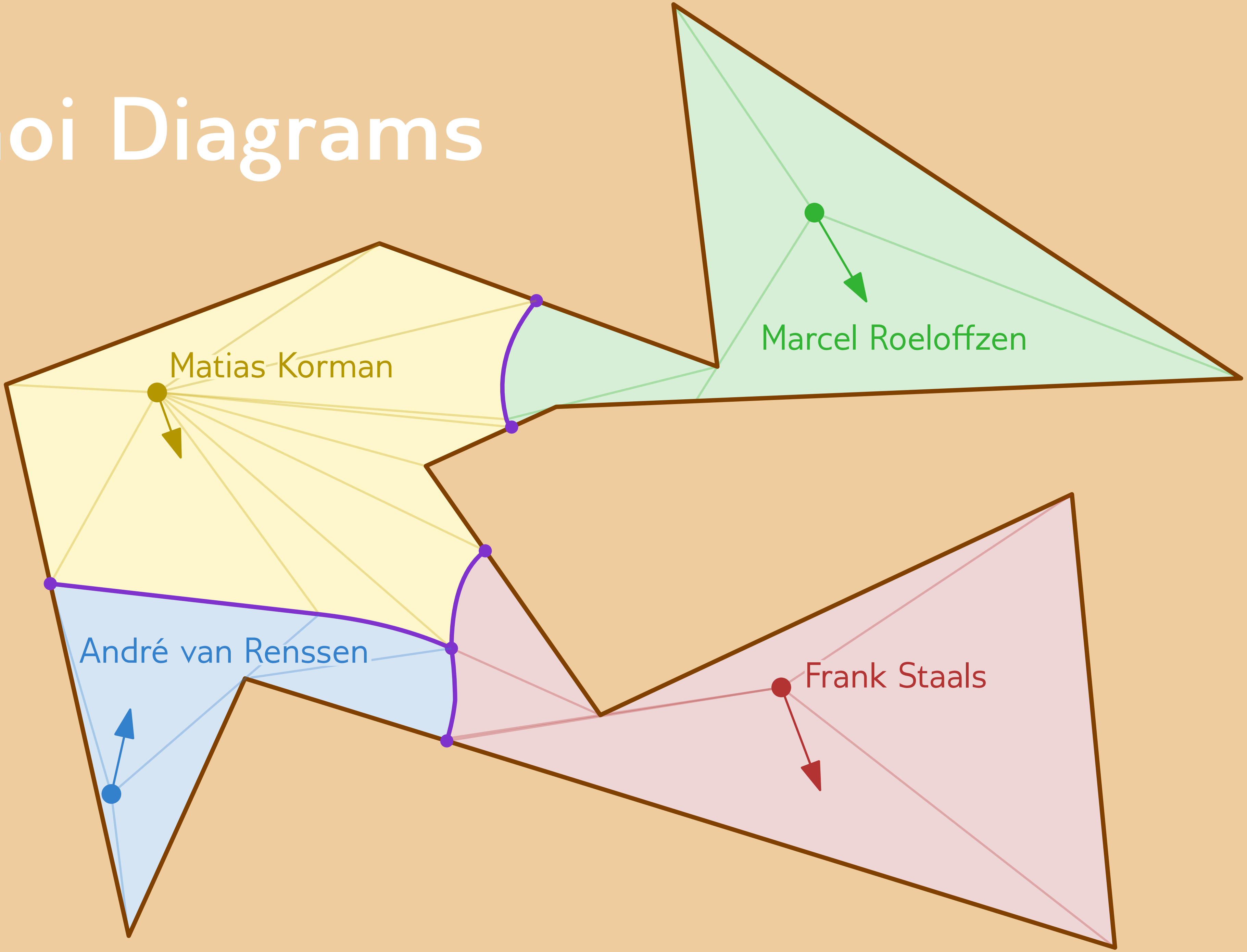
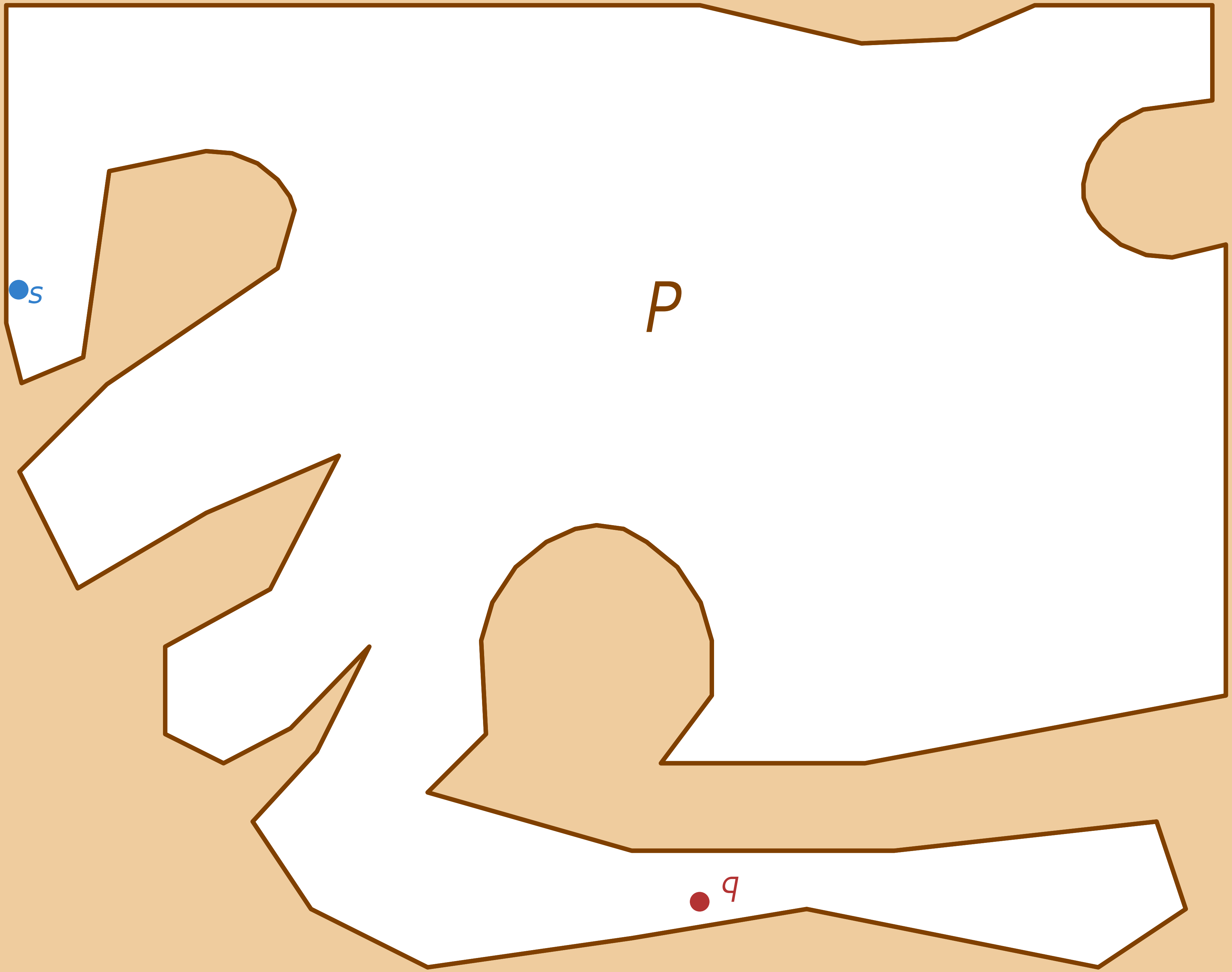
Kinetic Geodesic Voronoi Diagrams in a Simple Polygon



Motivation

Find shortest path from q to s in P



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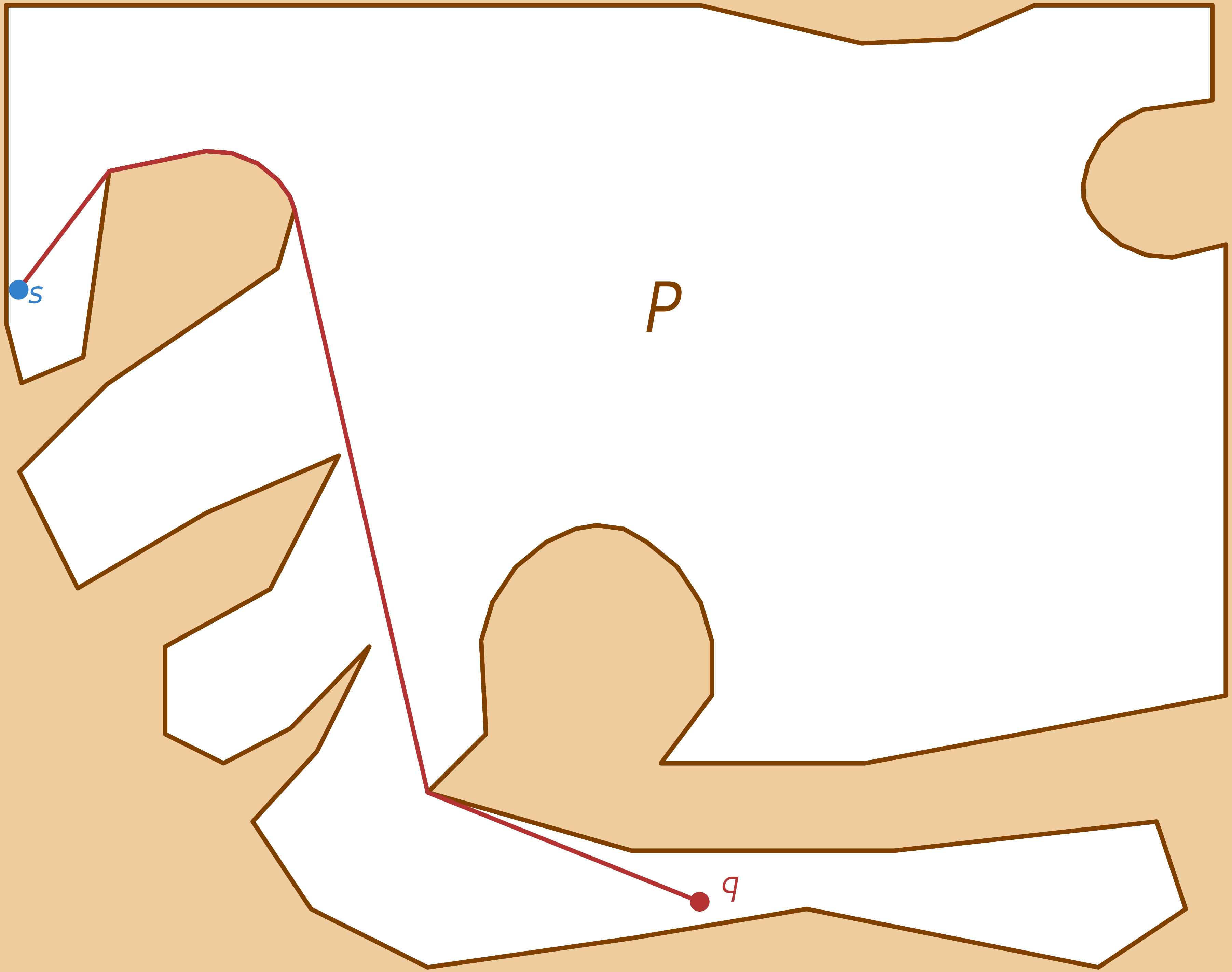

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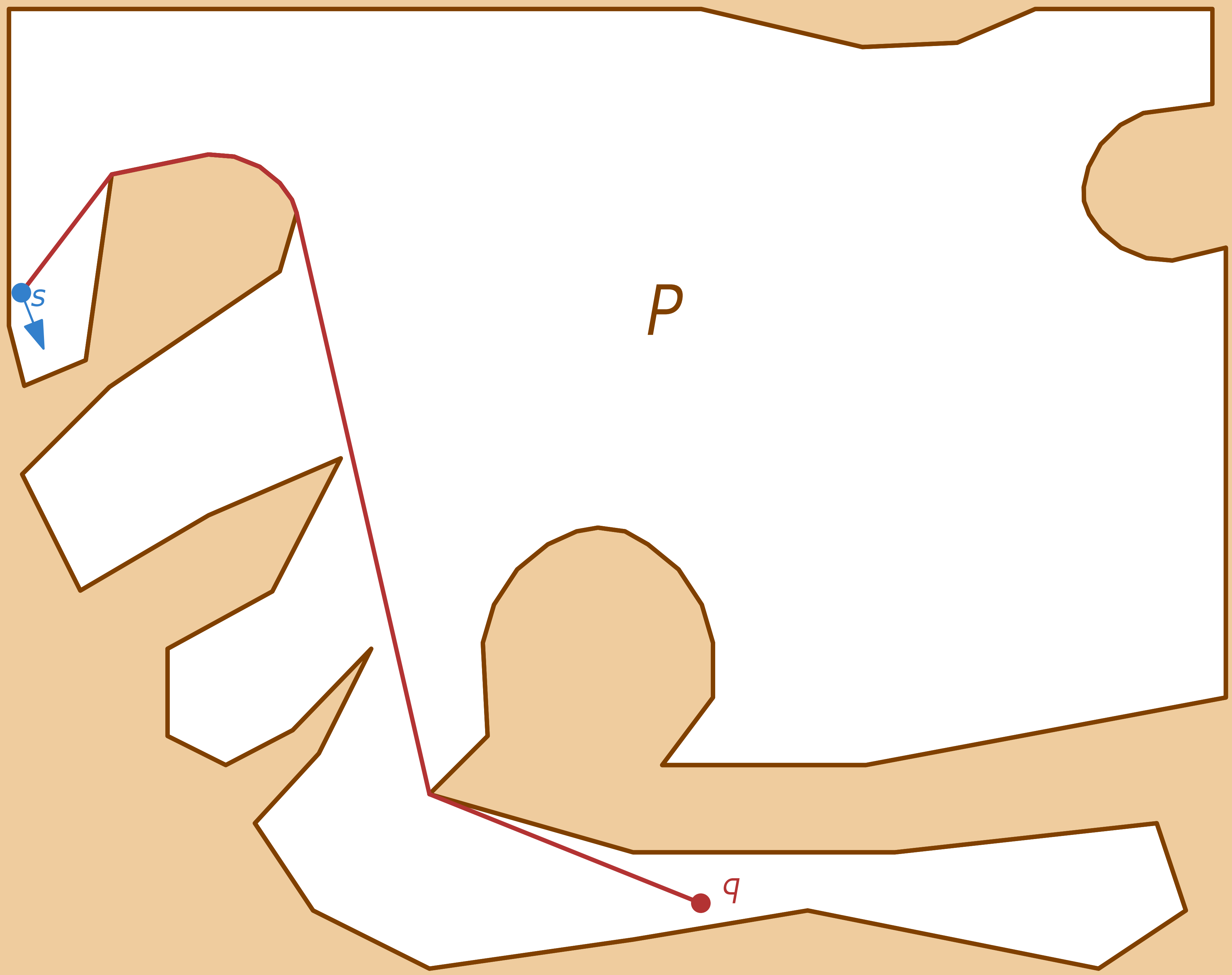
Find shortest path from q to s in P

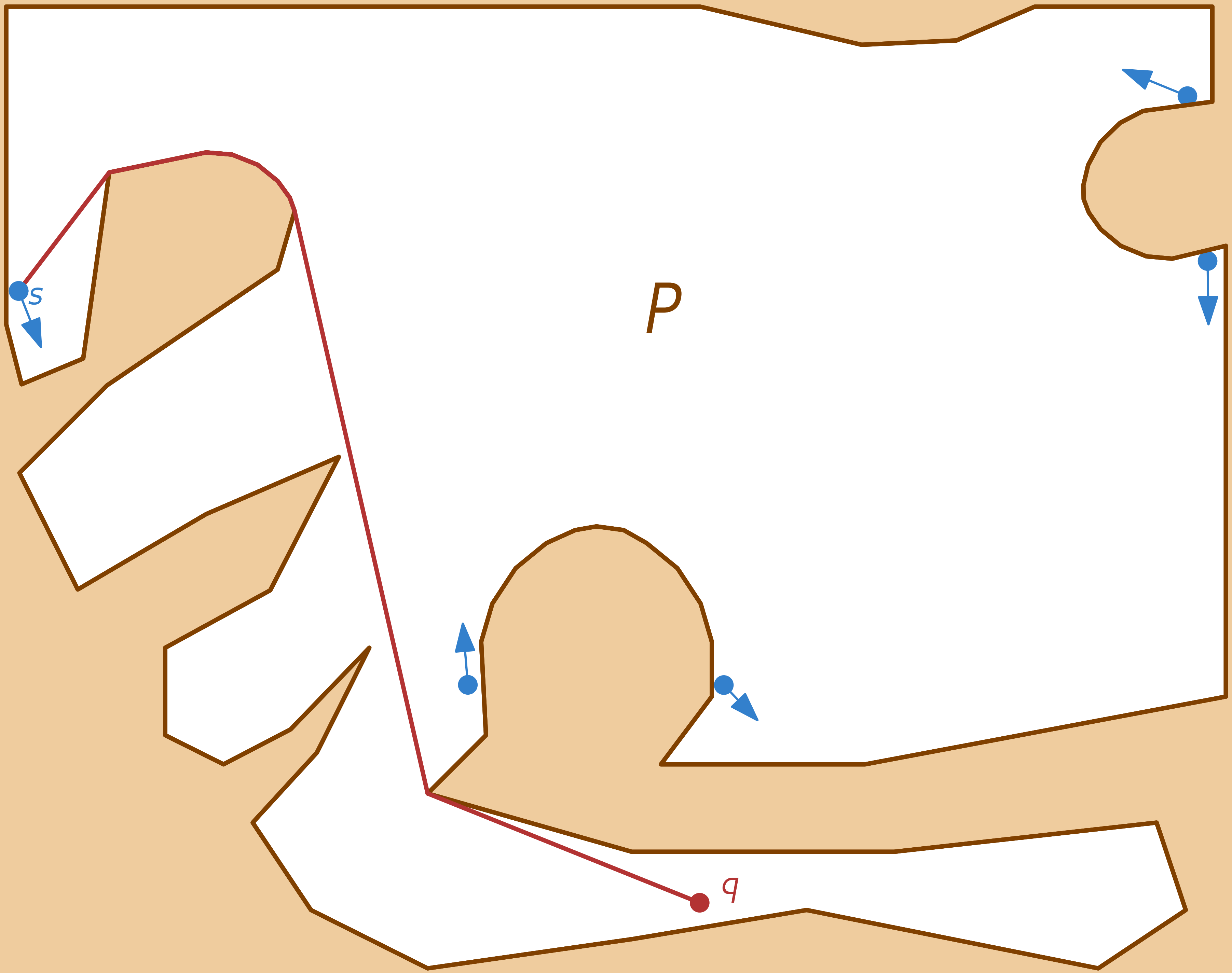
s may move



Motivation

Find shortest path from q to S in P

sites in S may move



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Find shortest path from q to S in P

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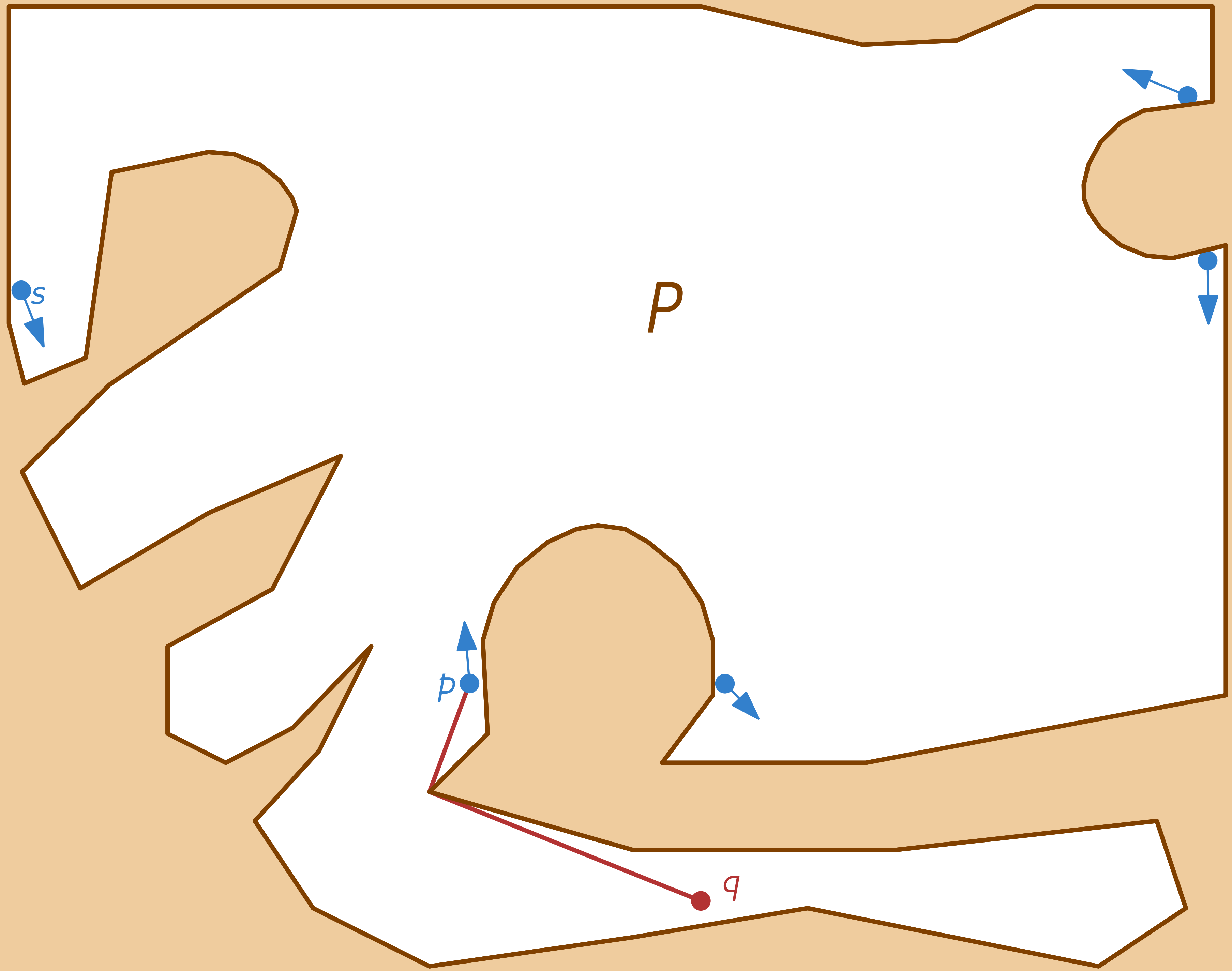

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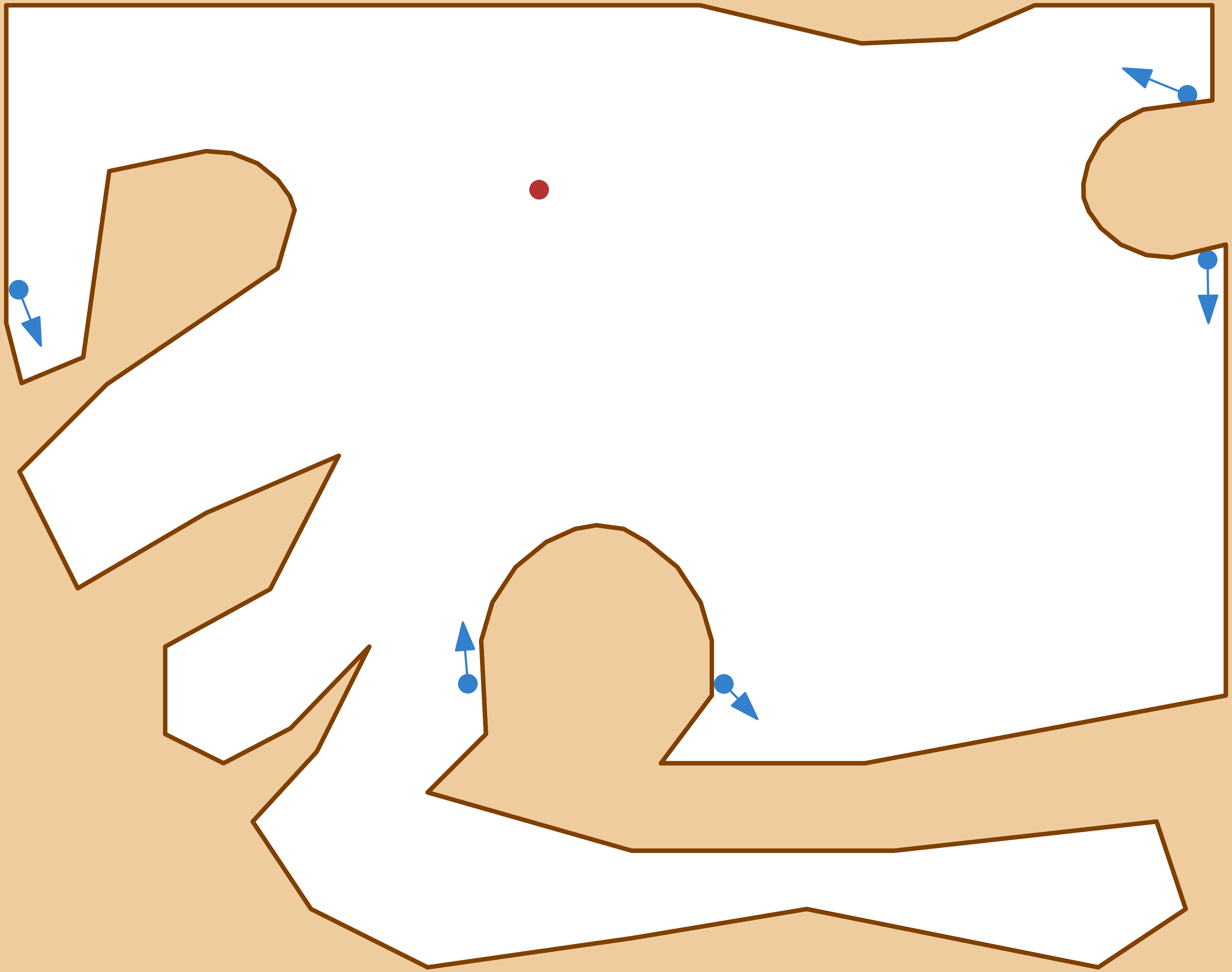
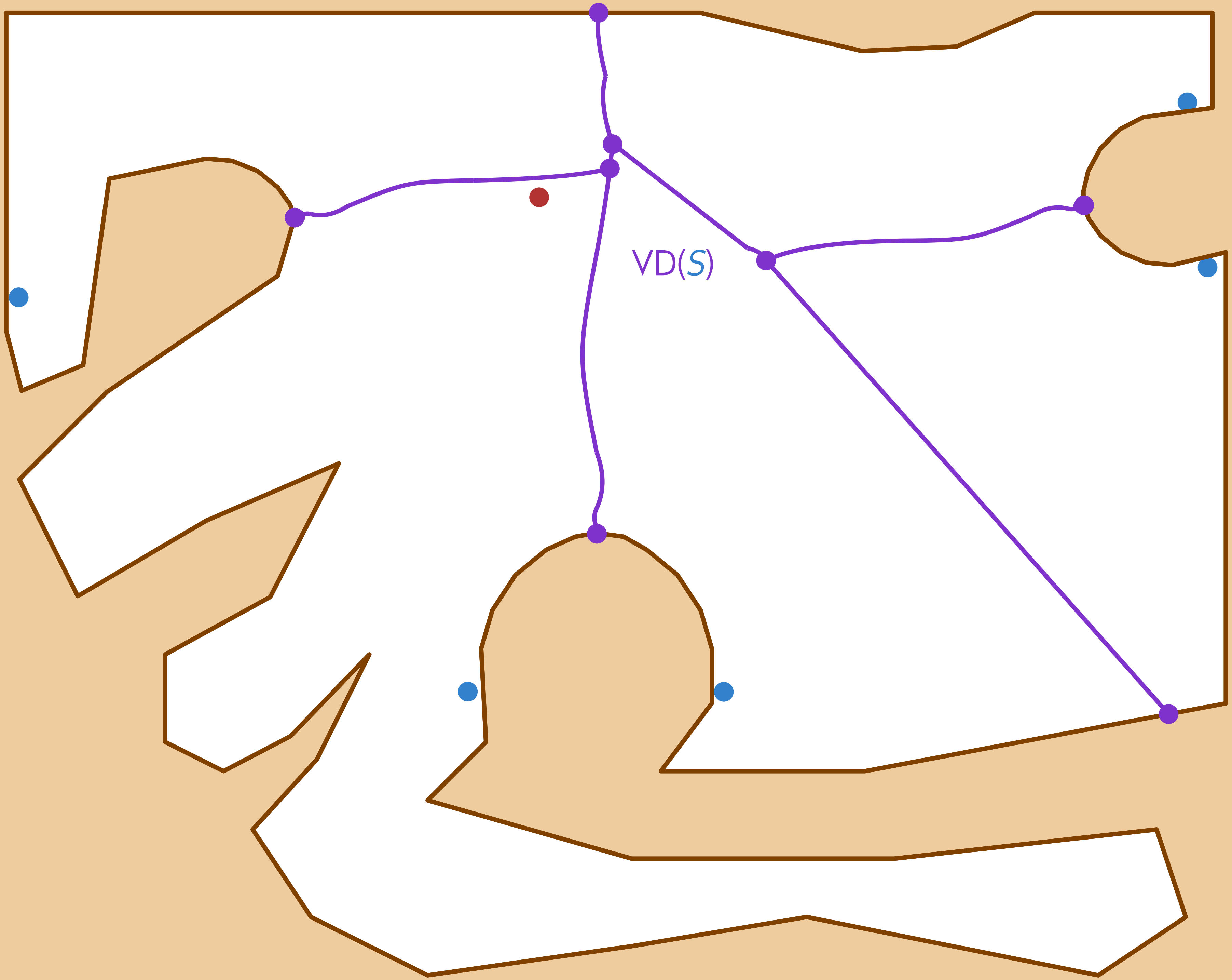


Motivation

Find shortest path from q to S in P

VD(S): (geodesic) Voronoi diagram



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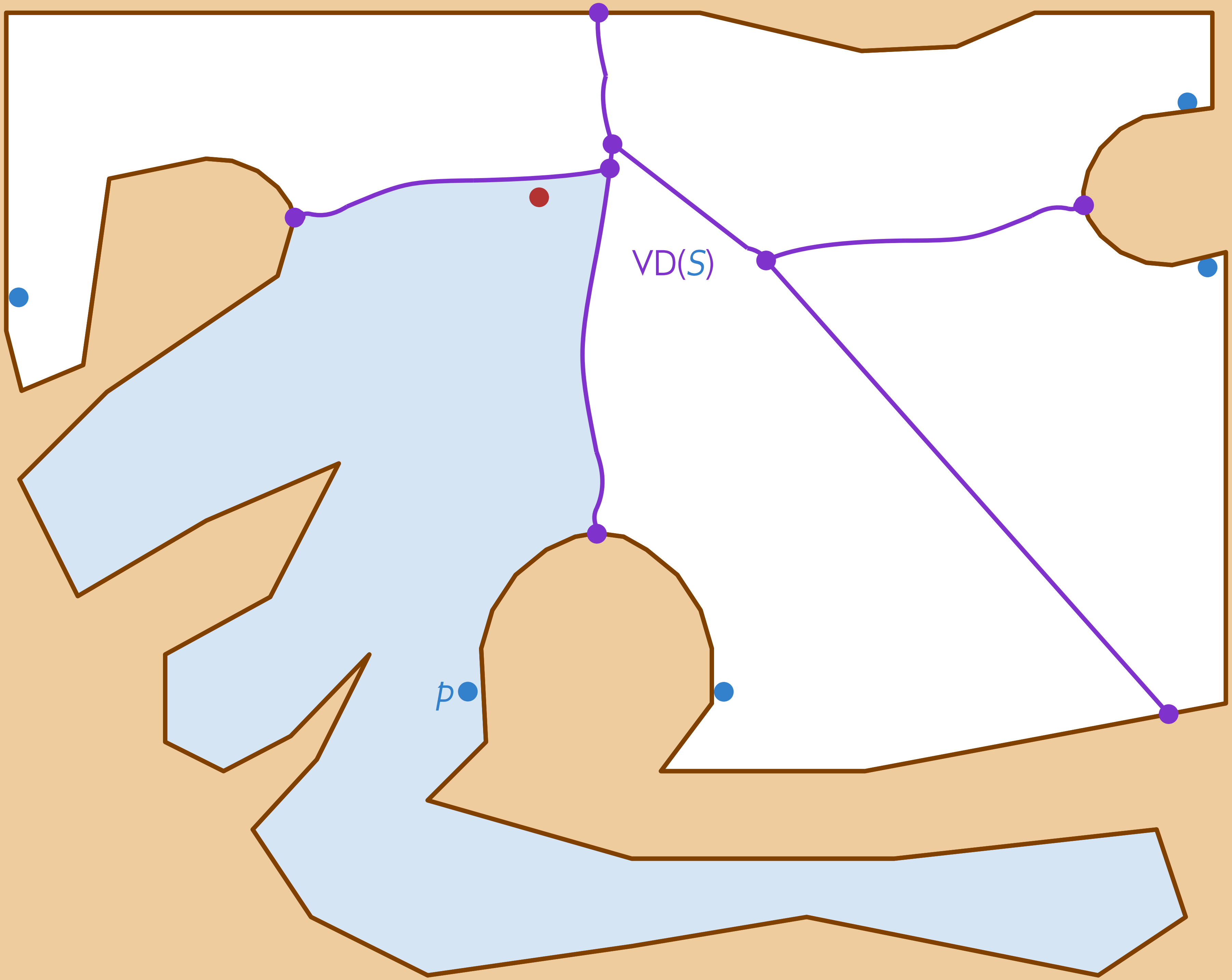

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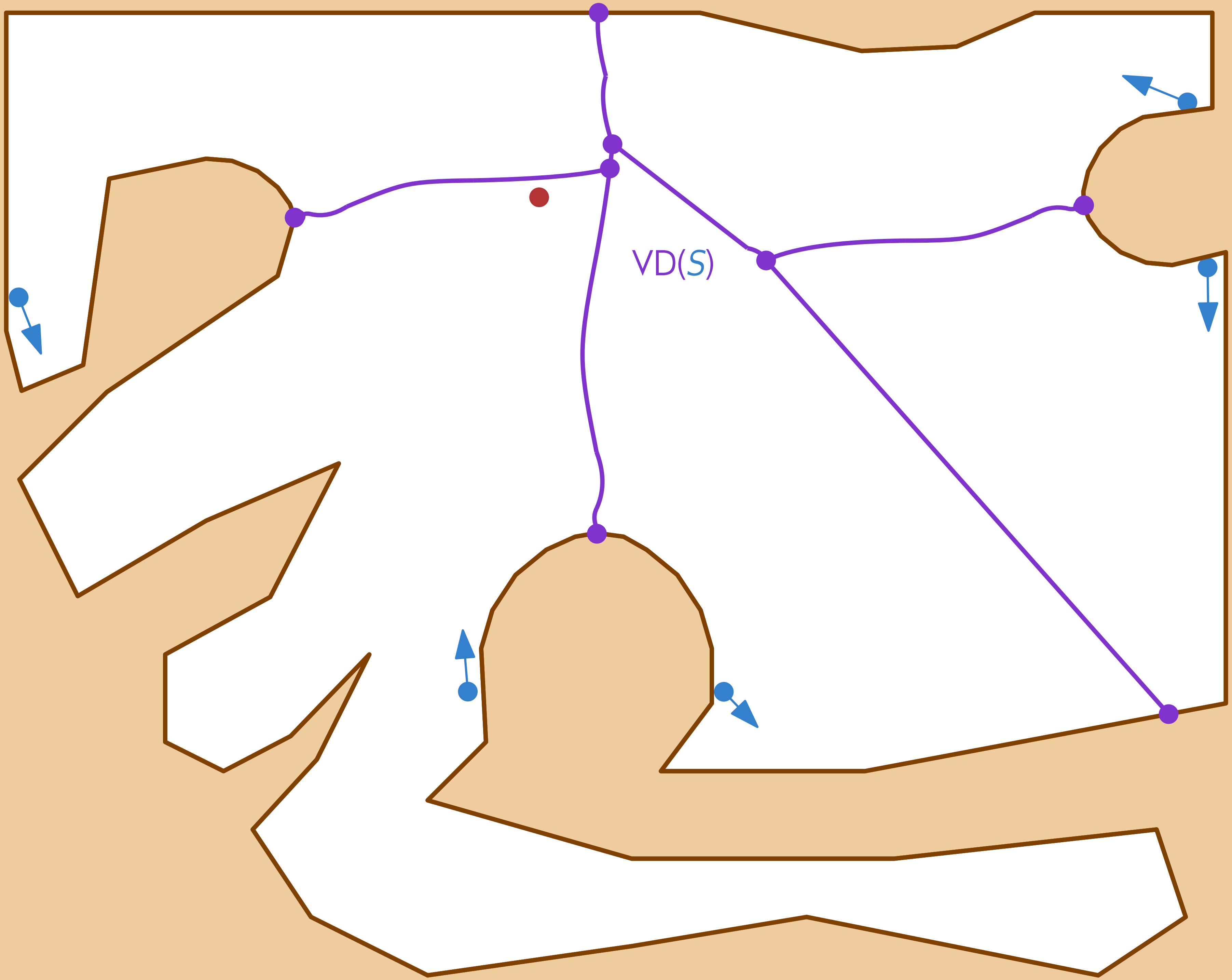

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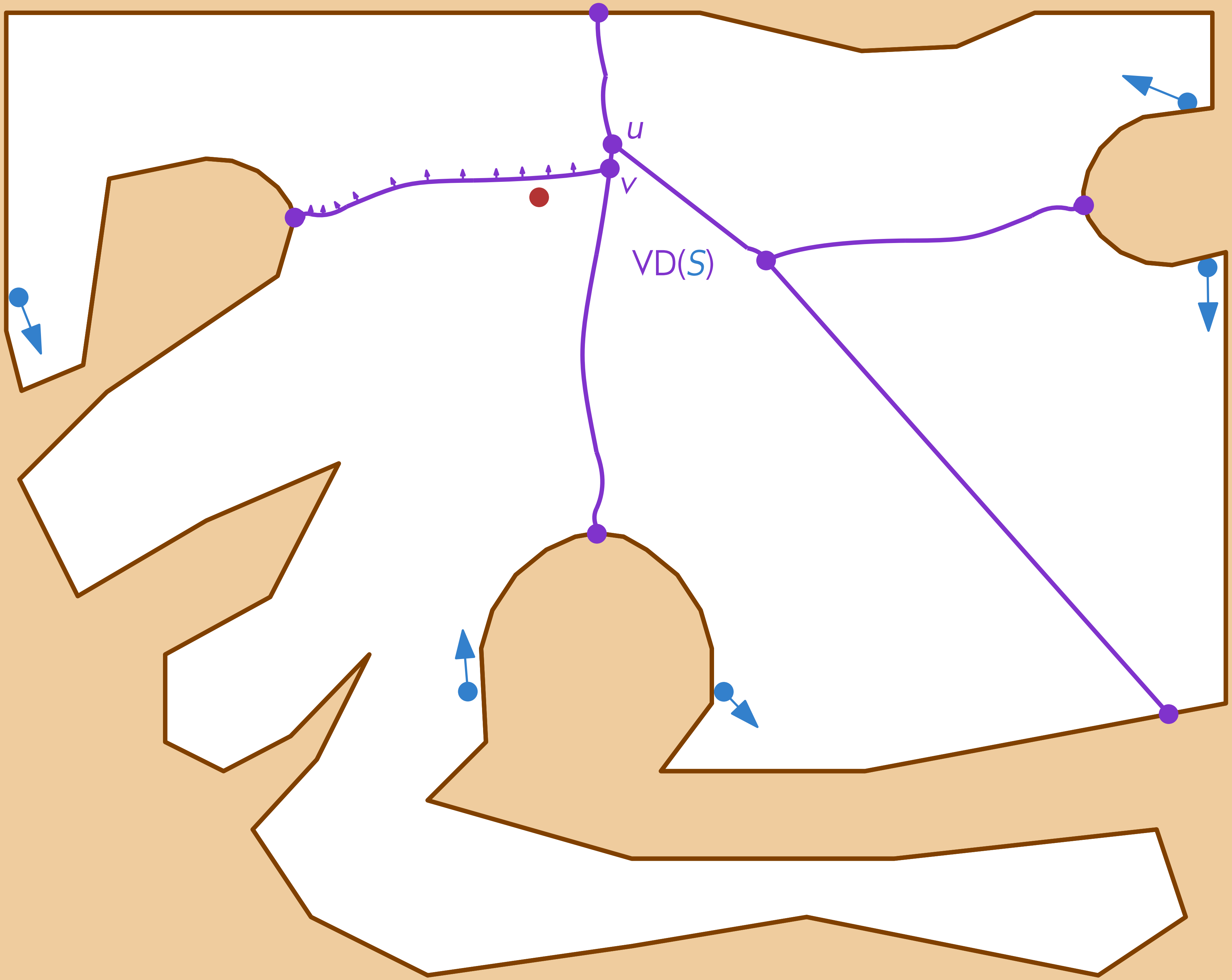

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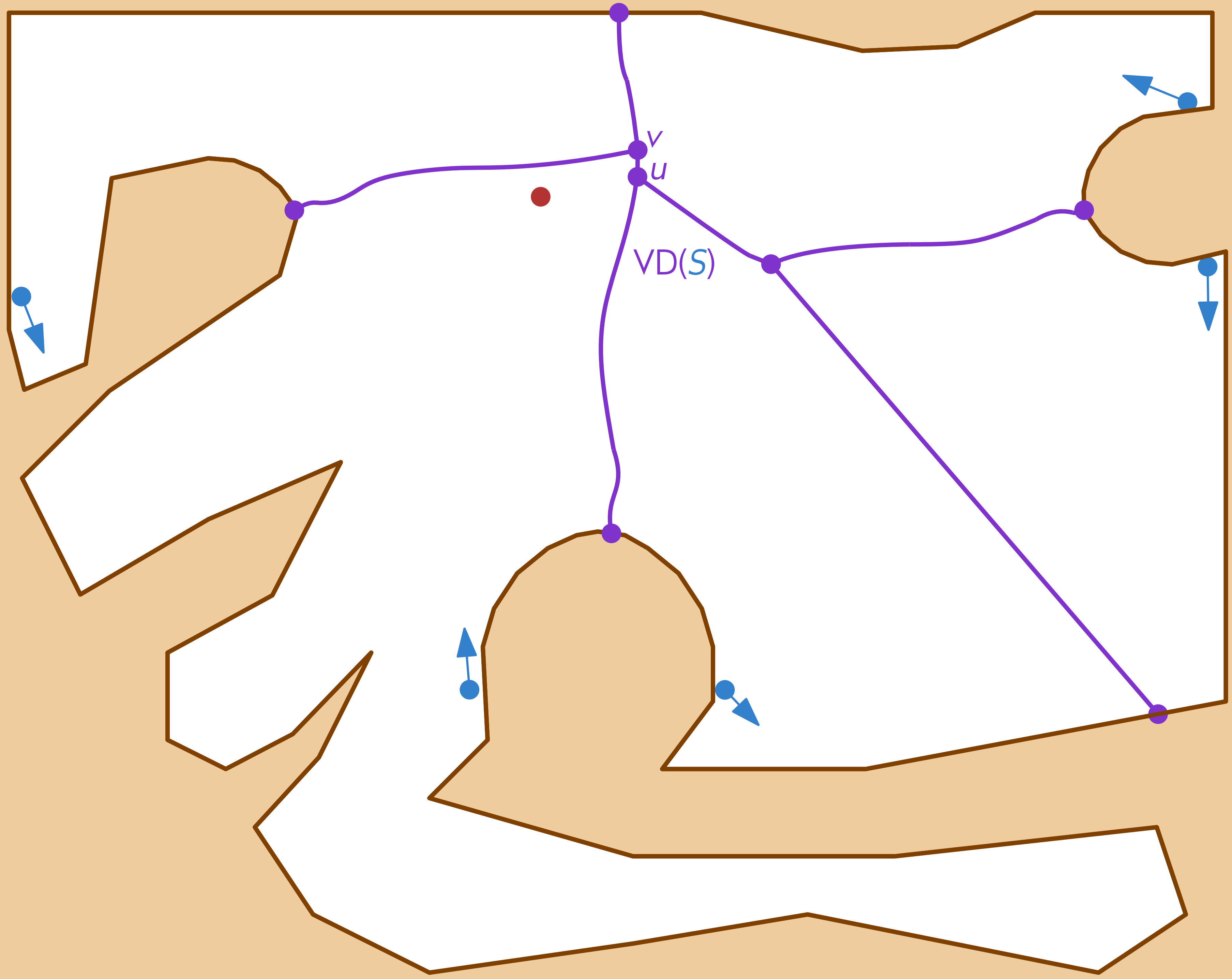
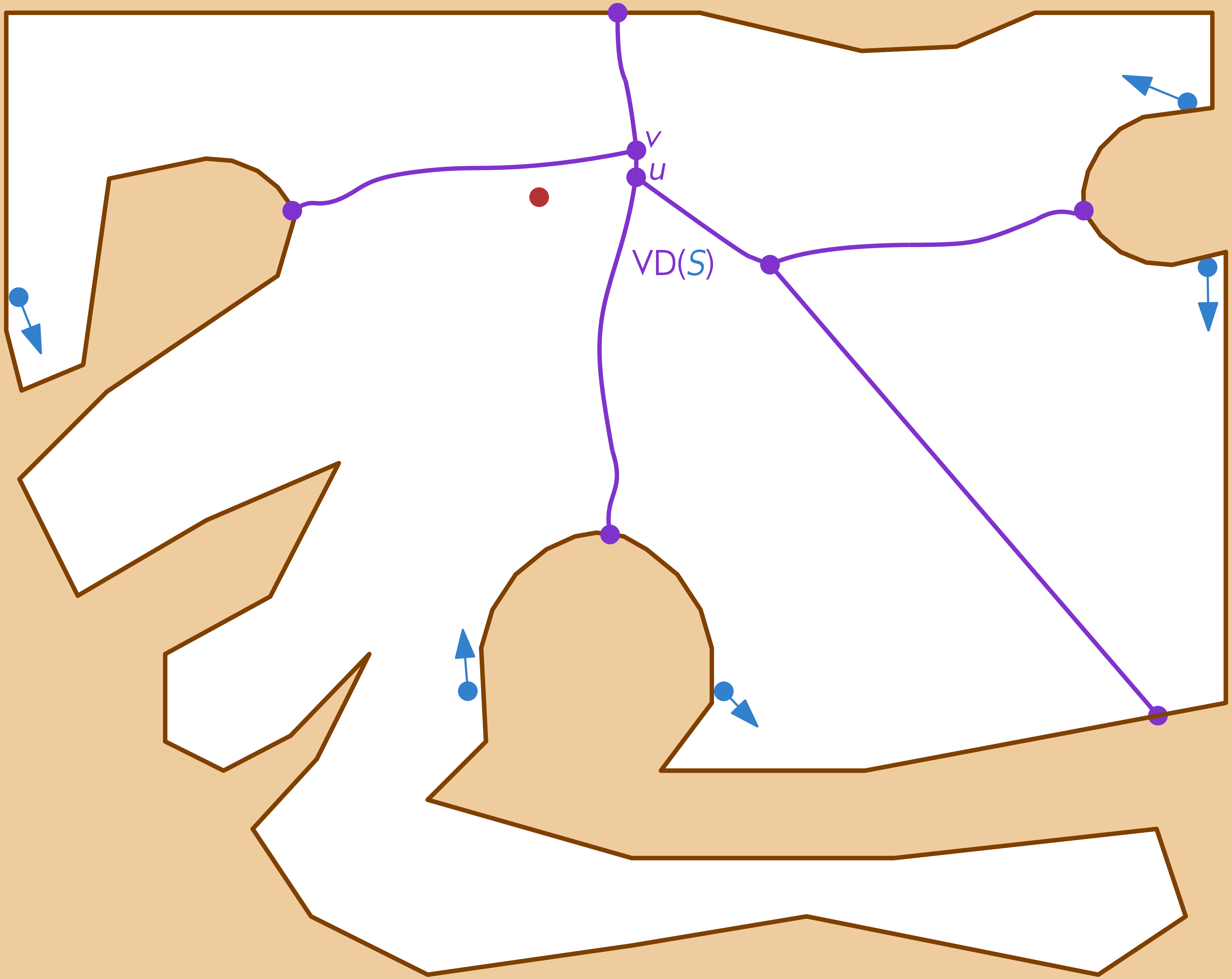


Motivation

Find shortest path from q to S in P

VD(S): (geodesic) Voronoi diagram

1. When does VD(S) change?
2. How many changes?
3. Can we maintain VD(S) efficiently?



Motivation

Find shortest path from q to S in P

Maintain MST(S), NN(S), etc.

VD(S): (geodesic) Voronoi diagram

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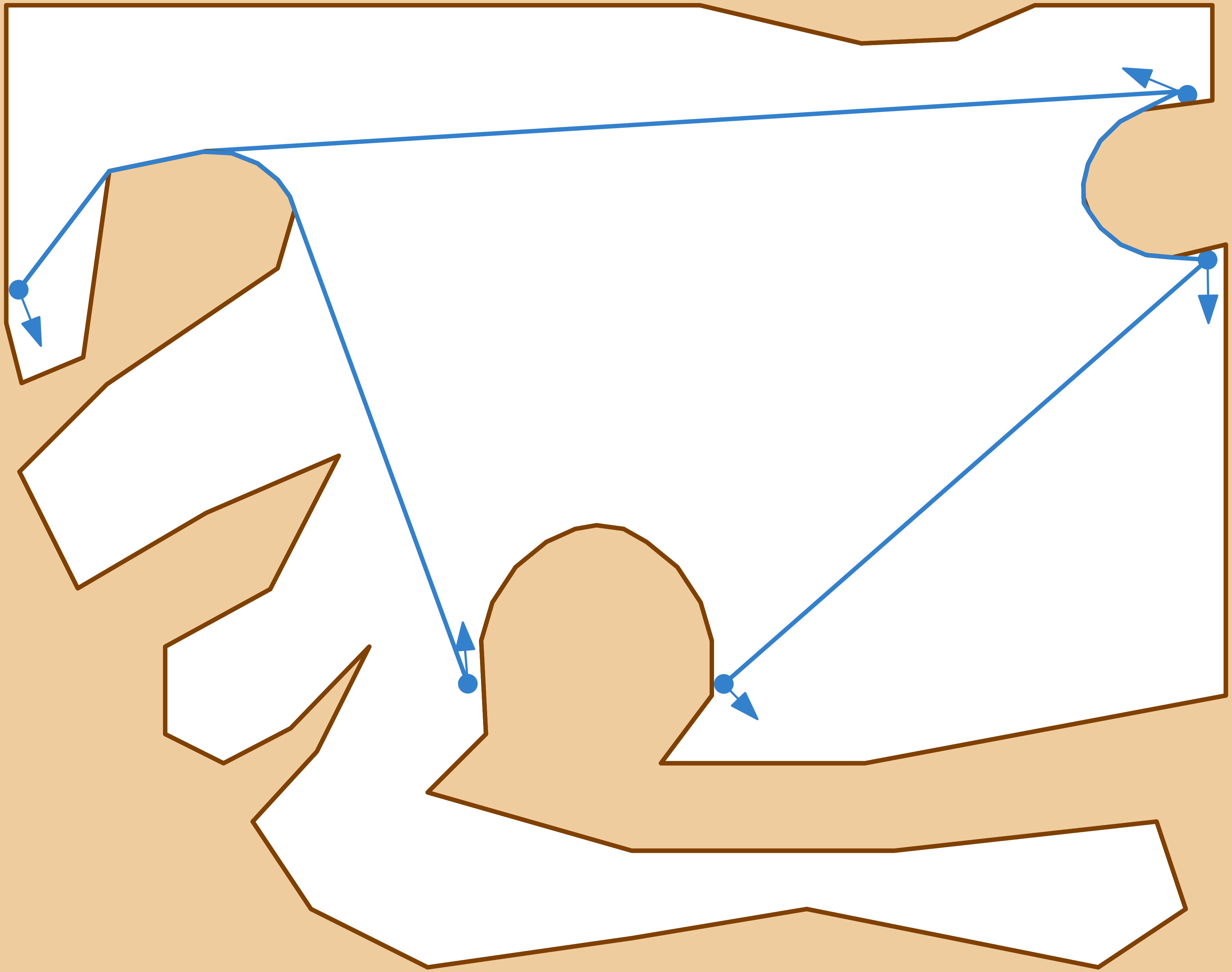
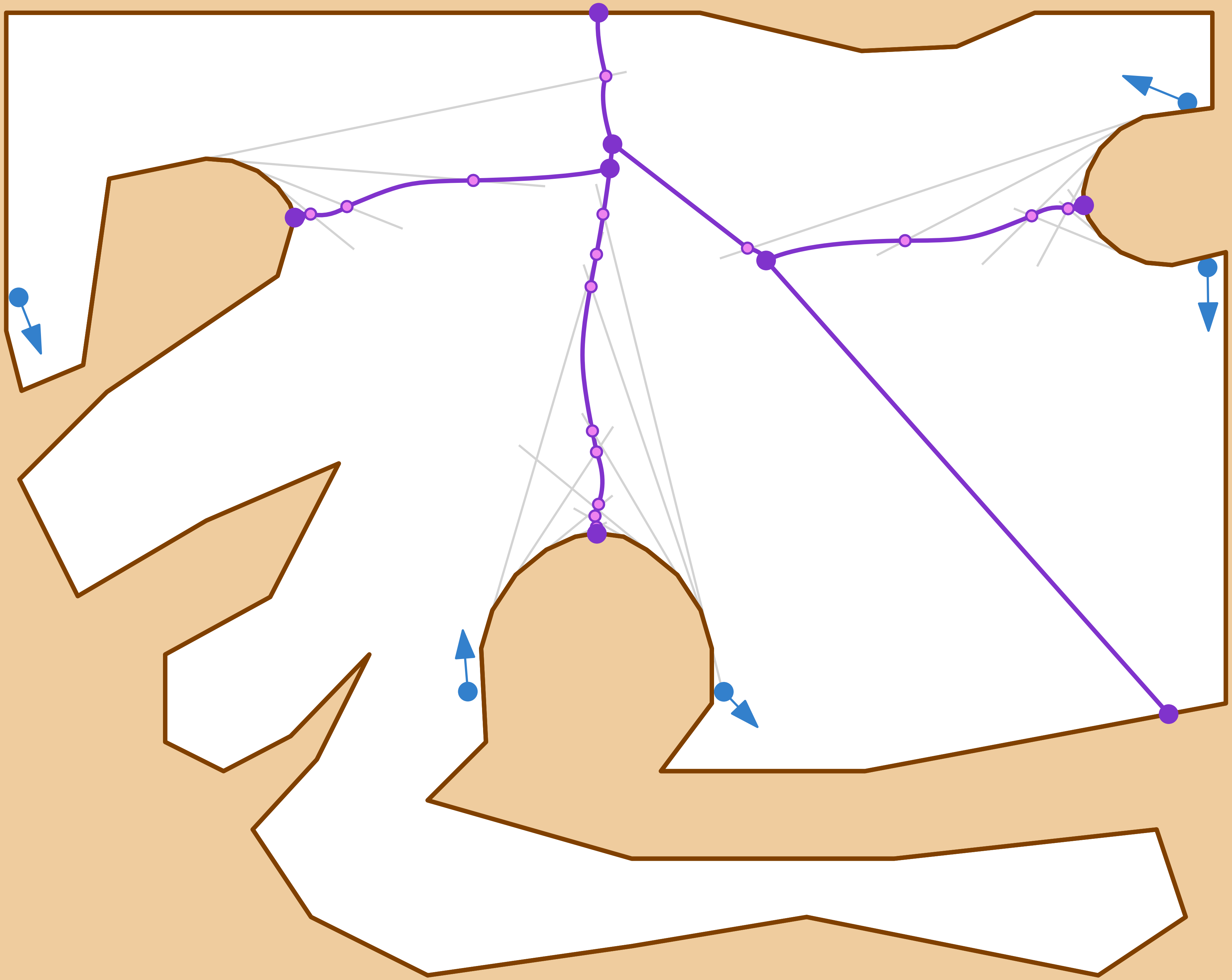


Problem



VD(S): (geodesic) Voronoi diagram

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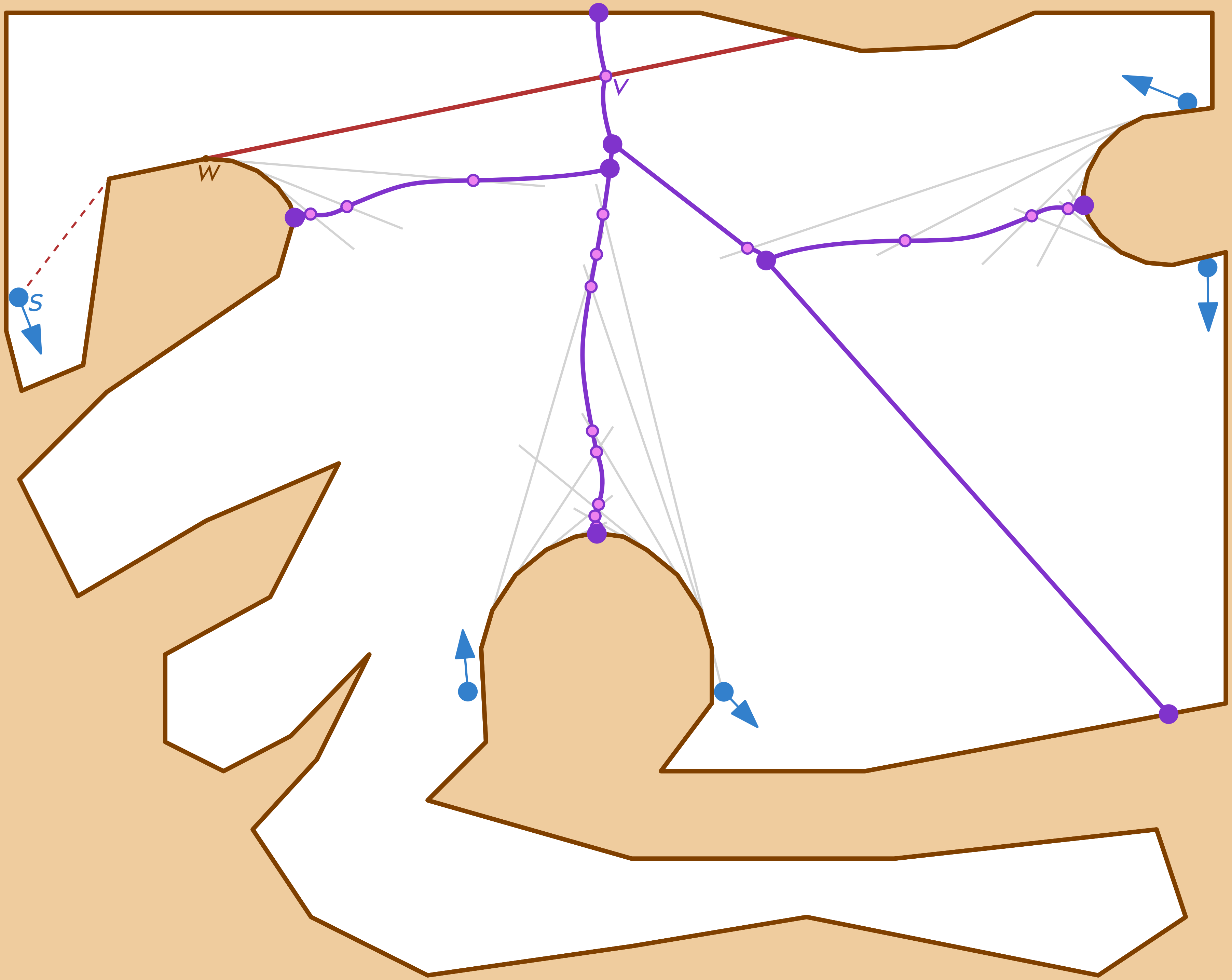
VD(S) has size $\Theta(n + m)$

Problem



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VD(S): (geodesic) Voronoi diagram

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2. How many changes?
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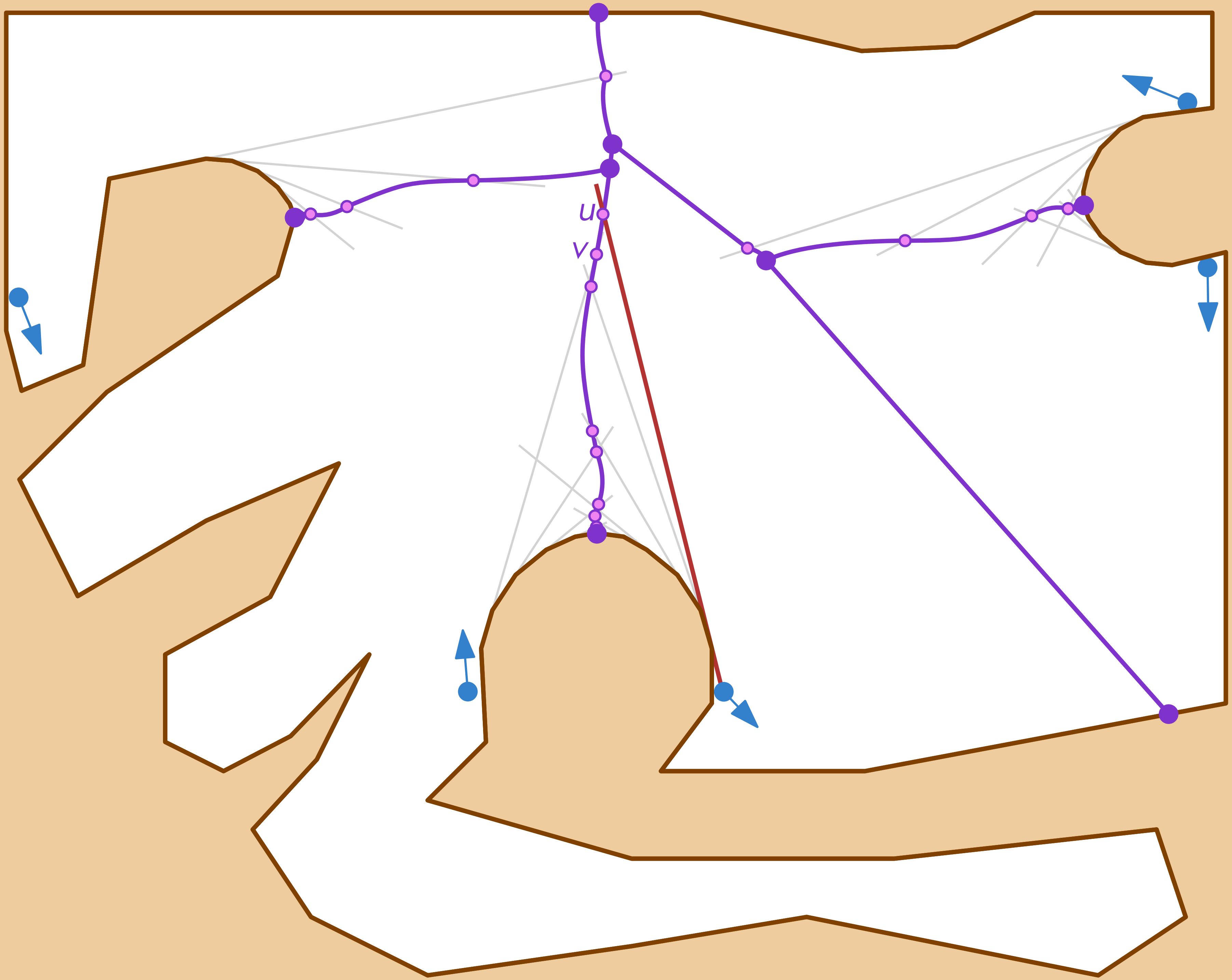
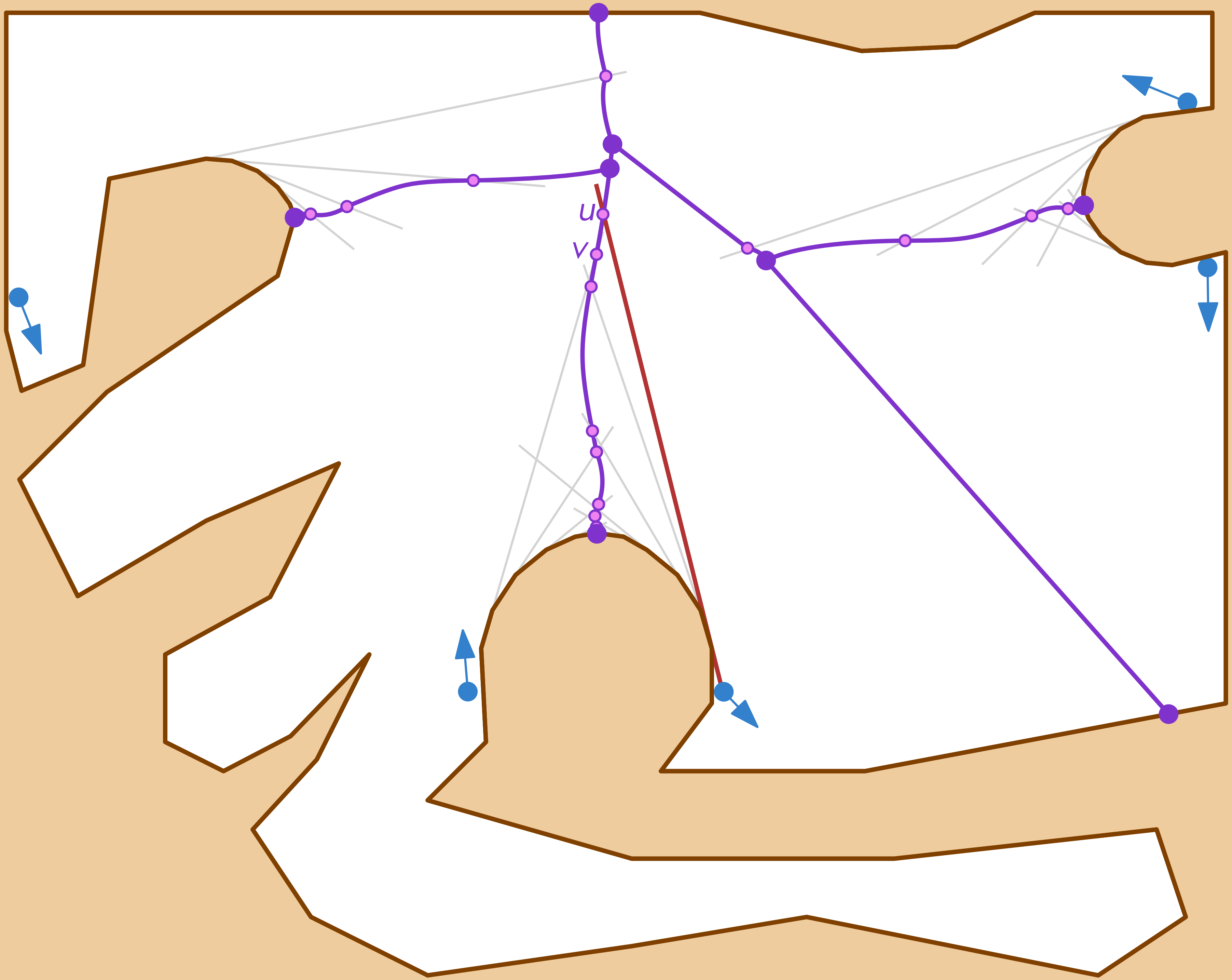


Problem

VD(S): (geodesic) Voronoi diagram

1. When does VD(S) change?
2. How many changes?
3. Can we maintain VD(S) **efficiently**?



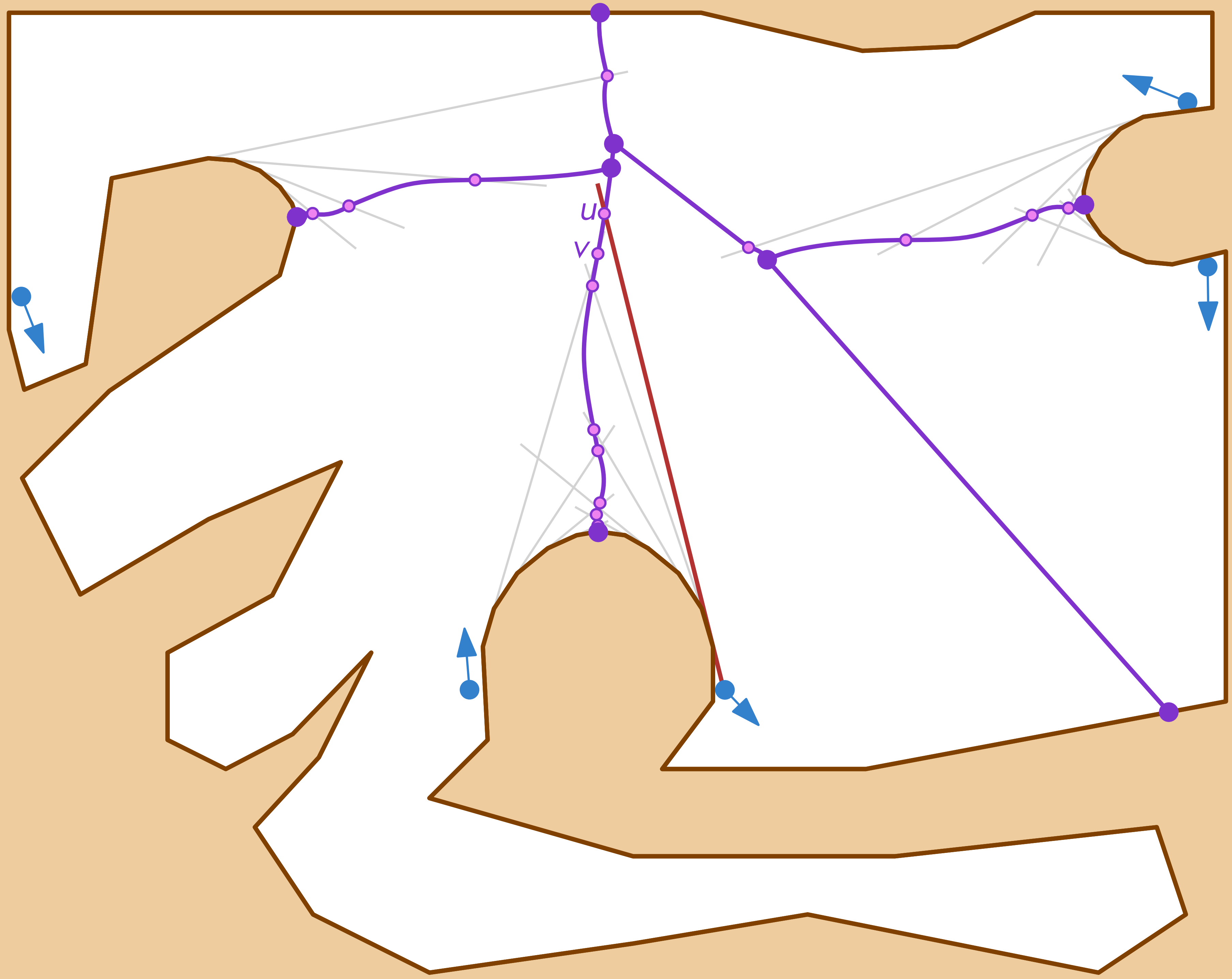
Problem

KDS framework:

compactness = space usage

responsiveness = time to handle event

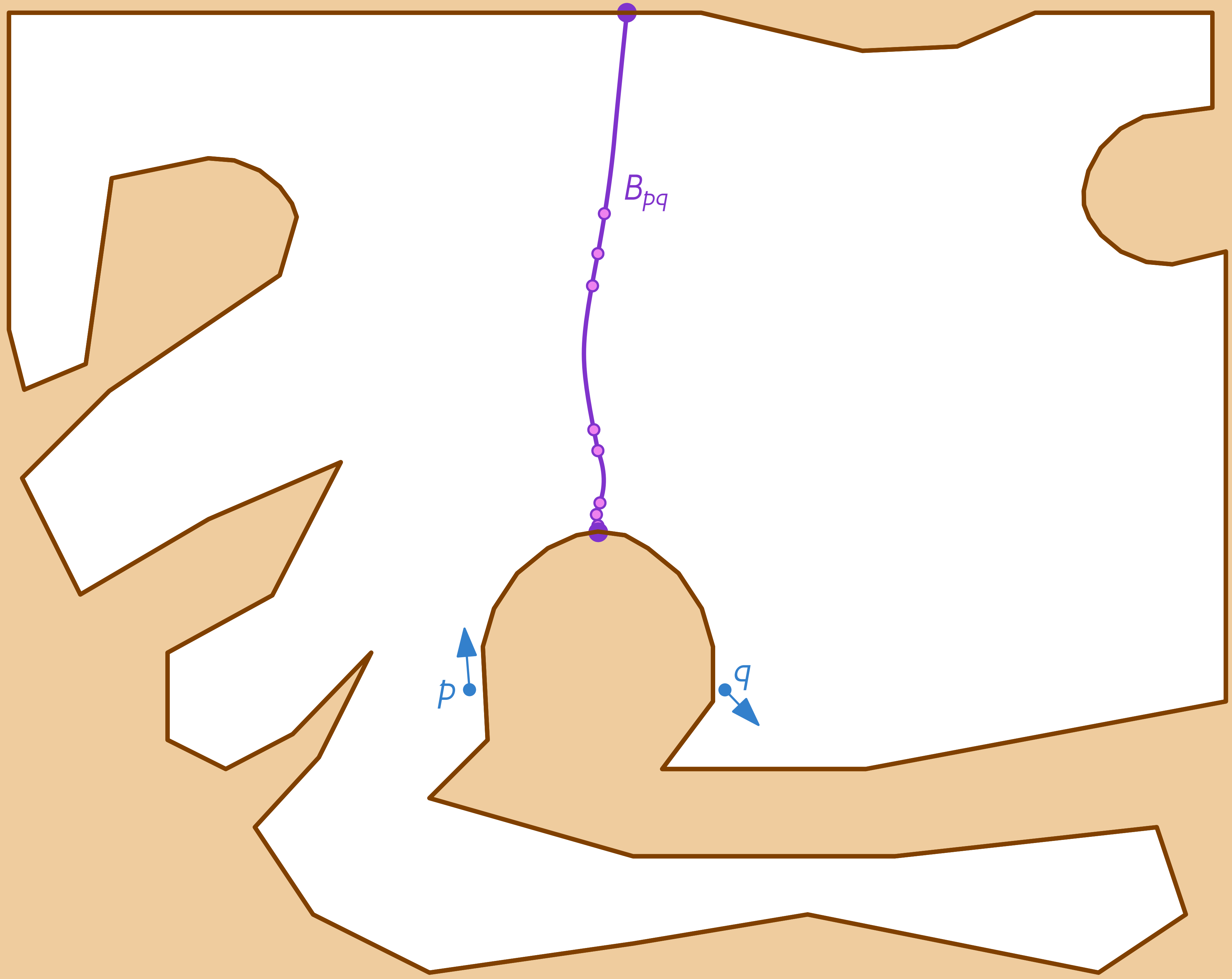
efficiency = $\frac{\text{\# events processed}}{\text{\# observable events}}$



Results

Bisector B_{pq}:

size: $\Theta(m)$

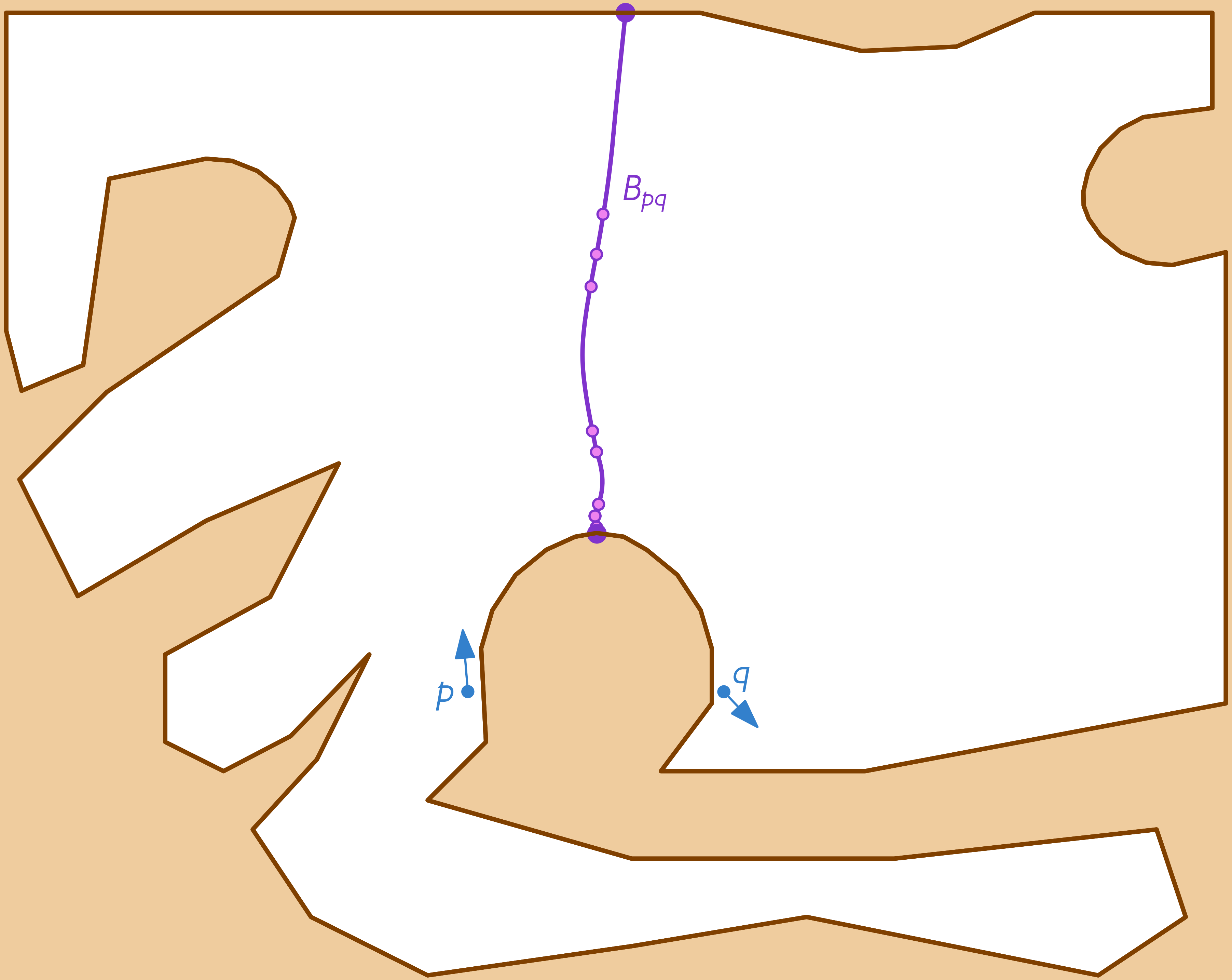


Results

Bisector B_{pq}:

size: $\Theta(m)$

#changes: $\Theta(m^3)$



Results

Bisector B_{pq}:

size: $\Theta(m)$

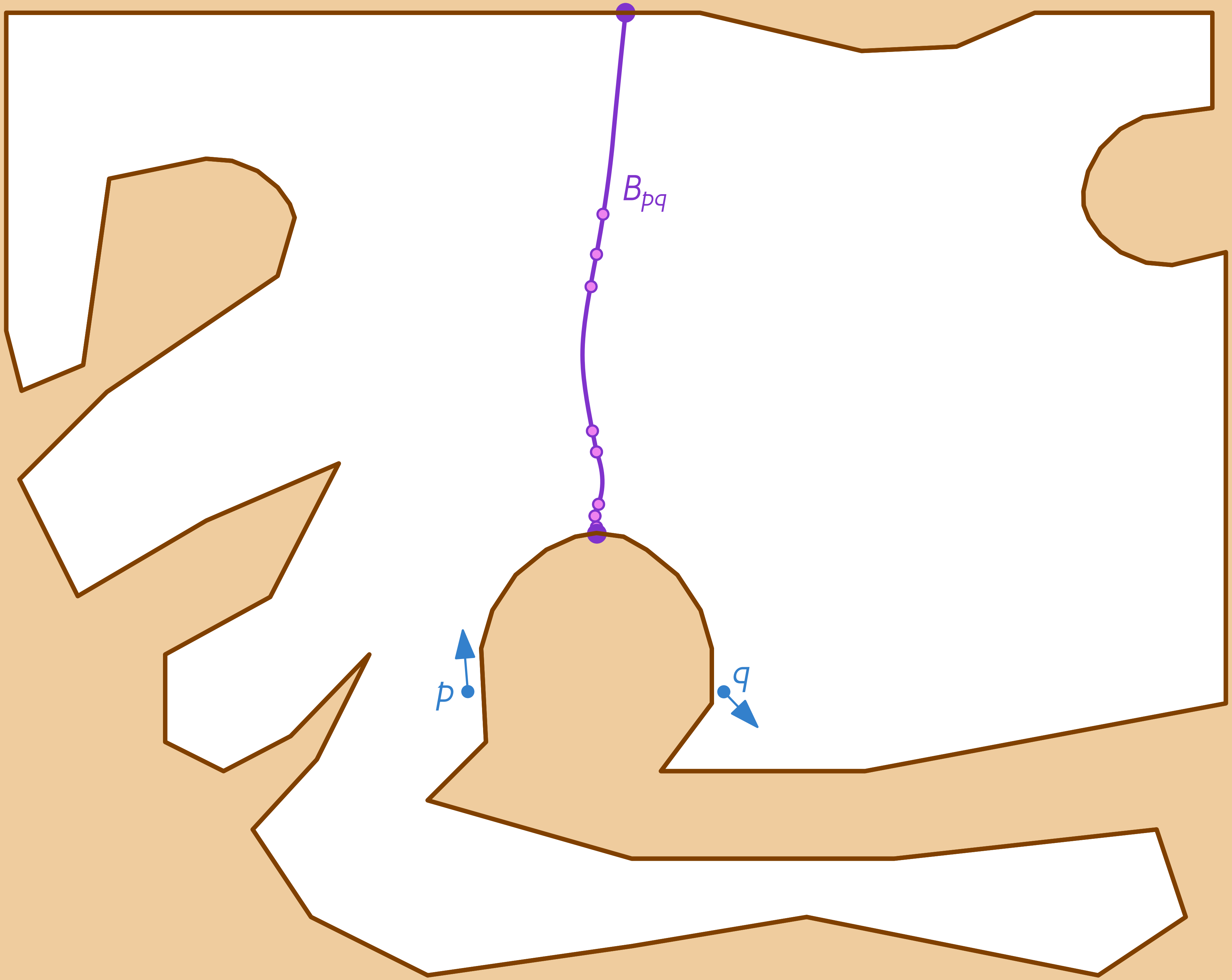
#changes: $\Theta(m^3)$

KDS:

$O(m)$ size

$O(m^3)$ events

$O(\log m)$ time per event



Results

Bisector B_{pq}:

size: $\Theta(m)$

#changes: $\Theta(m^3)$

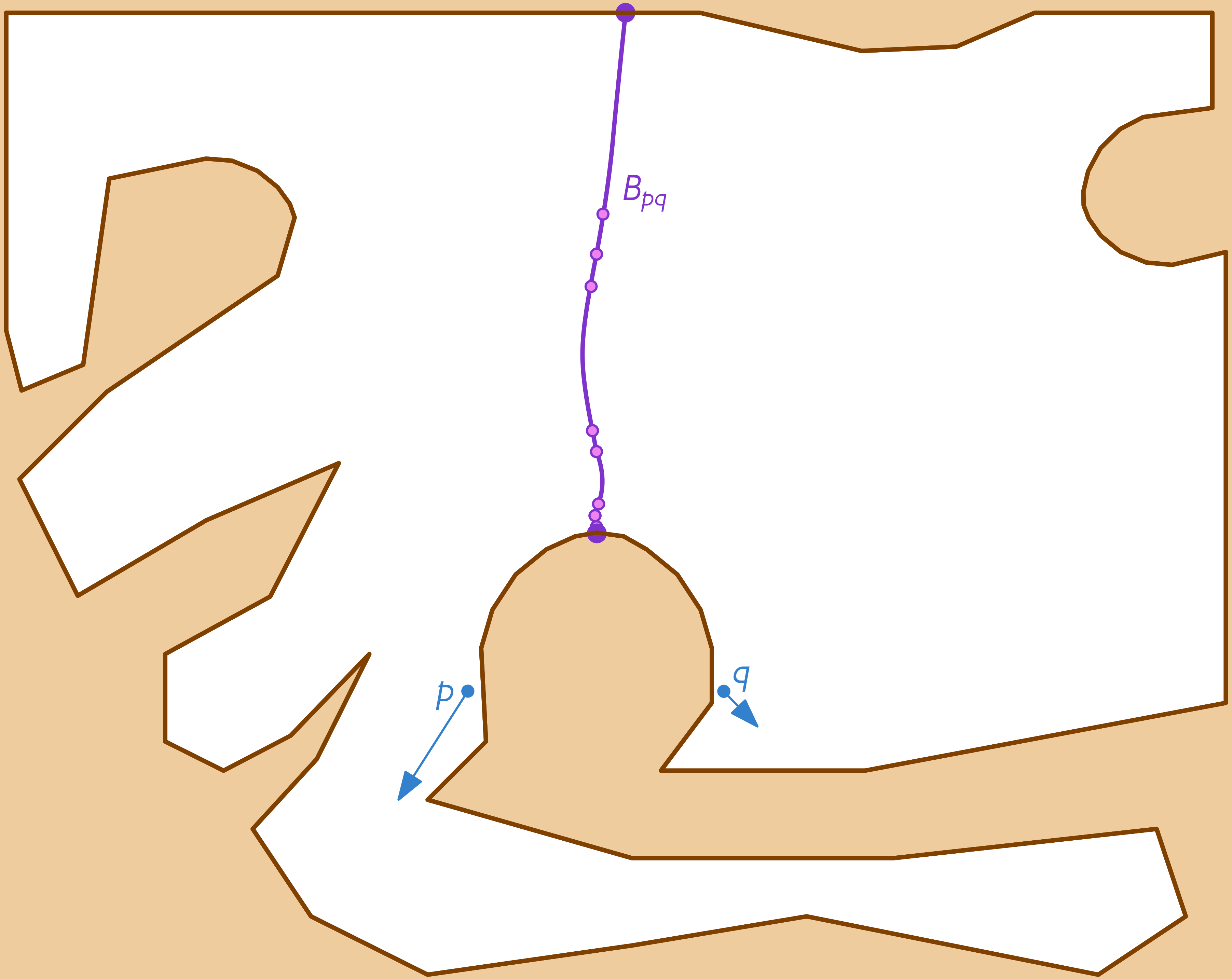
KDS:

$O(m)$ size

$O(m^3)$ events

$O(\log m)$ time per event

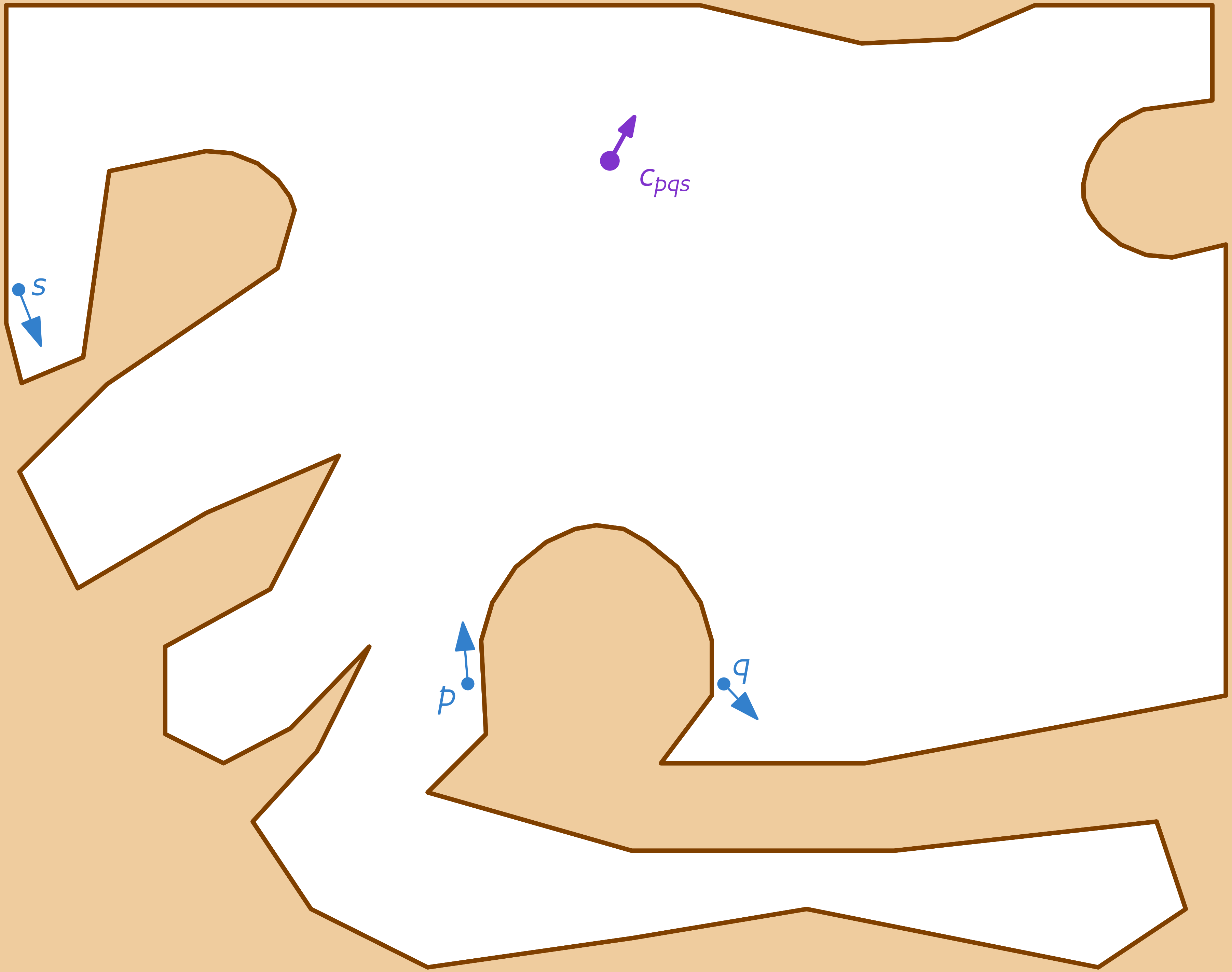
$O(\log m)$ time update



Results

Voronoi center c_{pqs}:

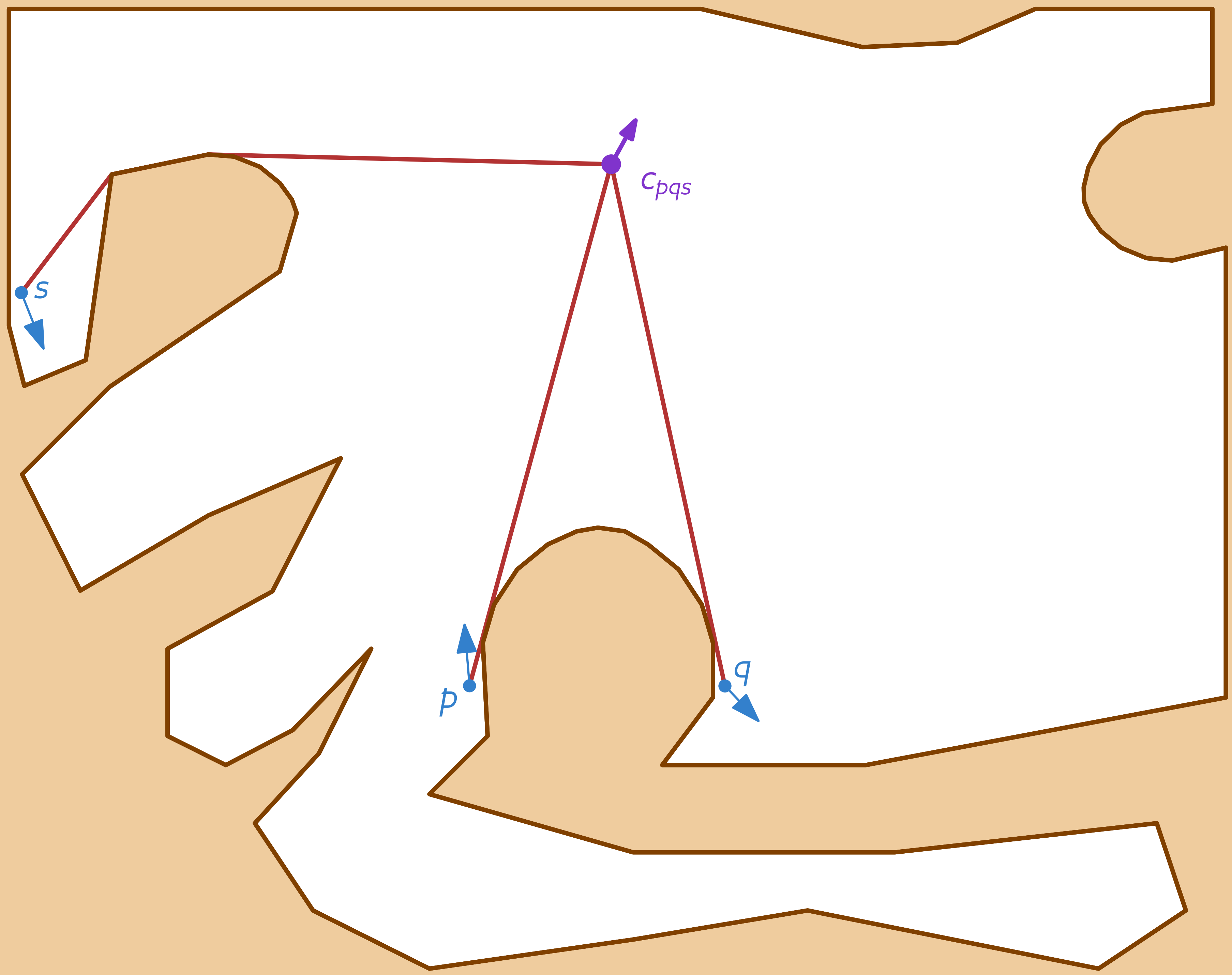
size: $\Theta(1)$



Results

Voronoi center c_{pqs}:

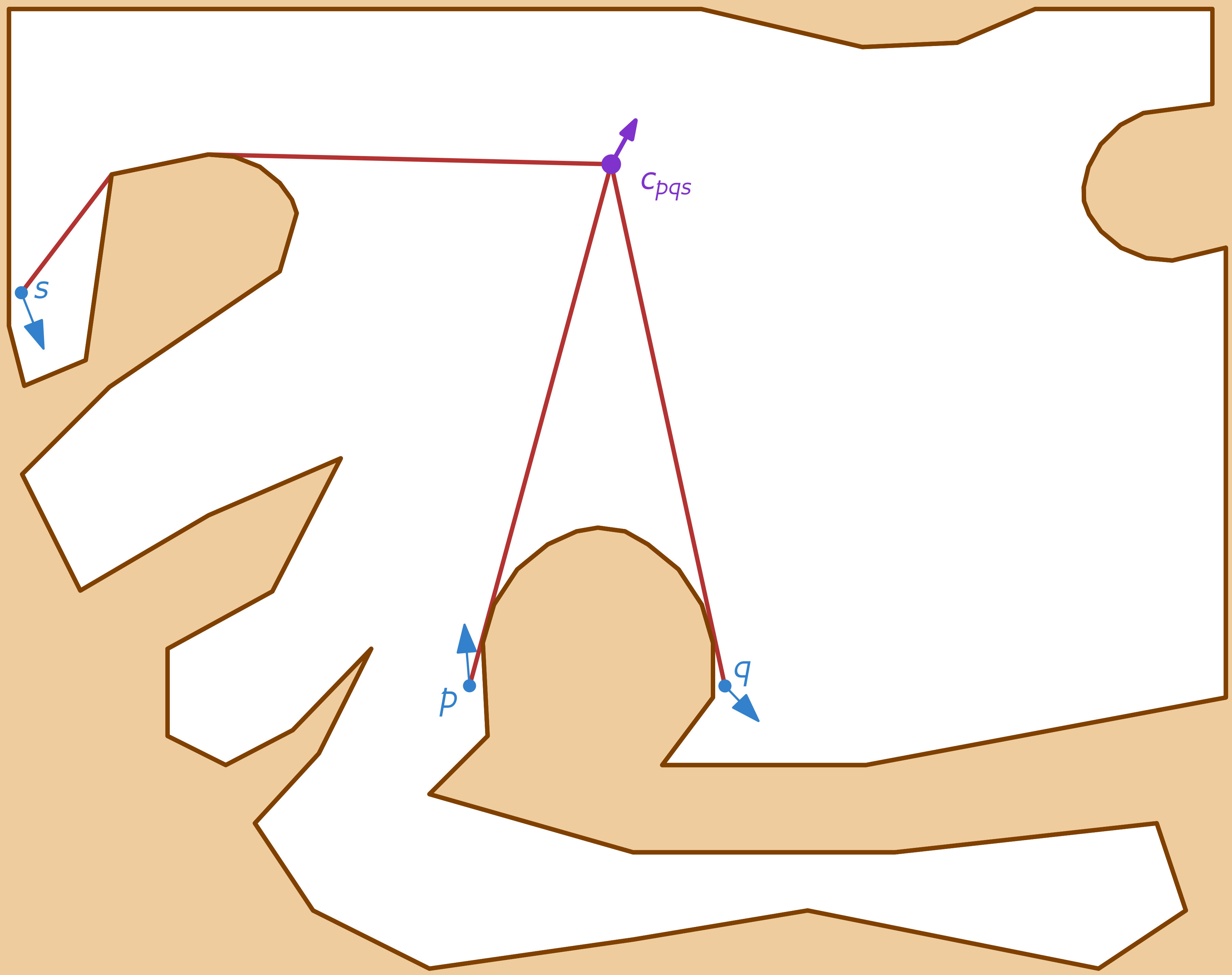
size: $\Theta(1)$



Results

Voronoi center c_{pqs}:

- size: $\Theta(1)$
- #changes: $\Theta(m^3)$



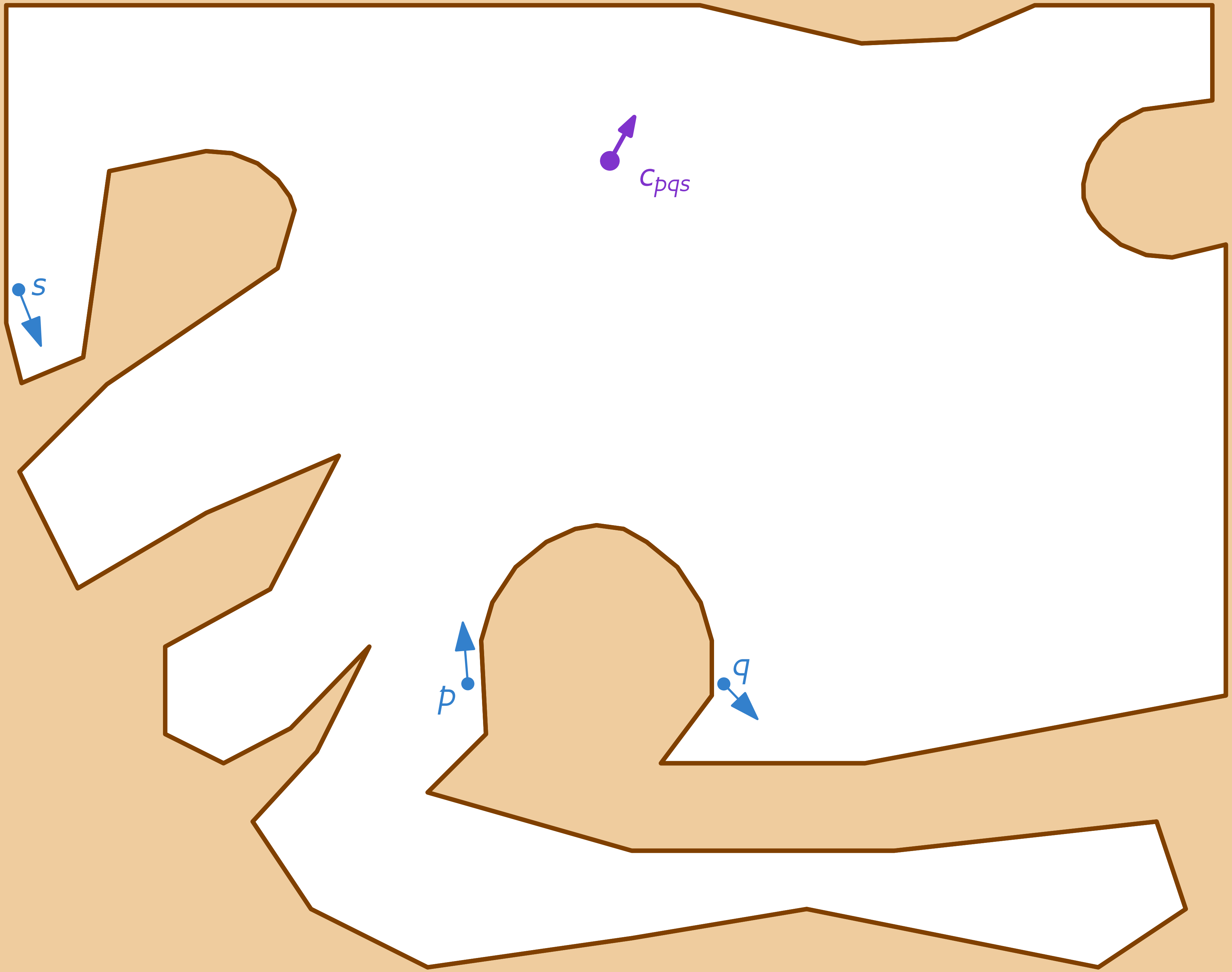
Results

Voronoi center c_{pqs}:

- size: $\Theta(1)$
- #changes: $\Theta(m^3)$

KDS:

- $O(m)$ size
- $O(m^3)$ events
- $O(\log^2 m)$ time per event
- $O(\log^2 m)$ time update



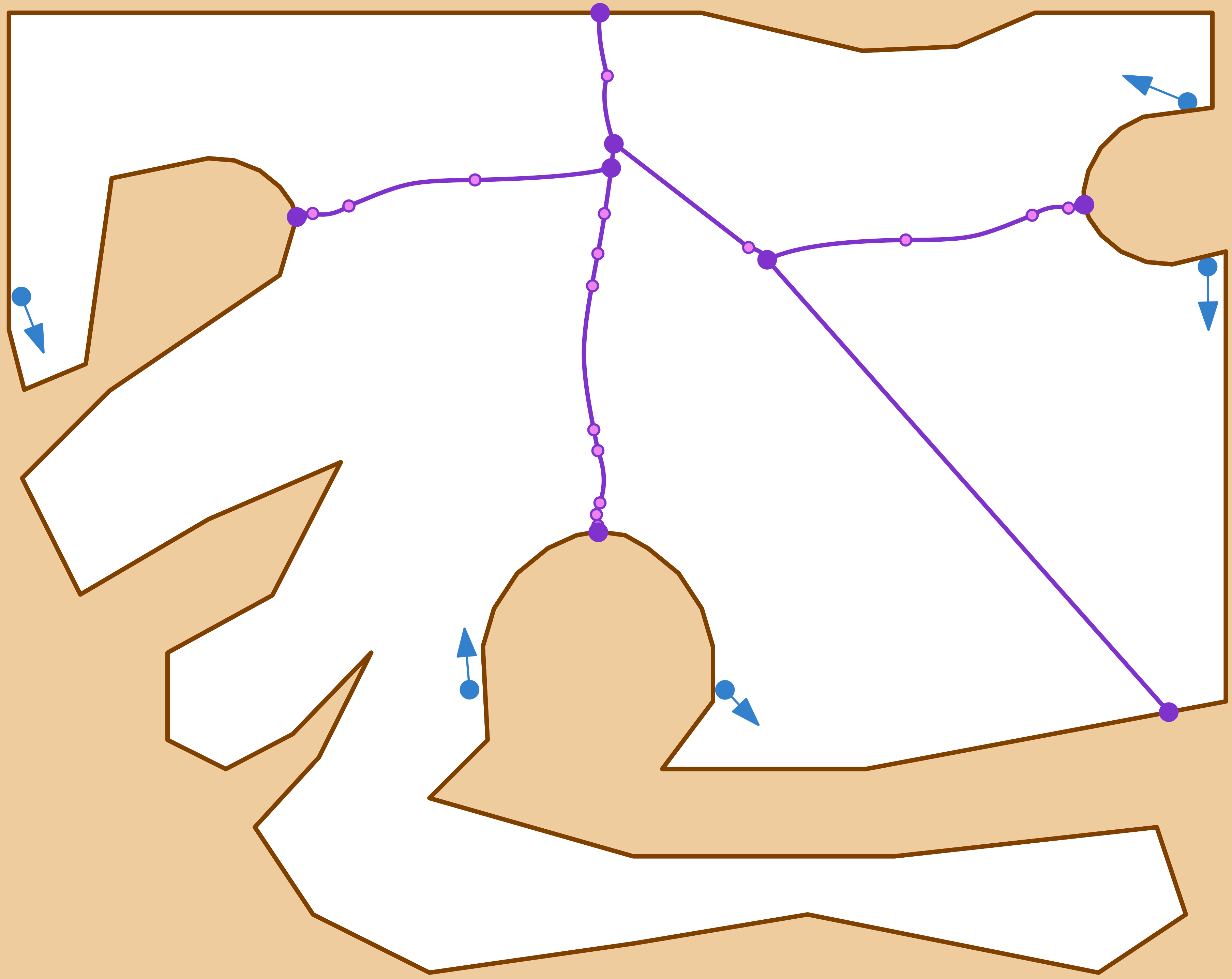
Results

VD (S):

size: $\Theta(n + m)$

#changes: $\tilde{O}(n^3 m^3)$

$\Omega(nm^3 + n^2 m)$



Results

VD (S):

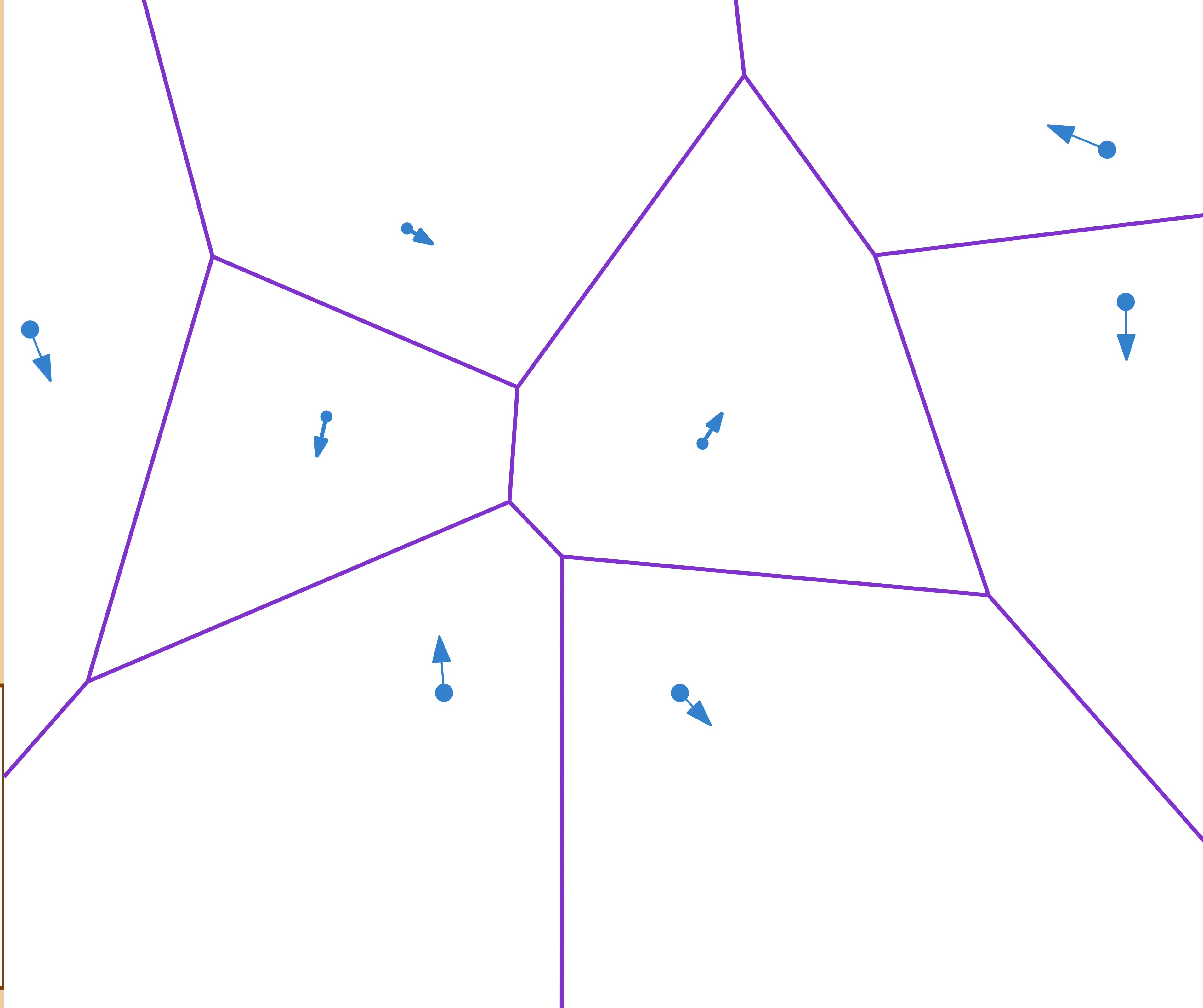
size: $\Theta(n+m)$

#changes: $\tilde{O}(n^3 m^3)$

$\Omega(nm^3 + n^2 m)$

In $\mathbb{R}^2$:

#changes: $\tilde{O}(n^3)$



Results

VD (S):

size: $\Theta(n + m)$

#changes: $\tilde{O}(n^3 m^3)$

$\Omega(nm^3 + n^2 m)$

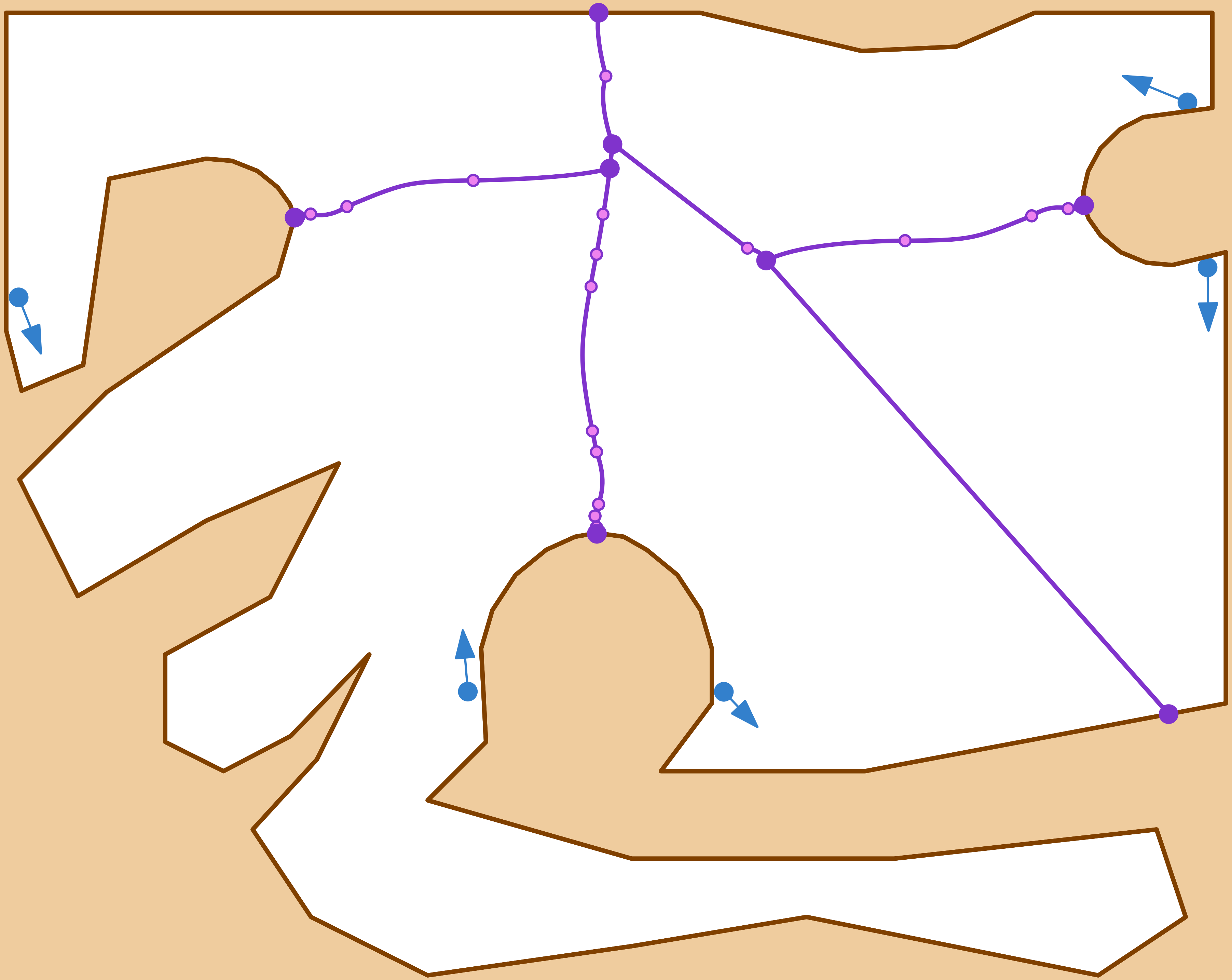
KDS:

$O(n + m)$ size

$\tilde{O}(n^3 m^3)$ events

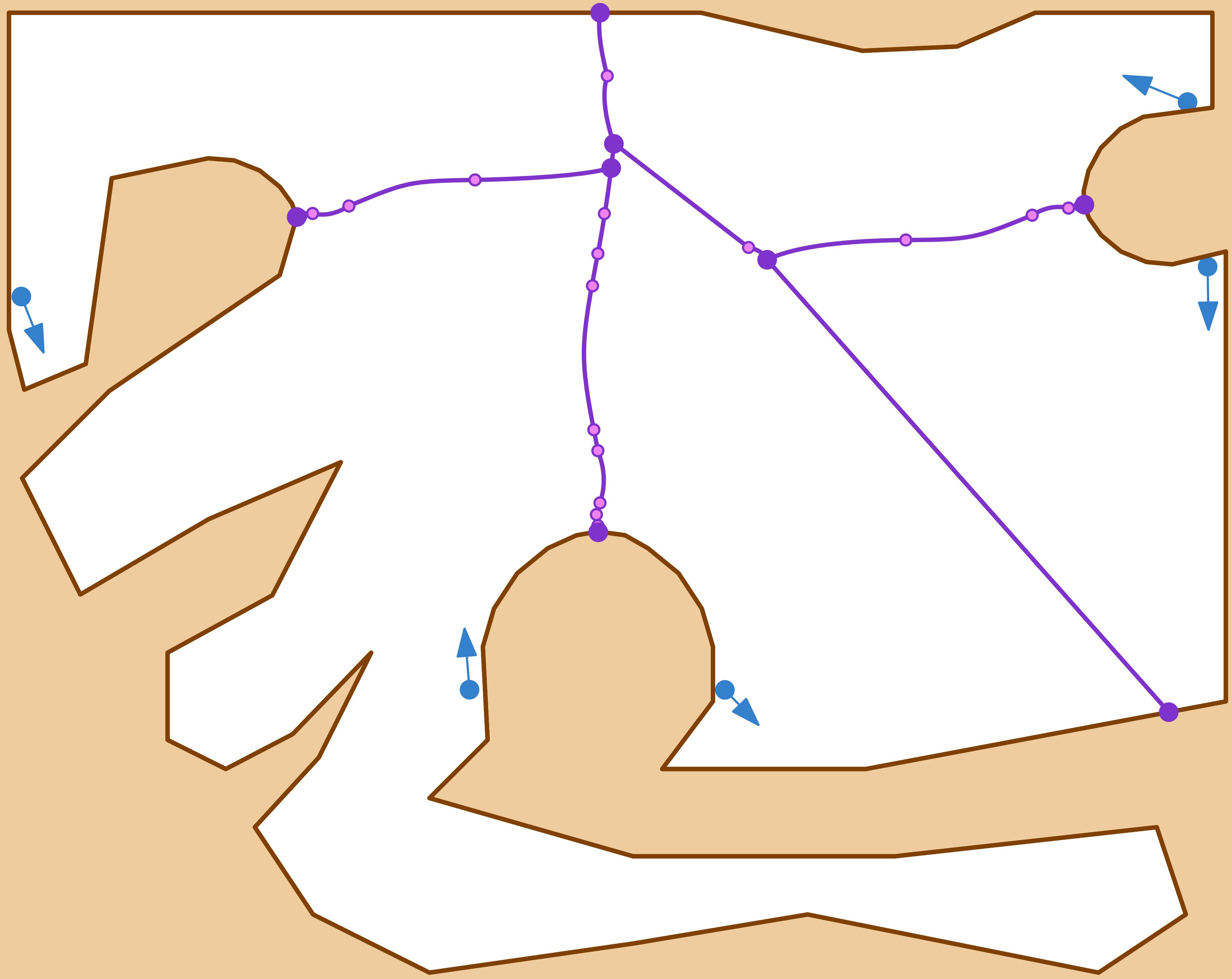
$O(k \log^2 m)$ time per event

k = #affected neighbours



Overview

1. $\Omega(m^3)$ lowerbound B_{pq}
2. main complications KDS



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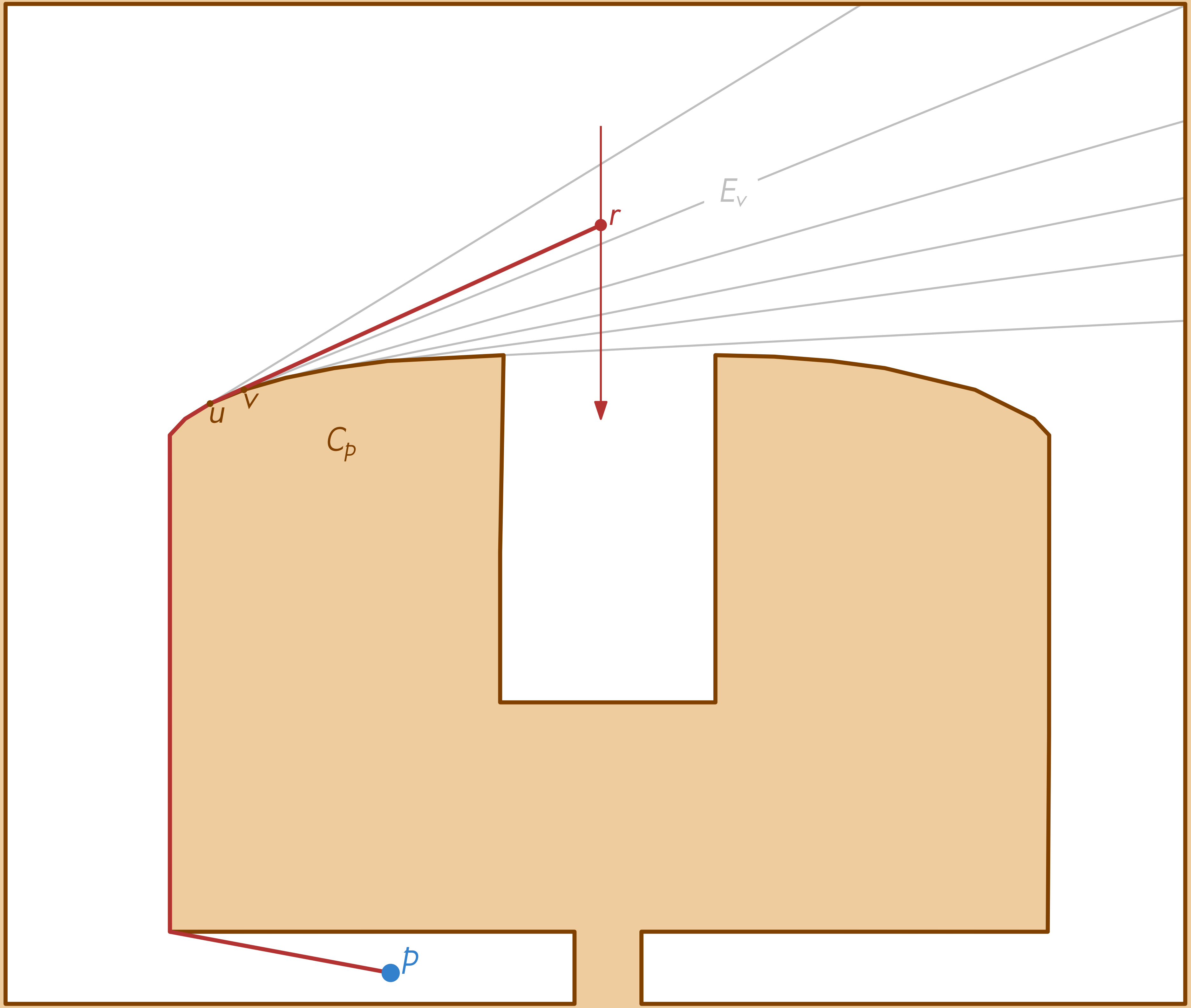

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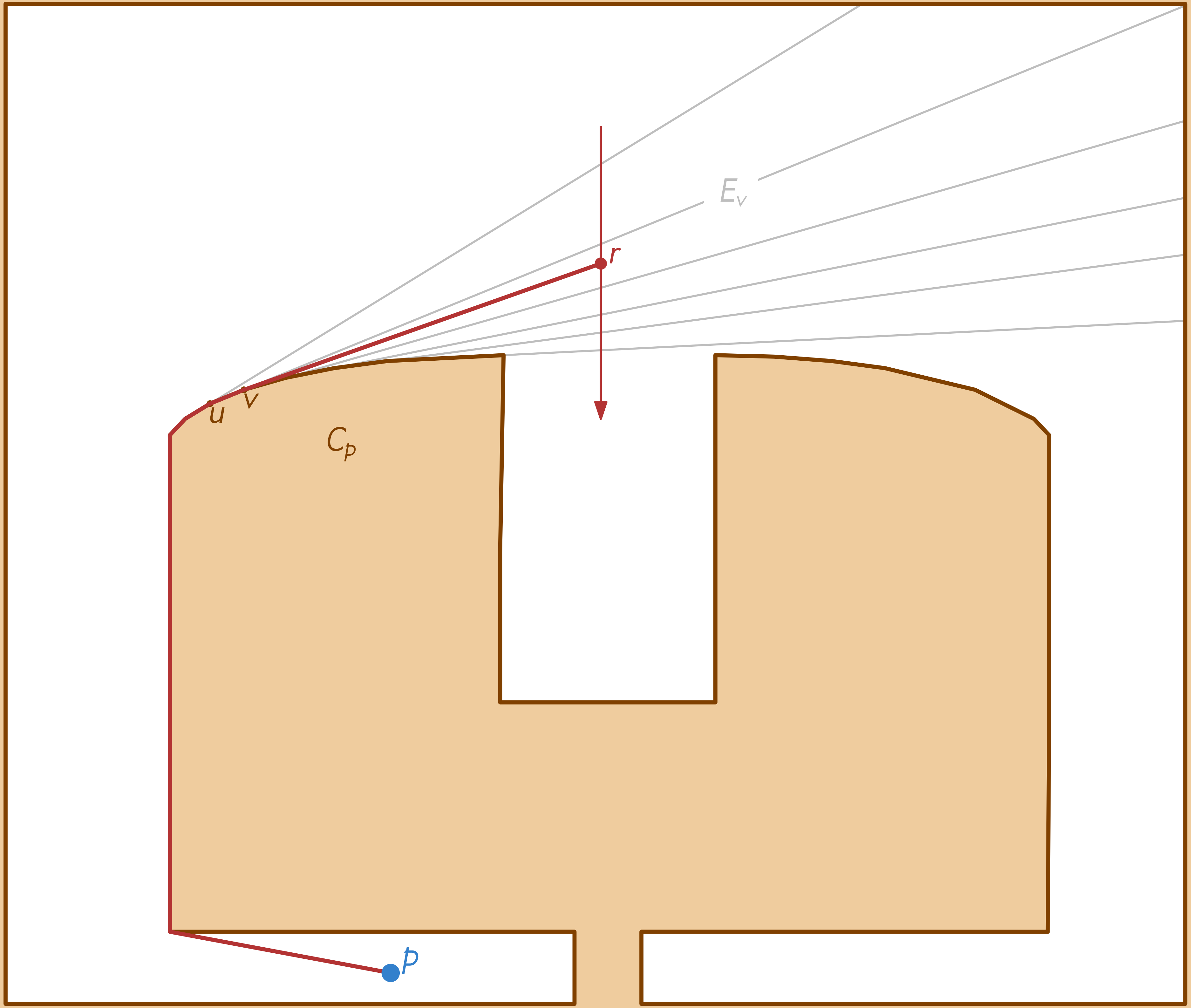

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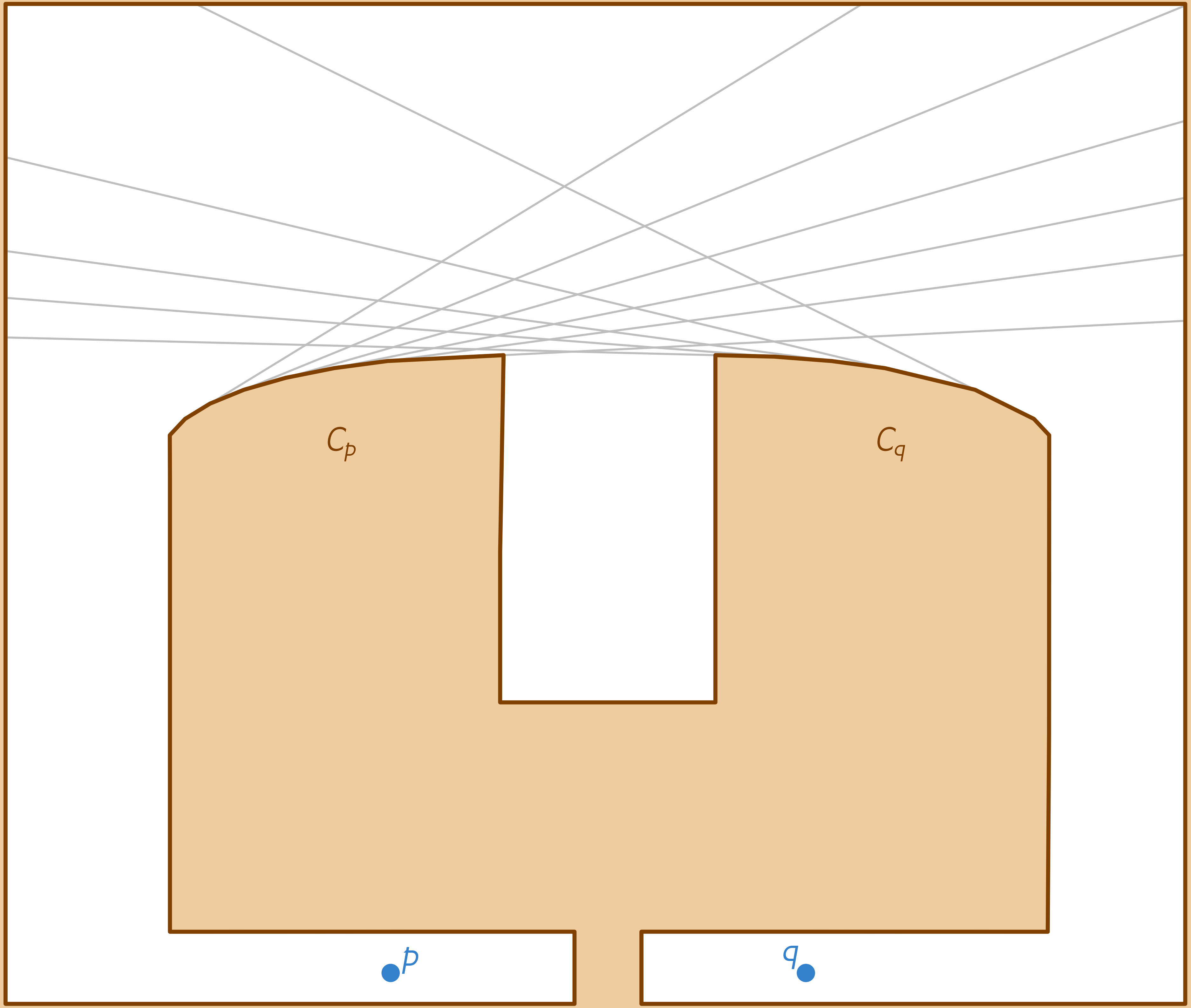

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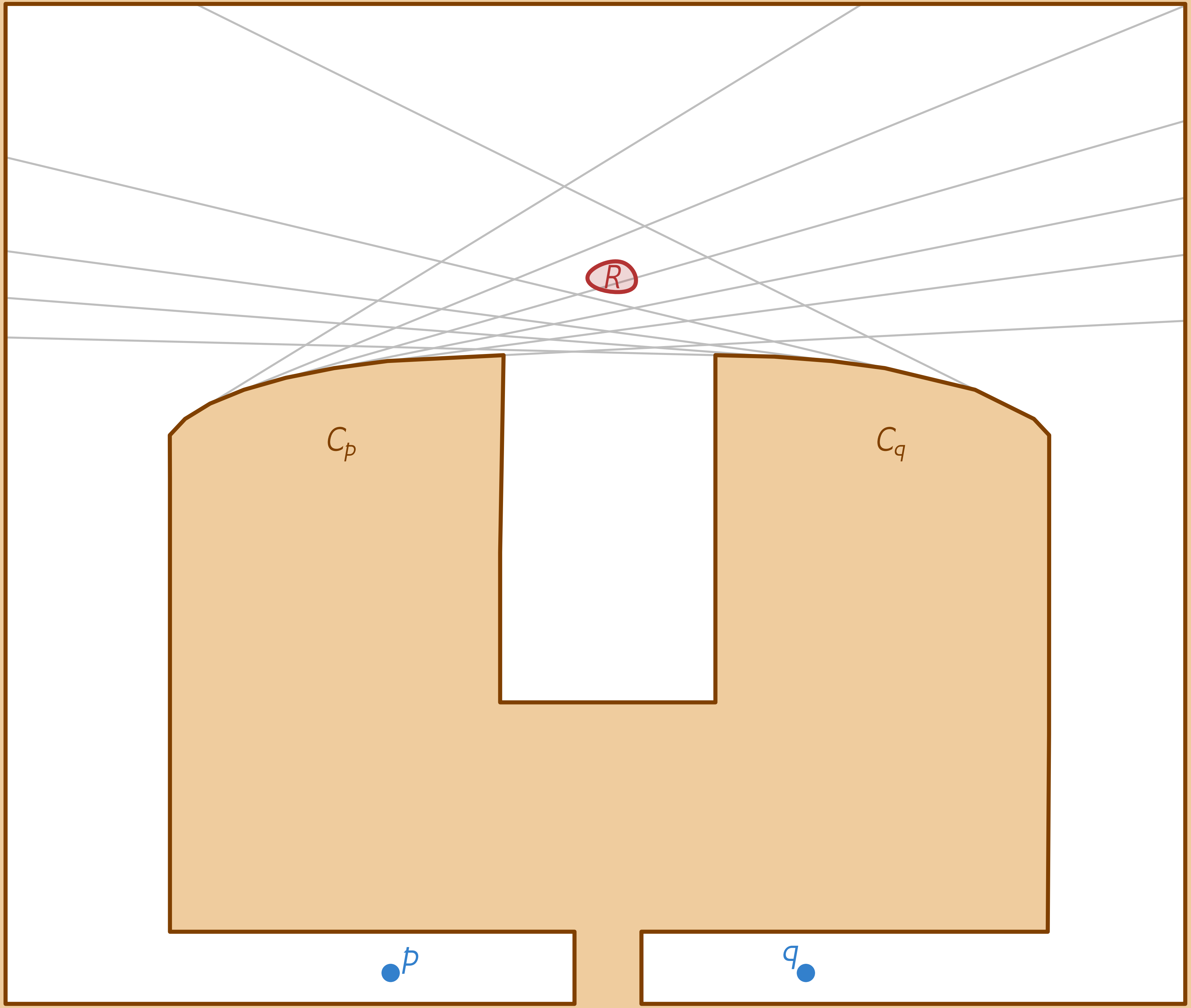

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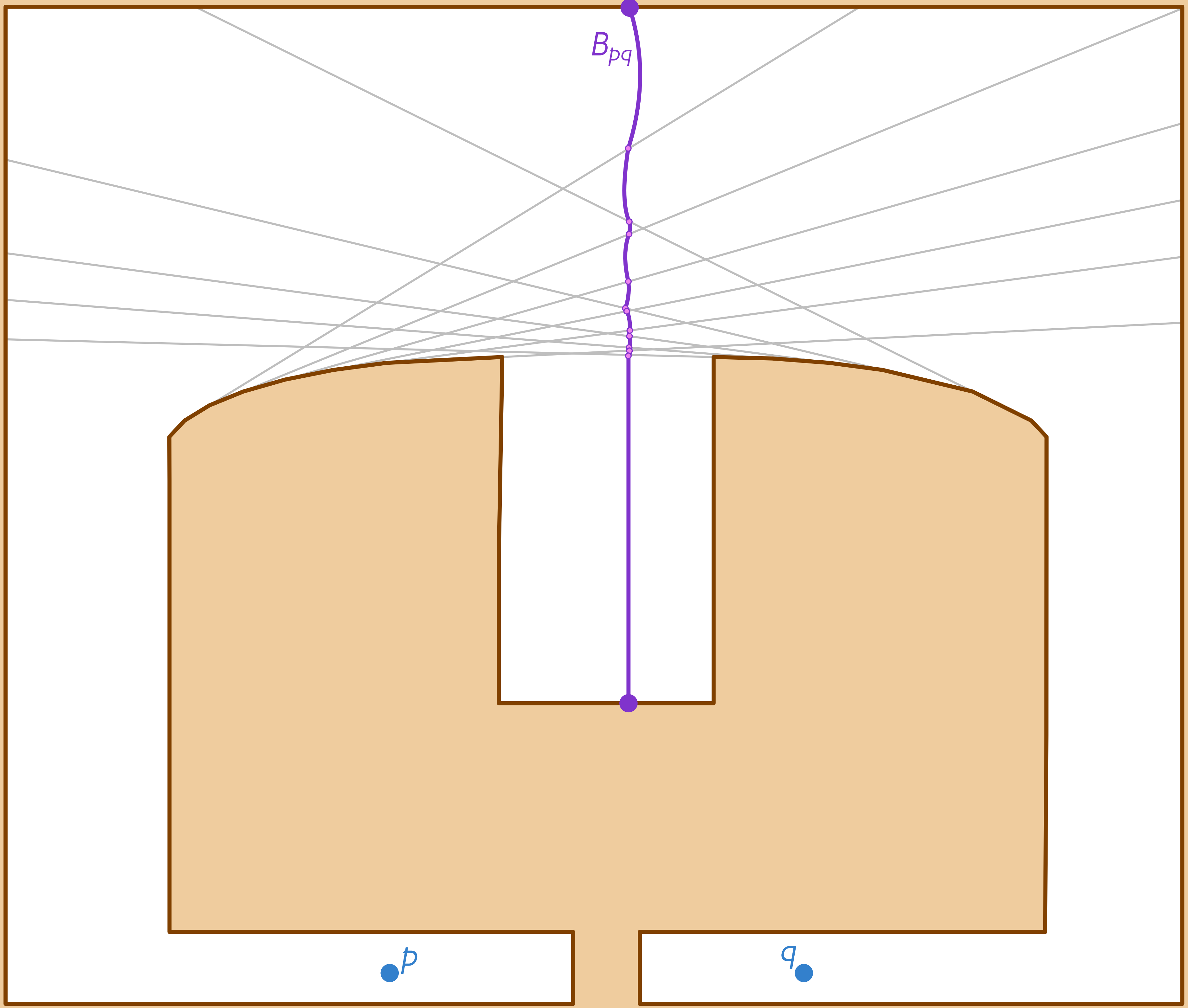

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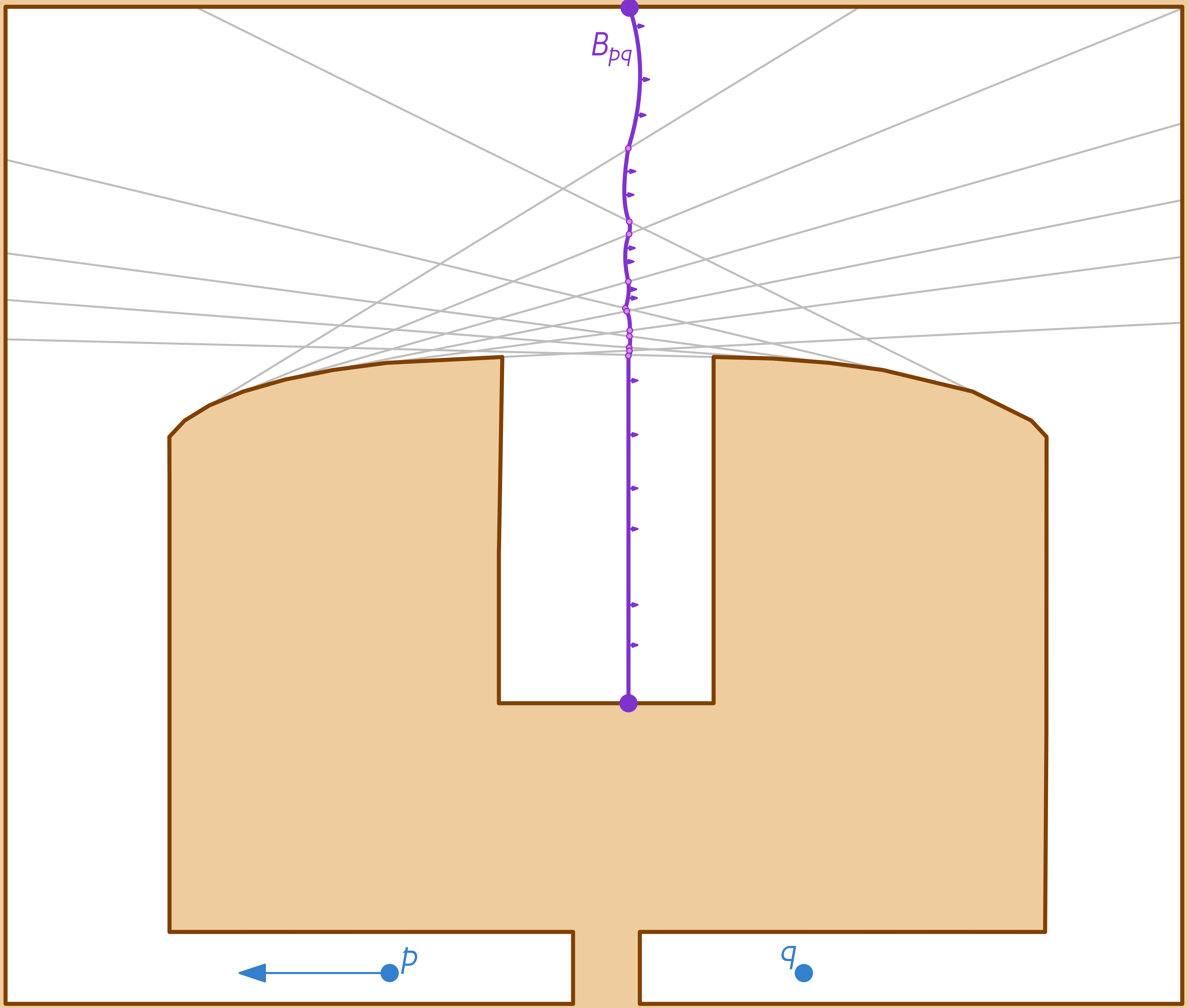

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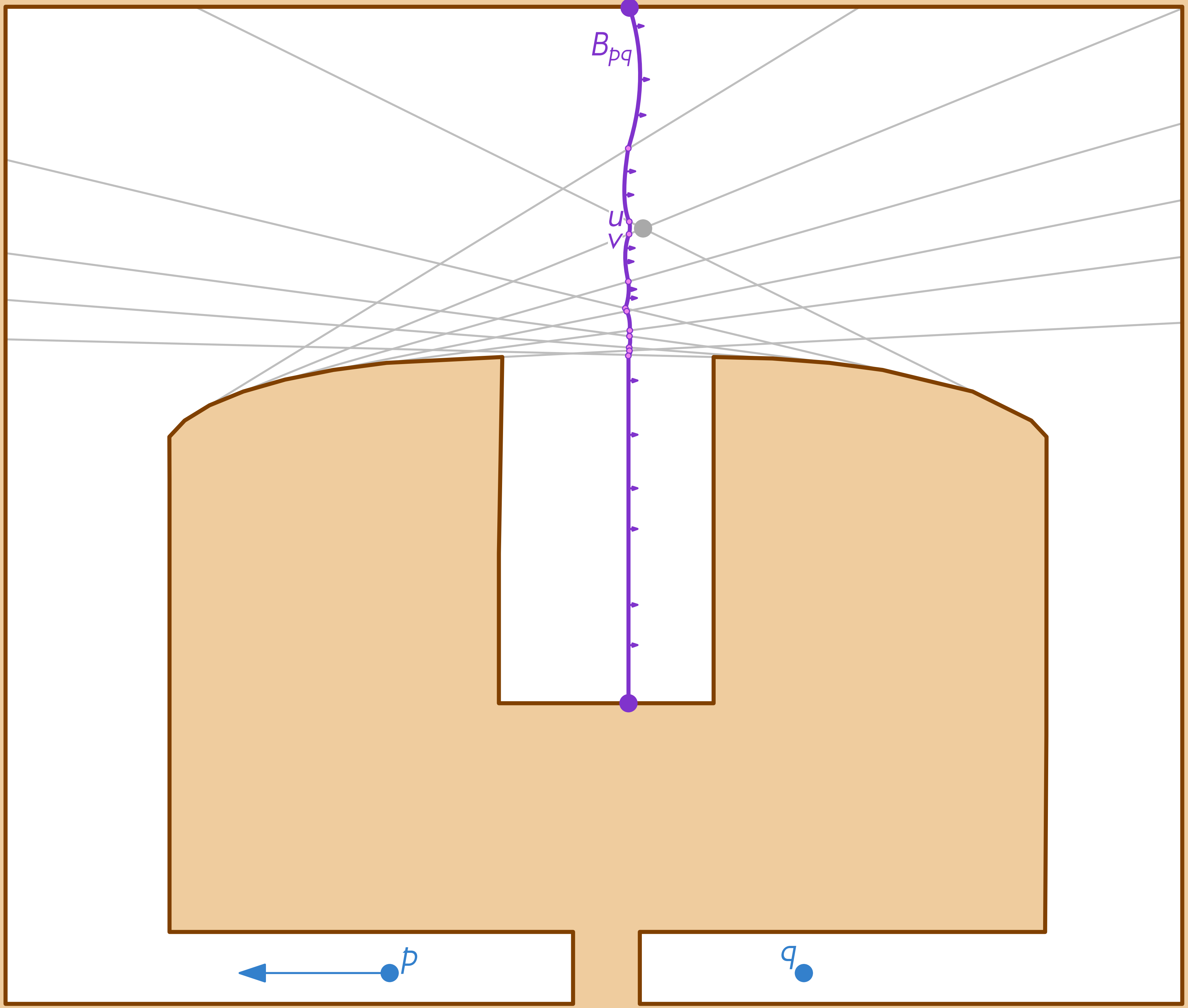
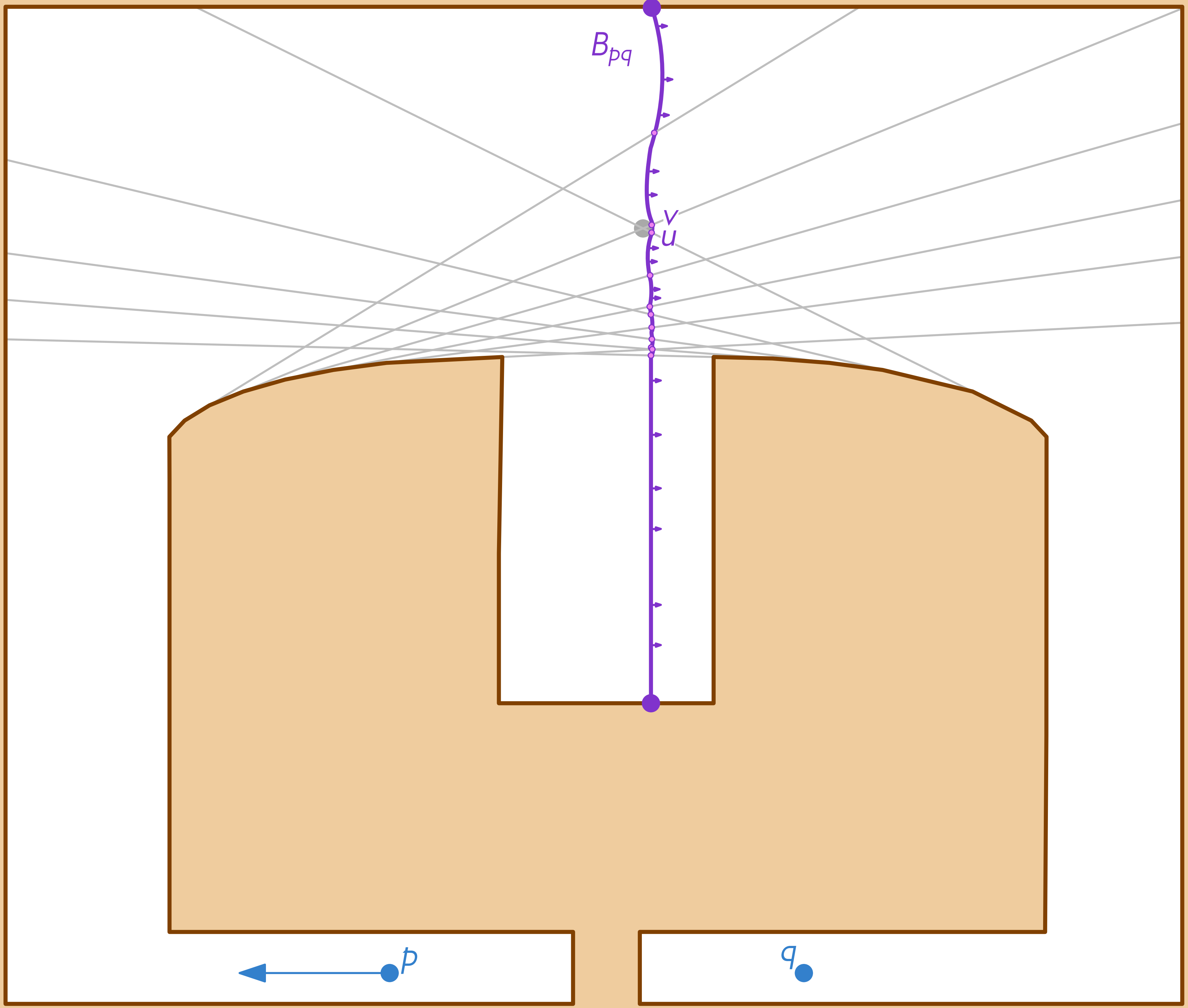


Lowerbound B_{pq}

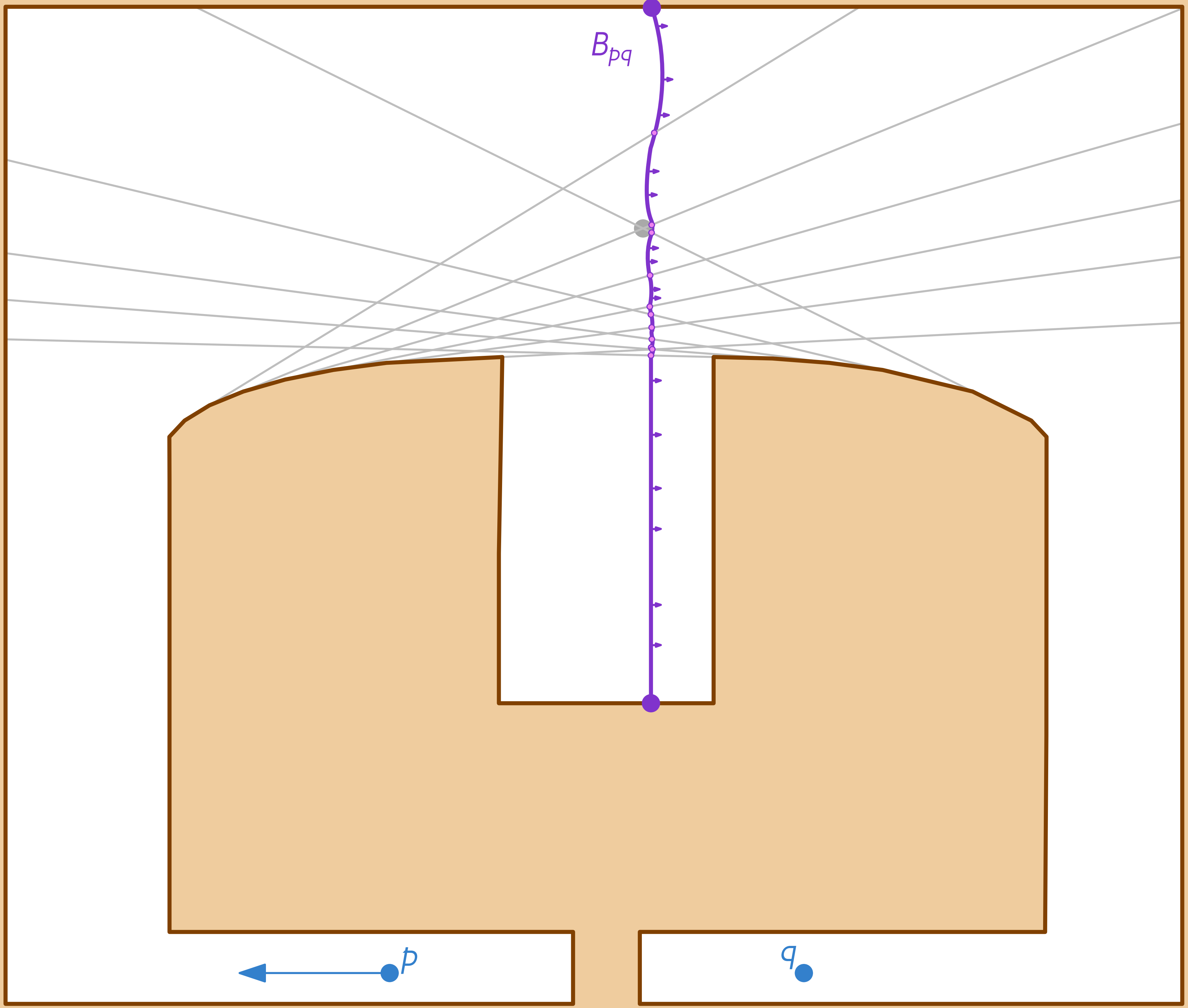
1. B_{pq} left to right
   $\implies \Omega(m^2)$ events



Lowerbound B_{pq}

1. B_{pq} left to right
   $\implies \Omega(m^2)$ events
2. Move B_{pq} left to right $\Omega(m)$ times.

   $\implies \Omega(m^3)$ events



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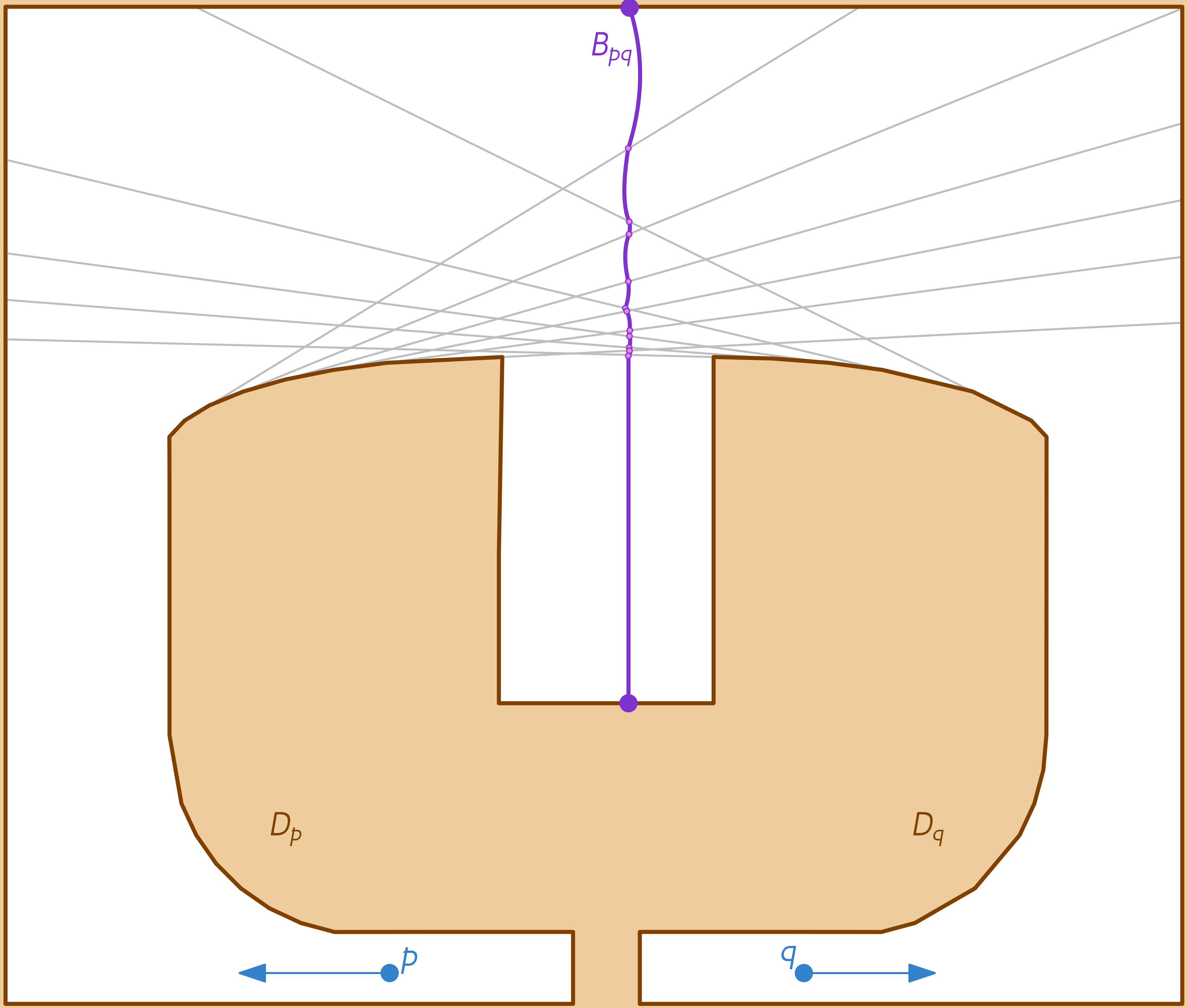

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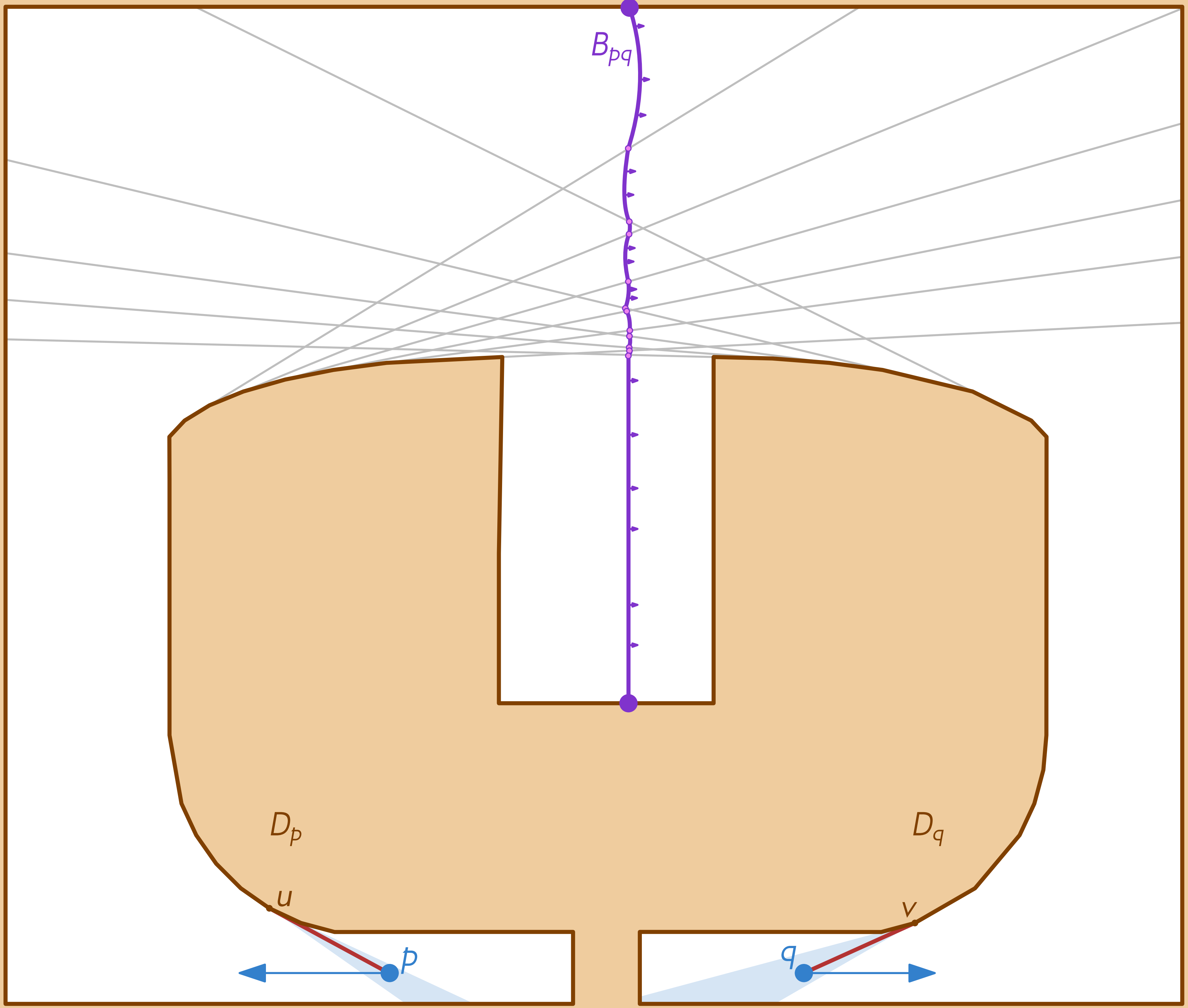

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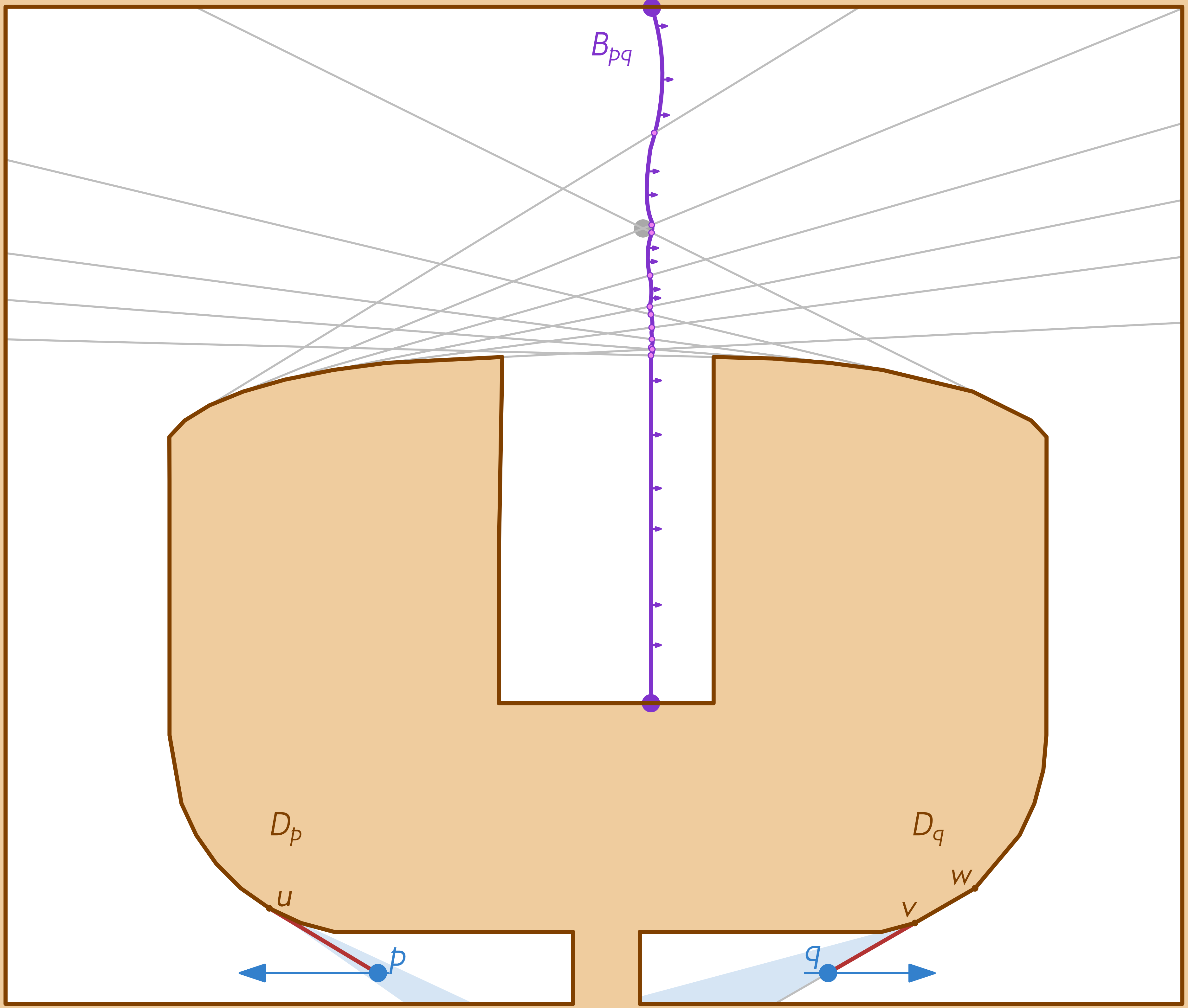

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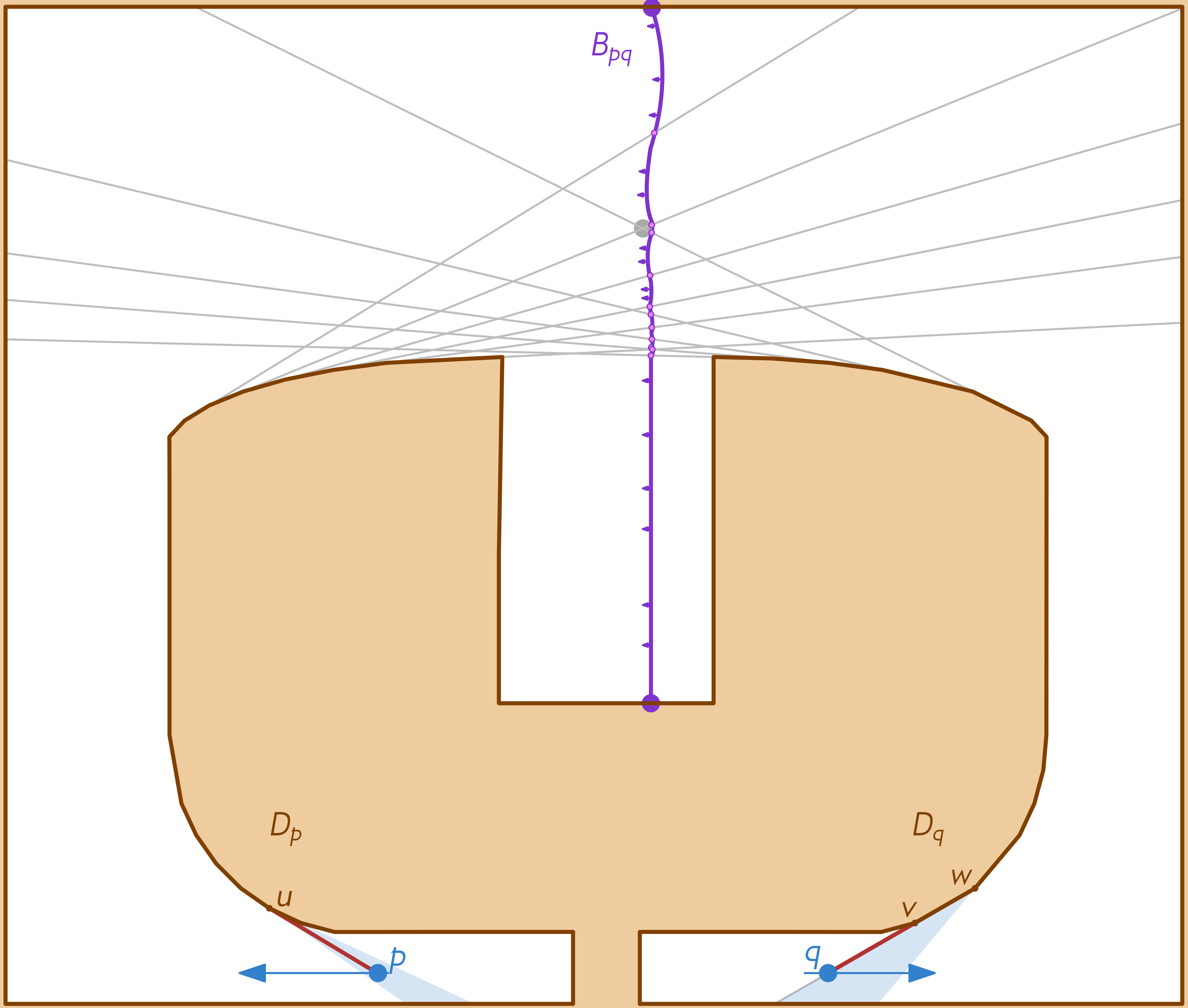

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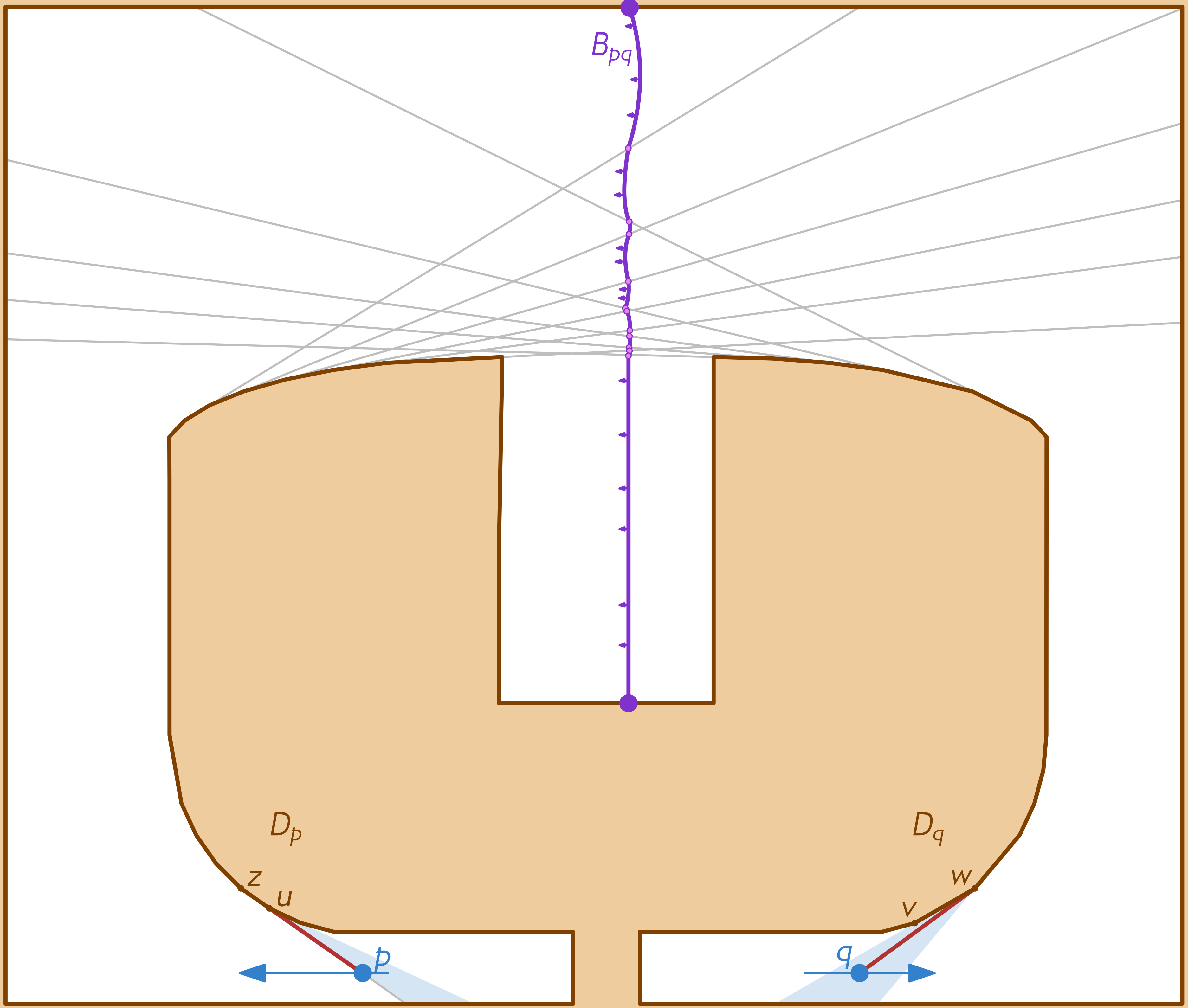

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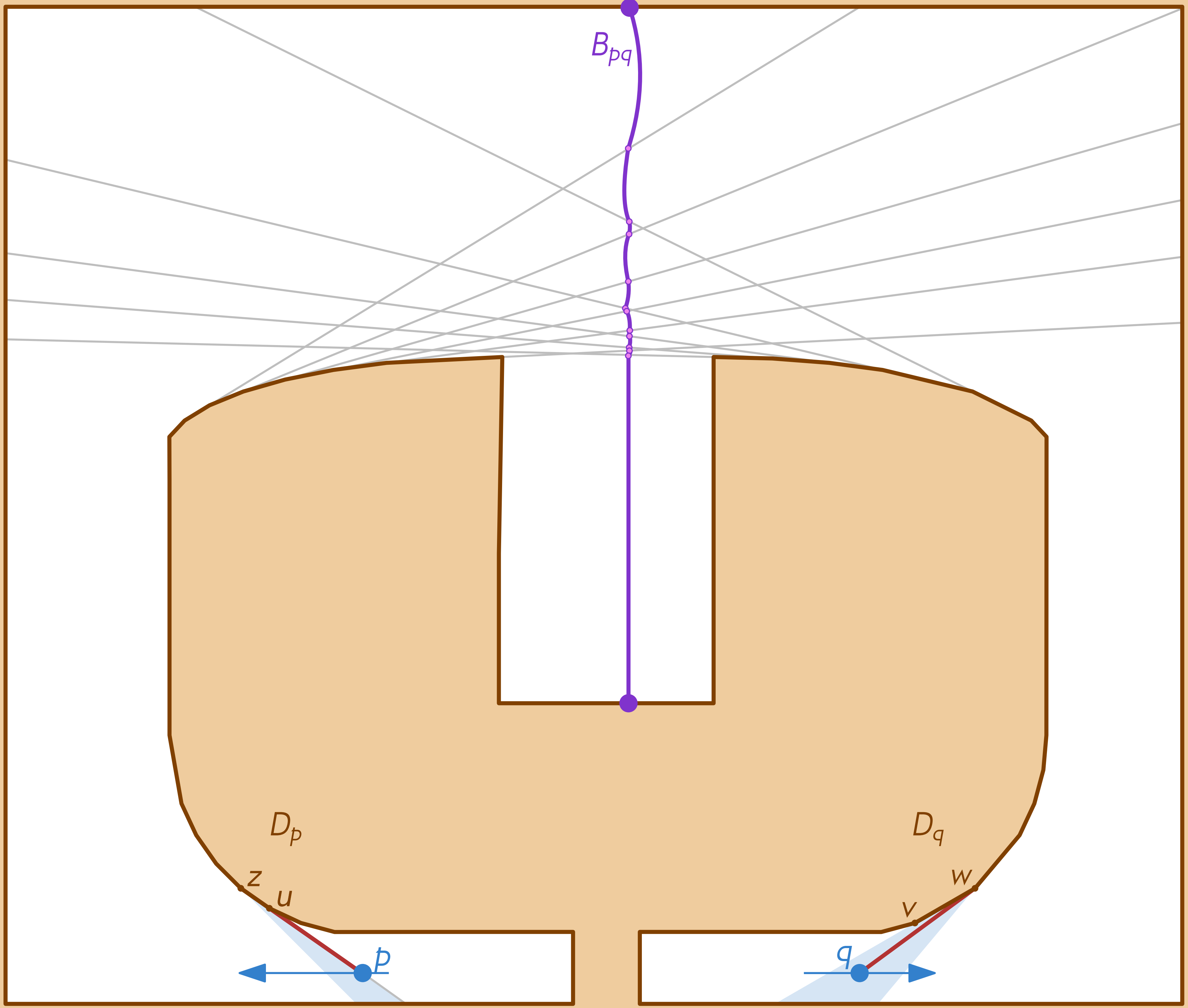

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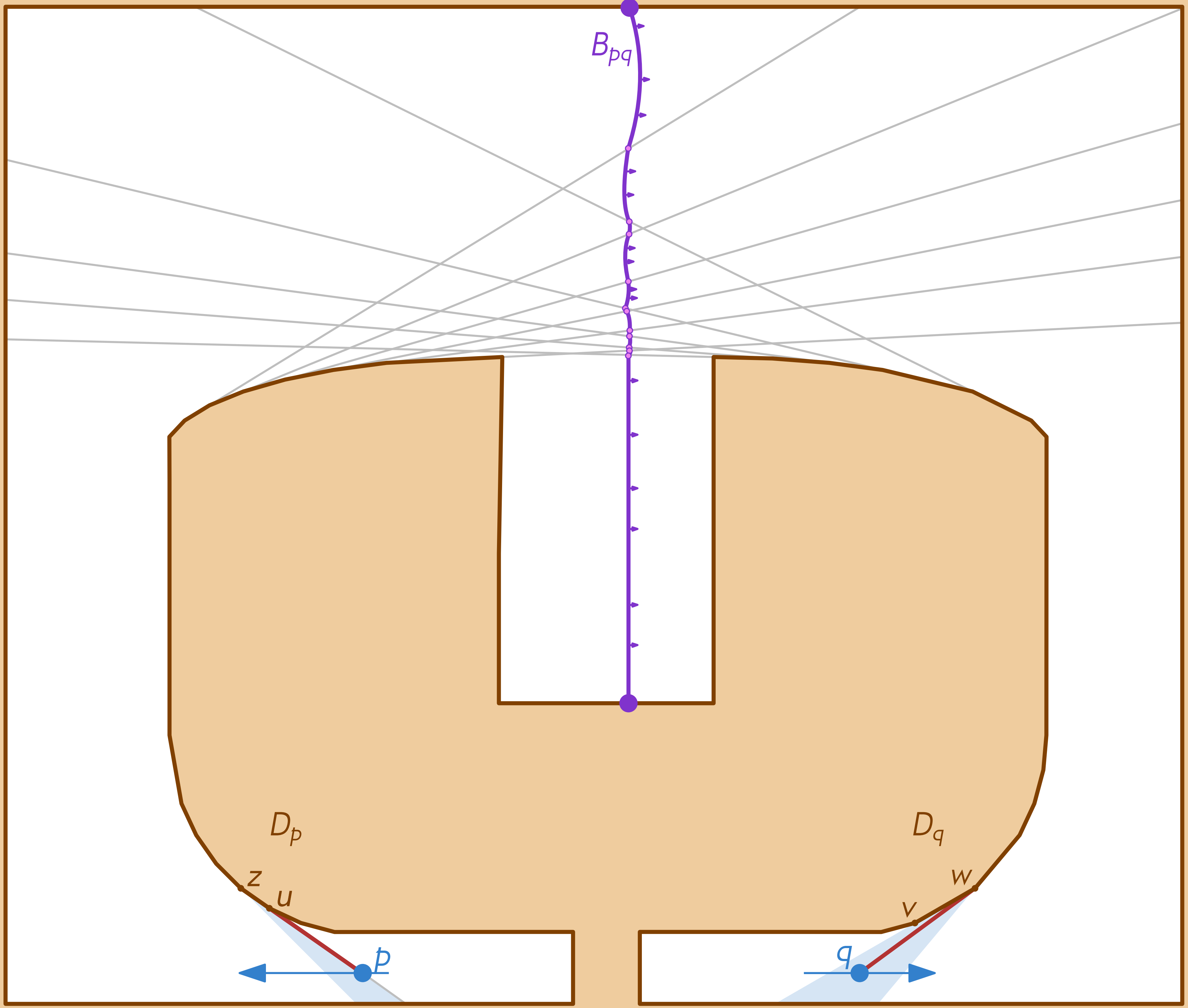
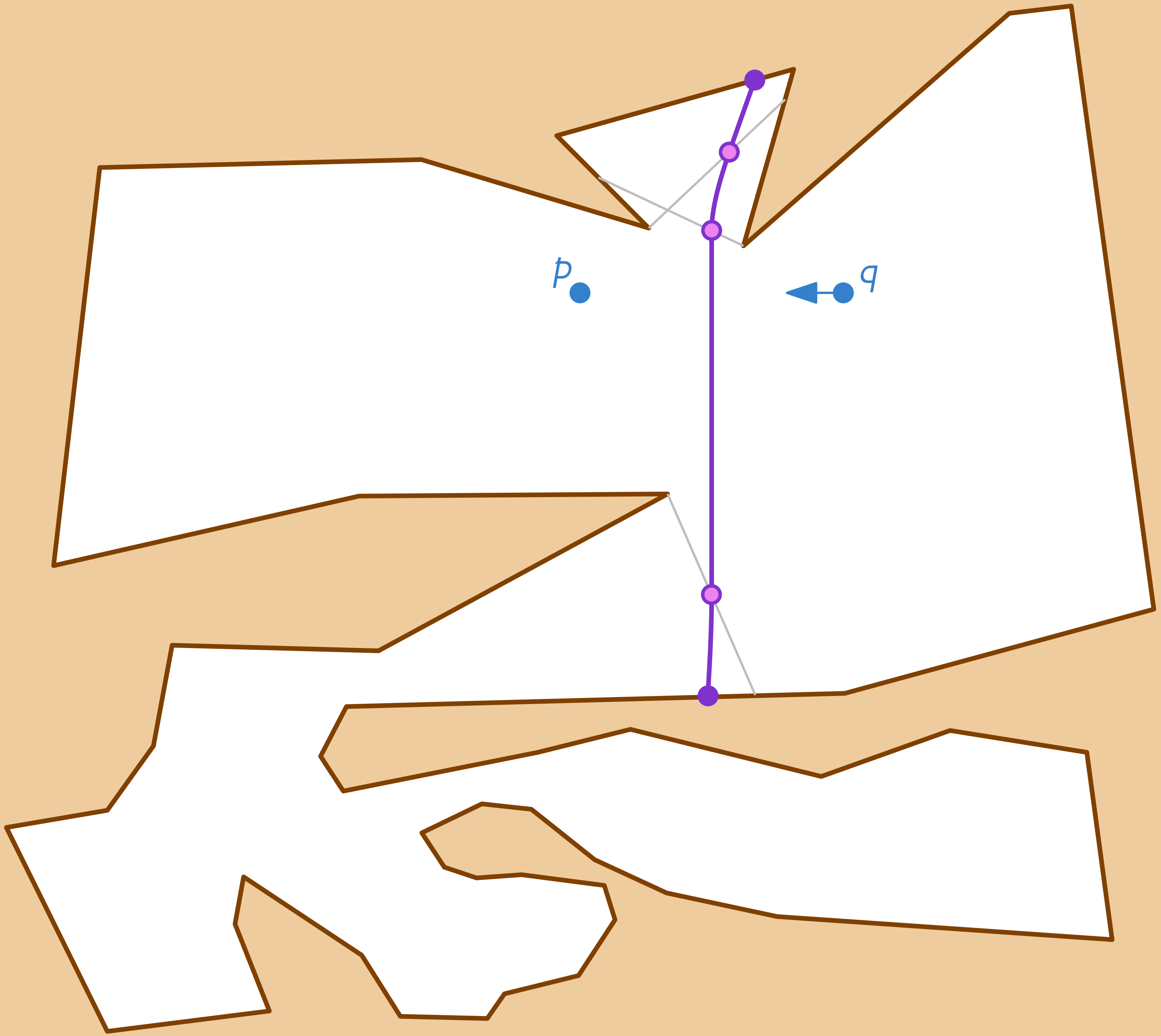


KDS Complications



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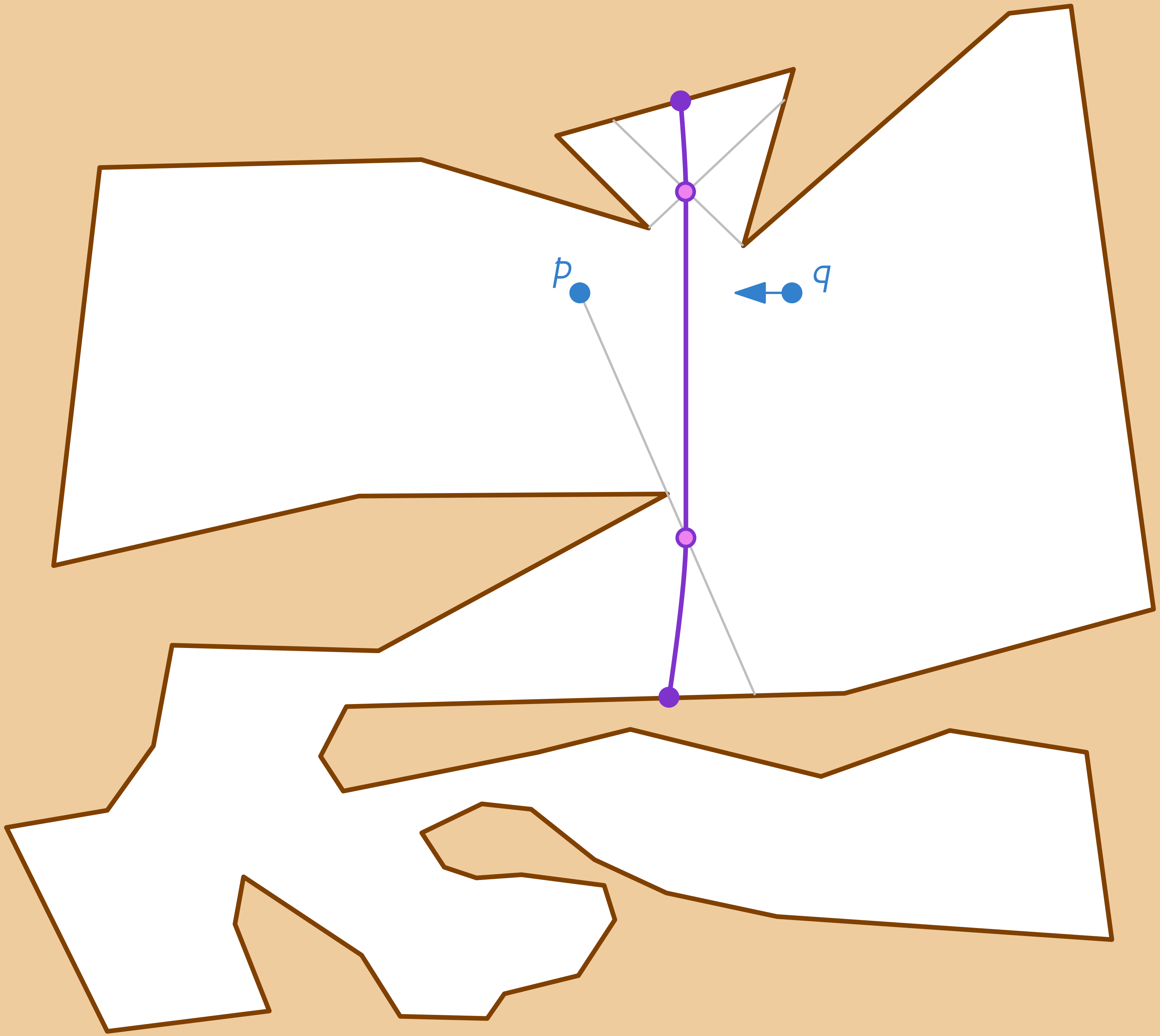
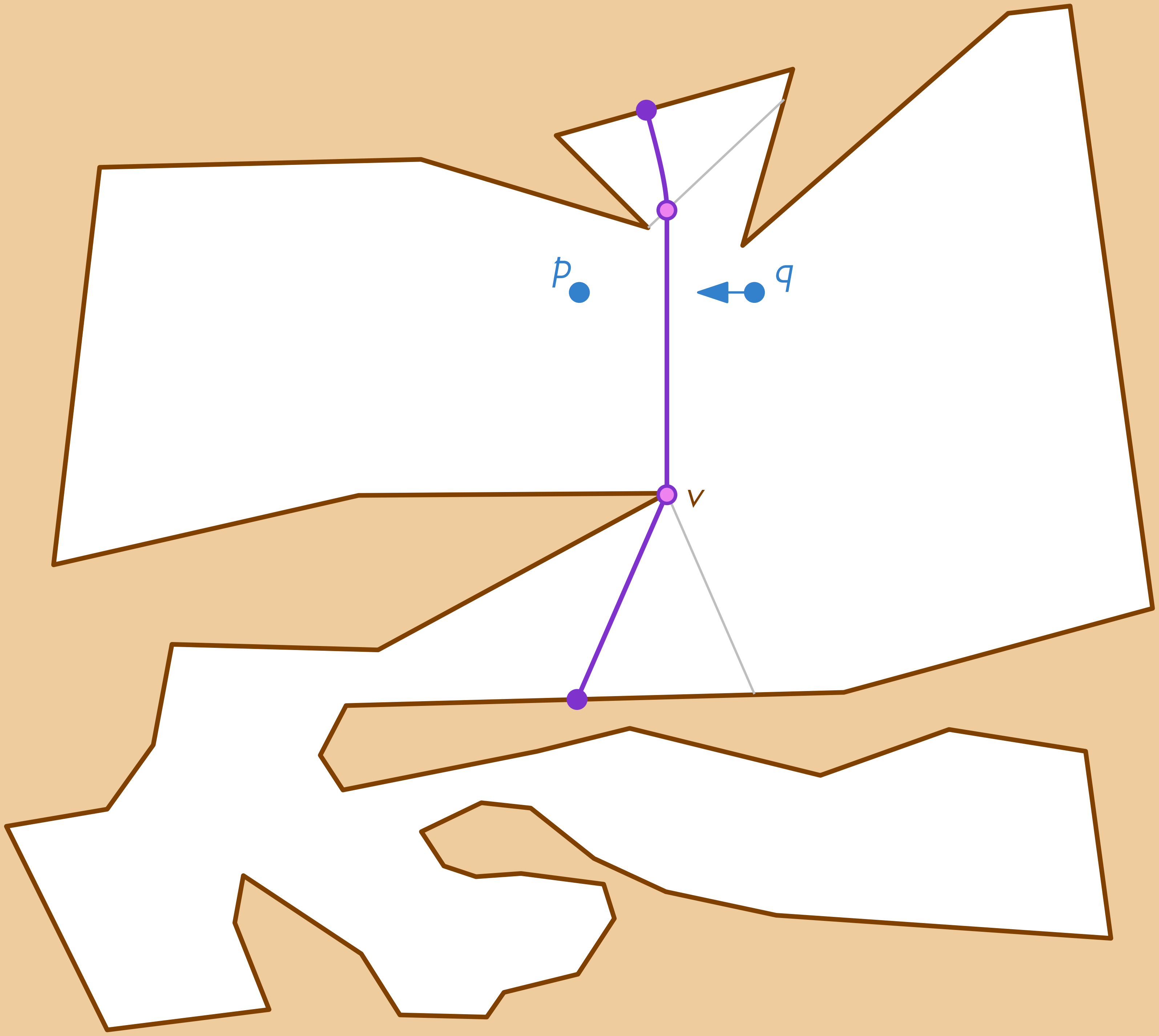


KDS Complications

1. Combined Events



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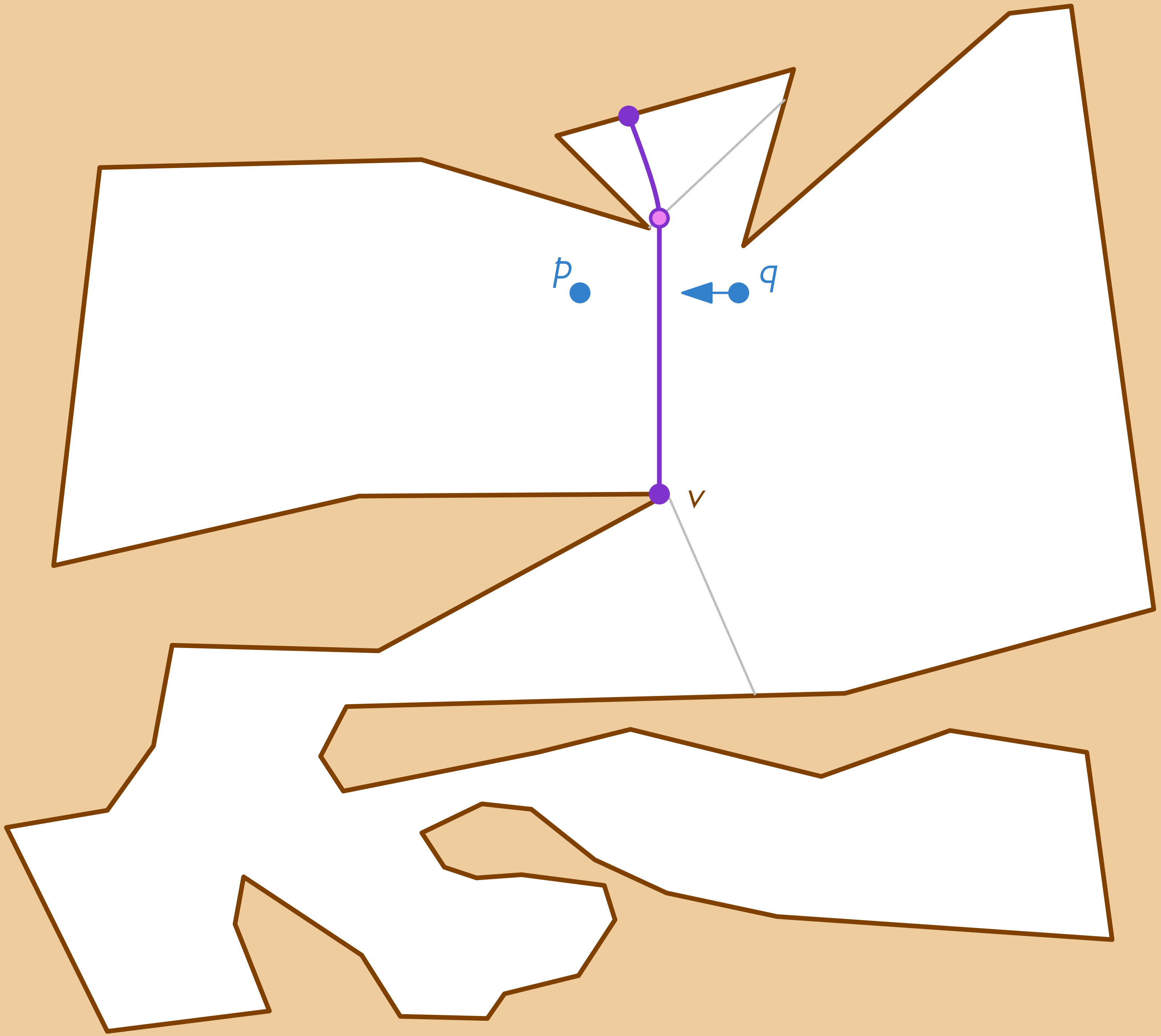

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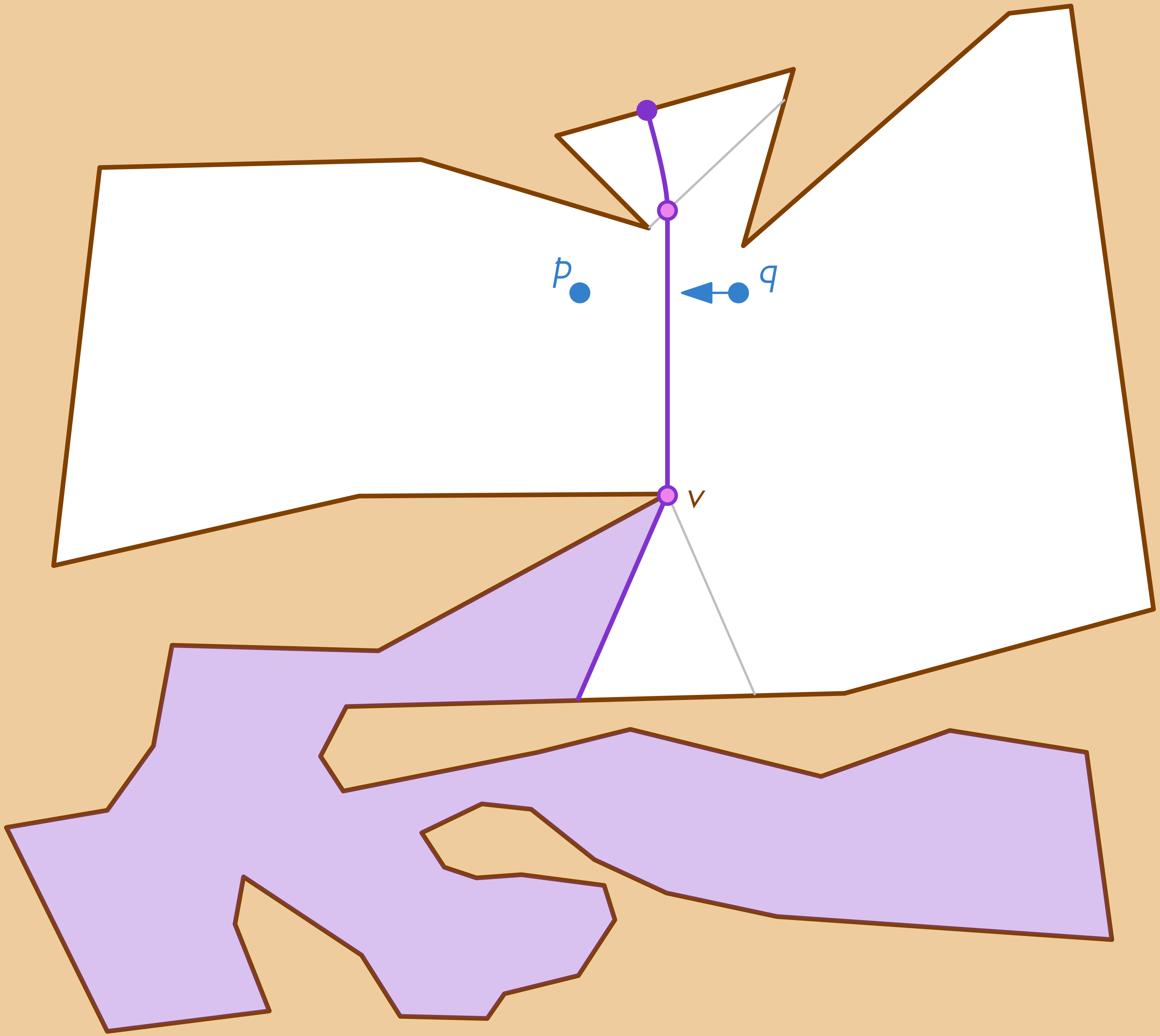

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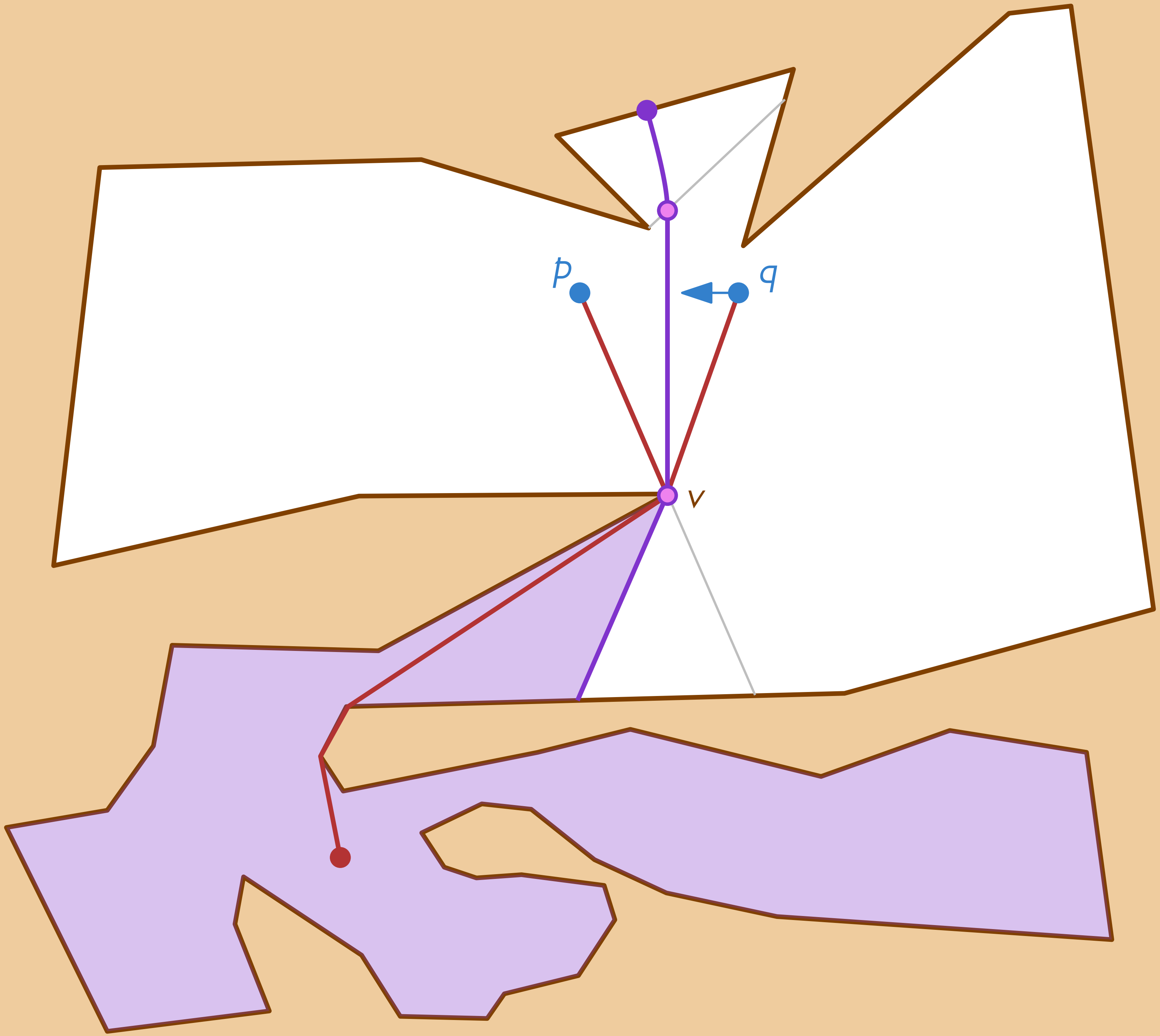

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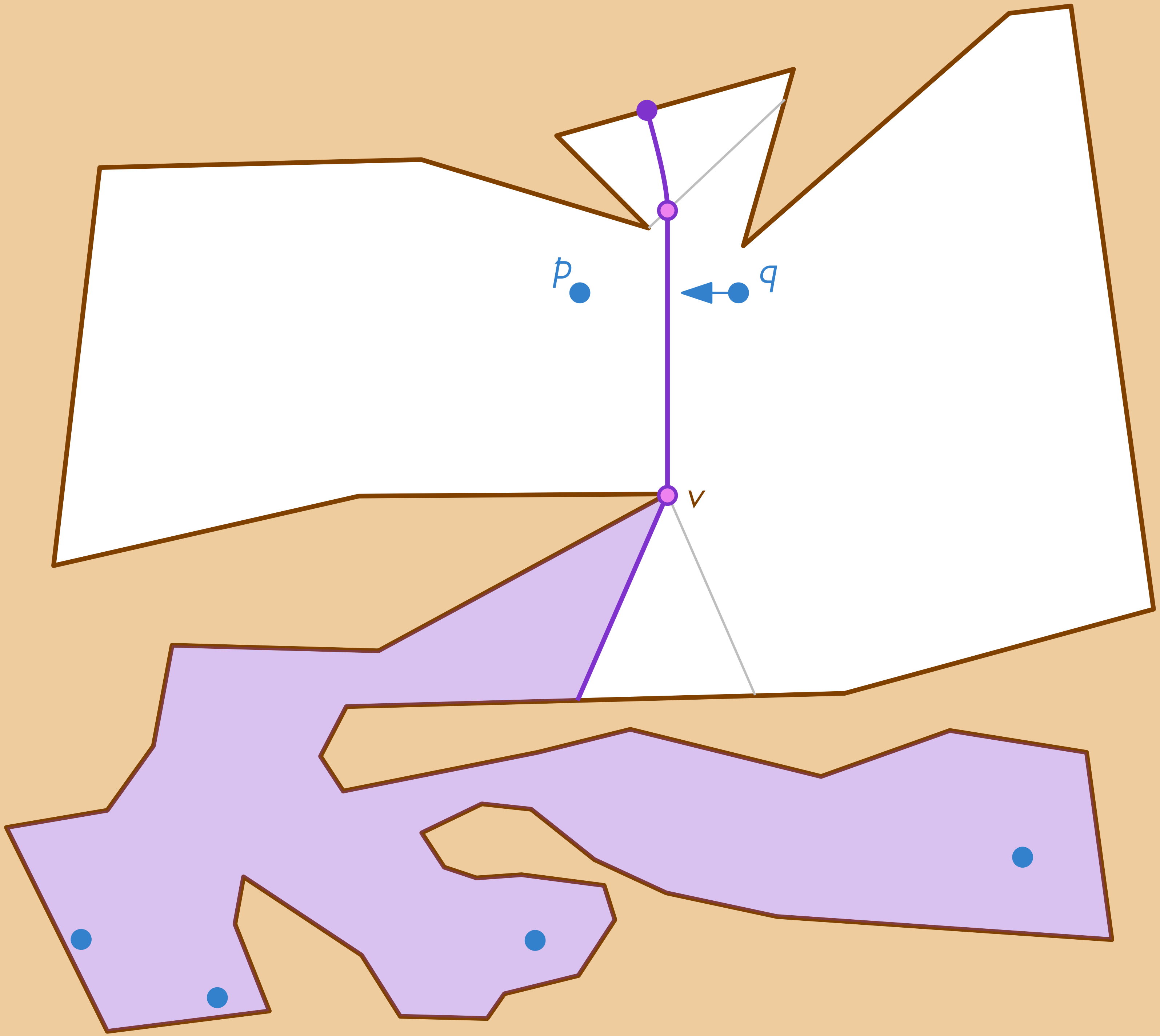
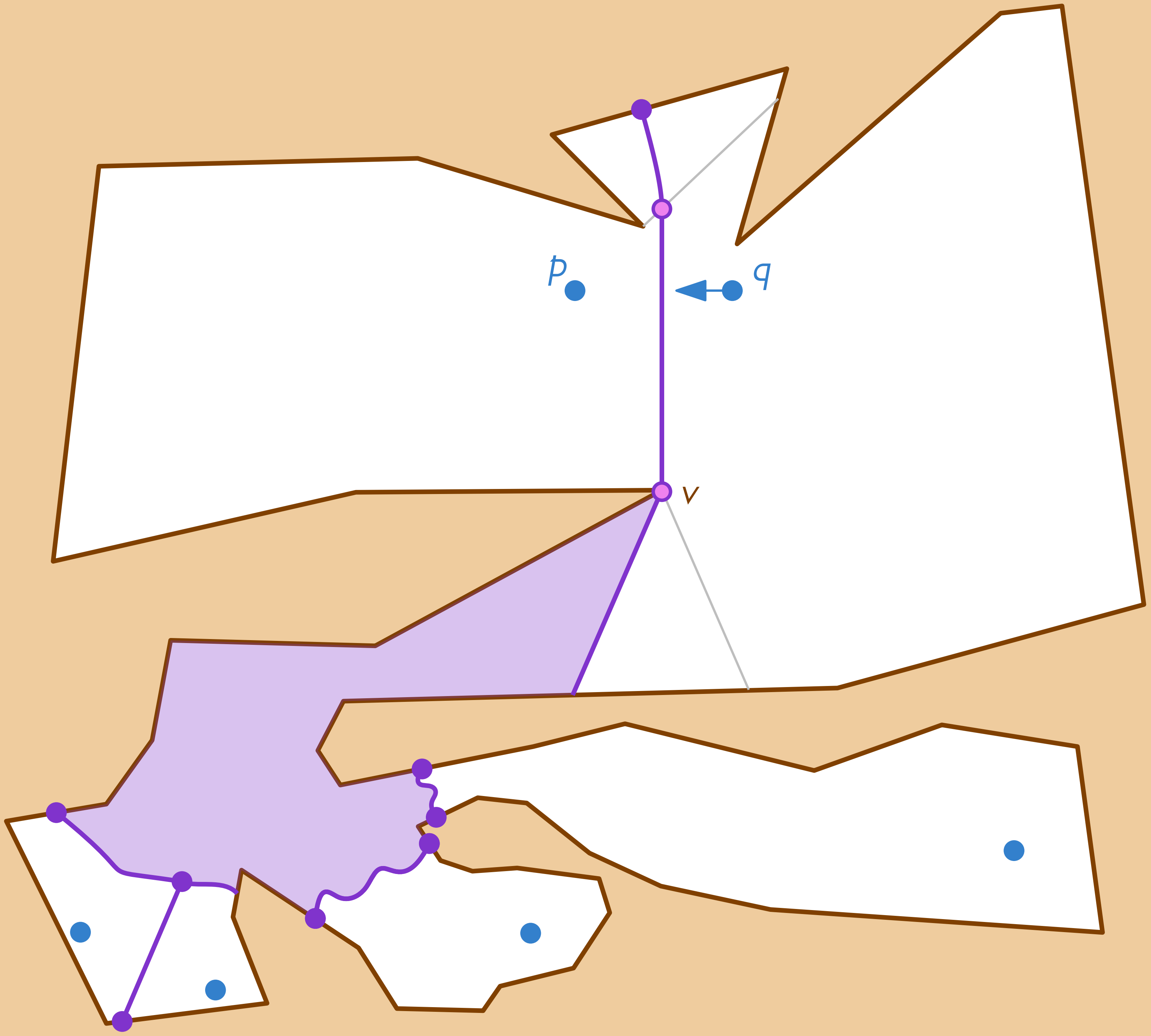


KDS Complications

1. Combined Events
2. Implicit Bisector Certificates



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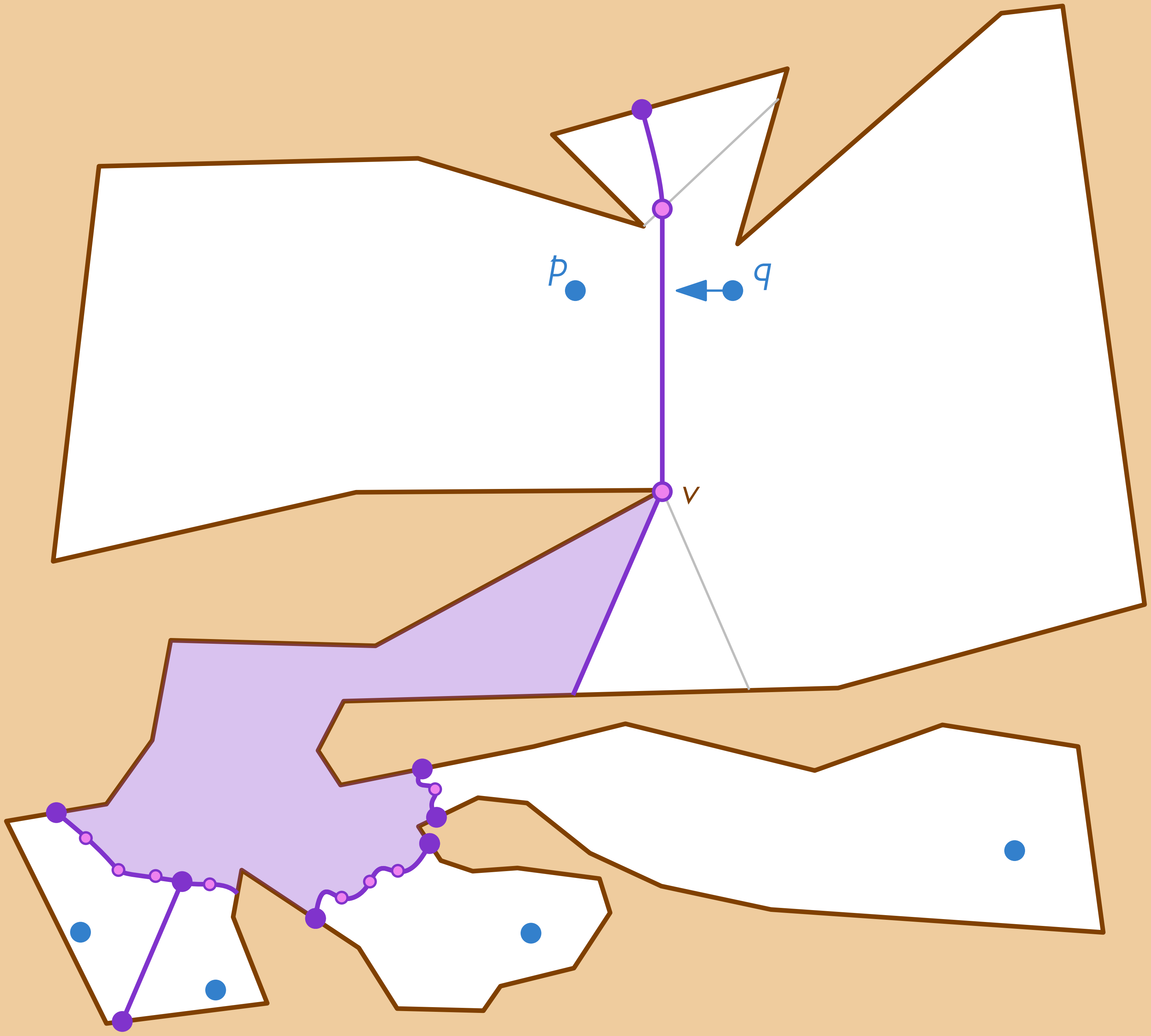

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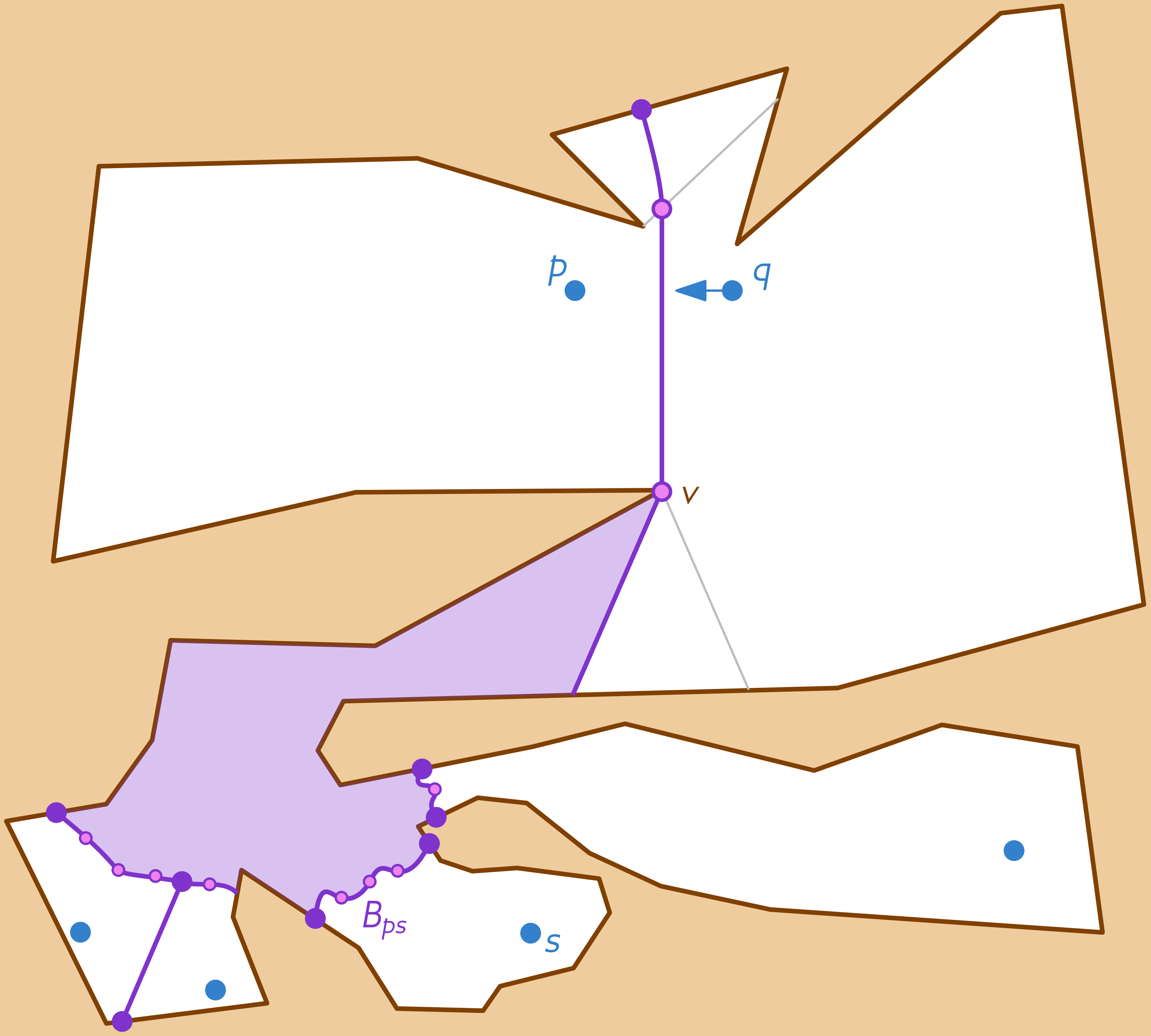

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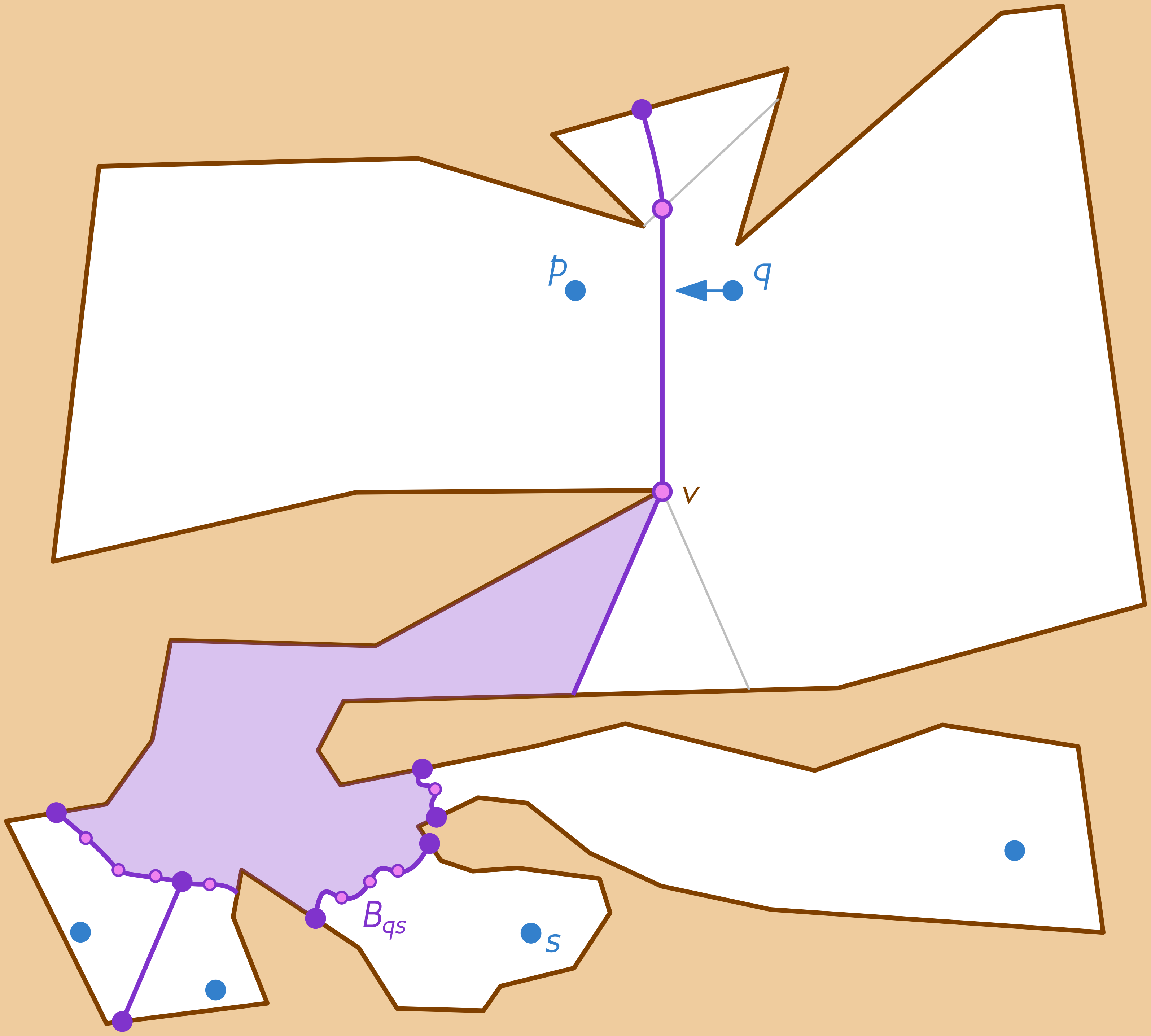

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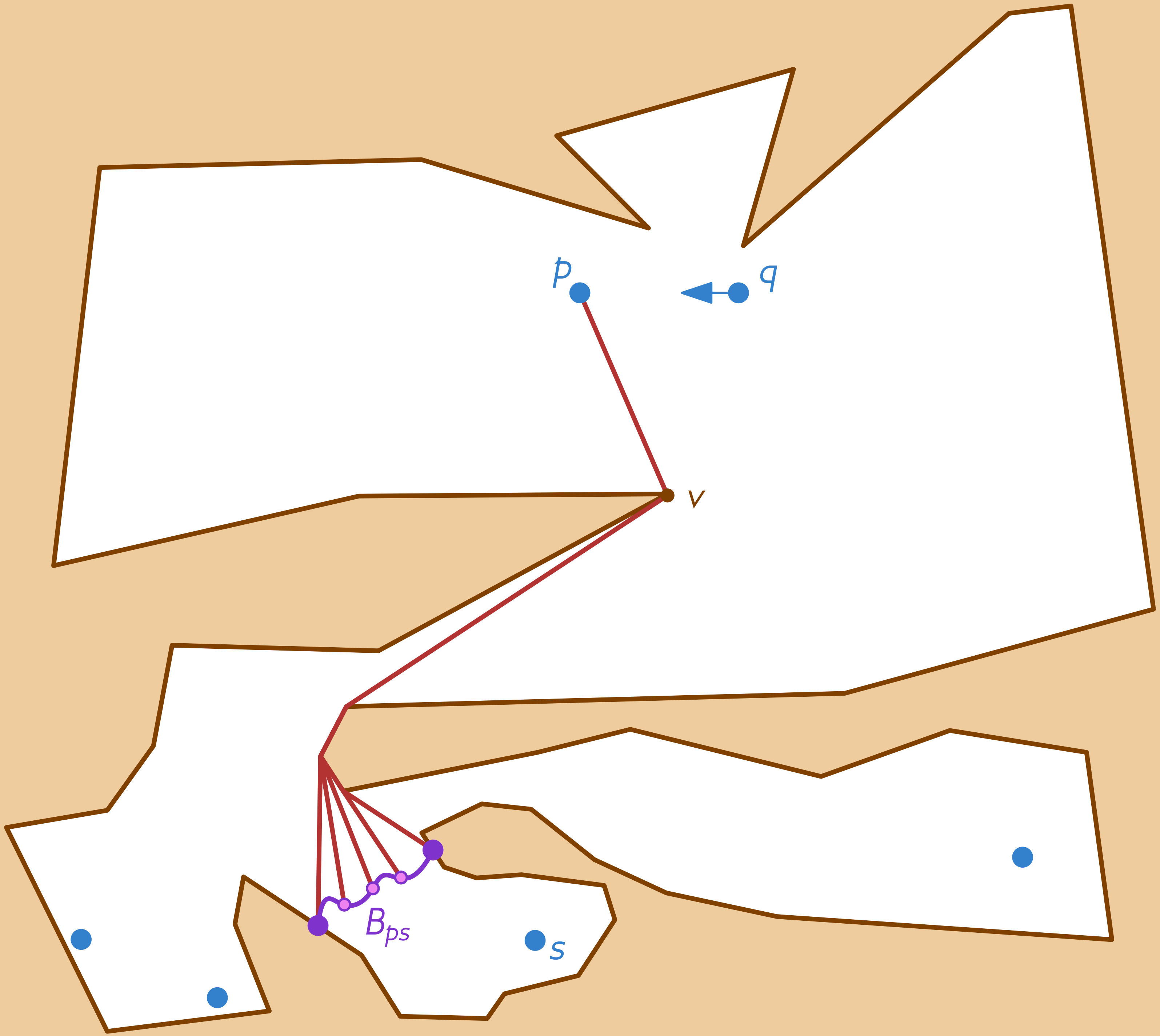
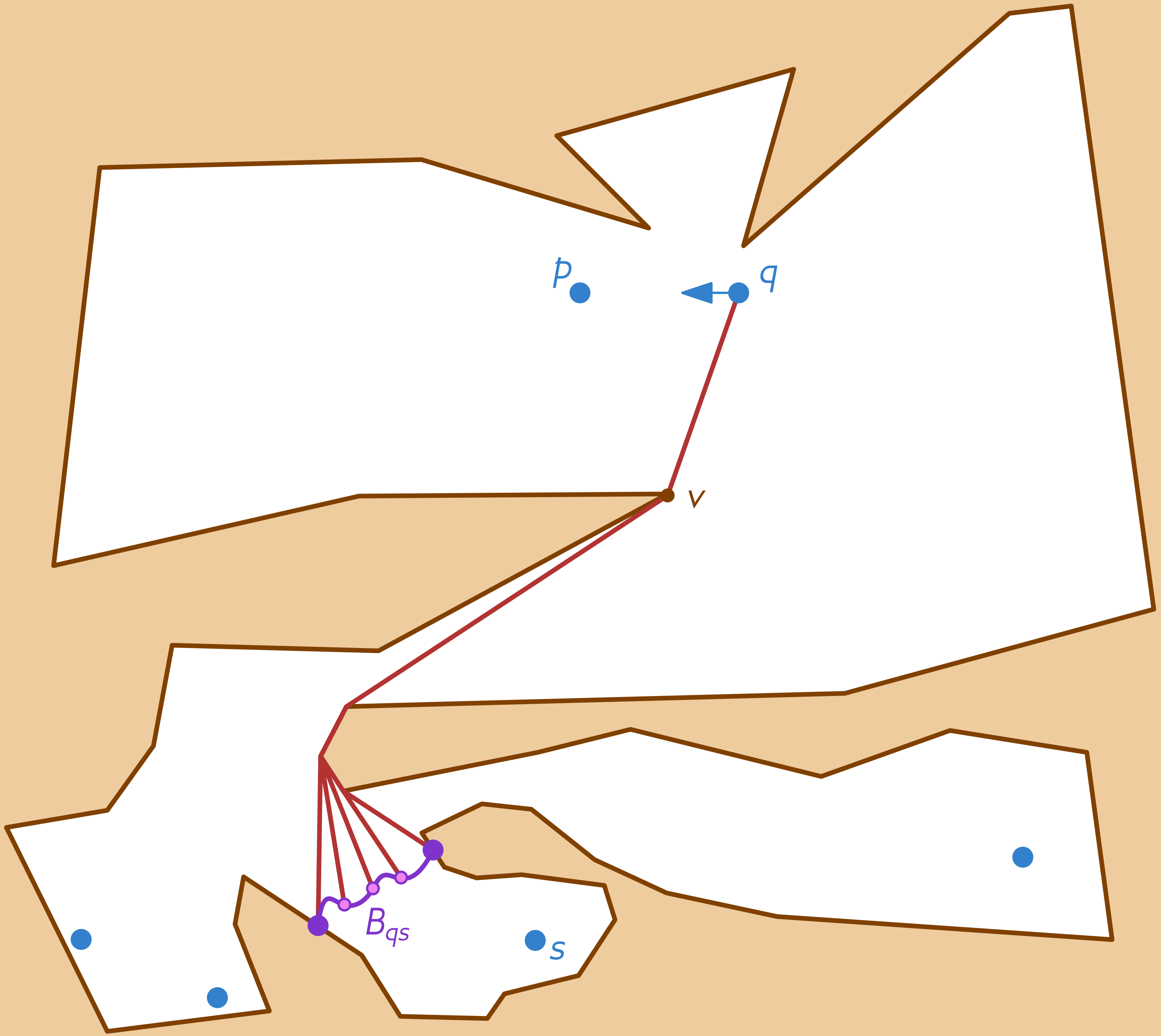
1. Combined Events
2. Implicit Bisector Certificates



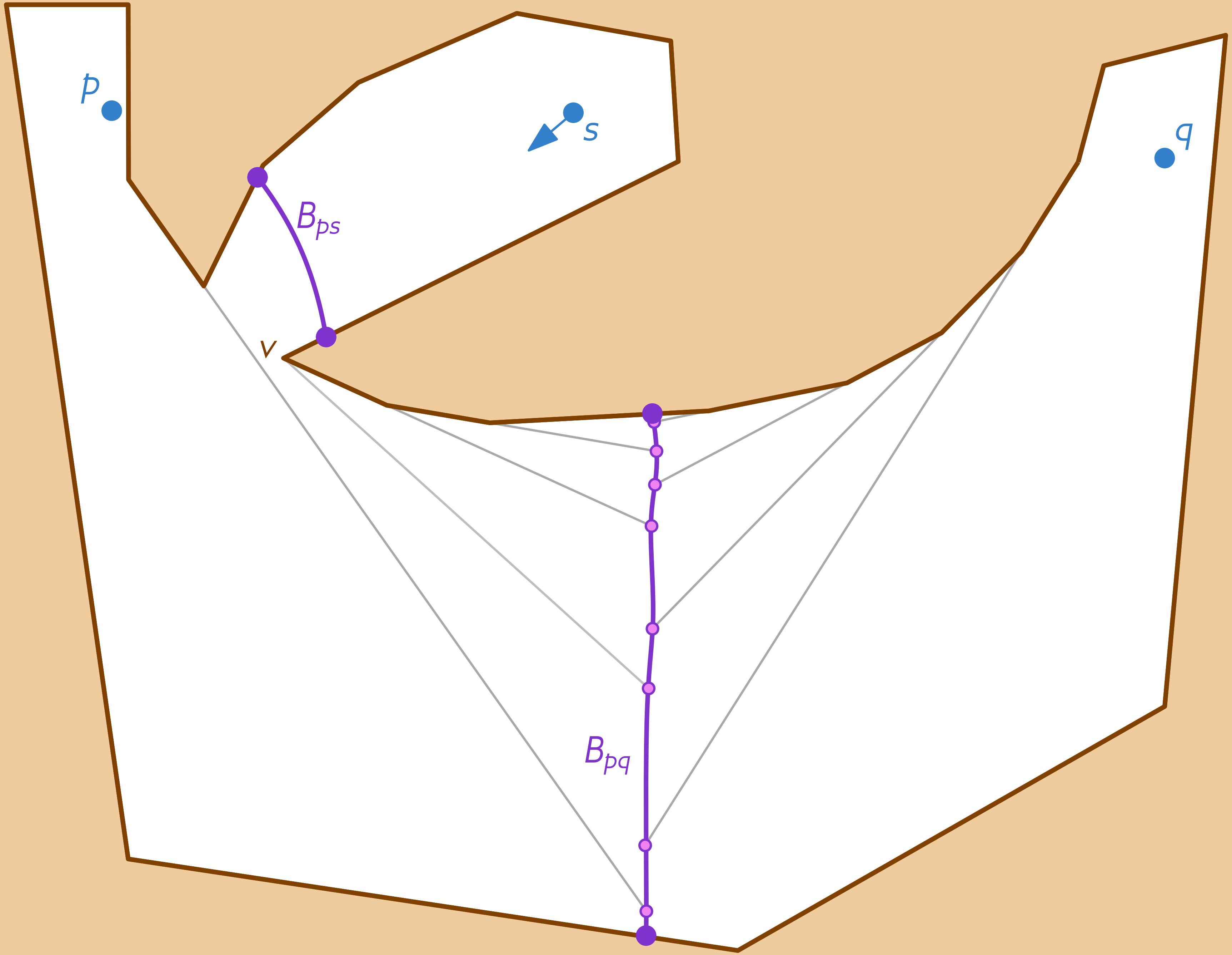
KDS Complications

1. Combined Events
2. Implicit Bisector Certificates



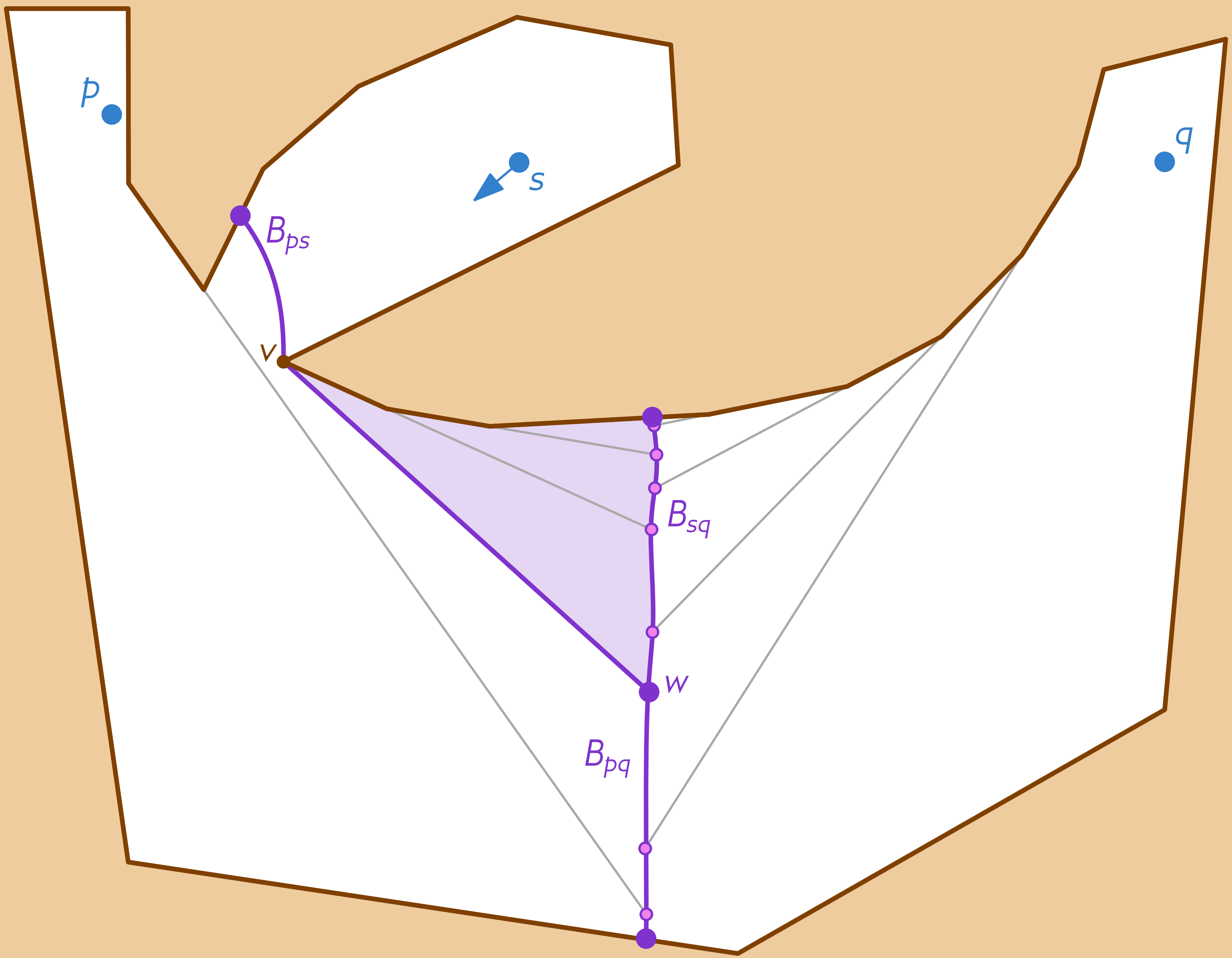
KDS Complications

1. Combined Events
2. Implicit Bisector Certificates
3. Bisector Splits



KDS Complications

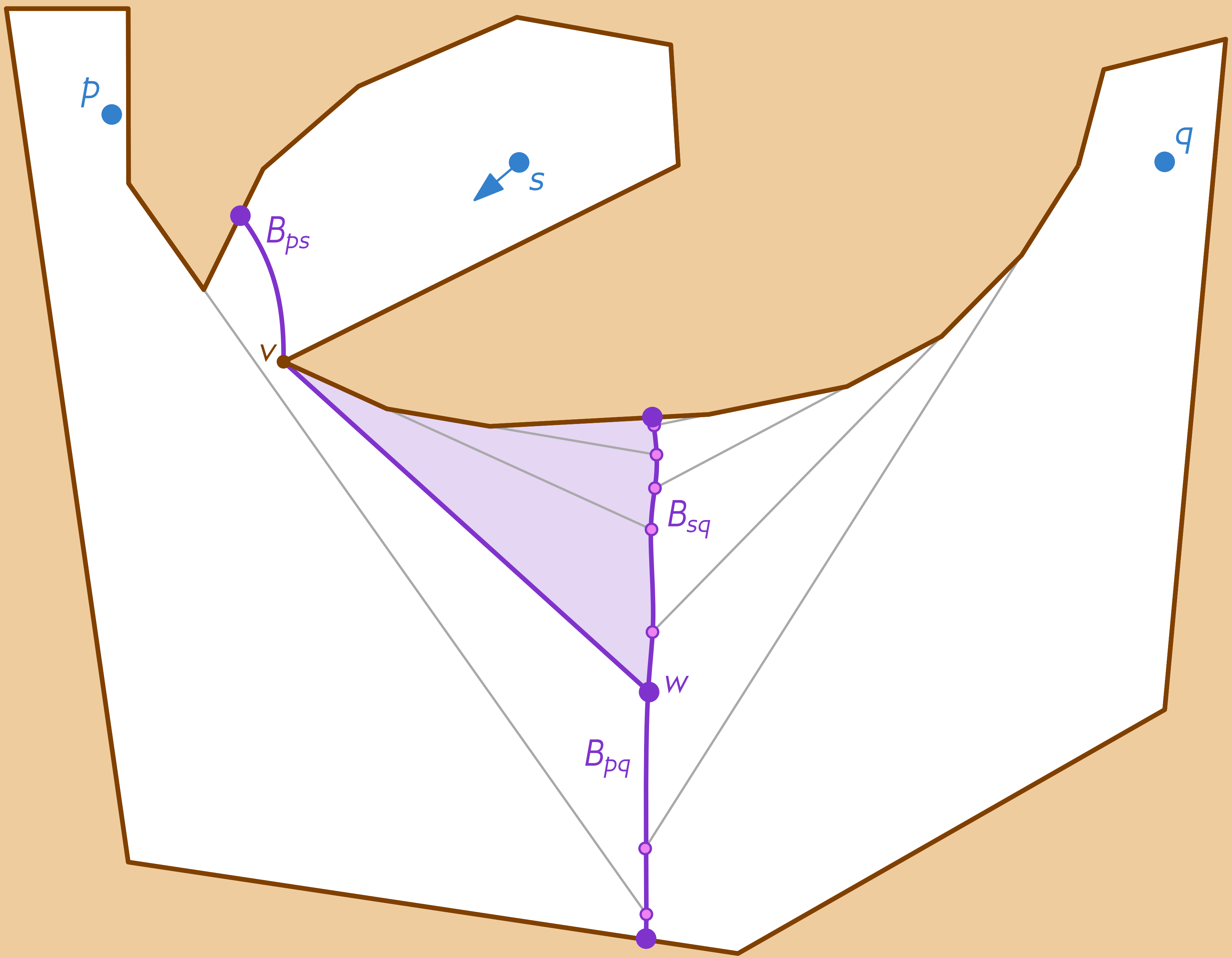
1. Combined Events
2. Implicit Bisector Certificates
3. Bisector Splits



KDS Complications

1. Combined Events
2. Implicit Bisector Certificates
3. Bisector Splits

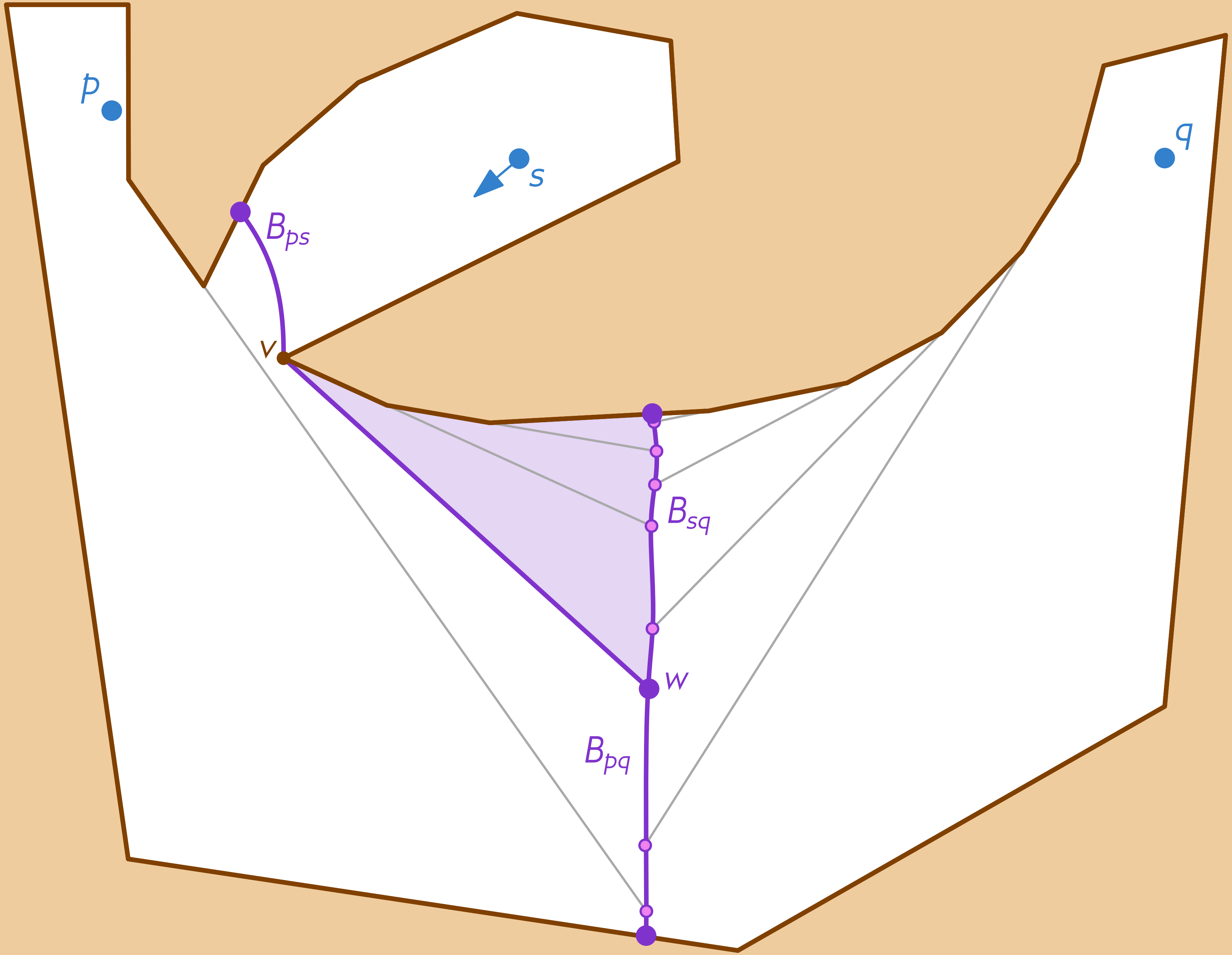
Thm. ∃ KDS $O(m)$ size to maintain B_{pq} that handles events and updates in $O(\log m)$ time each, and supports splits in $O(\log^2 m)$ time.



KDS Complications

1. Combined Events
2. Implicit Bisector Certificates
3. Bisector Splits

Thm. ∃ KDS $O(n+m)$ size to maintain VD(S) that handles events in $O(k(\log^2 m + \log n))$ time each.

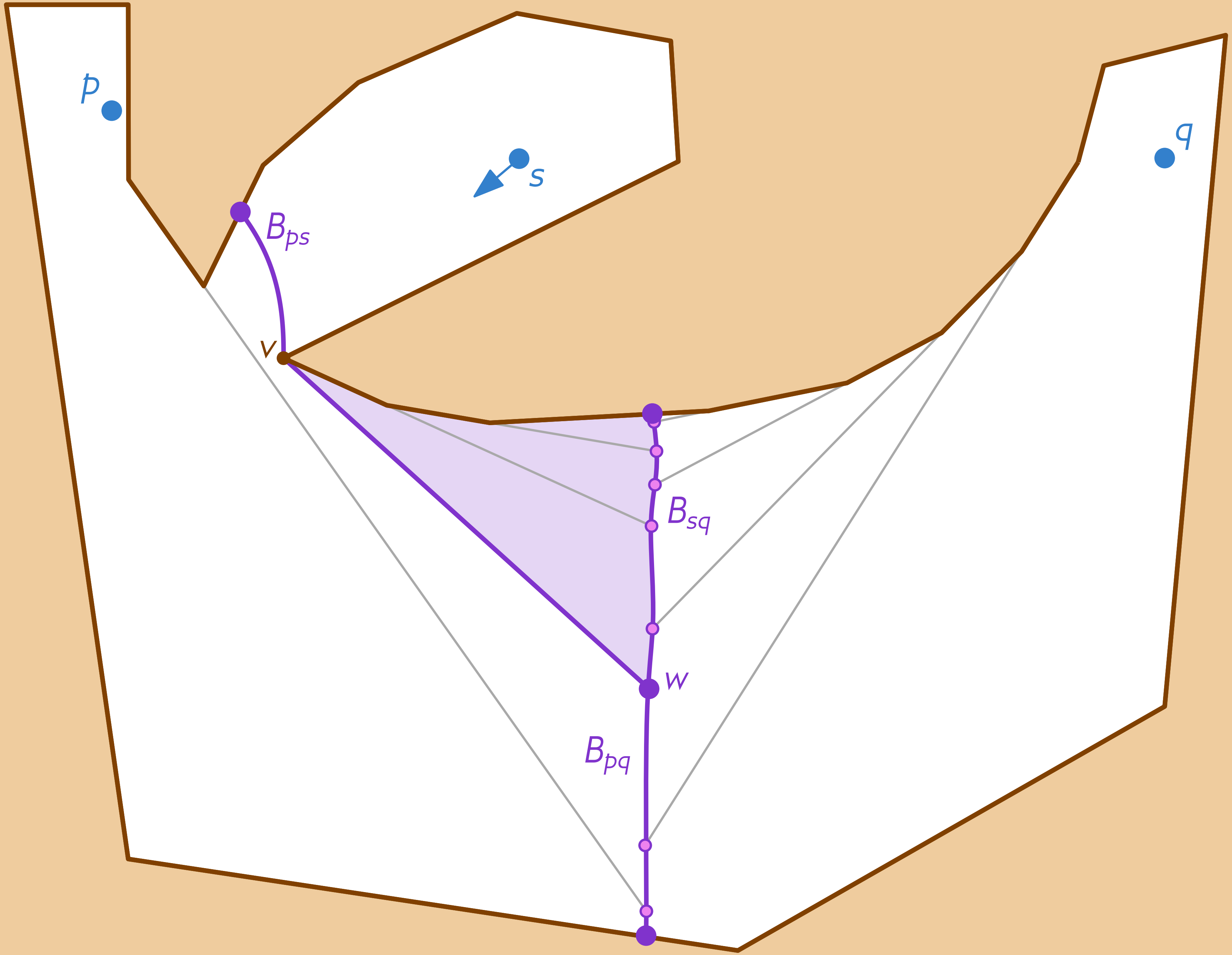


Future Work

1. Improve $\tilde{O}(n^3 m^3)$ bound on #events

   Can we get $\Theta(nm^3 + n^2 m)$?

Thm. $\exists$ KDS $O(n+m)$ size to maintain VD(S) that handles events in $O(k(\log^2 m + \log n))$ time each.



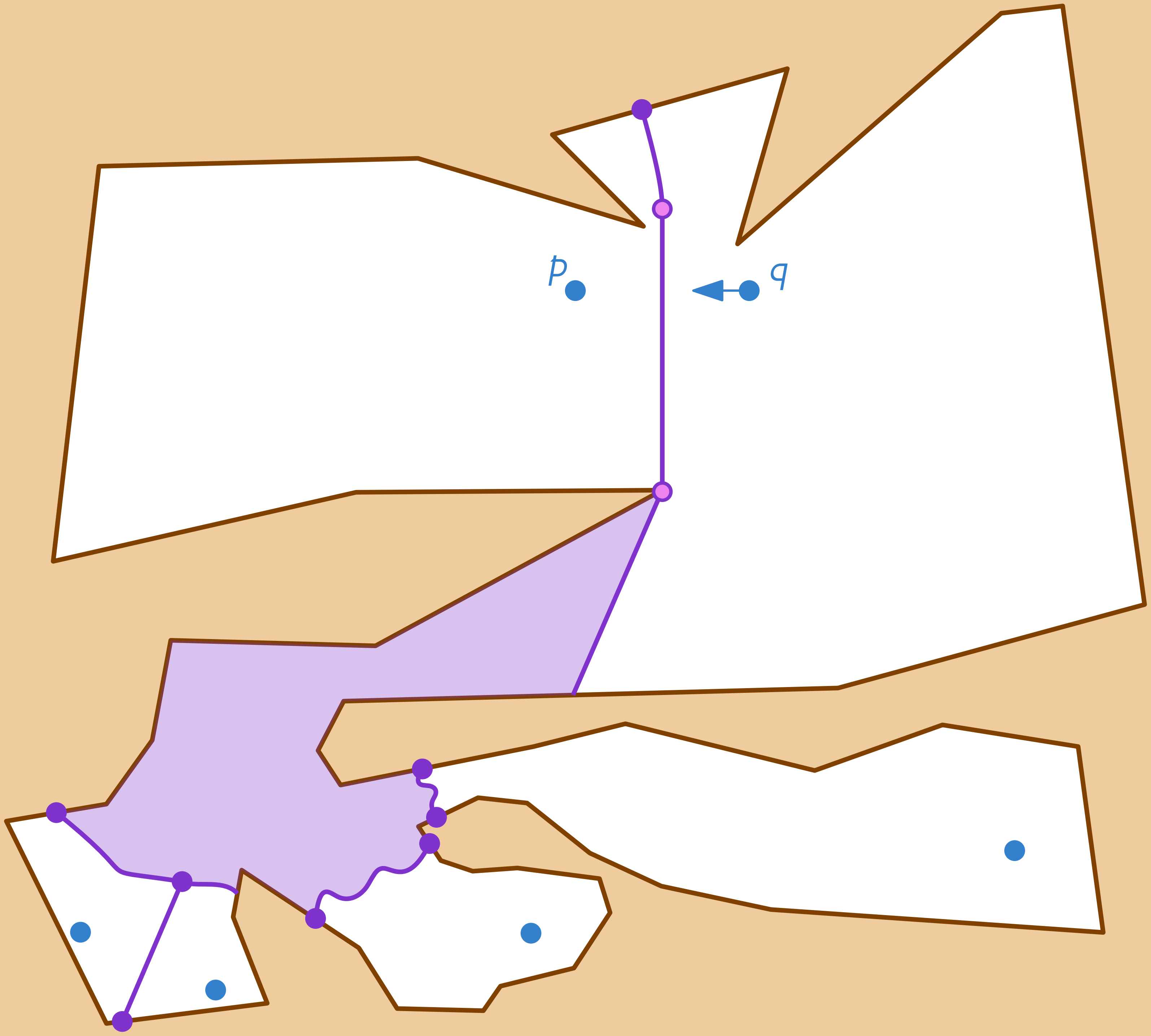
Future Work

1. Improve $\tilde{O}(n^3 m^3)$ bound on #events

    Can we get $\Theta(nm^3 + n^2 m)$?

2. events in $O(\text{polylog}\, nm)$ time?

Thm. $\exists$ KDS $O(n+m)$ size to maintain VD(S) that handles events in $O(k(\log^2 m + \log n))$ time each.

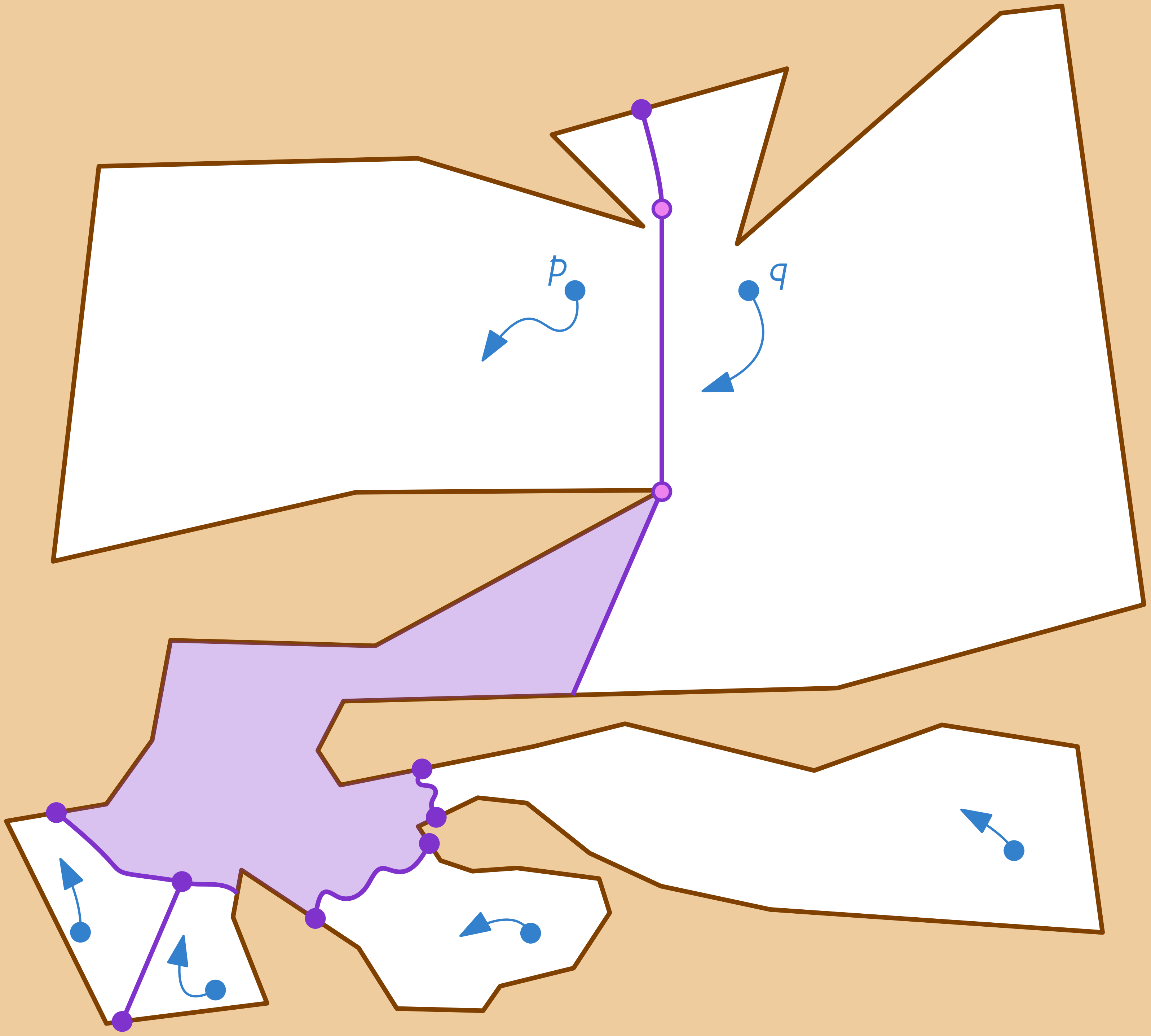


Future Work

1. Improve $\tilde{O}(n^3 m^3)$ bound on #events

   Can we get $\Theta(nm^3 + n^2 m)$?
2. events in $O(\text{polylog } nm)$ time?
3. more general movement?

Thm. $\exists$ KDS $O(n + m)$ size to maintain VD(S) that handles events in $O(k(\log^2 m + \log n))$ time each.



Future Work

1. Improve $\tilde{O}(n^3 m^3)$ bound on #events

   Can we get $\Theta(nm^3 + n^2 m)$?
2. events in $O(\text{polylog } nm)$ time?
3. more general movement?
4. holes?

Thm. $\exists$ KDS $O(n+m)$ size to maintain VD(S) that handles events in $O(k(\log^2 m + \log n))$ time each.