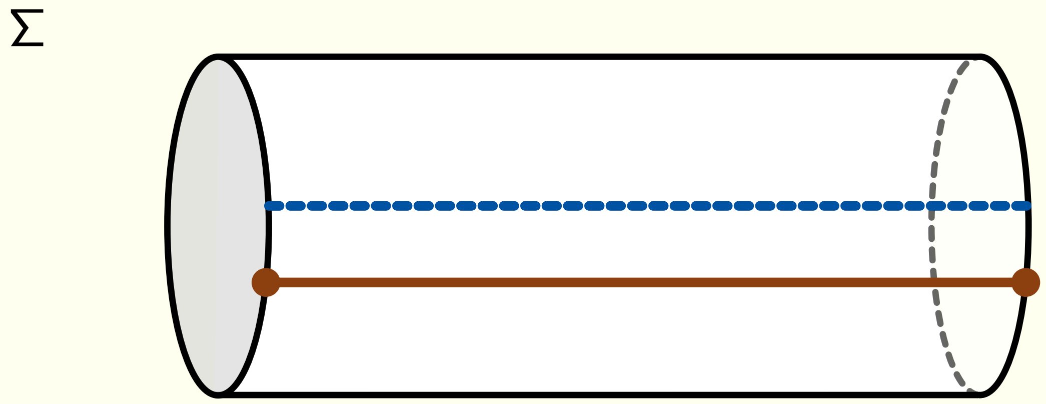
Main Question: Given a portalgon $\mathcal{P}$ representing Σ

a) is there a $O(1)$-happy equivalent portalgon $\mathcal{P}'$ representing Σ?

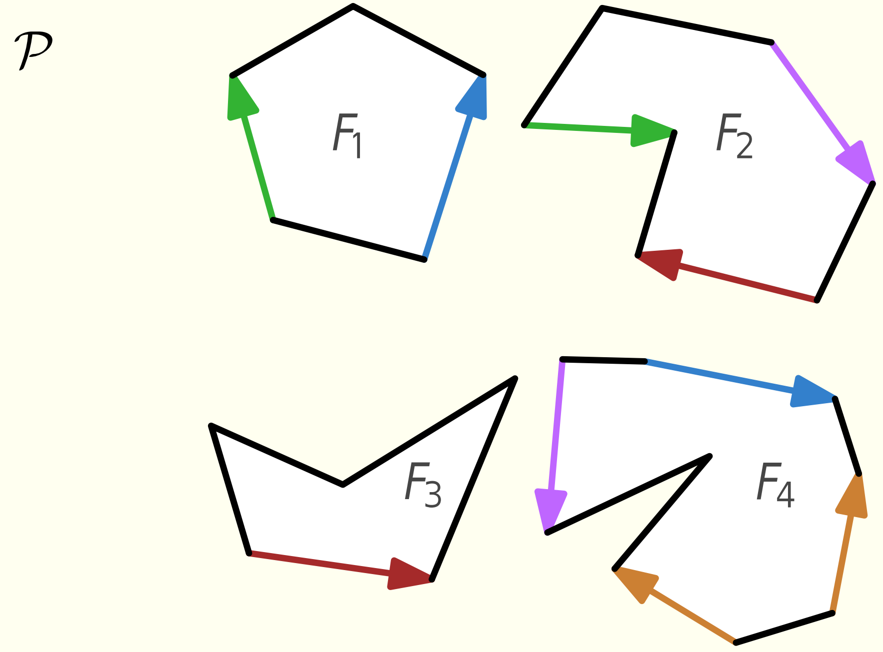
b) can we compute $\mathcal{P}'$ efficiently? OPEN

Thm. The **intrinsic Delaunay Triangulation** $\mathcal{P}'$ of $\mathcal{P}$ is $O(1)$ happy (and equivalent to $\mathcal{P}$).

Thm. If fragment graph of $\mathcal{P}$ has ≤ 1 simple cycle, we can compute an equivalent $\mathcal{P}'$ in $O(n + \log h)$ time.



Portalgons?



Main Question: Given a portalgon $\mathcal{P}$ representing Σ

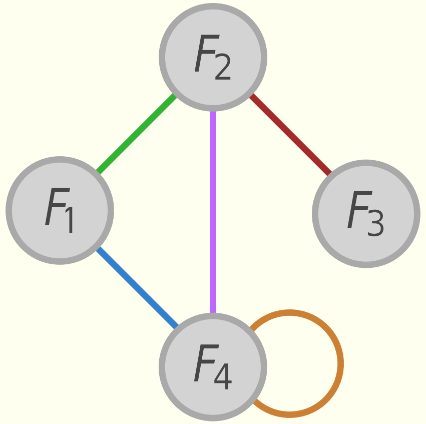
a) is there a $O(1)$-happy equivalent portalgon $\mathcal{P}'$ representing Σ?

b) can we compute $\mathcal{P}'$ efficiently? OPEN

Thm. The **intrinsic Delaunay Triangulation** $\mathcal{P}'$ of $\mathcal{P}$ is $O(1)$ happy (and equivalent to $\mathcal{P}$).

Thm. If fragment graph of $\mathcal{P}$ has ≤ 1 simple cycle, we can compute an equivalent $\mathcal{P}'$ in $O(n + \log h)$ time.

Fragment graph:



Results

Obs. Complexity $\pi(s,t)$ unbounded in terms of n.

Thm. Complexity of $\pi(s,t)$ in a h-happy $\mathcal{P}$ is $\Theta(n+hm)$

Thm. Complexity of SPM(s) in a h-happy $\mathcal{P}$ is $O(n^2h)$, and can be computed in $O(\lambda_4(n^2h)\log^2(nh))$ time.

Thm. The **intrinsic Delaunay Triangulation** $\mathcal{P}'$ of $\mathcal{P}$ is $O(1)$ happy (and equivalent to $\mathcal{P}$).

Thm. If fragment graph of $\mathcal{P}$ has ≤ 1 simple cycle, we can compute an equivalent $\mathcal{P}'$ in $O(n+\log h)$ time.

Given $\mathcal{P}$, can we compute an $O(1)$-happy $\mathcal{P}'$ equivalent to $\mathcal{P}$ efficiently? OPEN

Results

Obs. Complexity $\pi(s,t)$ unbounded in terms of n.

Thm. Complexity of $\pi(s,t)$ in a h-happy $\mathcal{P}$ is $\Theta(n+hm)$

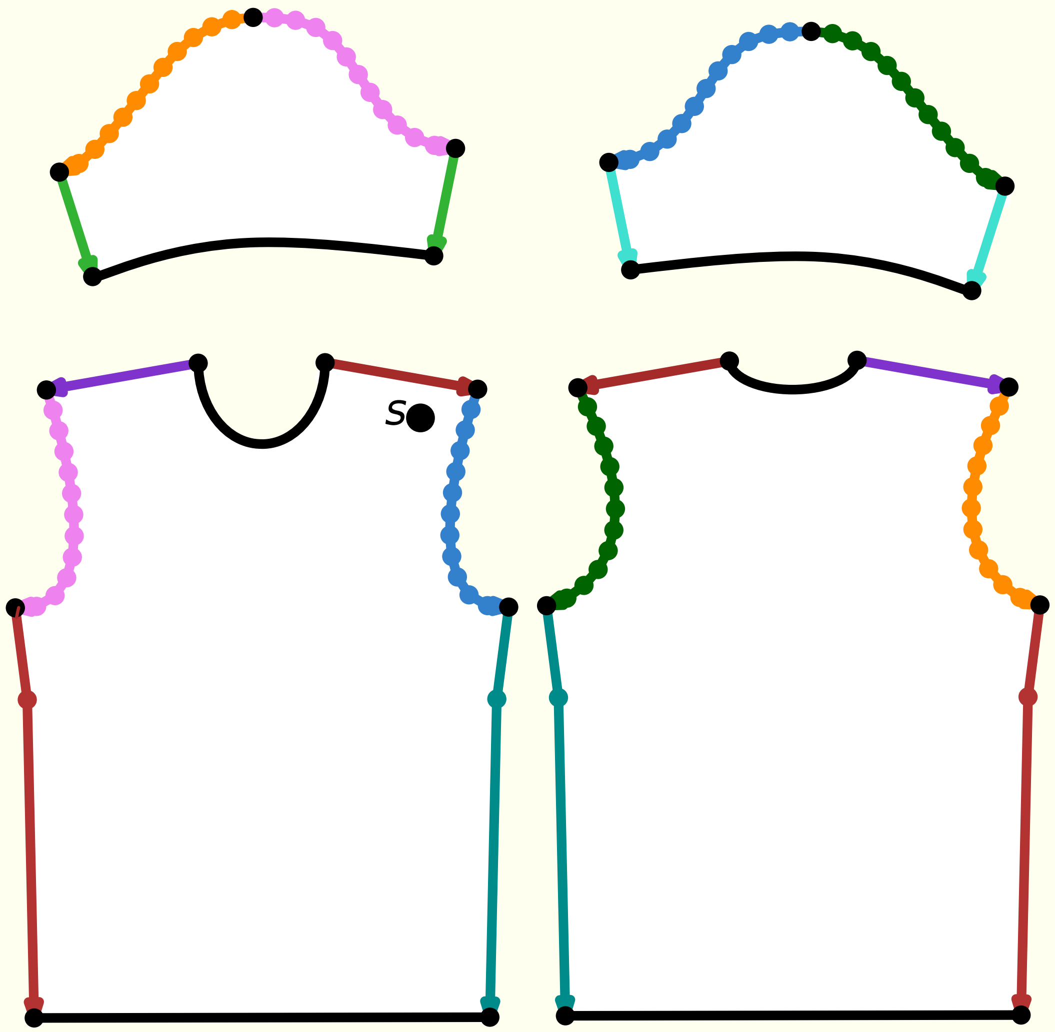
Thm. Complexity of SPM(s) in a h-happy $\mathcal{P}$ is $O(n^2h)$, and can be computed in $O(\lambda_4(n^2h)\log^2(nh))$ time.

Thm. The **intrinsic Delaunay Triangulation** $\mathcal{P}'$ of $\mathcal{P}$ is $O(1)$ happy (and equivalent to $\mathcal{P}$).

Thm. If fragment graph of $\mathcal{P}$ has ≤ 1 simple cycle, we can compute an equivalent $\mathcal{P}'$ in $O(n+\log h)$ time.

Given $\mathcal{P}$, can we compute an $O(1)$-happy $\mathcal{P}'$ equivalent to $\mathcal{P}$ efficiently? OPEN

Main Idea: use continuous dijkstra



Results

Obs. Complexity $\pi(s,t)$ unbounded in terms of n.

Thm. Complexity of $\pi(s,t)$ in a h-happy $\mathcal{P}$ is $\Theta(n+hm)$

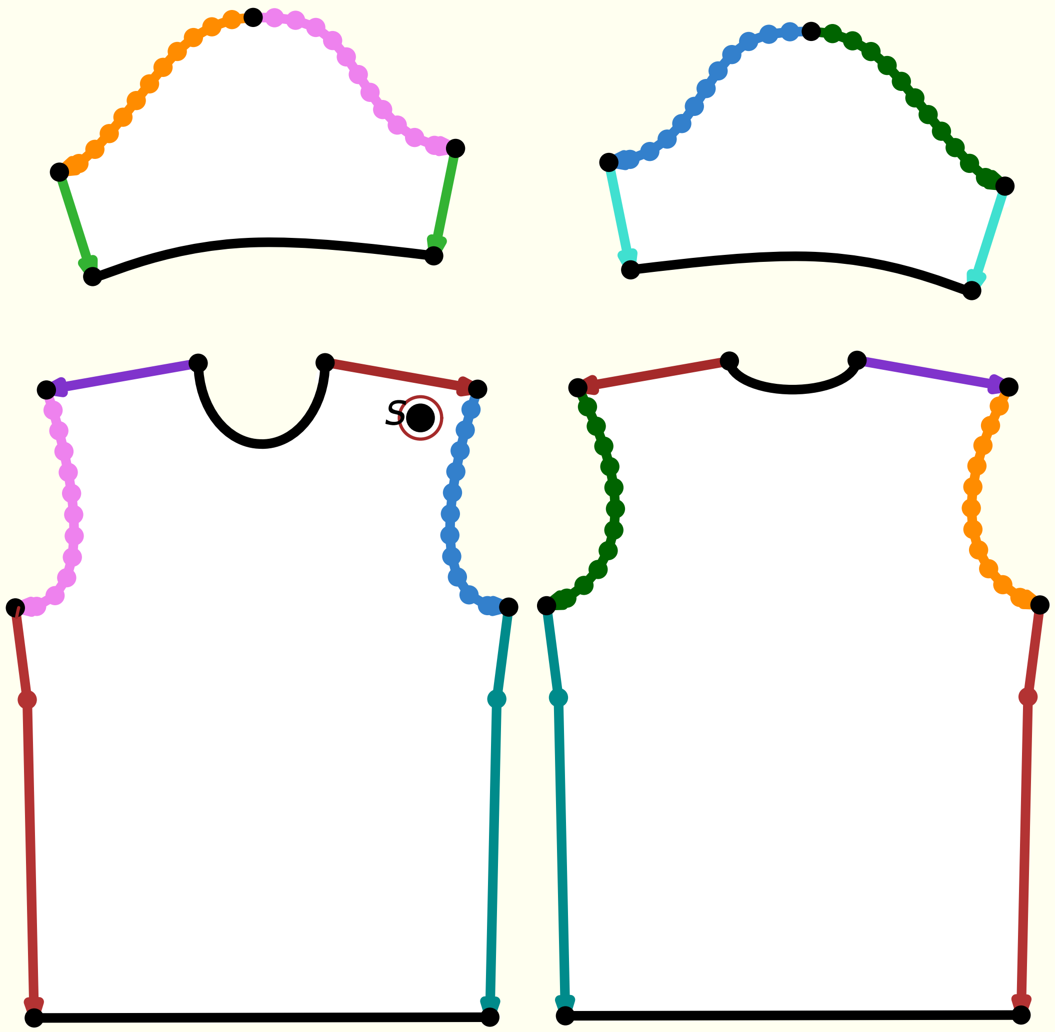
Thm. Complexity of SPM(s) in a h-happy $\mathcal{P}$ is $O(n^2h)$, and can be computed in $O(\lambda_4(n^2h)\log^2(nh))$ time.

Thm. The **intrinsic Delaunay Triangulation** $\mathcal{P}'$ of $\mathcal{P}$ is $O(1)$ happy (and equivalent to $\mathcal{P}$).

Thm. If fragment graph of $\mathcal{P}$ has ≤ 1 simple cycle, we can compute an equivalent $\mathcal{P}'$ in $O(n+\log h)$ time.

Given $\mathcal{P}$, can we compute an $O(1)$-happy $\mathcal{P}'$ equivalent to $\mathcal{P}$ efficiently? OPEN

Main Idea: use continuous dijkstra



Results

Obs. Complexity $\pi(s,t)$ unbounded in terms of n.

Thm. Complexity of $\pi(s,t)$ in a h-happy $\mathcal{P}$ is $\Theta(n + hm)$

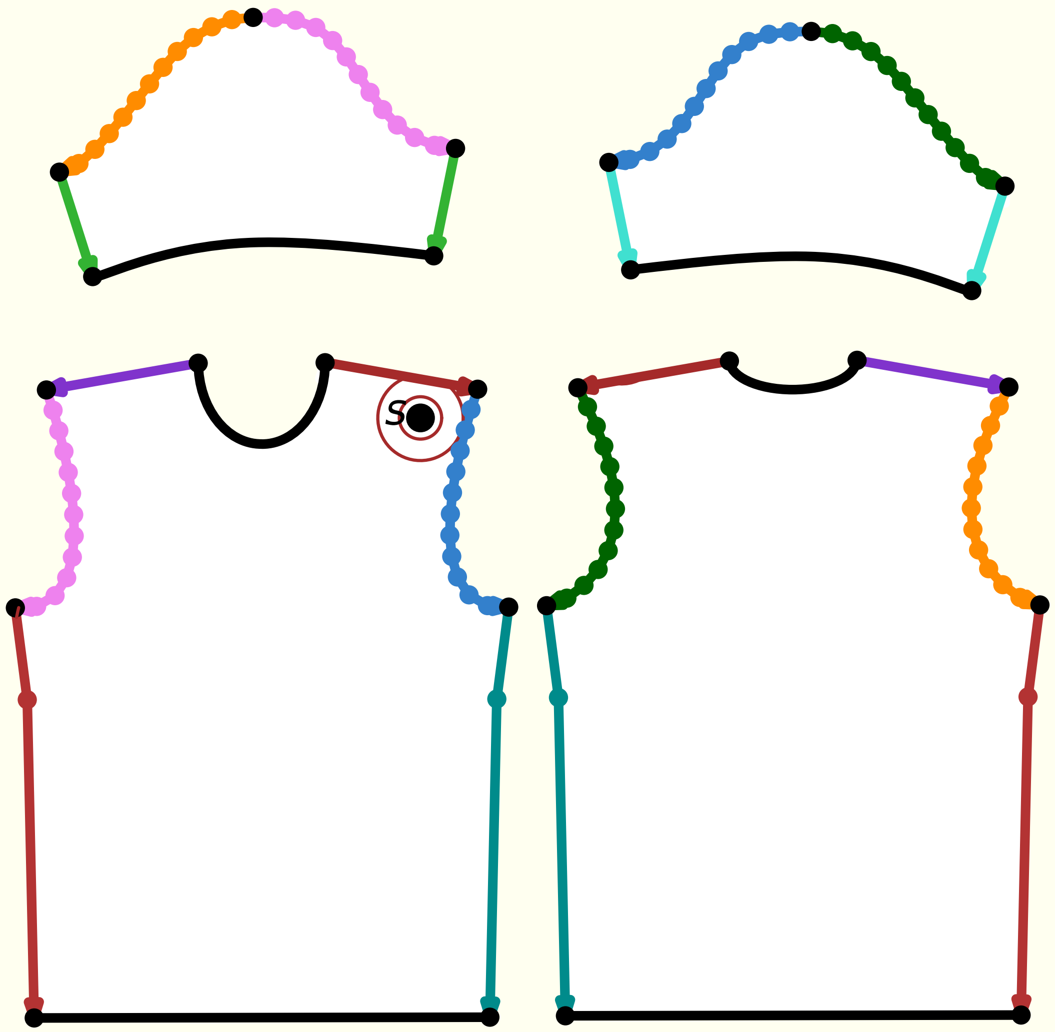
Thm. Complexity of SPM(s) in a h-happy $\mathcal{P}$ is $O(n^2 h)$, and can be computed in $O(\lambda_4(n^2 h) \log^2(nh))$ time.

Thm. The **intrinsic Delaunay Triangulation** $\mathcal{P}'$ of $\mathcal{P}$ is $O(1)$ happy (and equivalent to $\mathcal{P}$).

Thm. If fragment graph of $\mathcal{P}$ has ≤ 1 simple cycle, we can compute an equivalent $\mathcal{P}'$ in $O(n + \log h)$ time.

Given $\mathcal{P}$, can we compute an $O(1)$-happy $\mathcal{P}'$ equivalent to $\mathcal{P}$ efficiently? OPEN

Main Idea: use continuous dijkstra



Results

Obs. Complexity $\pi(s,t)$ unbounded in terms of n.

Thm. Complexity of $\pi(s,t)$ in a h-happy $\mathcal{P}$ is $\Theta(n+hm)$

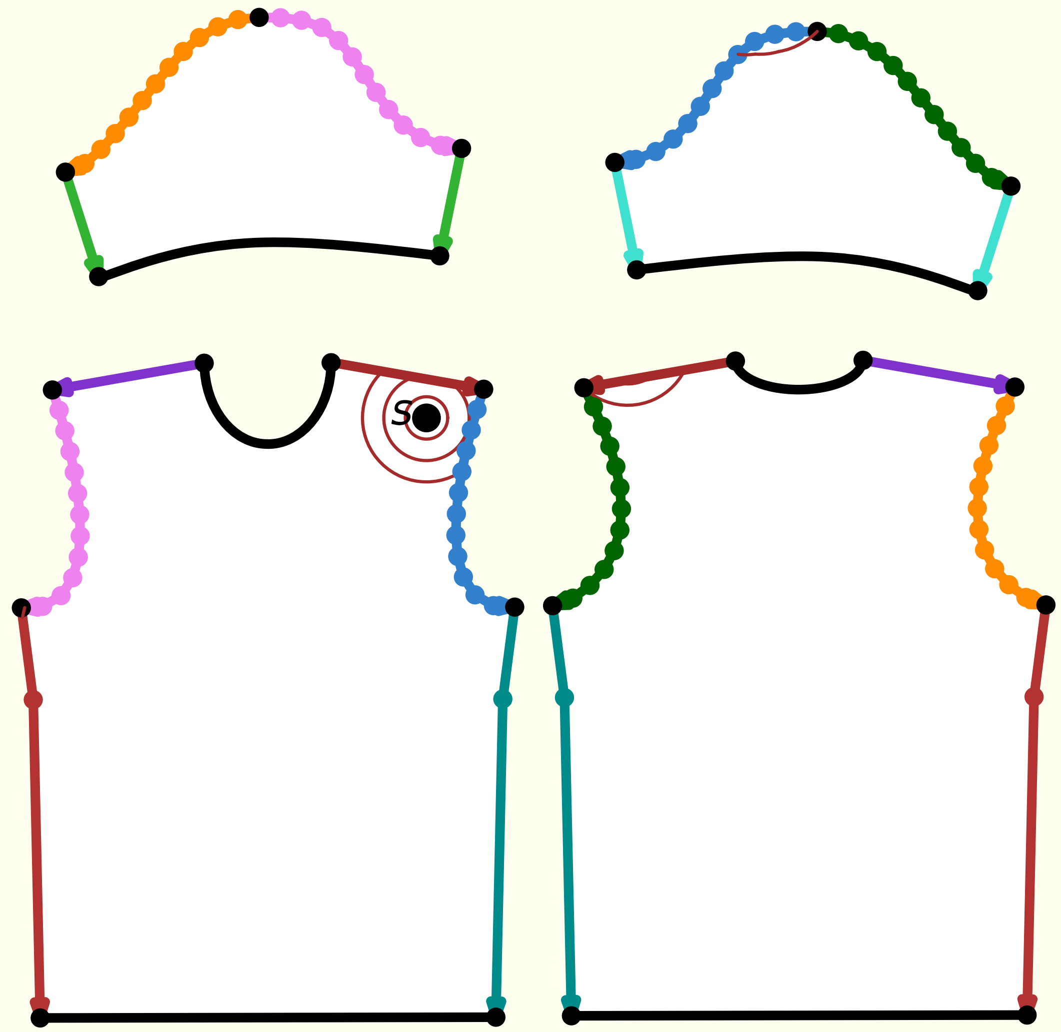
Thm. Complexity of SPM(s) in a h-happy $\mathcal{P}$ is $O(n^2 h)$, and can be computed in $O(\lambda_4(n^2 h)\log^2(nh))$ time.

Thm. The intrinsic Delaunay Triangulation $\mathcal{P}'$ of $\mathcal{P}$ is $O(1)$ happy (and equivalent to $\mathcal{P}$).

Thm. If fragment graph of $\mathcal{P}$ has ≤ 1 simple cycle, we can compute an equivalent $\mathcal{P}'$ in $O(n + \log h)$ time.

Given $\mathcal{P}$, can we compute an $O(1)$-happy $\mathcal{P}'$ equivalent to $\mathcal{P}$ efficiently? OPEN

Main Idea: use continuous dijkstra



Results

Obs. Complexity $\pi(s, t)$ unbounded in terms of n.

Thm. Complexity of $\pi(s, t)$ in a h-happy $\mathcal{P}$ is $\Theta(n + hm)$

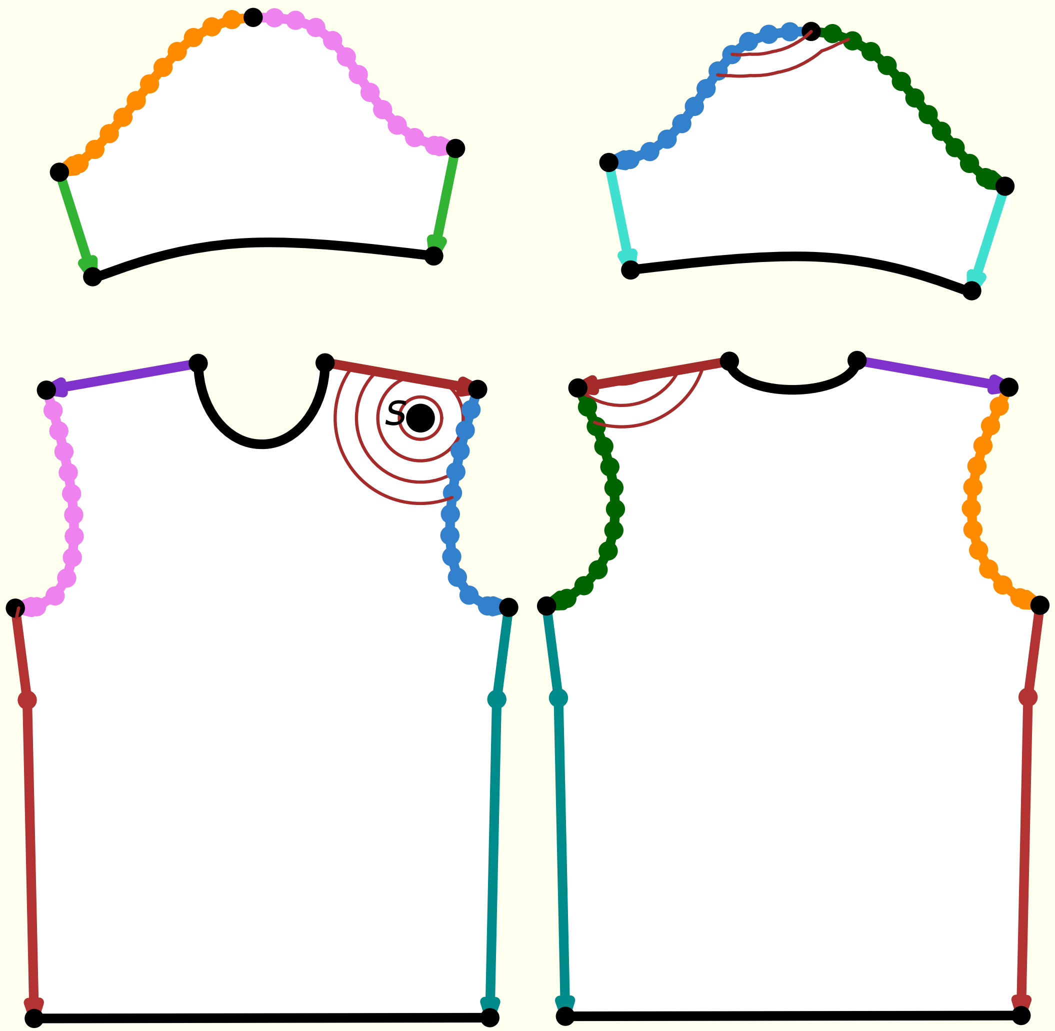
Thm. Complexity of SPM(s) in a h-happy $\mathcal{P}$ is $O(n^2 h)$, and can be computed in $O(\lambda_4(n^2 h) \log^2(nh))$ time.

Thm. The **intrinsic Delaunay Triangulation** $\mathcal{P}'$ of $\mathcal{P}$ is $O(1)$ happy (and equivalent to $\mathcal{P}$).

Thm. If fragment graph of $\mathcal{P}$ has ≤ 1 simple cycle, we can compute an equivalent $\mathcal{P}'$ in $O(n + \log h)$ time.

Given $\mathcal{P}$, can we compute an $O(1)$-happy $\mathcal{P}'$ equivalent to $\mathcal{P}$ efficiently? OPEN

Main Idea: use continuous dijkstra



Results

Obs. Complexity $\pi(s,t)$ unbounded in terms of n.

Thm. Complexity of $\pi(s,t)$ in a h-happy $\mathcal{P}$ is $\Theta(n+hm)$

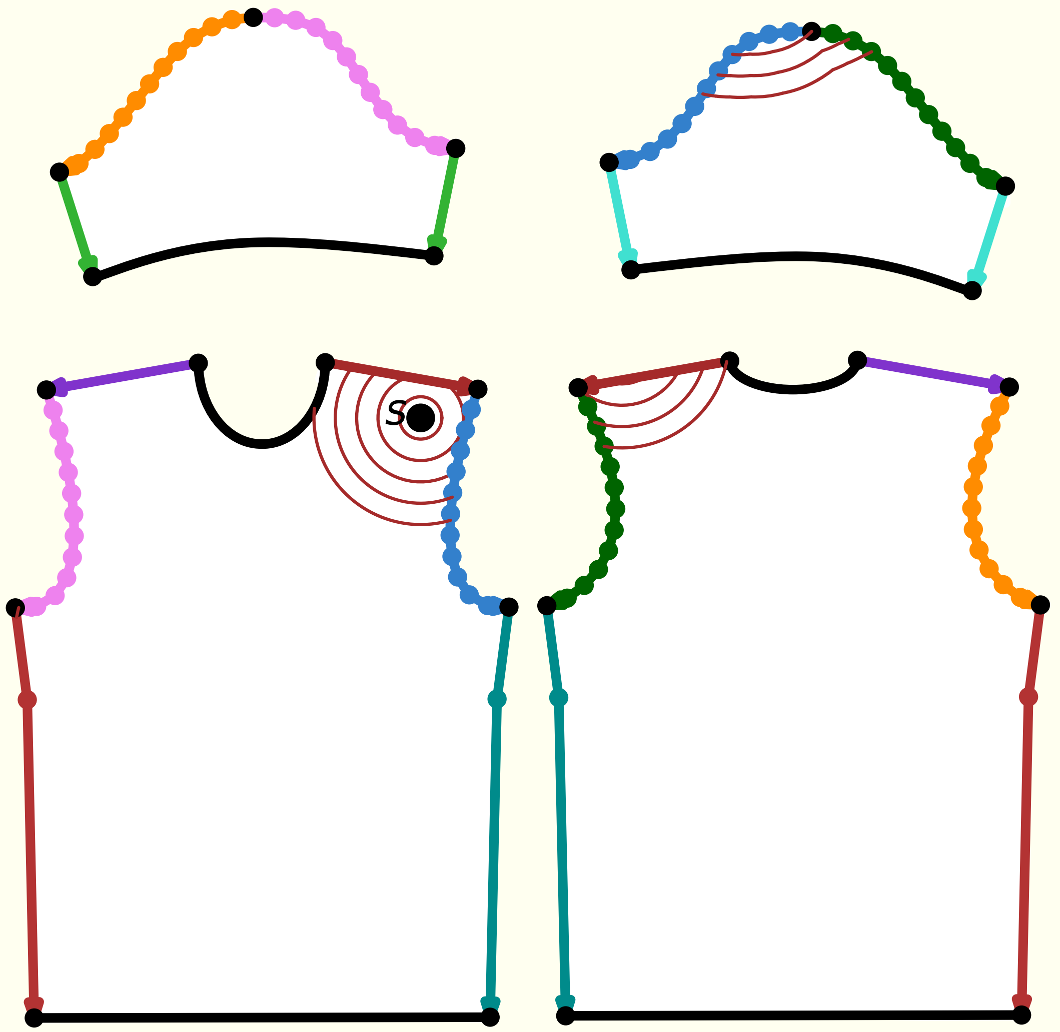
Thm. Complexity of SPM(s) in a h-happy $\mathcal{P}$ is $O(n^2h)$, and can be computed in $O(\lambda_4(n^2h)\log^2(nh))$ time.

Thm. The **intrinsic Delaunay Triangulation** $\mathcal{P}'$ of $\mathcal{P}$ is $O(1)$ happy (and equivalent to $\mathcal{P}$).

Thm. If fragment graph of $\mathcal{P}$ has ≤ 1 simple cycle, we can compute an equivalent $\mathcal{P}'$ in $O(n+\log h)$ time.

Given $\mathcal{P}$, can we compute an $O(1)$-happy $\mathcal{P}'$ equivalent to $\mathcal{P}$ efficiently? OPEN

Main Idea: use continuous dijkstra



Results

Obs. Complexity $\pi(s, t)$ unbounded in terms of n.

Thm. Complexity of $\pi(s, t)$ in a h-happy $\mathcal{P}$ is $\Theta(n + hm)$

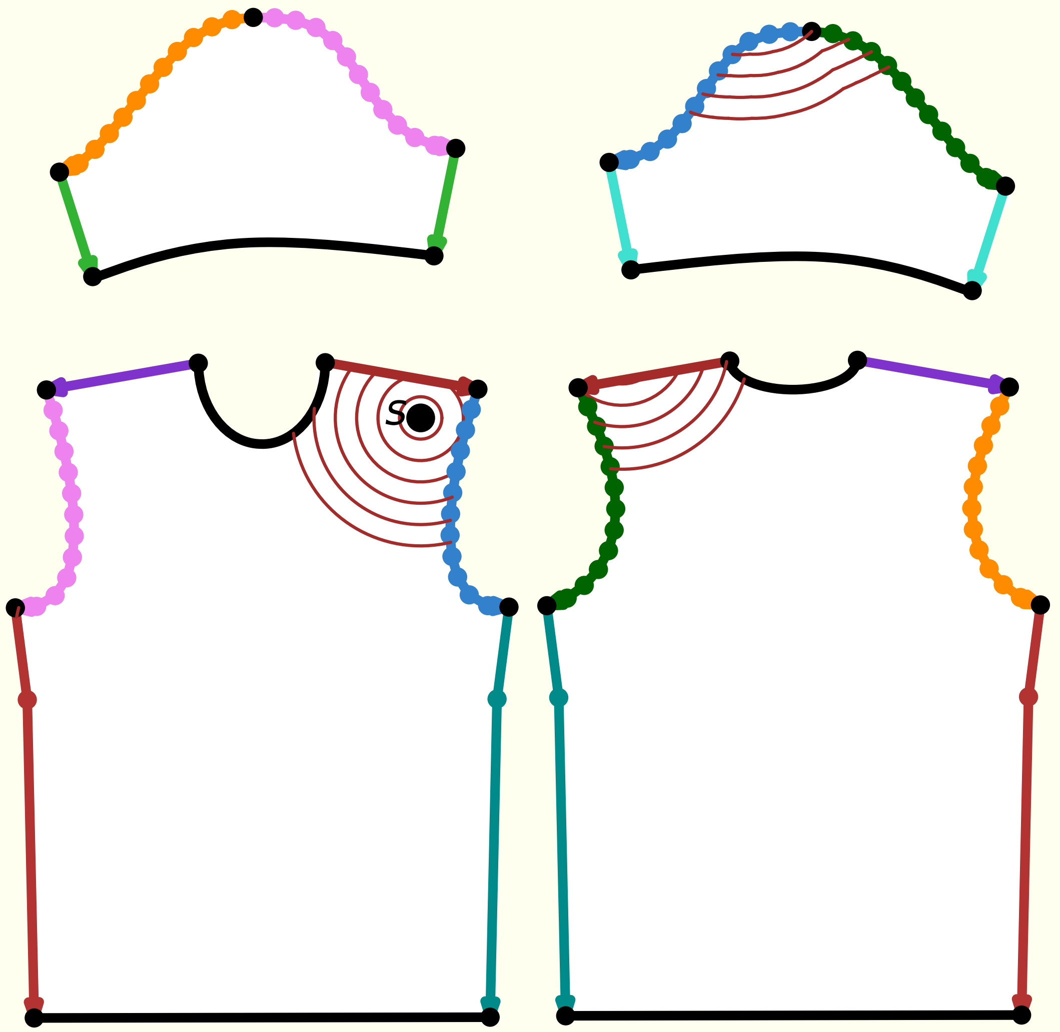
Thm. Complexity of SPM(s) in a h-happy $\mathcal{P}$ is $O(n^2 h)$, and can be computed in $O(\lambda_4(n^2 h) \log^2(nh))$ time.

Thm. The **intrinsic Delaunay Triangulation** $\mathcal{P}'$ of $\mathcal{P}$ is $O(1)$ happy (and equivalent to $\mathcal{P}$).

Thm. If fragment graph of $\mathcal{P}$ has ≤ 1 simple cycle, we can compute an equivalent $\mathcal{P}'$ in $O(n + \log h)$ time.

Given $\mathcal{P}$, can we compute an $O(1)$-happy $\mathcal{P}'$ equivalent to $\mathcal{P}$ efficiently? OPEN

Main Idea: use continuous dijkstra



Results

Obs. Complexity $\pi(s,t)$ unbounded in terms of n.

Thm. Complexity of $\pi(s,t)$ in a h-happy $\mathcal{P}$ is $\Theta(n+hm)$

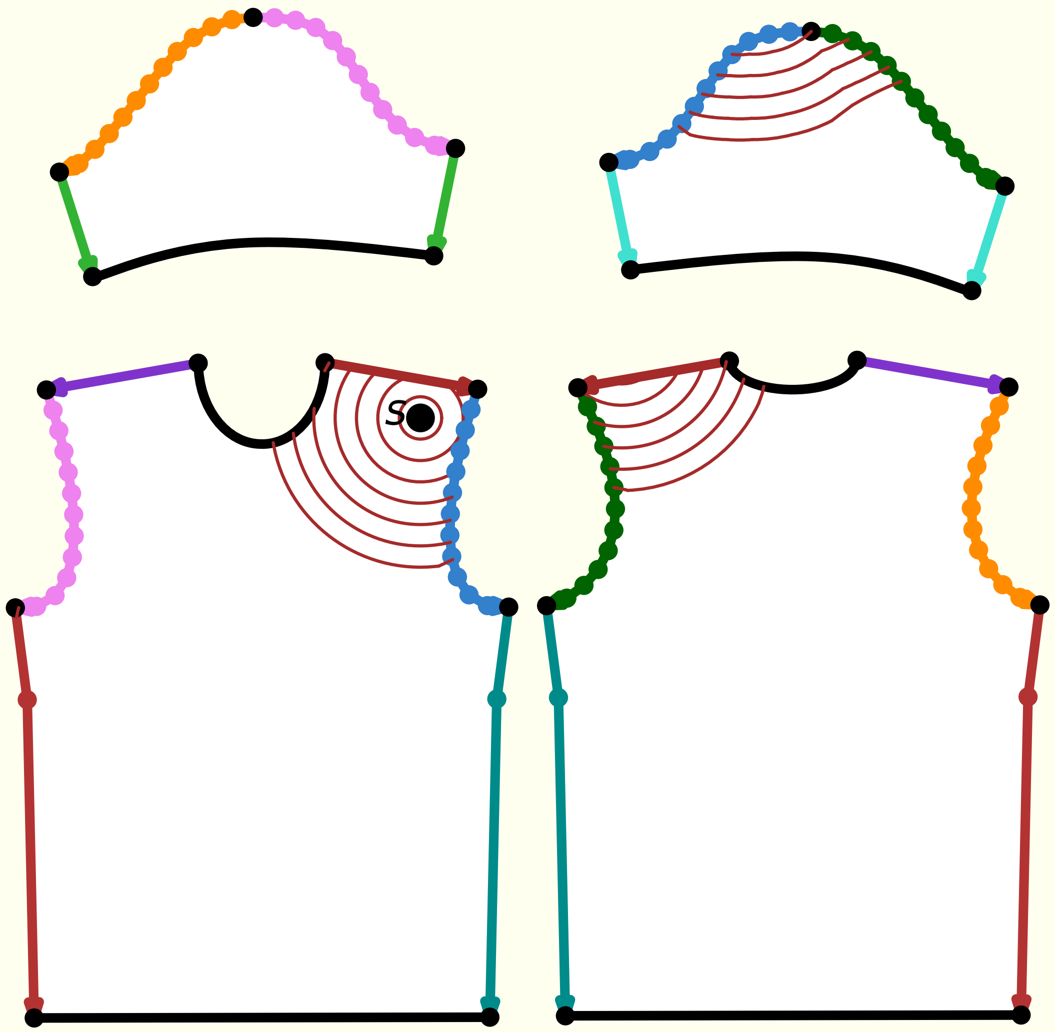
Thm. Complexity of SPM(s) in a h-happy $\mathcal{P}$ is $O(n^2h)$, and can be computed in $O(\lambda_4(n^2h)\log^2(nh))$ time.

Thm. The **intrinsic Delaunay Triangulation** $\mathcal{P}'$ of $\mathcal{P}$ is $O(1)$ happy (and equivalent to $\mathcal{P}$).

Thm. If fragment graph of $\mathcal{P}$ has ≤ 1 simple cycle, we can compute an equivalent $\mathcal{P}'$ in $O(n+\log h)$ time.

Given $\mathcal{P}$, can we compute an $O(1)$-happy $\mathcal{P}'$ equivalent to $\mathcal{P}$ efficiently? OPEN

Main Idea: use continuous dijkstra



Results

Obs. Complexity $\pi(s,t)$ unbounded in terms of n.

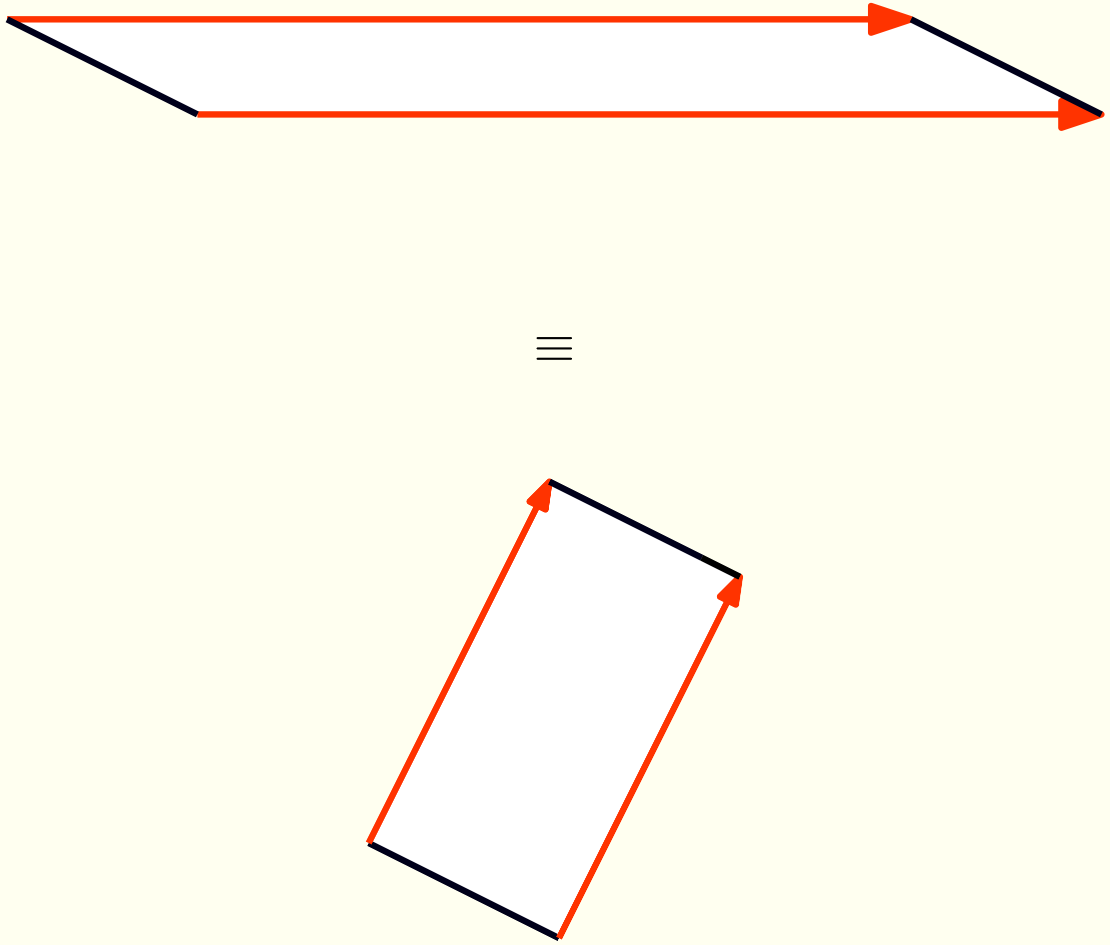
Thm. Complexity of $\pi(s,t)$ in a h-happy $\mathcal{P}$ is $\Theta(n+hm)$

Thm. Complexity of SPM(s) in a h-happy $\mathcal{P}$ is $O(n^2h)$, and can be computed in $O(\lambda_4(n^2h)\log^2(nh))$ time.

Thm. The **intrinsic Delaunay Triangulation** $\mathcal{P}'$ of $\mathcal{P}$ is $O(1)$ happy (and equivalent to $\mathcal{P}$).

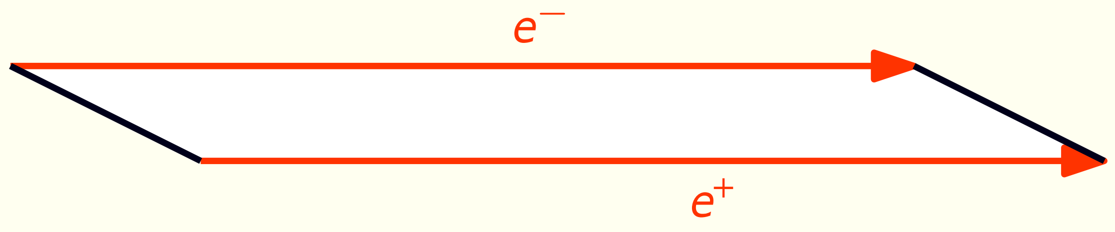
Thm. If fragment graph of $\mathcal{P}$ has ≤ 1 simple cycle, we can compute an equivalent $\mathcal{P}'$ in $O(n+\log h)$ time.

Given $\mathcal{P}$, can we compute an $O(1)$-happy $\mathcal{P}'$ equivalent to $\mathcal{P}$ efficiently? OPEN



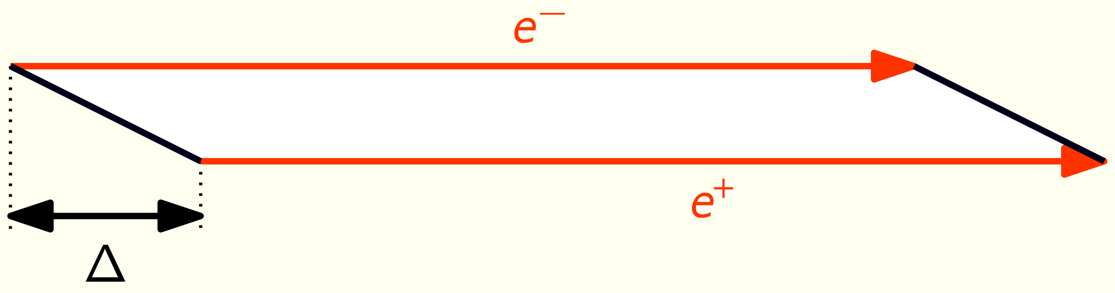
Making portalgons happy

Consider a parallelogram $F \in \mathcal{F}$ with a horizontal portal (e^-, e^+)



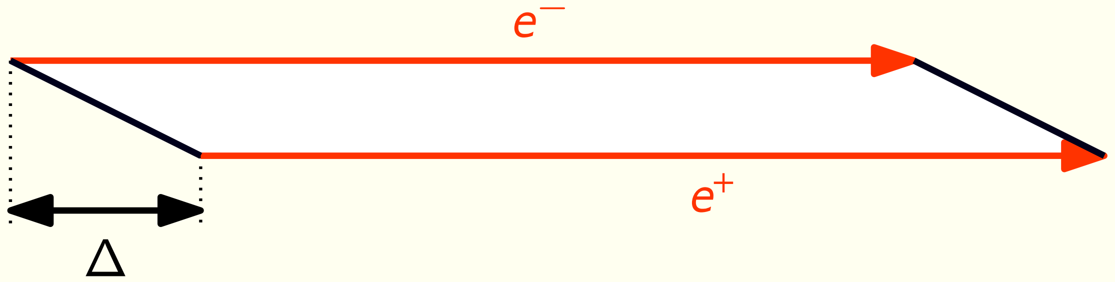
Making portalgons happy

Consider a parallelogram $F \in F$ with a horizontal portal (e^-, e^+)



Making portalgons happy

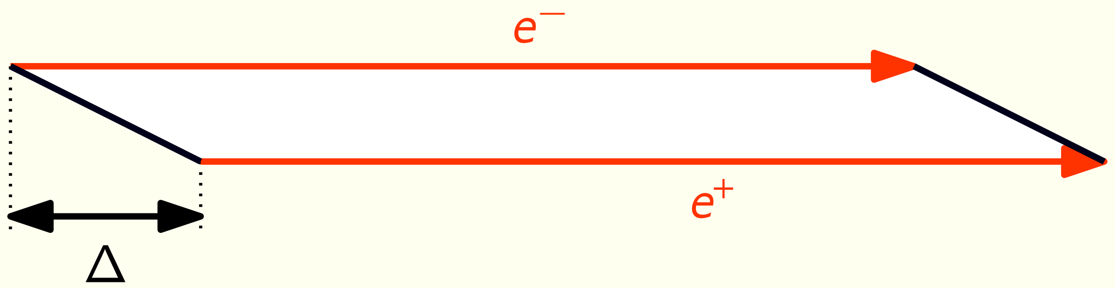
Consider a parallelogram $F \in \mathcal{F}$ with a horizontal portal (e^-, e^+)



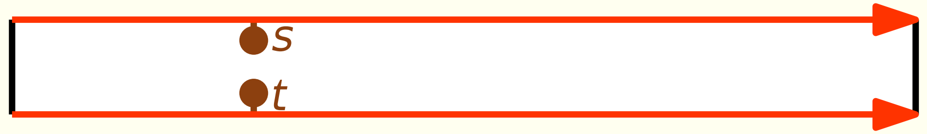
Lem. If the **shift** $\Delta = 0$ then F is 2-happy.

Making portalgons happy

Consider a parallelogram $F \in F$ with a horizontal portal (e^-, e^+)

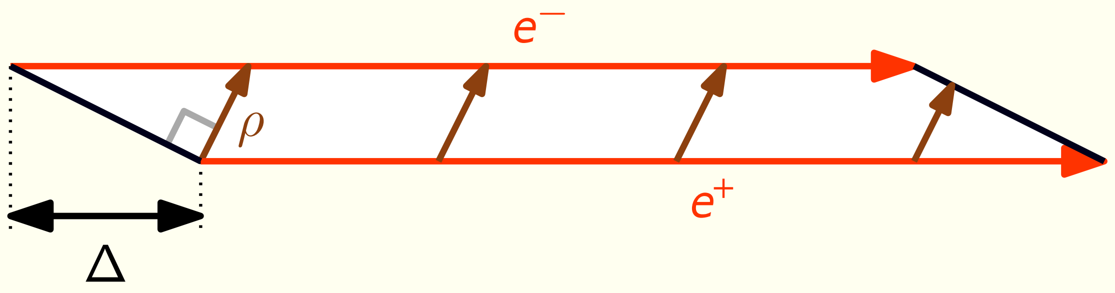


Lem. If the **shift** $\Delta = 0$ then F is 2-happy.



Making portalgons happy

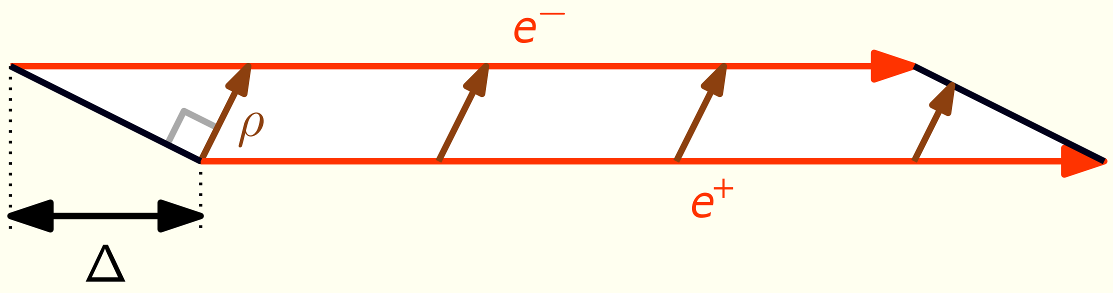
Consider a parallelogram $F \in F$ with a horizontal portal (e^-, e^+)



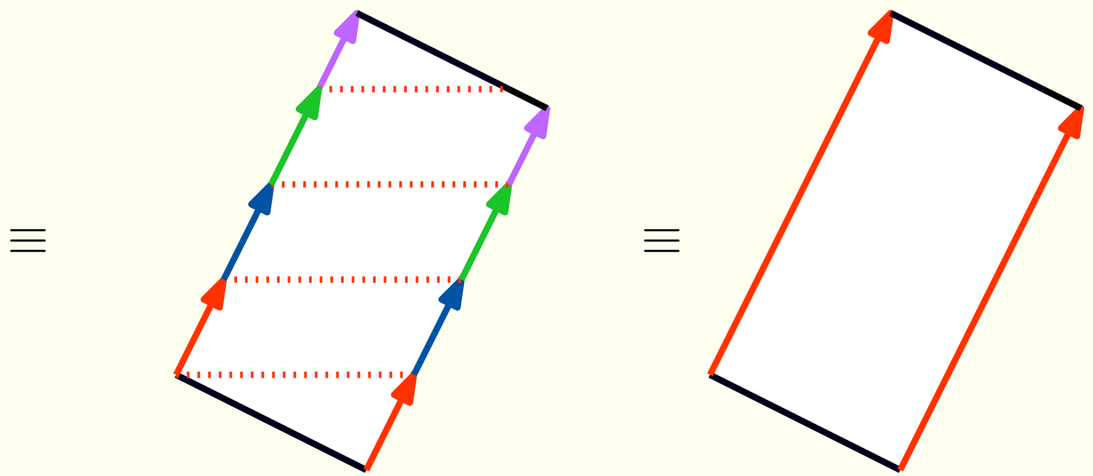
Lem. If the **shift** $\Delta = 0$ then F is 2-happy.

Making portalgons happy

Consider a parallelogram $F \in \mathcal{F}$ with a horizontal portal (e^-, e^+)

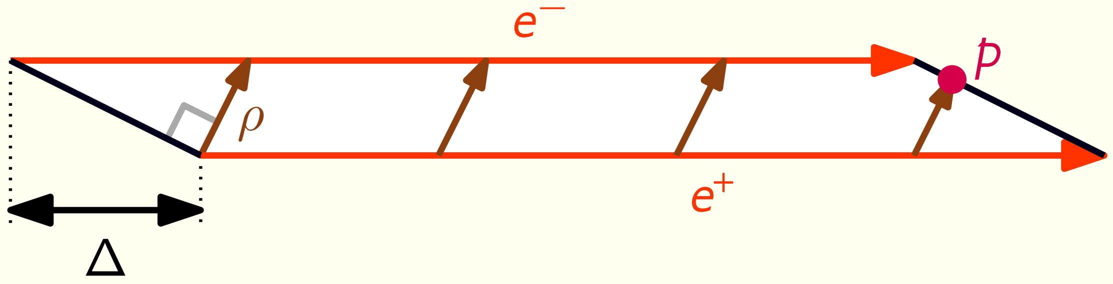


Lem. If the **shift** $\Delta = 0$ then F is 2-happy.

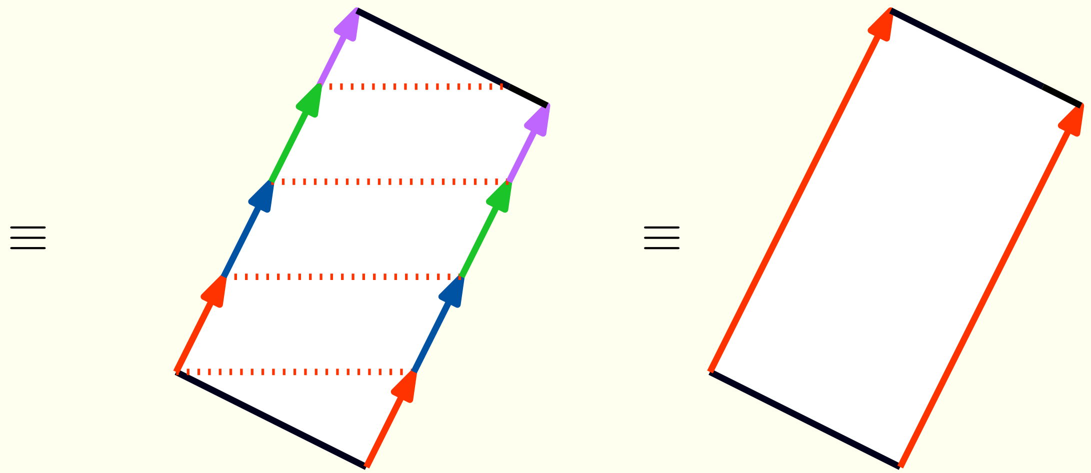


Making portalgons happy

Consider a parallelogram $F \in \mathcal{F}$ with a horizontal portal (e^-, e^+)

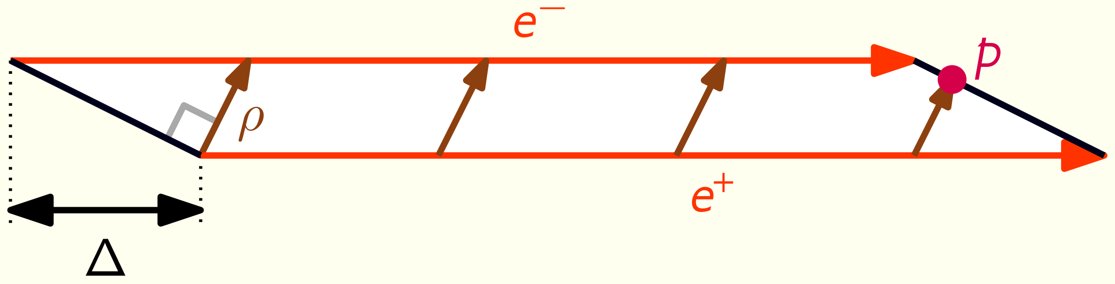


Lem. If the **shift** $\Delta = 0$ then F is 2-happy.



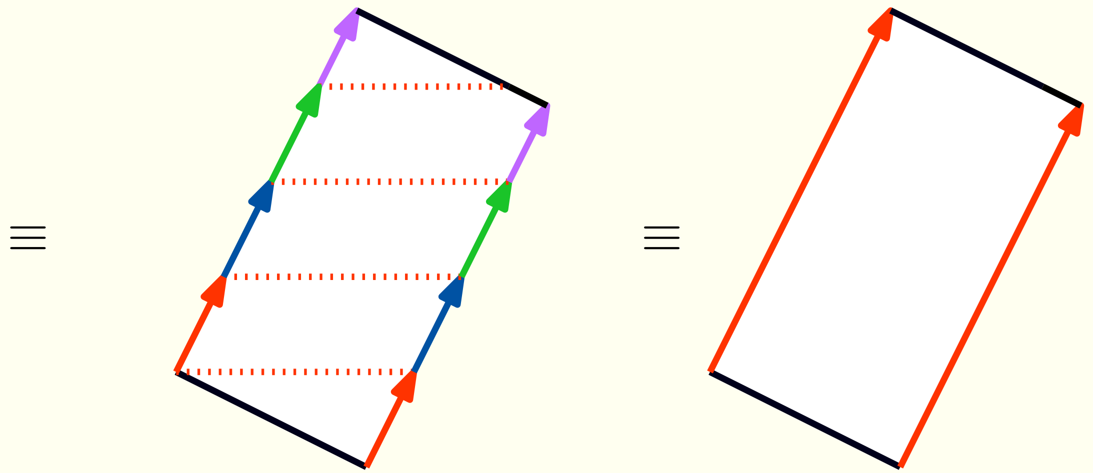
Making portalgons happy

Consider a parallelogram $F \in F$ with a horizontal portal (e^-, e^+)



Lem. If the **shift** $\Delta = 0$ then F is 2-happy.

Lem. A parallelogram F can be made 2-happy in $O(1)$ time.



Making portalgons happy

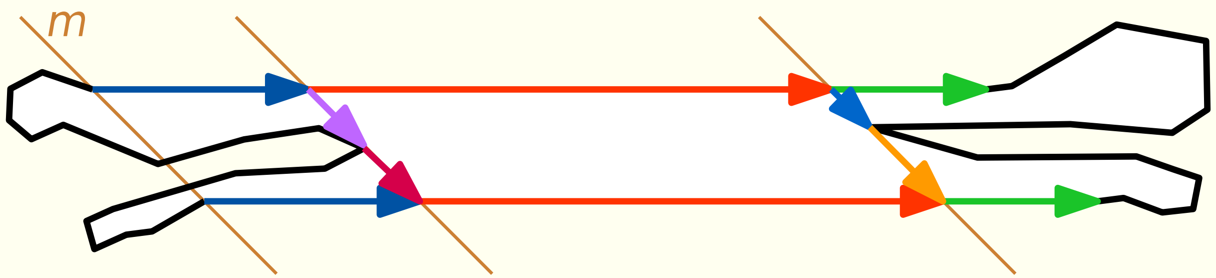
Consider a $F \in F$ with a horizontal portal (e^{-}, e^{+})

Lem. If the **shift** $\Delta = 0$ then F is 2-happy.

Making portalgons happy

Consider a $F \in \mathcal{F}$ with a horizontal portal (e^-, e^+)

Lem. If the **shift** $\Delta = 0$ then F is 2-happy.

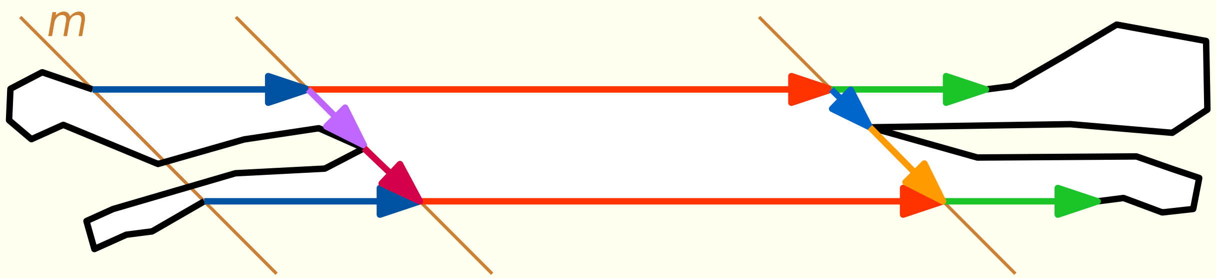


Making portalgons happy

Consider a $F \in \mathcal{F}$ with a horizontal portal (e^-, e^+)

Lem. If the **shift** $\Delta = 0$ then F is 2-happy.

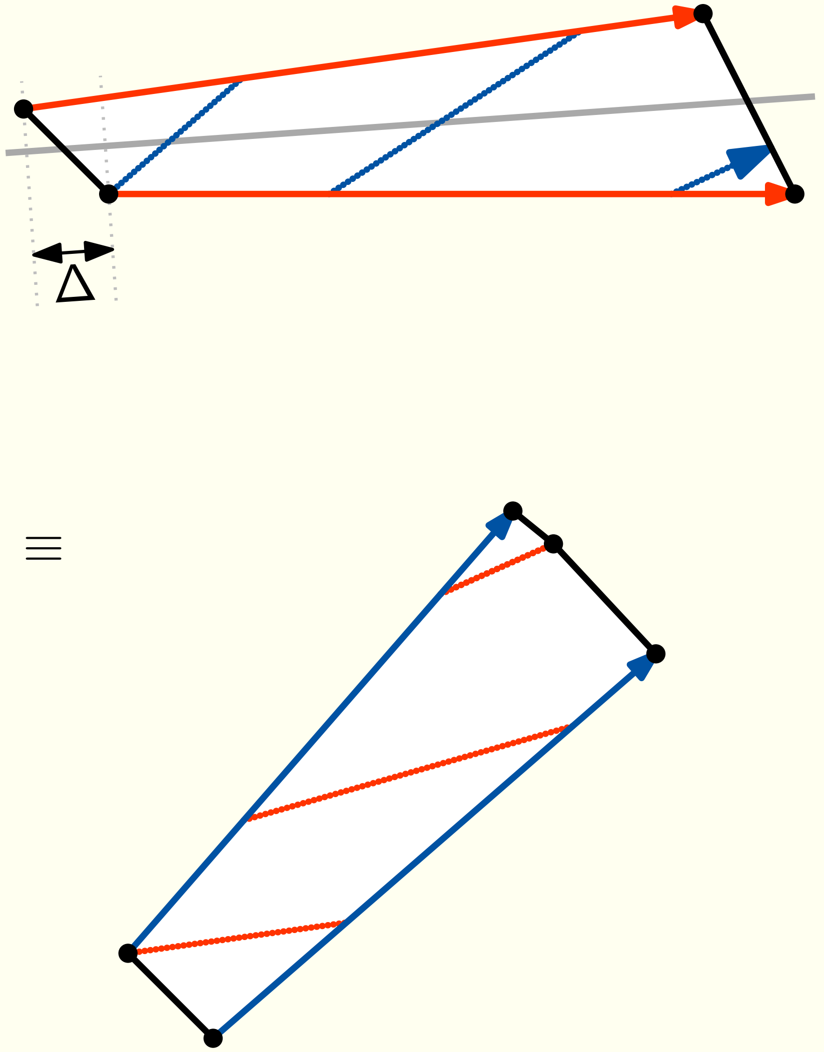
Lem. We can compute $\mathcal{P}'$ equivalent to F in $O(n)$ time.



Making portalgons happy

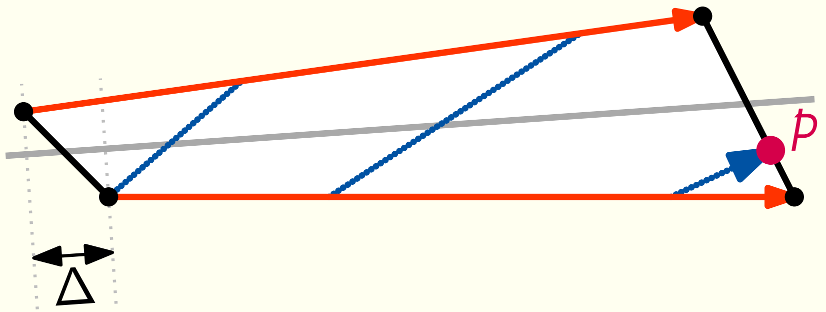
Consider a $F \in F$ with a portal (e^{-}, e^{+})

Lem. If the **shift** $\Delta = 0$ then F is 2-happy.



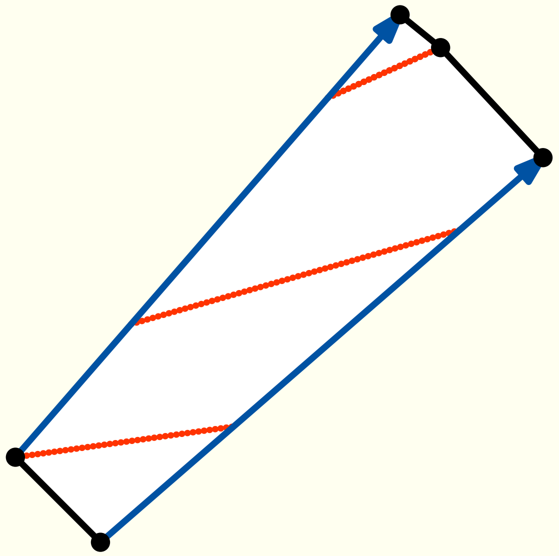
Making portalgons happy

Consider a $F \in F$ with a portal (e^-, e^+)



Lem. If the **shift** $\Delta = 0$ then F is 2-happy.

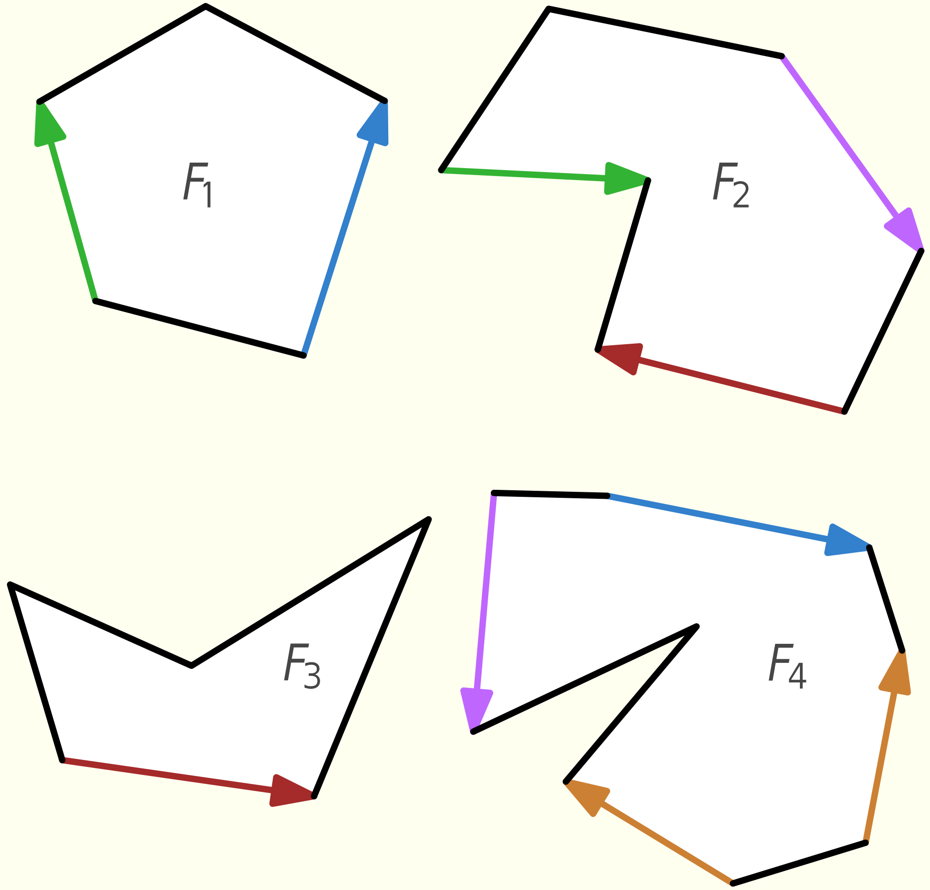
$\equiv$



Lem. We can compute $\mathcal{P}'$ equivalent to F in $O(n + \log h)$ time.

Concluding Remarks

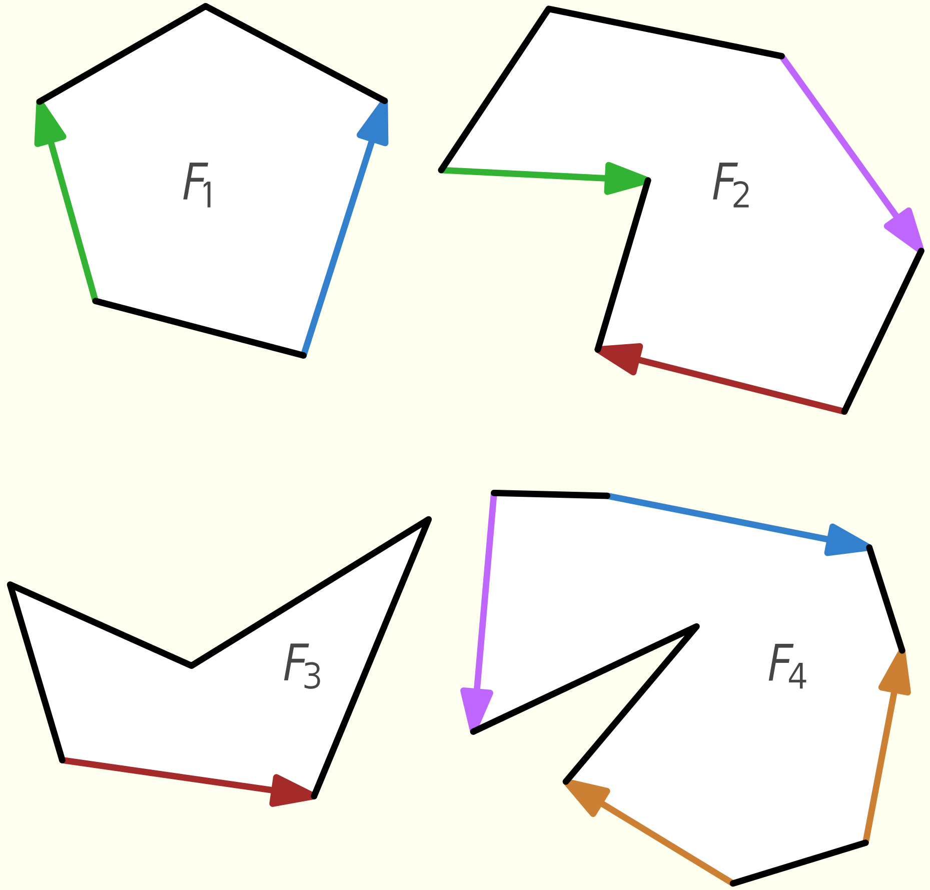
Thm. Complexity of SPM(s) in a h-happy $\mathcal{P}$ is $O(n^2 h)$, and can be computed in $O(\lambda_4(n^2 h) \log^2(nh))$ time.



Concluding Remarks

Thm. Complexity of SPM(s) in a h-happy $\mathcal{P}$ is $O(n^2 h)$, and can be computed in $O(\lambda_4(n^2 h) \log^2(nh))$ time.

Thm. The intrinsic Delaunay Triangulation $\mathcal{P}'$ of $\mathcal{P}$ is $O(1)$ happy (and equivalent to $\mathcal{P}$).



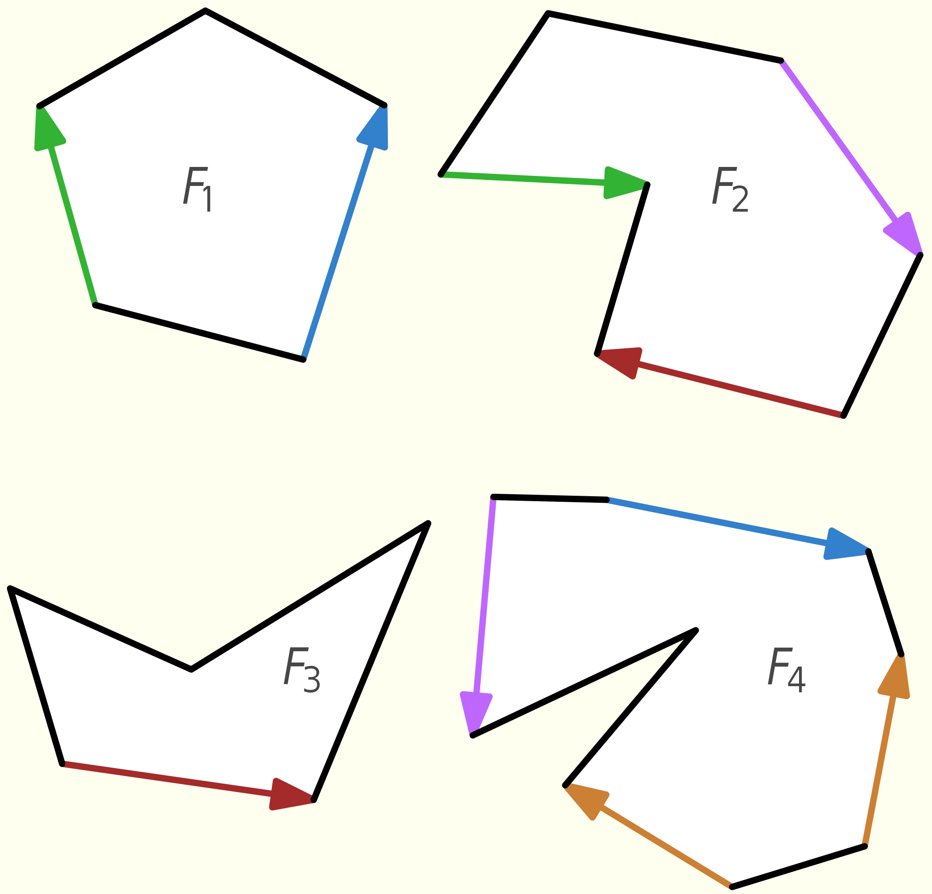
Concluding Remarks

Thm. Complexity of SPM(s) in a h-happy $\mathcal{P}$ is $O(n^2 h)$, and can be computed in $O(\lambda_4(n^2 h)\log^2(nh))$ time.

Can we compute SPM(s) in $O(nmh)$ time? OPEN

Thm. The intrinsic Delaunay Triangulation $\mathcal{P}'$ of $\mathcal{P}$ is $O(1)$ happy (and equivalent to $\mathcal{P}$).

Given $\mathcal{P}$, can we compute an $O(1)$-happy $\mathcal{P}'$ equivalent to $\mathcal{P}$ efficiently? OPEN



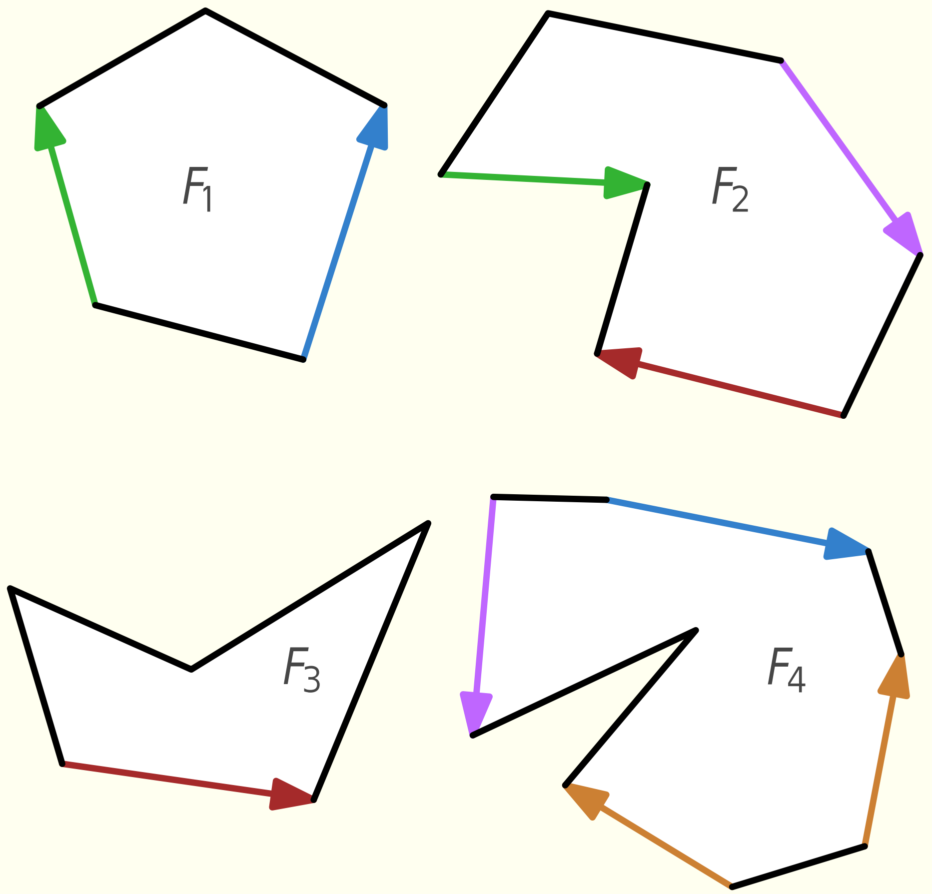
Concluding Remarks

Thm. Complexity of SPM(s) in a h-happy $\mathcal{P}$ is $O(n^2 h)$, and can be computed in $O(\lambda_4(n^2 h) \log^2(nh))$ time.

Can we compute SPM(s) in $O(nmh)$ time? OPEN

Thm. The intrinsic Delaunay Triangulation $\mathcal{P}'$ of $\mathcal{P}$ is $O(1)$ happy (and equivalent to $\mathcal{P}$).

Given $\mathcal{P}$, can we compute an $O(1)$-happy $\mathcal{P}'$ equivalent to $\mathcal{P}$ efficiently? OPEN



Thank You!